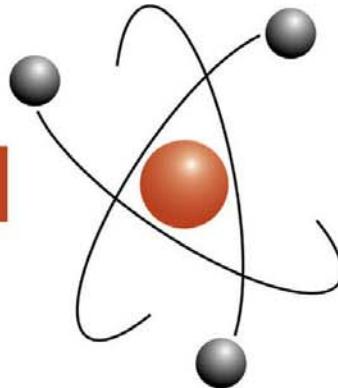


**S U S T A I N**



**A B I L I T Y**

**ACTUARIES AND THE FUTURE**

# **Insurance Applications of Fuzzy Logic**

**Arnold F. Shapiro  
Penn State University**

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**Institute of Actuaries of Australia**



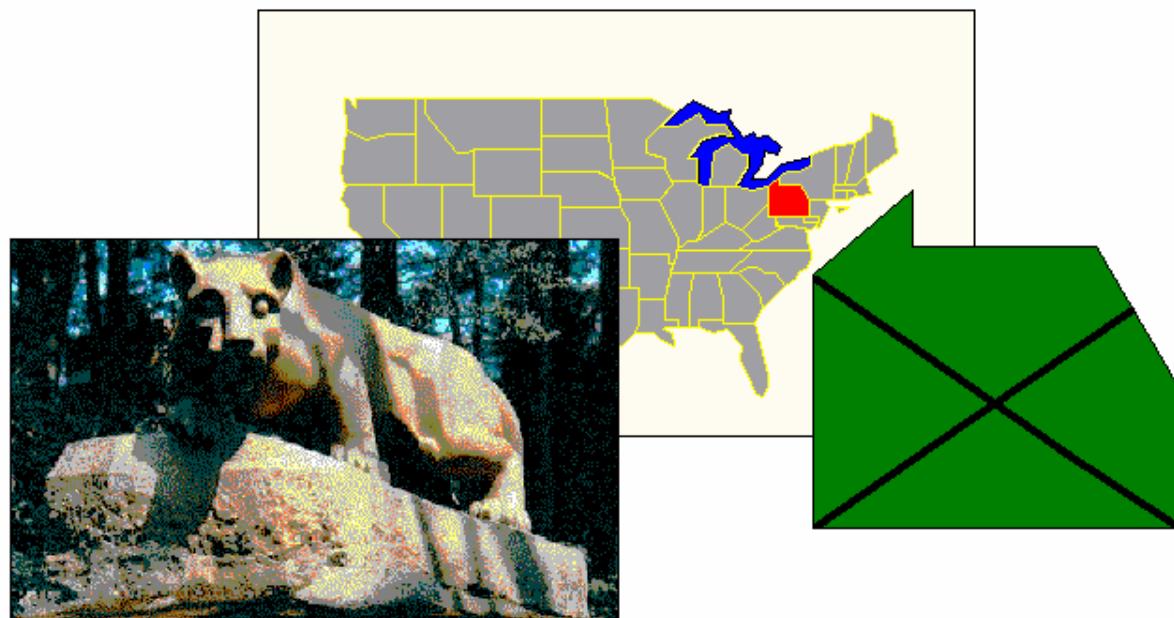
112



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# Penn State University



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## Questions

What are fuzzy systems?

How are fuzzy systems conceptualized?

What are representative insurance applications of fuzzy systems?

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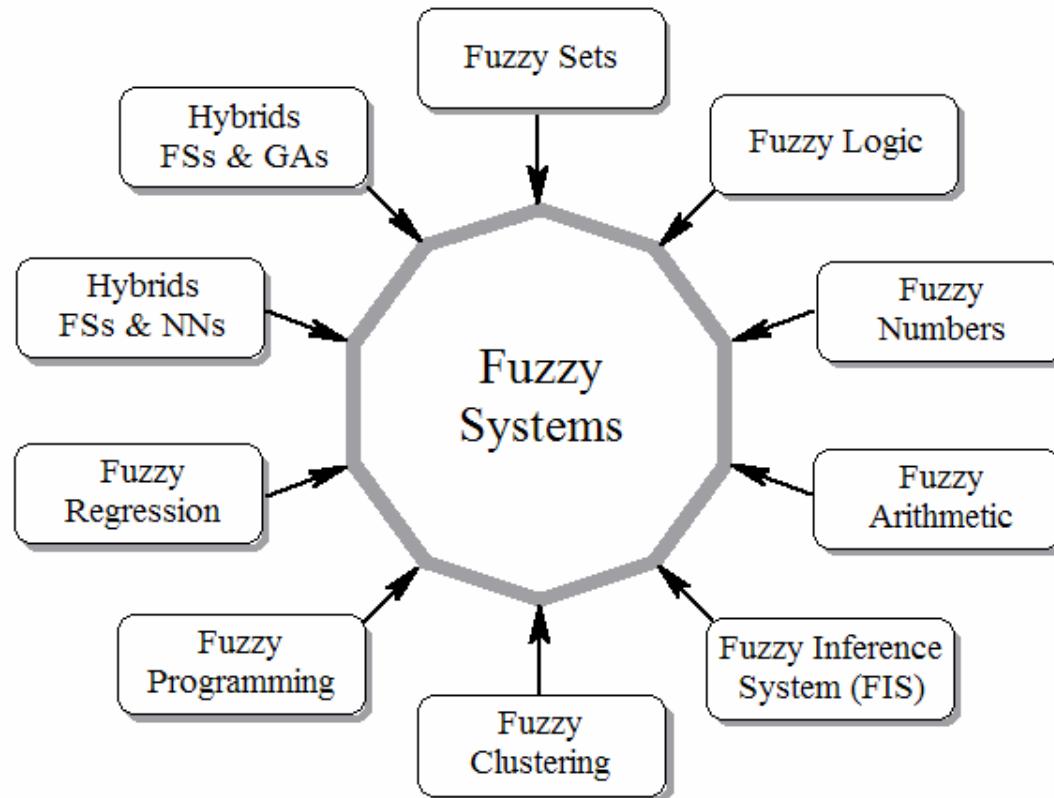
## Fuzzy Logic (Fuzzy Systems)

Fuzzy logic, FL, has 4 principal facets (Zadeh):

- 1 The logical facet, FL
  - The logic of approximate reasoning
  - Fuzzy logic in its narrow sense
- 2 The set-theoretic facet, FLs
  - Fuzzy sets
  - Concerned with classes having unsharp boundaries
- 3 The relational facet, FLr
  - Concerned with linguistic variables, fuzzy if-then rules and fuzzy relations
  - Underlies FL in control, decision analysis, industrial systems, & consumer products
- 4 The epistemic facet, FLe
  - Concerned with knowledge, meaning, and linguistics



## Fuzzy Systems (Fuzzy Logic)



H

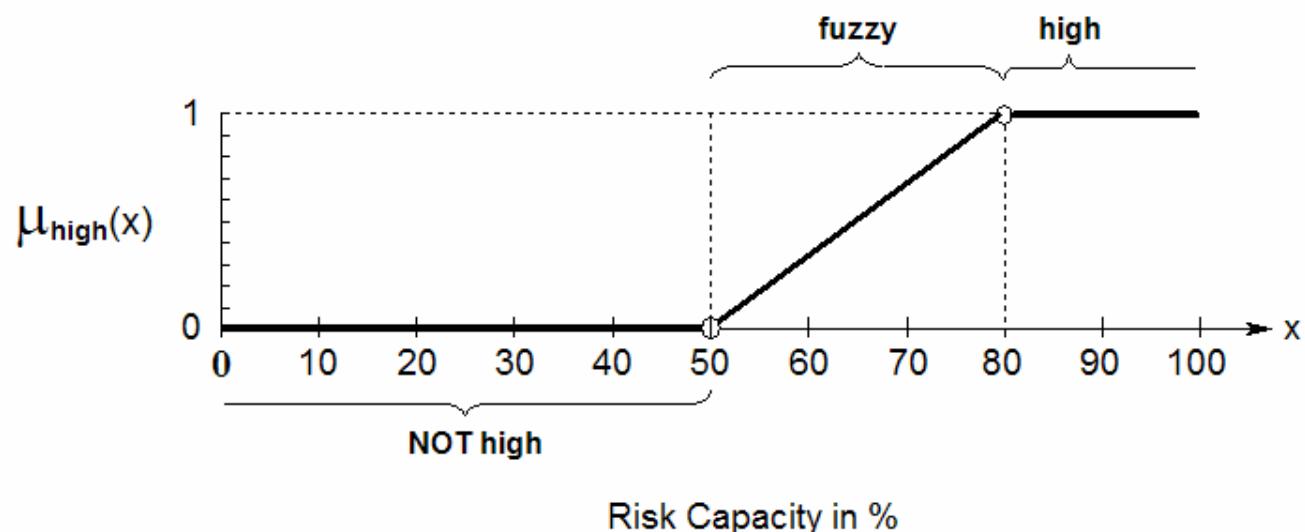


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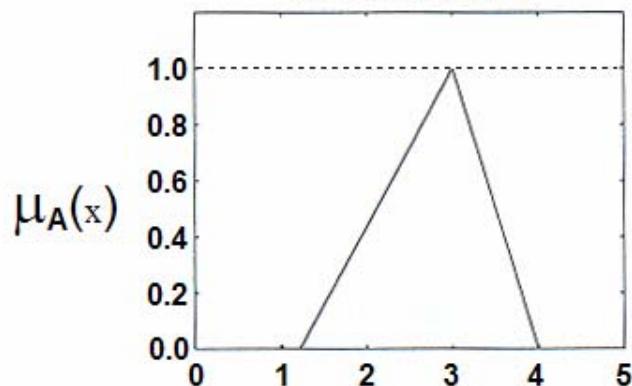


