Financial Services Forum

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A new approach to multiple curve Market Models of Interest Rates

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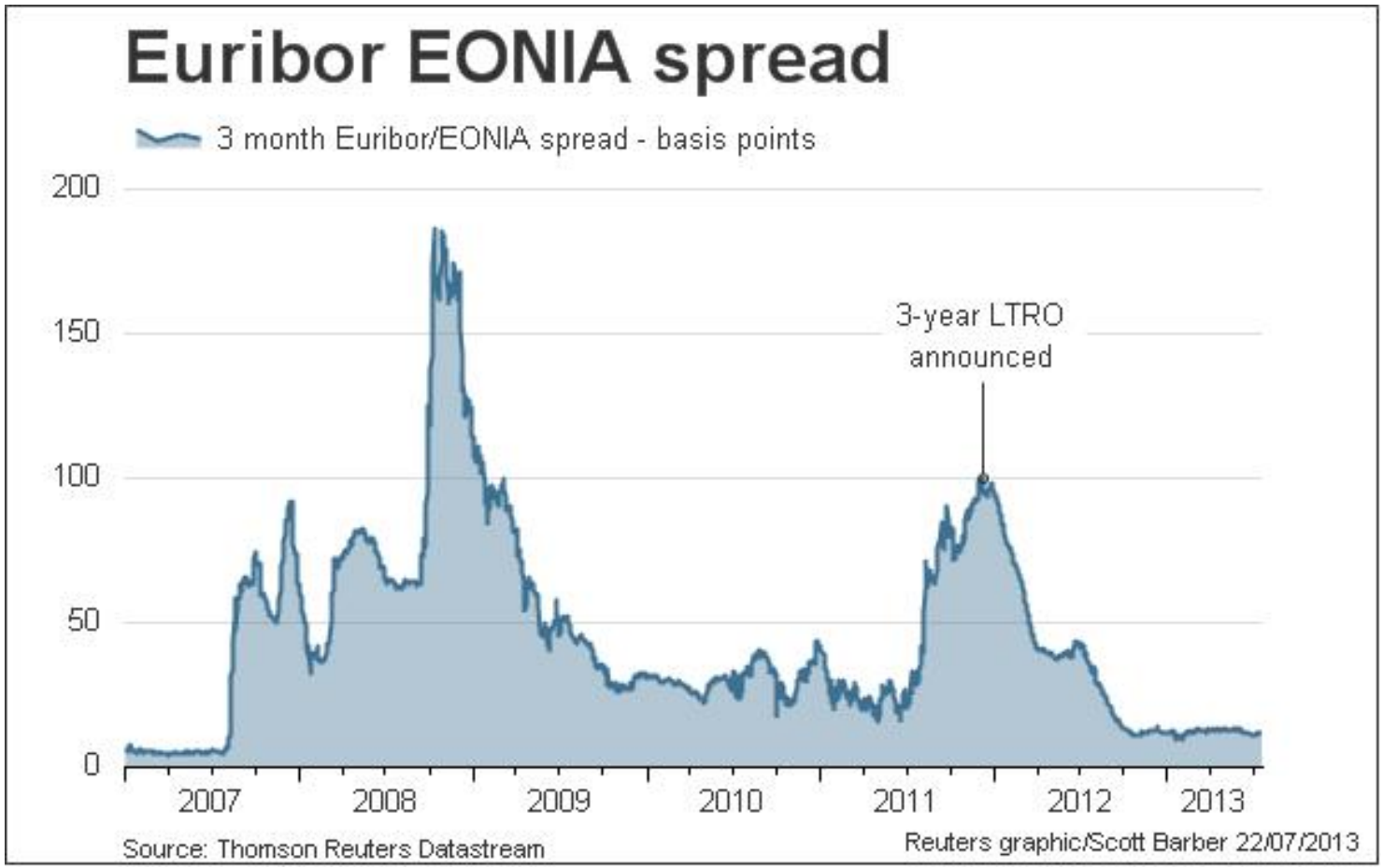
Agenda

