“Errors and misconceptions”

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The Real World
Vs
Stochastic Heaven

- Fitzh 2001 challenged empirical studies about β and portfolio investment returns

- Sherris and Wong (2003)
  - assume iid, lognormality etc
  - derive
  - ignore limitations of assumptions

What was said

Fitzherbert 2001 (from abstract):

"If the appropriate definition of mean return for **long-term investment or asset modelling** *is mean continuously compounded return or* **its equivalent***, then much of the* **empirical support** *for a positive relationship between beta values and return may need to be re-evaluated."*

What S&W thought was said

"Fitzherbert claims that if the expected return used in empirical studies were the average continuously compounded return then the CAPM relationship between expected return and β would be questionable. We <u>demonstrate that an arithmetic average of returns should be used to estimate the expected return in the standard CAPM</u> and show that using a geometric average is incorrect."

What are the differences?

- Fitzherbert 2001
  - long term investment return
  - geometric mean determines actual outcome
  - empirical results have been misinterpreted for 30 years
- Sherris & Wong 2003
  - standard CAPM - a single period model
  - ***assuming independence***, arithmetic mean return determines expected outcome

In Stochastic Heaven

- *Rates of return are iid*
- $\log\{ 1 + r(i) \} \sim N(\mu, \sigma^2)$

- $E [r(i)] = \exp \{\mu + 0.5 \sigma^2 \} - 1$

- $E [X(n) / X(0)] = \exp \{n \mu + 0.5 n \sigma^2 \}$

- *But* $GMR(n) \rightarrow \exp\{\mu\} - 1$ *almost surely*

Back in the Real World

- ***Numerous factors determine Earnings, dividends etc.***

- ***Market participants determine valuation basis through P/E ratios, dividend yields etc***

- ***plus 'noise'***

RW dual process models

X(t) *price index at time t*

V(t) *earnings dividends or book value*

PV(t) *Market valuation basis*

P/E, 1 / [div yield] or price/book

$X(t) = V(t) \times PV(t)$ *so*

$\log\{X(t)\} = \log\{E(t)\} + \log\{PE(t)\}$ *or*

$\log\{X(t)\} = \log\{D(t)\} + \log\{PD(t)\}$

Features of dual processes

log { X(t) } = log { V(t) } + log { PV(t) }

Real world features

volatility of X(t) dominated by vol of PV(t)