## Membership Function

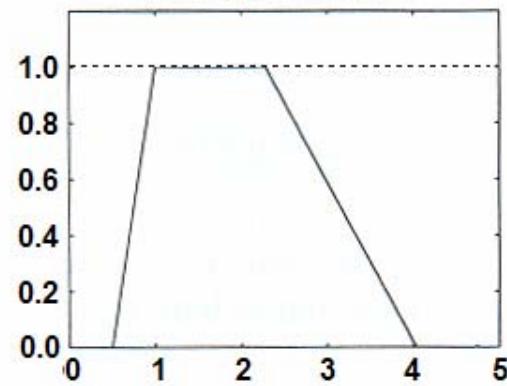


## Examples of MFs

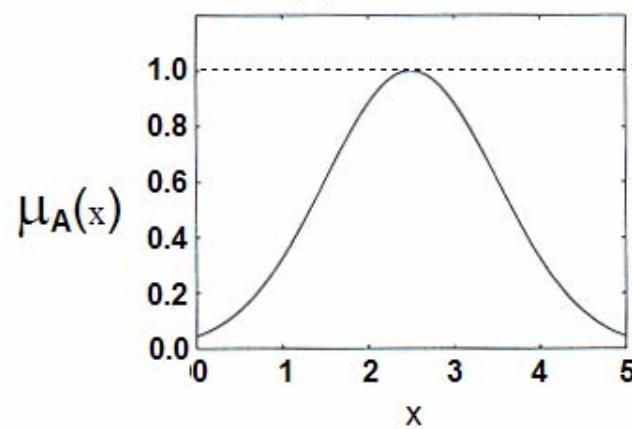
(a) Triangular MF



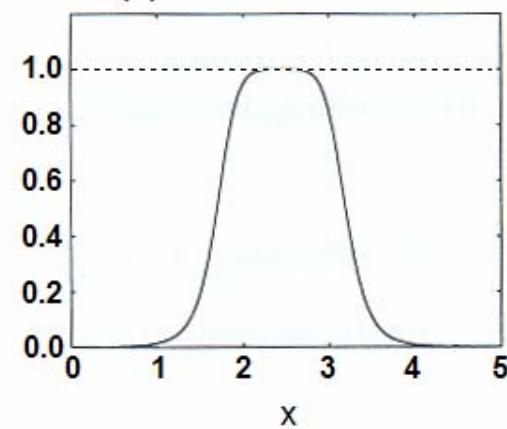
(b) Trapezoidal MF



(c) Gaussian MF



(d) Generalized Bell MF



Boissonnade (1984) 02

9



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# Earthquake

## Application

**Study Objectives:**

## Earthquake risk

Develop a model to estimate earthquake insurance premium rates and insurance strategies

**Data source:**

Damage data

**Technology:**

FST

**Benchmark:**

Bayesian discriminative function

**Conclusion:**

Boissonnade (1984) "Earthquake Damage and Insurance Risk."

44

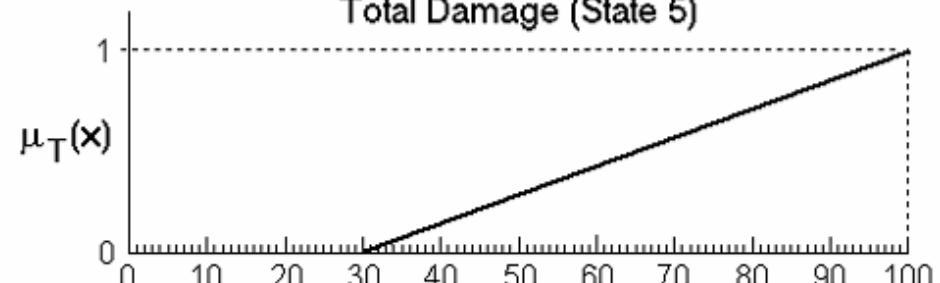
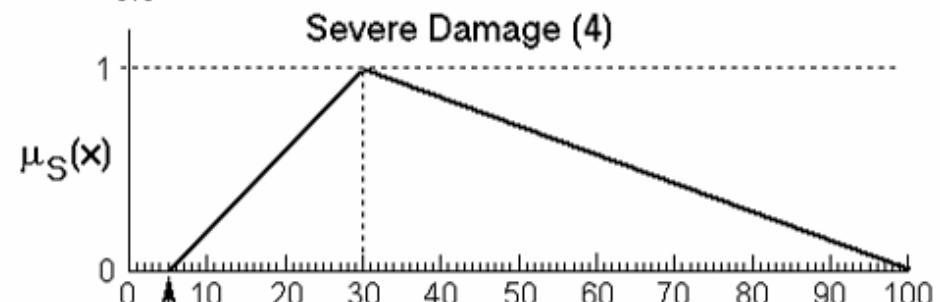
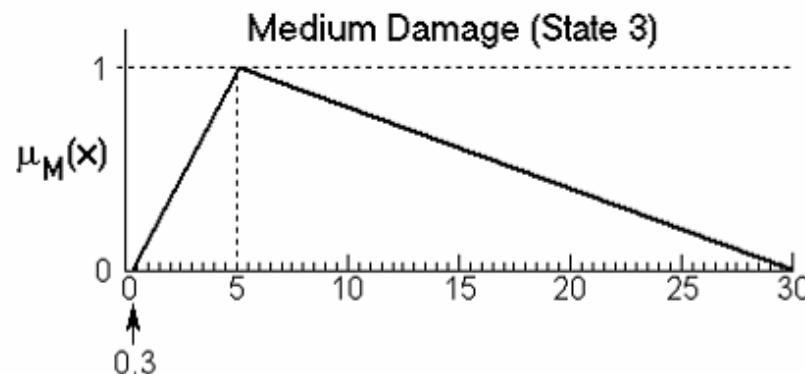
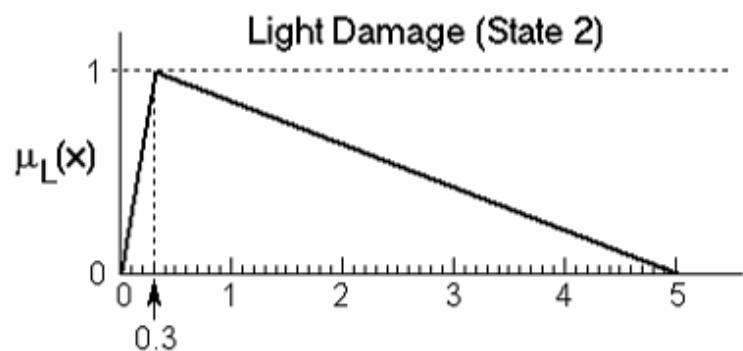
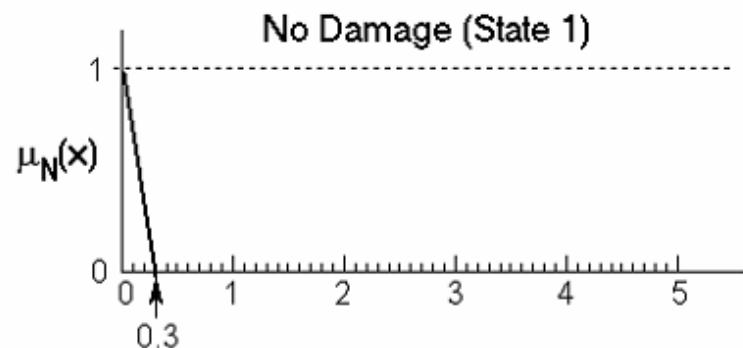


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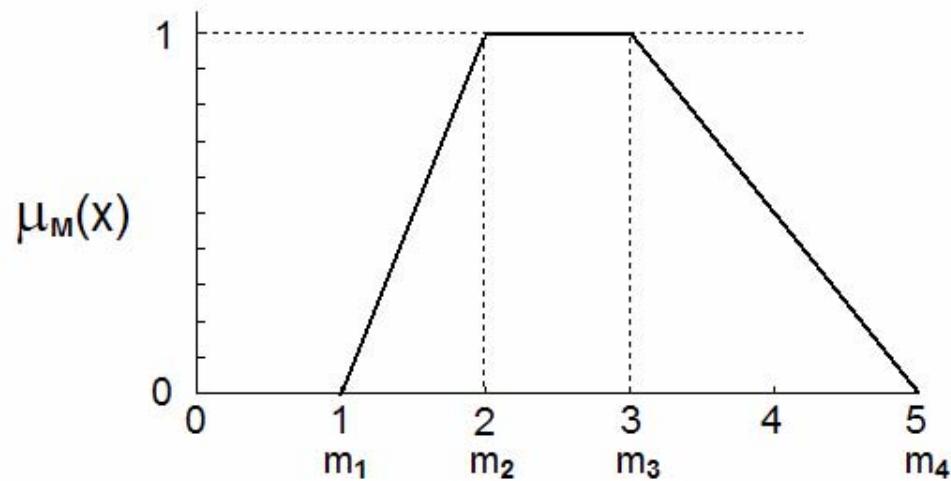
## States of Property Damage

% of building damaged

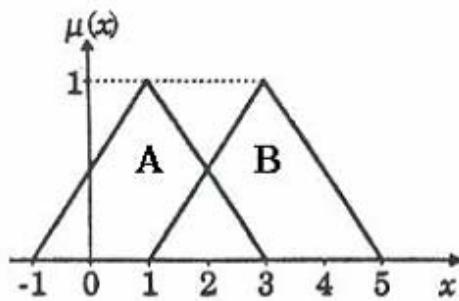


Boissonnade (1984) "Earthquake Damage and Insurance Risk."