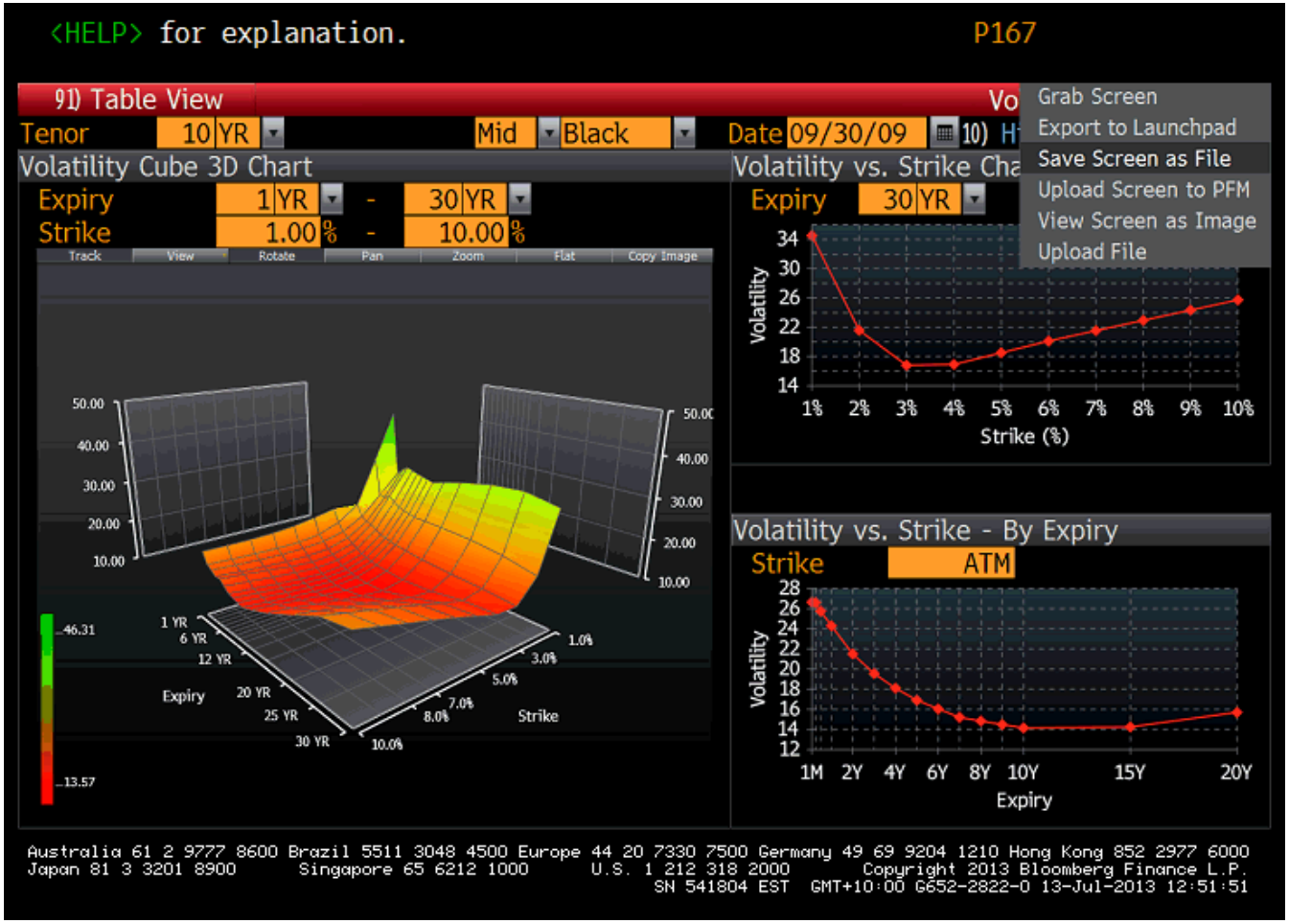
- Motivation/Context – current challenges in term structure modelling
- Volatility modelling
- New model with multiple regimes
- Model statics – implied volatility surface
- Market calibration example