log { PV(t) } is stationary

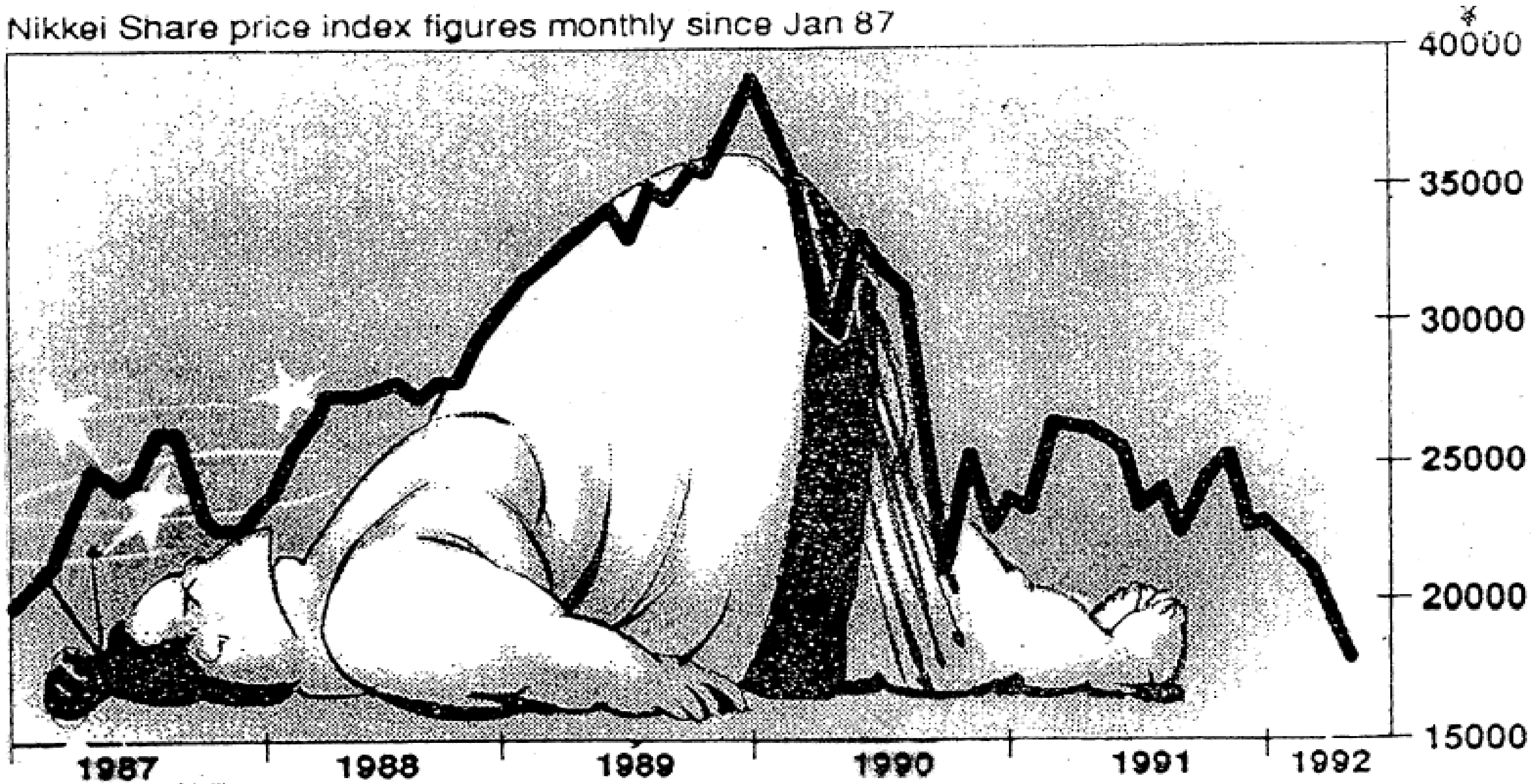
Glassman and Hassett (1999) "Dow 36000"

Nomura advert in 1989

Stock prices *trend-revert* (see MGWP)

Models which assume serial independence overestimate long-term variance

Nikkei 225 Jan 87 – May 92



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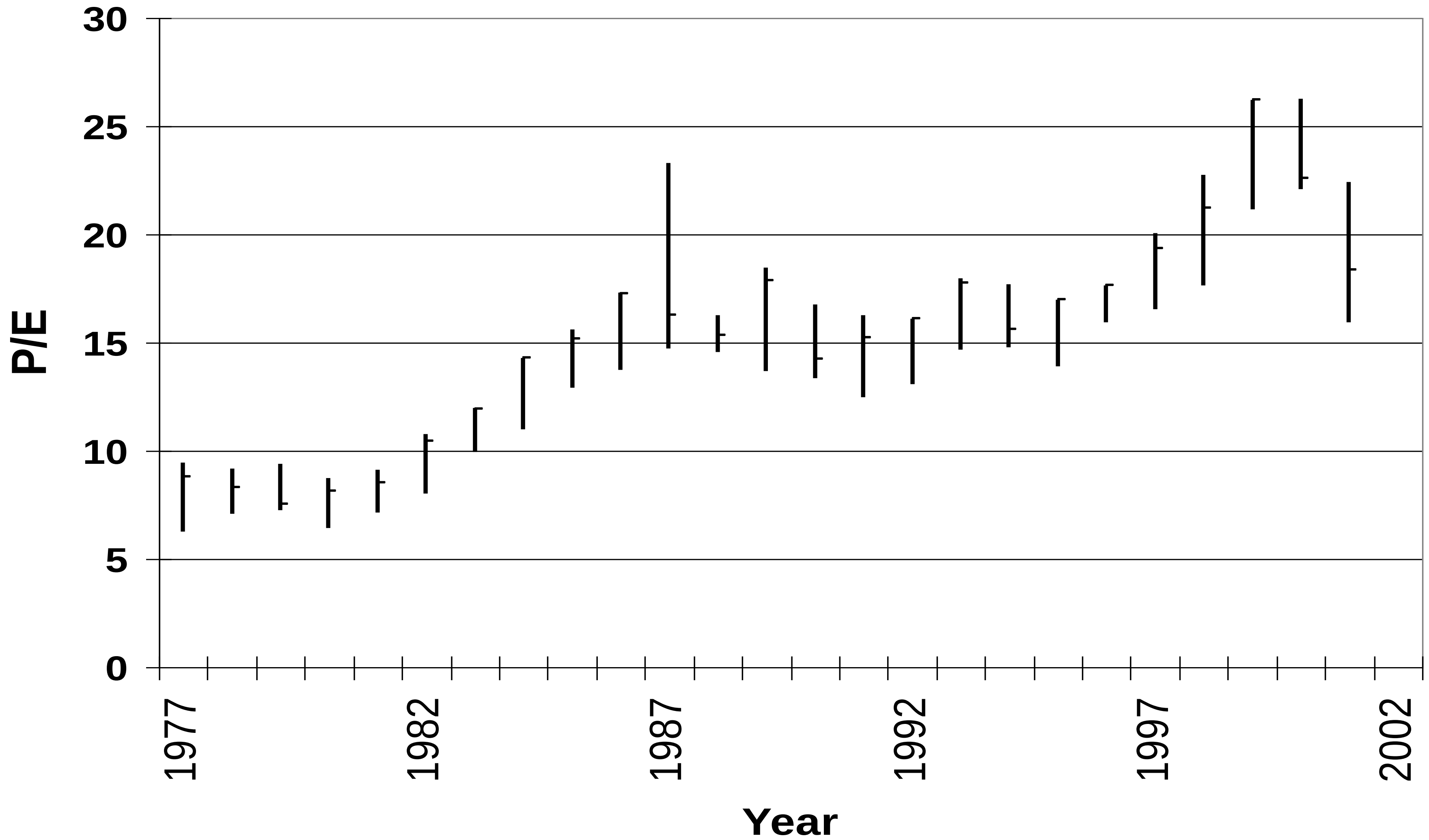
UK P/ [adjusted average E]