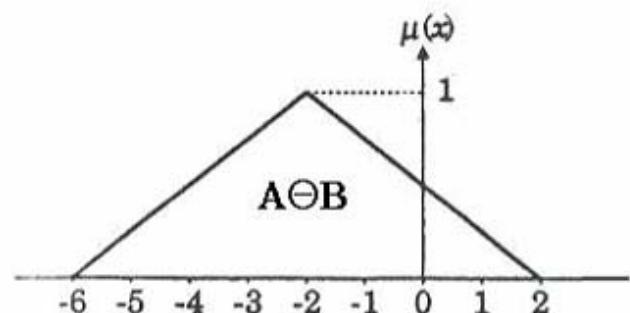
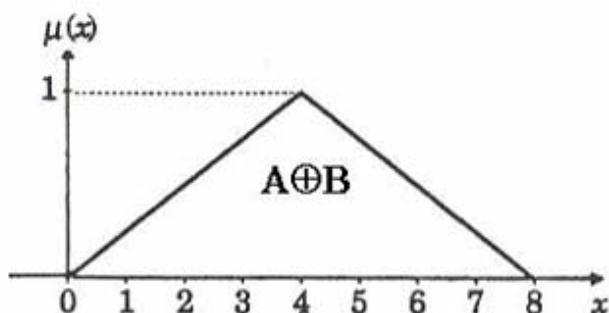
## Fuzzy Numbers



## Fuzzy Arithmetic

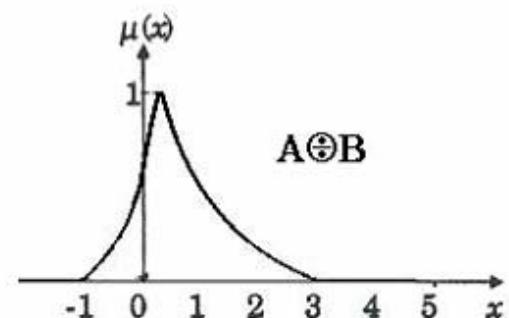
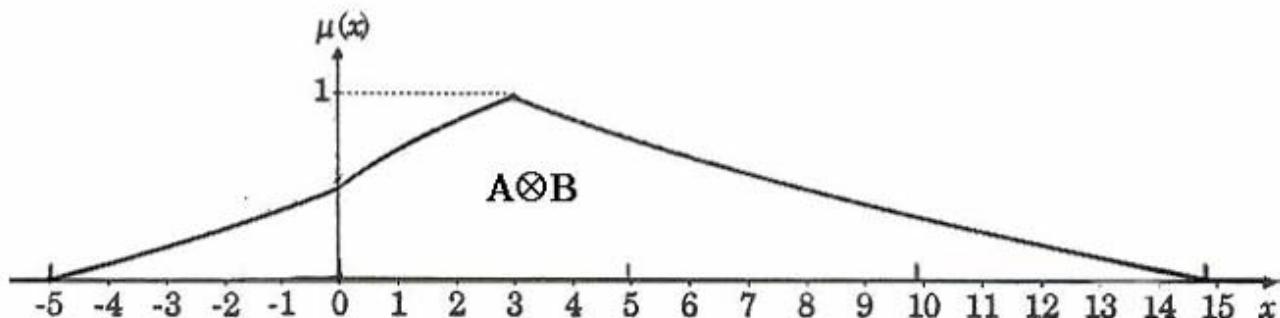


$$\mu_z(w) = \vee_{uv} \mu_x(u) \wedge \mu_y(v), \quad u,v,w \in \Re, \quad w = u * v$$



### Operations

- Addition
- Subtraction
- Multiplication
- Division



Lemaire (1990) 01

22



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## Net Single Premiums

Application	Life Insurance
Study Objectives:	Develop a fuzzy model of a net single premium
Data source:	Survey
Technology:	Fuzzy logic, fuzzy numbers
Conclusions:	Fuzzy set theory could provide decision procedures that are much more flexible

Lemaire (1990) "Fuzzy Insurance," ASTIN Bulletin

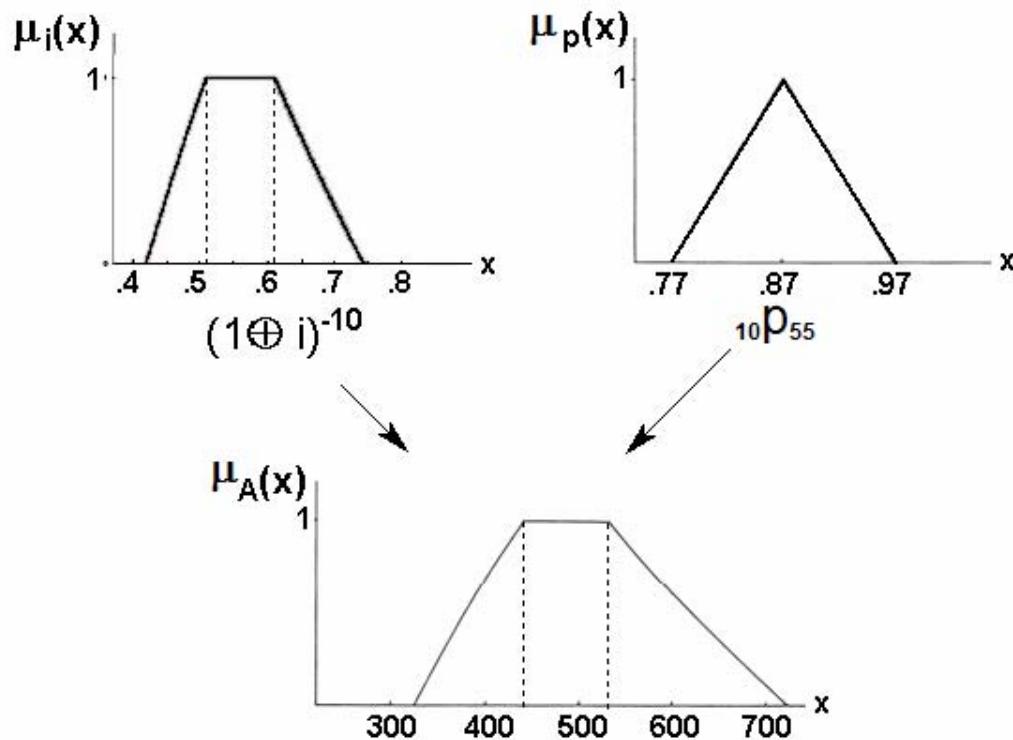
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## Net Single Premium for a Pure Endowment



Lemaire (1990) "Fuzzy Insurance," ASTIN Bulletin

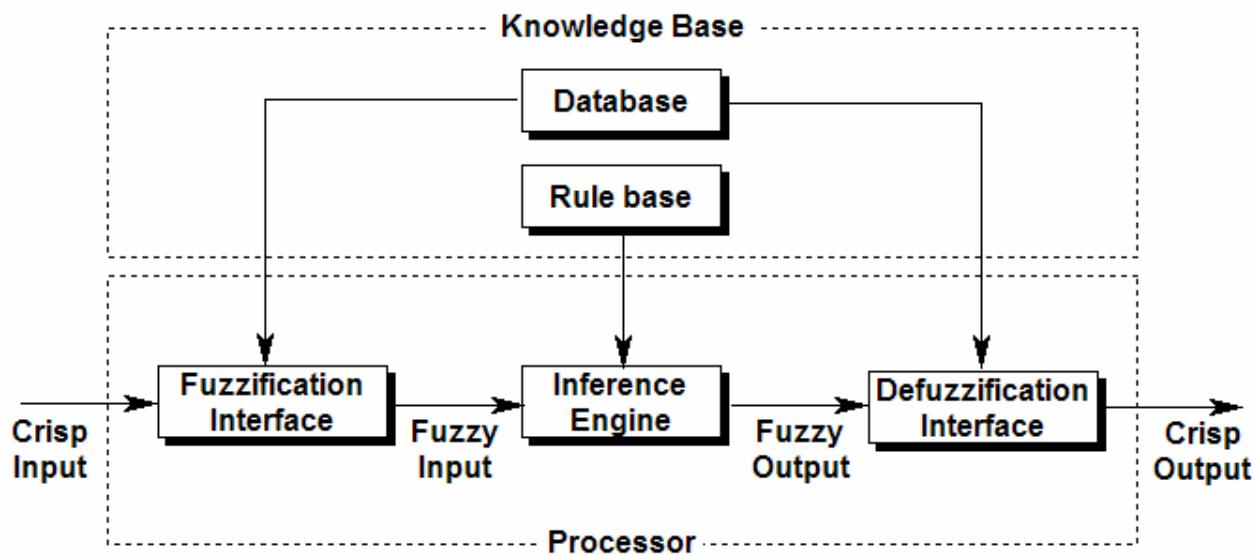
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## Fuzzy Inference System (FIS)



Young (1993) 01

14



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# Group Health Underwriting

Application	Group Health Insurance
Study Objectives:	Improved the model of the selection process
Data source:	Hypothetical
Technology:	Fuzzy sets
Benchmark:	The debits and credits method of underwriting
Conclusion:	Fuzzy set theory provides greater flexibility
Software:	PC spreadsheet

Young (1993) "The Application of Fuzzy Sets to Group Health Underwriting," TSA

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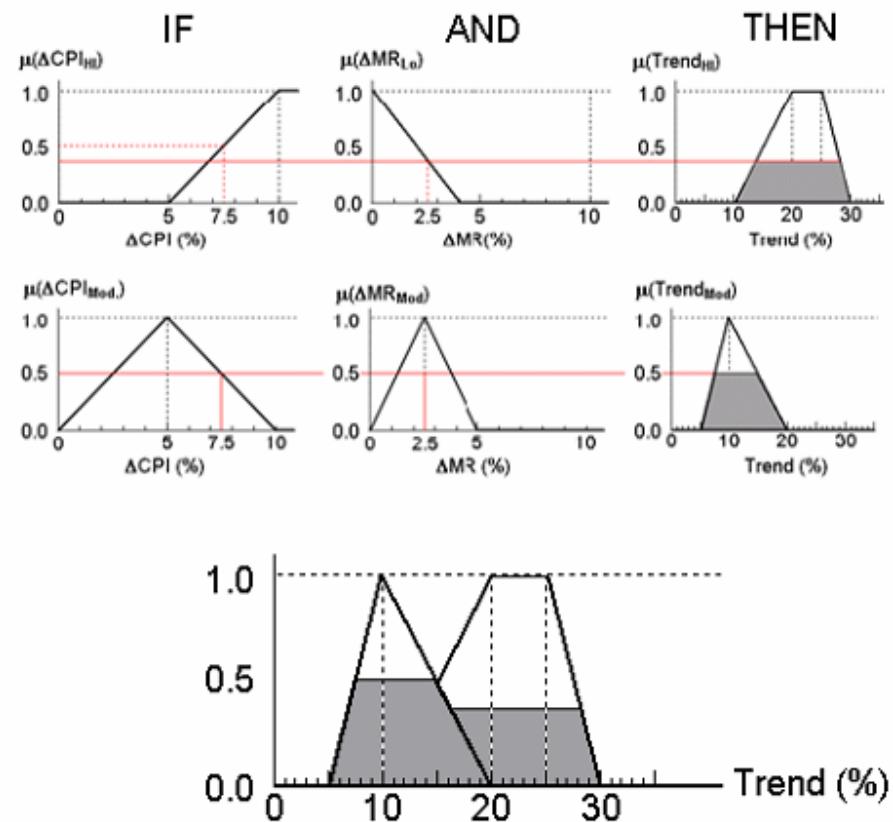


# Trend in Medicare Reimbursement

Medicare Reimbursement Matrix

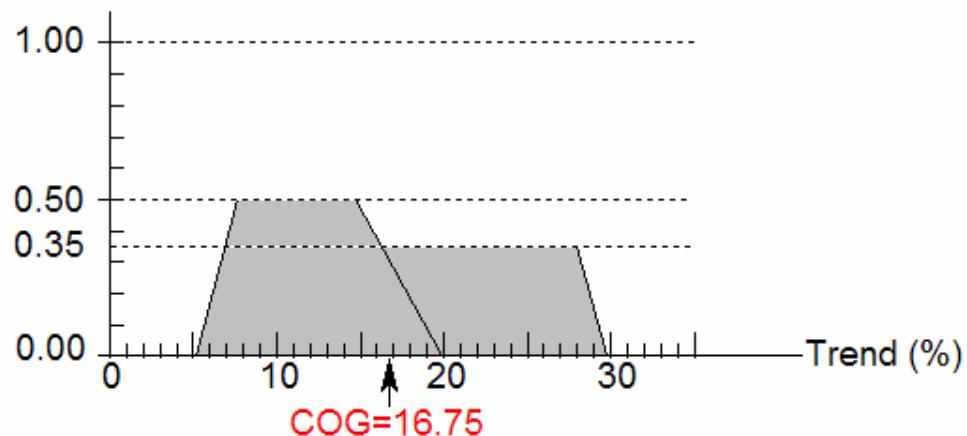
$\Delta(\text{Medicare reimbursement})$		
	Low	Mod.
Low		
Mod.		Mod.
High	High	

Fuzzy Inference System (FIS)



Young (1993). "The Application of Fuzzy Sets to Group Health Underwriting," TSA

## Defuzzification (COG)



$\mu$	Trend (%)	
(1)	(2)	(1)*(2)
0.350	7	2.45
0.500	9	4.50
0.500	11	5.50
0.500	13	6.50
0.475	15	7.13
0.350	17	5.95
0.350	19	6.65
0.350	21	7.35
0.350	23	8.05
0.350	25	8.75
0.350	27	9.45
0.150	29	4.35
4.58	216	76.63
		16.75

Young (1993) "The Application of Fuzzy Sets to Group Health Underwriting," TSA

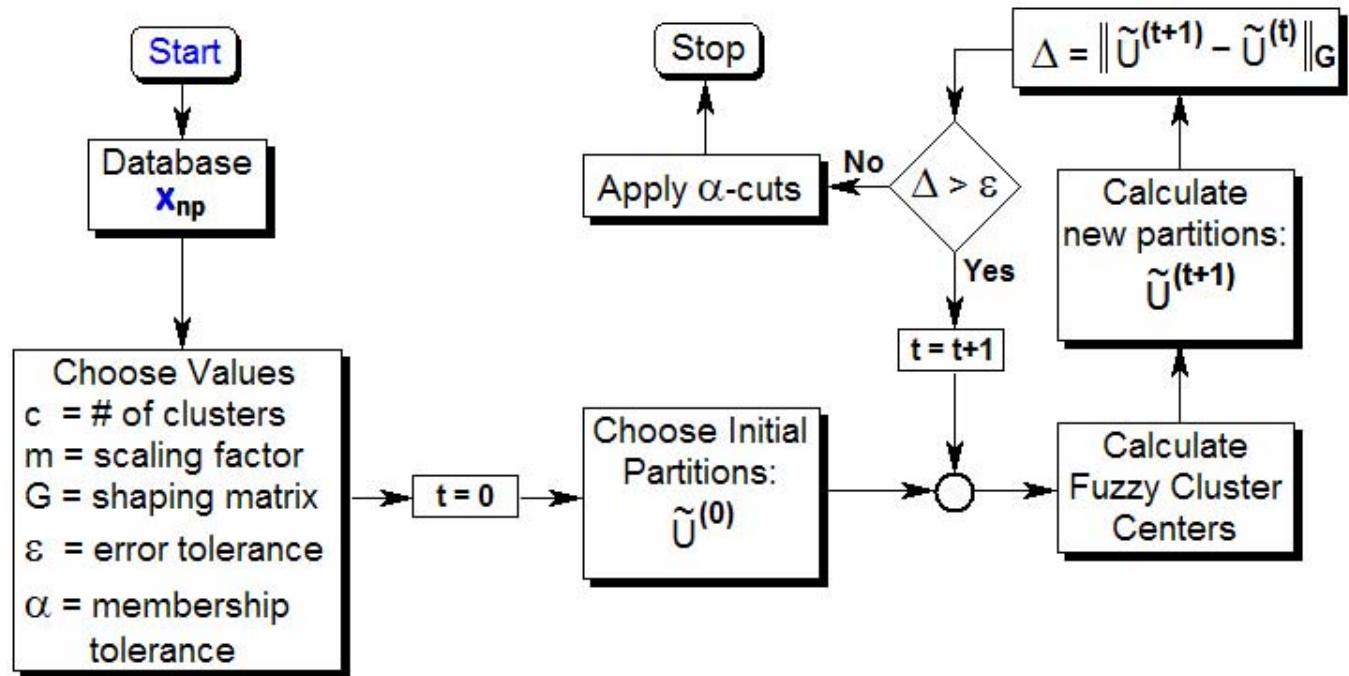
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## Fuzzy Clustering: Fuzzy c-Means Algorithm



Ostaszewski (1993) 02

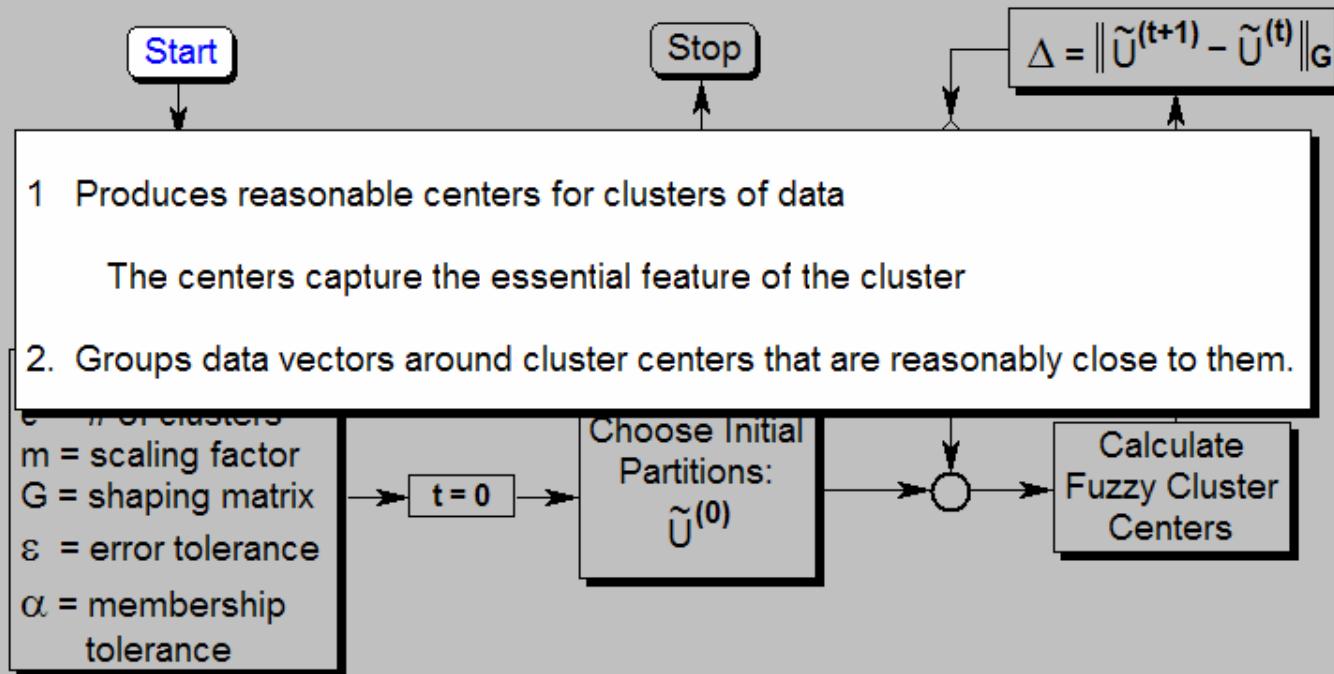
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## Fuzzy Clustering: Fuzzy c-Means Algorithm