Continuous compounding ???

- $\delta(i) = \log\{ 1 + r(i) \}$
  - where *r(i)* is return in period *i*
  - refer para 3.6
  - CCMR = mean of $\delta(i)$ [*also = log (1 + gmr)*]
- $E[A \times B] \neq E[A] \times E[B]$
  - *unless A&B are independent*
- *but* $E[\log \{A \times B\}] = E[\log \{A\}] + E[\log\{B\}]$

log-return Vs force of return ?

Non-SP versions of CAPM

- Sherris and Wong ‘derive’
  - with what assumptions (Stochastic Heaven?)
  - Merton’s continuous model ‘tested’ by Jensen (1972)

- What did Jensen say?
  - see Fitzh (2001) para 3.9

  “the [continuously compounded] model does not fit the data”

The BJS (1972) study

- Fitzh (2001) commented
  - BJS study was based on arithmetic means of monthly discrete rates of ‘excess return’
  - study period ran from bust to bubble
  - low β portfolios dominated by utilities
  - widely misinterpreted as evidence of a link between β and <u>long-term</u> return

The low P/E effect

- Sherris and Wong say

  "higher risk portfolios will have lower P/E ratios [and therefore higher returns] regardless of how the risk is measured. .... The lower the median P/E the higher the expected return."

<u>and according to Basu</u> the lower the β

This argument contradicts EMH and CAPM!

What are the limits of IID?

- long-term stock prices ***trend*** revert
  - refer MGWP report (and others)

- extreme valuation ratios tend to revert
  - ask AMP and defined benefit scheme consultants

Trend reversion Vs IID

simple binomial model

r(i) = +30% prob 0.5 AMR= 10%

= - 10% prob 0.5 StdDev = 20%

If IID *AMR* $\Rightarrow$ *E[X(10)/X(0)] = 1.1* ^{10} = 2.59

Now let r(i+1) = 0.2 – r(i) r(i) has same distn

but series is trend reverting and not indep

***E[X(10)/X(0)]* = 2.19 approx 2% less per unit**

Are S&W conclusions justified?

- **Use of arithmetic mean depends on IID**
- **Lognormal expectations rely on extreme values on right tail**
- **No justification of Stochastic Heaven is offered**
- **Effect of approximations are unquantified**