20



## Rating Territories

Application	Automobile insurance
Study Objectives:	Develop rating territories & cluster fraudulent claims
Data source:	Massachusetts automobile insurance data
Technology:	Clustering, c-means technique
Benchmark:	Prior studies
Conclusions:	Fuzzy clustering is a valuable method for risk and claim classification

Derrig & Ostaszewski (1995) "Fuzzy Techniques of Pattern Recognition in Risk and Claim Classification," JRI

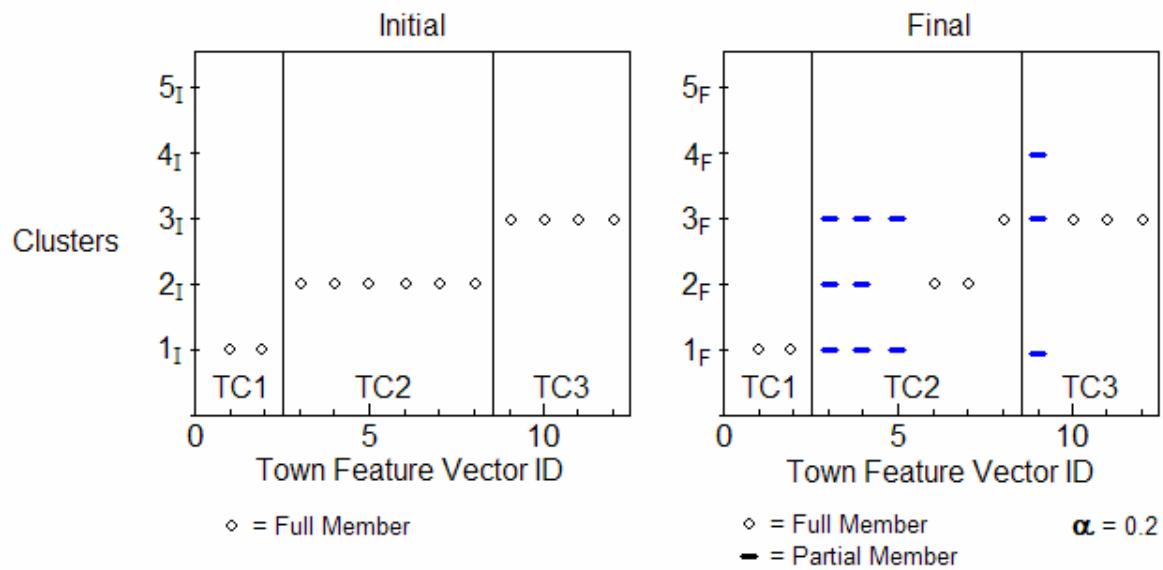
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## Fuzzy Town Clustering



Derrig & Ostaszewski (1995) "Fuzzy Techniques of Pattern Recognition in Risk and Claim Classification," JRI

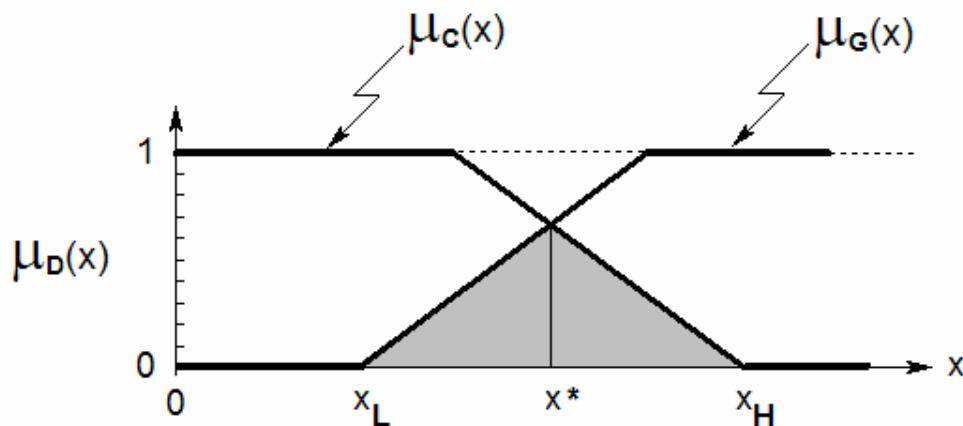
65



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## Decision Making



$$x^* = \arg \max_x \min \{\mu_G(x), \mu_C(x)\}$$

Lemaire (1990-a) 01

10



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# Optimal Retention in an XL Reinsurance Program

Application	Reinsurance
Study Objectives:	Use fuzzy decision-making to select the optimal retention for a pure excess of loss (XL) treaty
Data source:	Subjective
Benchmark:	Decision procedures based on conventional set theory
Conclusion:	FST could provide decision procedures that are much more flexible

Lemaire (1990) "Fuzzy Insurance," ASTIN Bulletin

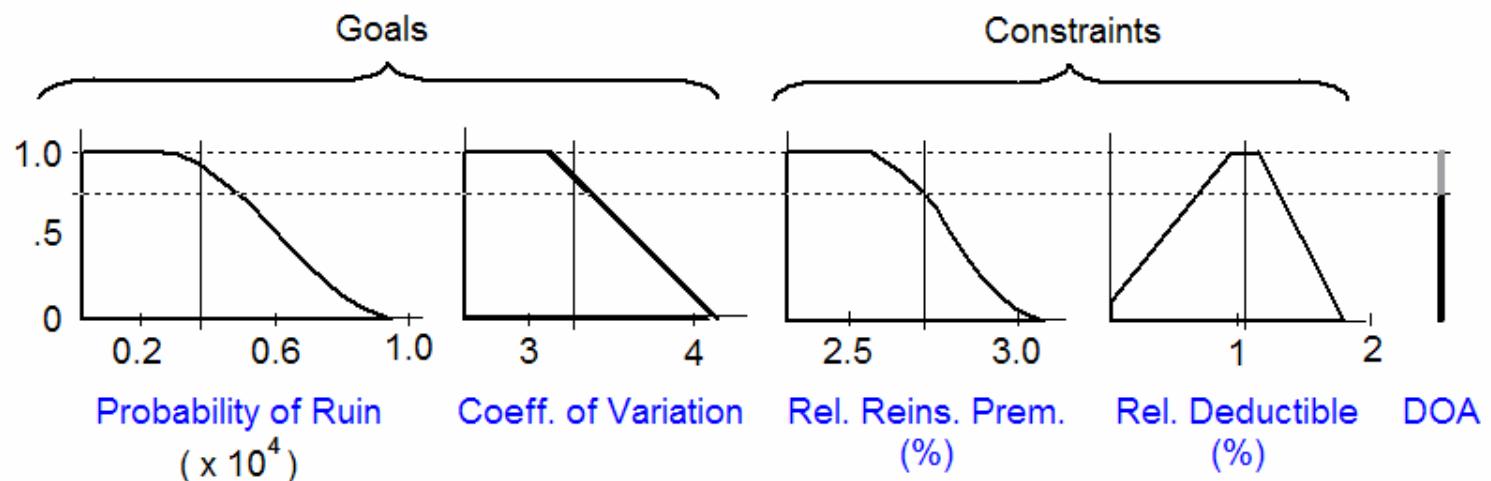
87



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## Optimal Retention in an XL Reinsurance Program



Lemaire (1990) "Fuzzy Insurance," ASTIN Bulletin

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## The Fuzzy LP Problem

Find  $x_{ij}$  such that:

$$C = \sum_{ij} c_{ij} x_{ij} \stackrel{\sim}{\leq} C_0$$

$$z_i = \sum_j a_{ij} x_{ij} \stackrel{\sim}{\geq} b_i$$

$$x_{ij} \geq 0$$

$a_{ij}$ ,  $b_i$  and  $c_{ij}$  may be fuzzy #'s

Maximize  $\alpha$  ( $\alpha$ -cut) subject to:

$$C \leq C_0 + \lambda - \lambda\alpha$$

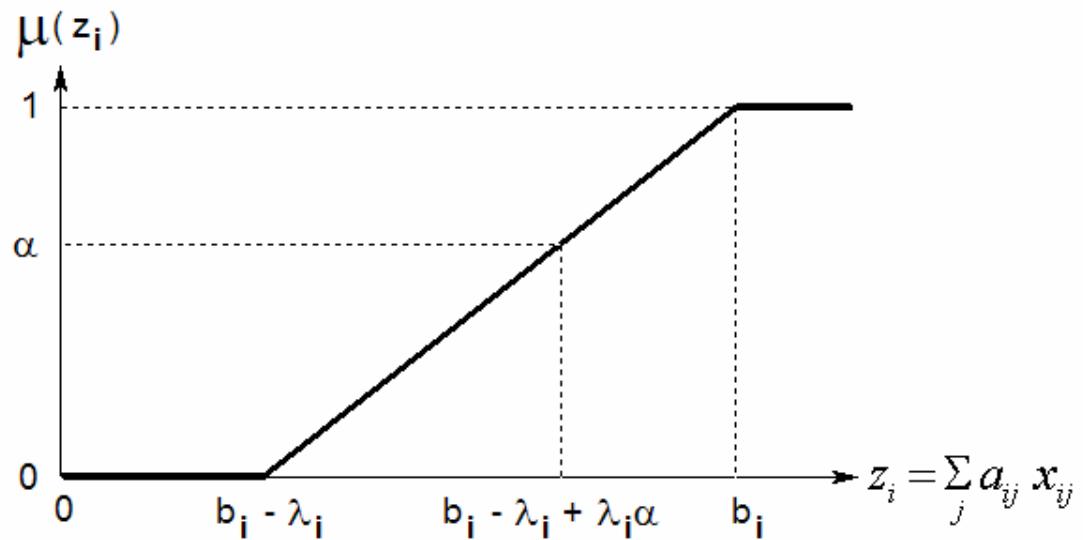
$$z_i \geq b_i - \lambda_i + \lambda_i \alpha$$

$$0 \leq \alpha \leq 1$$

11



## Defuzzify the LP constraint



$$z_i = \sum_j a_{ij} x_{ij} \geq b_i \Rightarrow z_i \geq b_i - \lambda_i + \lambda_i \alpha$$

Guo & Huang (1996) 02

12



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# Asset Allocation

Application	Portfolio Selection
<b>Study Objectives:</b>	Simultaneously maximize the portfolio return, minimize the portfolio risk, and maximize the possibility of reaching higher returns
<b>Technology:</b>	Zimmermann's (1978) fuzzy programming method
<b>Conclusions:</b>	The algorithm provided maximal flexibility for decision makers to effectively balance the portfolio's return and risk

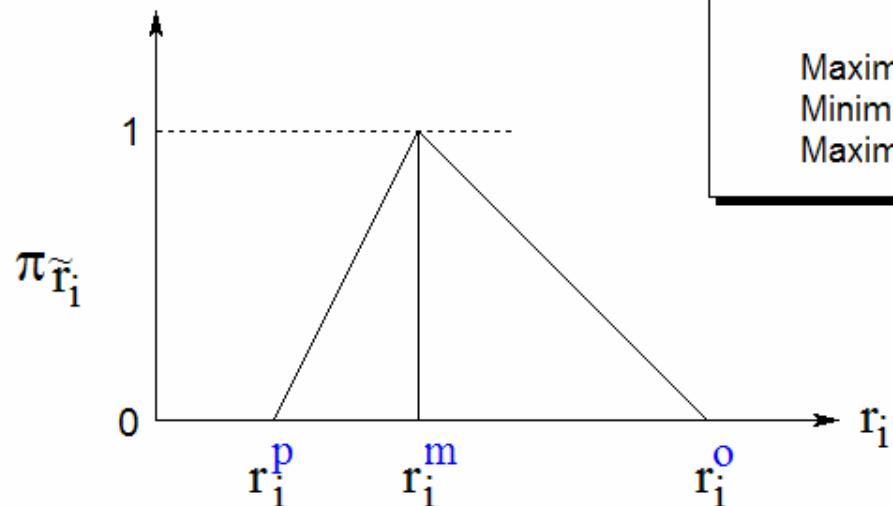
Guo and Huang (1996) "A possibilistic linear programming method for asset allocation"



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## Triangular Possibility Distribution of Entire Portfolio rate of return, $\tilde{r}$



Linear Programming Problem

For a random rate of return

Maximize the mean  
Minimize the risk  
Maximize the upside

Guo and Huang (1996) "A possibilistic linear programming method for asset allocation"

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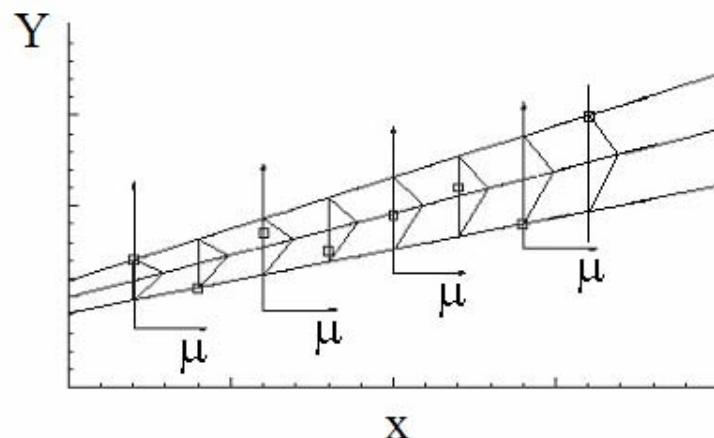


## Fuzzy Linear Regression

$$\tilde{Y}(x_j) = \tilde{A}_0 + \tilde{A}_1 x_{1j} + \cdots + \tilde{A}_n x_{nj}$$

where

Support of TFN  $\tilde{A}_i$  is minimized  
Support contains empirical points



Canz, T. (1996) "Fuzzy Linear Programming in DSS for Energy System Planning"

Sanchez and Gomez (2003) 01

13



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# Term Structure of Interest Rates

Application	Financial Pricing
Study Objectives:	Estimate interest rates
Data source:	Spanish fixed income securities
Technology:	Fuzzy regression
Benchmark:	Spline functions
Conclusion:	The advantage of fuzzy regression is that more information can be used

Sanchez and Gomez (2003) "Estimating a term structure of interest rates for fuzzy financial pricing by using fuzzy regression methods"

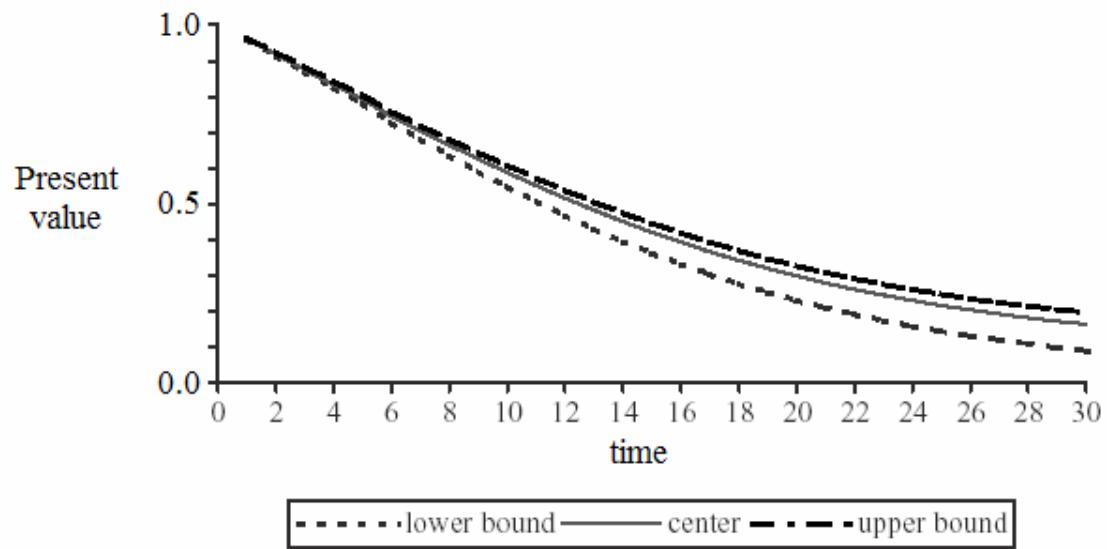
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## Fuzzy Discount Function



Sanchez and Gomez (2003) "Estimating a term structure of interest rates for fuzzy financial pricing by using fuzzy regression methods"

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## Risk Profile of Liability Losses

Application	Automobile liability losses
Study Objectives:	Develop a computerized fuzzy risk profile
Technology:	FST
Conclusions:	The result is a realistic approach to the formal analysis of risk

Jablonowski (1996) "A new perspective on risk," CPCU Journal

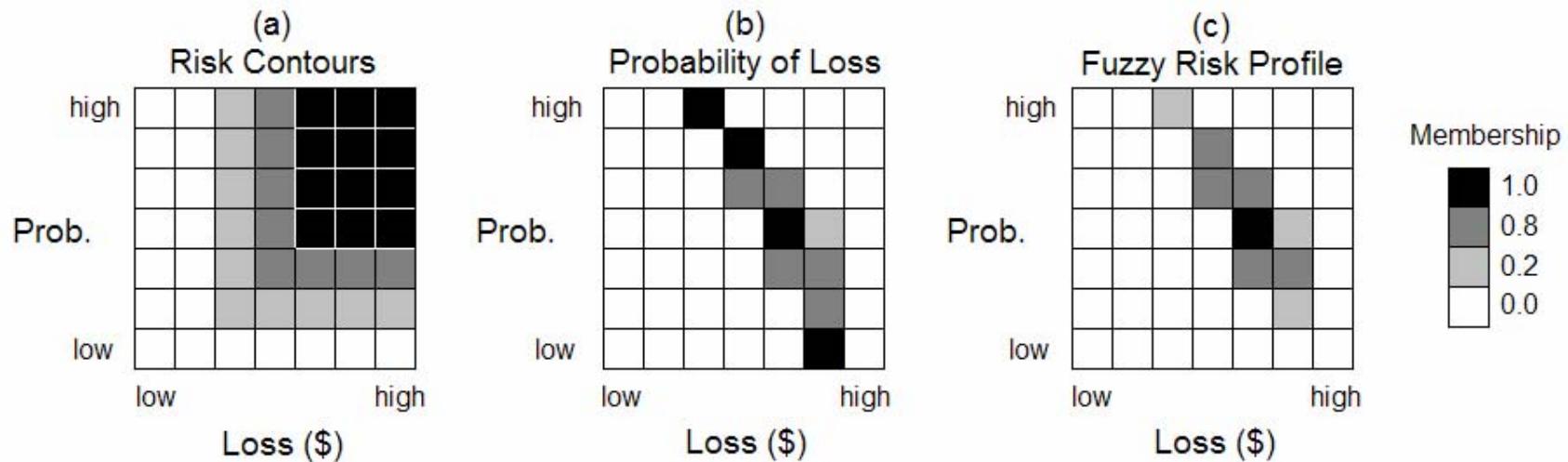
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## Building a Fuzzy Risk Profile



Jablonowski (1996) "A new perspective on risk," CPCU Journal



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# Cash-flow Matching

## Application

### Study Objectives:

Asset-liability management

Use fuzzy arithmetic to derive a cash-flow matching approximation

### Technology:

### Conclusion:

Fuzzy arithmetic, fuzzy zooming

Fuzzy arithmetic may be applied advantageously where there are uncertainties associated with cash flows

Buehlmann and Berliner (1992) Fuzzy zooming of cash flows, 24th TICA, 6:437-453.



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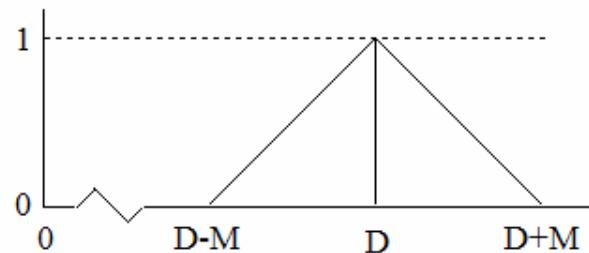
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## Fuzzy Zooming

A series of fixed real payments  $\{G_t\}$  can be replaced by a single payment of

$$(1+i)^D \left( \sum v^t G_t \right)$$

at time D (the duration), with associated triangular fuzzy value (D-M, D, D+M):



where  $M^2 (\sim \sigma^2)$  is the dispersion, and

$$D = \sum t w_t, \quad M = \sqrt{\sum t^2 w_t - D^2}, \quad \text{and} \quad w_t = v^t G_t / \sum v^t G_t$$

Buehlmann and Berliner (1992) Fuzzy zooming of cash flows, 24th TICA, 6:437-453.

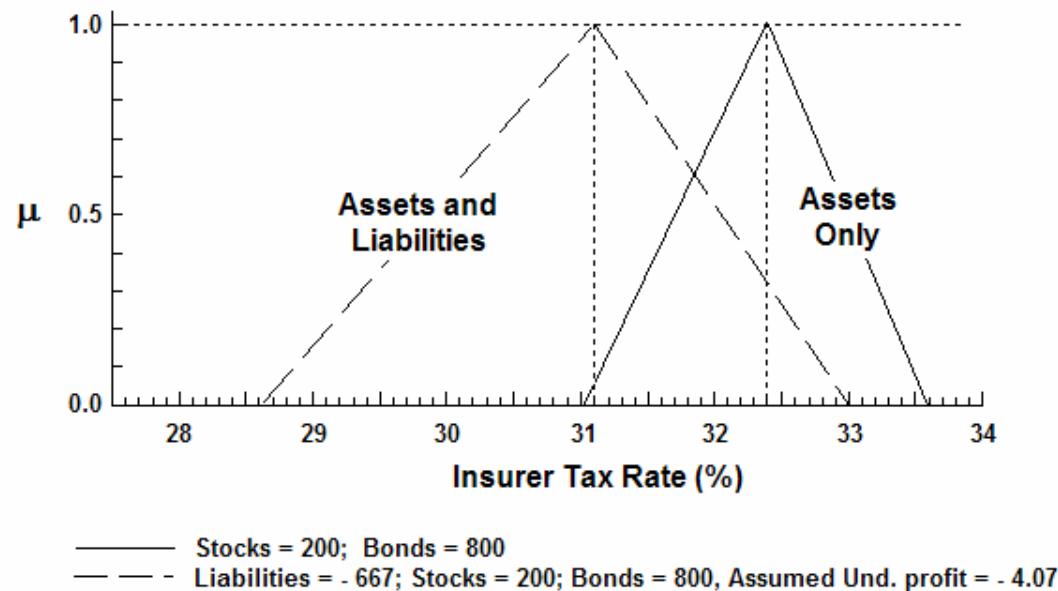
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## Fuzzy Investment Tax Rates: Effect of Liability Tax Shield



Derrig & Ostaszewski (1997) "Managing the Tax Liability of a Property-Liability Insurance Company," JRI

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## P-L Insurance Claim Costs

Application	Automobile BI liability
Study Objectives:	Use FST as a mechanism for combining forecasts using multiple fuzzy criteria
Data source:	Massachusetts PP auto quarterly pure prem data
Technology:	FST
Benchmark:	72 benchmark forecasting methods
Conclusion:	FST provides an effective method for combining statistical and judgement criteria

Cummins & Derrig, "Fuzzy Trends in Property-Liability Insurance Claim Costs," JRI (1993)



## A Good Forecast

The ultimate choice is a "good" forecast rather than a forecasting method

Historical accuracy -- reasonably good for past predictions

Unbiasedness -- reasonably unbiased

Reasonableness -- extreme choices should be avoided

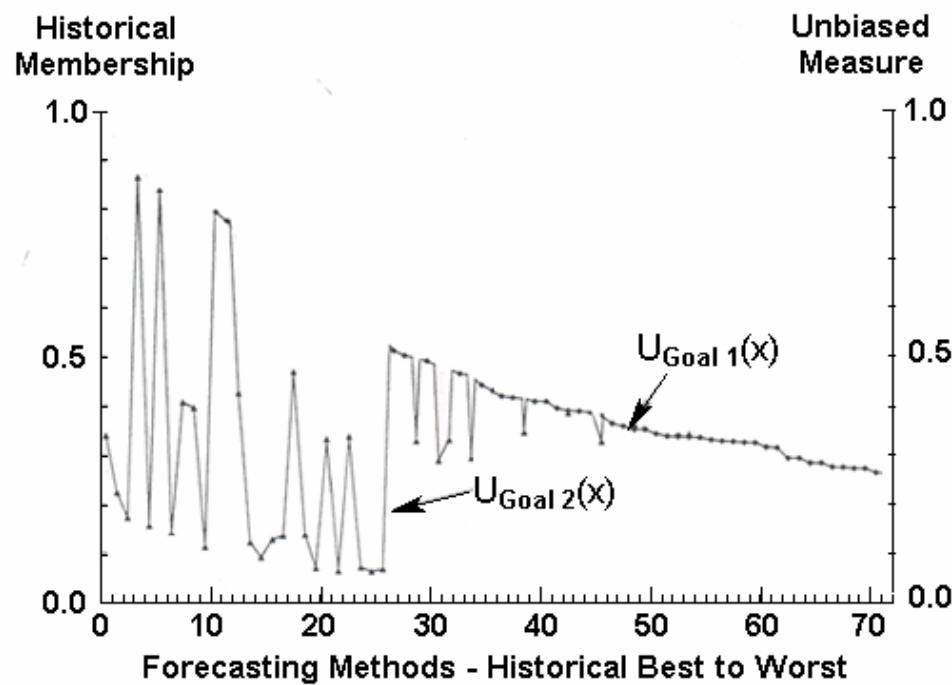
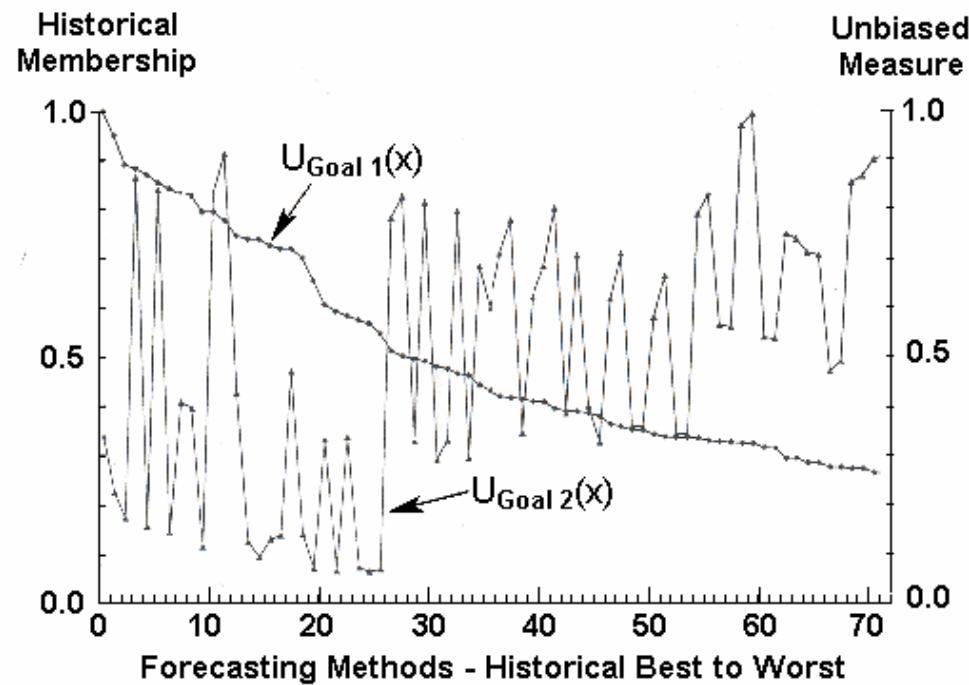
Cummins & Derrig, "Fuzzy Trends in Property-Liability Insurance Claim Costs," JRI (1993)

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## Policyholder Ages

Application	Material damage (MD) and bodily injury (BI)
Study Objectives:	Develop suitable age groupings from past data
Data source:	50,000 motor policies
Technology:	c-means clustering algorithm
Conclusion:	The flexibility of the fuzzy approach makes it most suitable for grouping policyholder age

Verrall and Yakoubov (1999) "A Fuzzy Approach to Grouping by Policyholder Age in General Insurance"



## Policyholder Ages

	Age Groupings						
Group	1	2	3	4	5	6	7
Risk Cluster	1	2	3	4	3	5	6
Age	- 25	26-27	28-31	32-47	48-51	52-68	69 -
Relative Risk	6.66	2.23	1.89	1.48	1.64	1.18	1.00

Verrall and Yakoubov (1999) "A Fuzzy Approach to Grouping by Policyholder Age in General Insurance"

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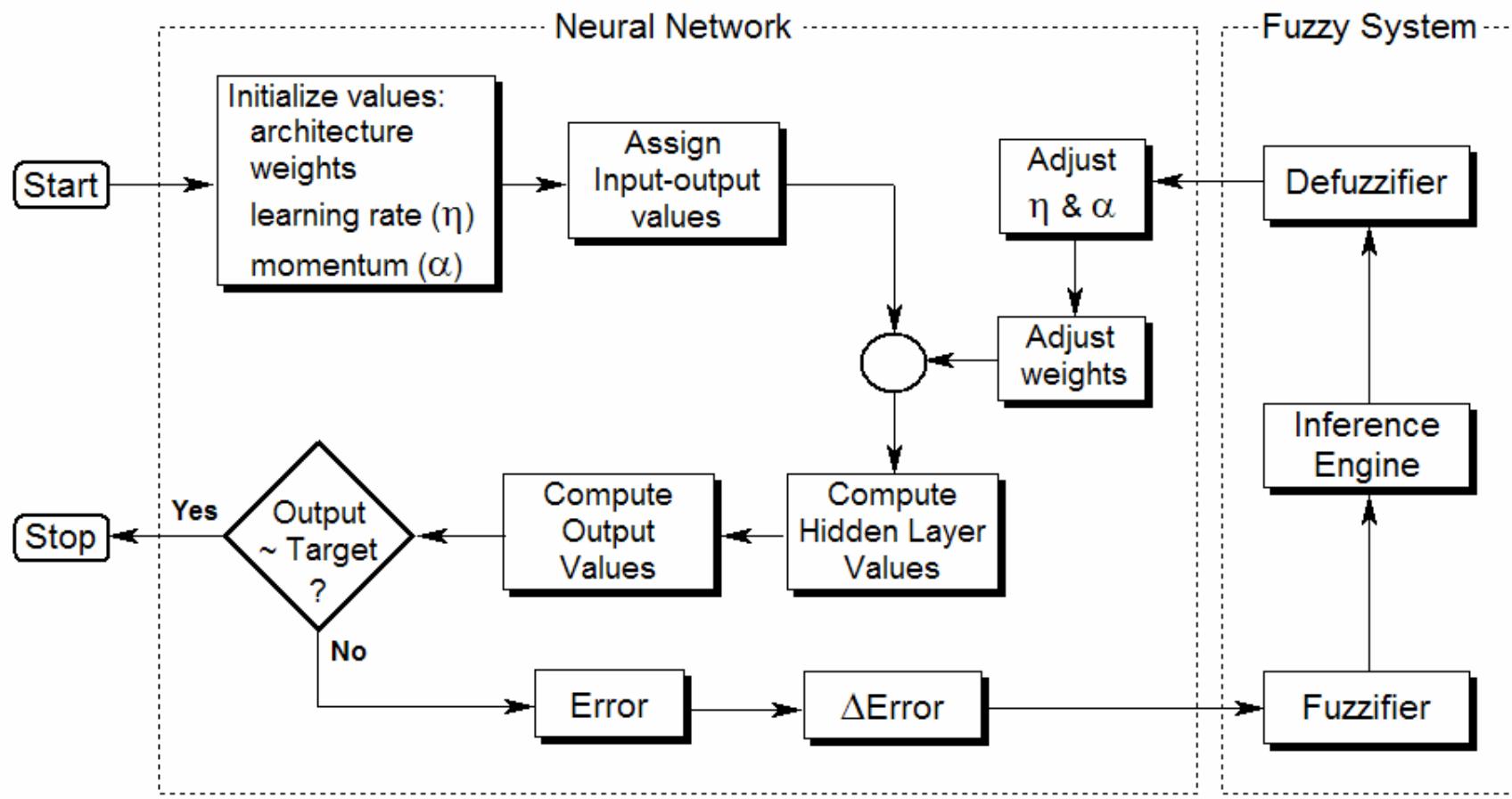


## Advantages and Disadvantages of the NNs, FL, and GAs

Technology	Advantage	Disadvantage
Neural Networks	Adaptation Learning Approximation	Slow convergence speed 'Black box' data processing structure.
Fuzzy Logic	Approximate reasoning	Difficult to tune. Lacks effective learning capability.
Genetic Algorithms	Systematic random search Derivative-free optimization	Difficult to tune No convergence criterion.



# Neural Network and Fuzzy Inference System



$$\Delta w(t) = -\eta \frac{\partial E}{\partial w(t)} + \alpha \Delta w(t-1)$$

26



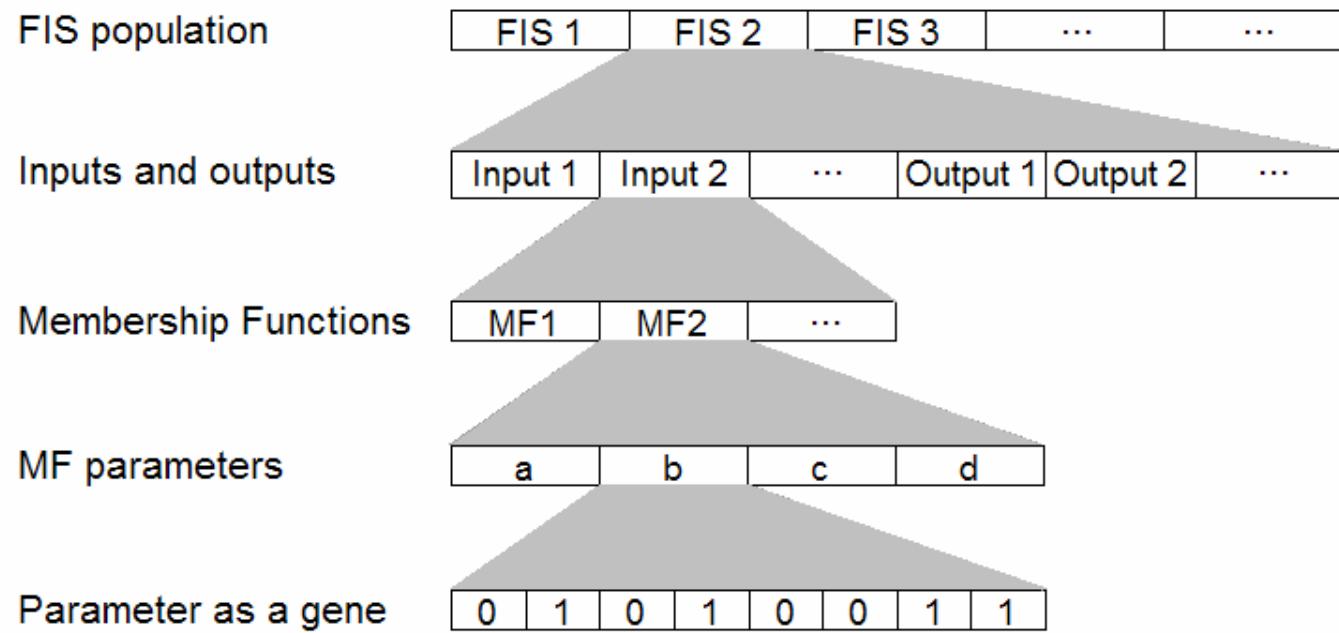
## Fuzzy Rule Table for $\Delta\eta$ and $\Delta\alpha$

		Training Error		
		Small	Medium	Big
$\Delta$ Error:	Small			
	Negative	Very small increase	Very small increase	Small increase
	Zero	No change	No change	Small increase
	Positive	Small decrease	Medium decrease	Large decrease

$$\Delta w(t) = - \eta \frac{\partial E}{\partial w(t)} + \alpha \Delta w(t-1)$$



## GA Hierarchical Representation of FIS



## Conclusions

Many of the FL techniques have been applied in actuarial science:

- Fuzzy arithmetic
- Fuzzy inference systems
- Fuzzy clustering

Actuarial areas where FL has been applied include:

- Classification
- Underwriting
- PV and FV
- Pricing
- Asset Allocation and Cash Flow
- Projections

FL in actuarial science is a fruitful area for exploration:

- Qualitative (fuzzy) situations needing mathematical solutions
- SC will extend previous FL studies