Motivation/Context

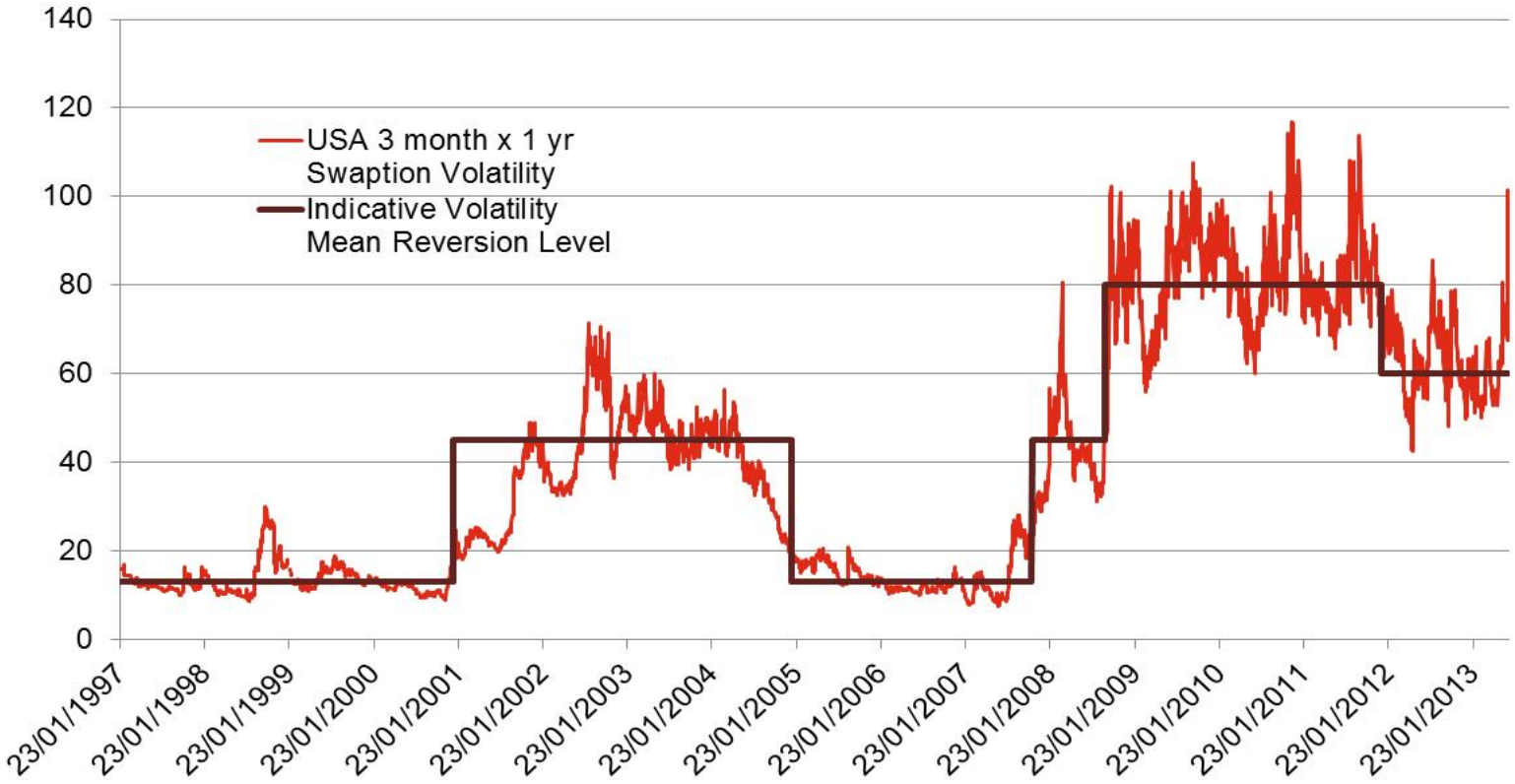
New spread dynamics



Long expiry smile



Option-implied volatility regimes



Modelling requirements

- Non-zero and non-constant spread – multiple curves
- Smile and smile evolution
- Arbitrage-free
- Tractable - rapid pricing and calibration formulas
- Capable of allowing for:
  - Counterparty credit risk
  - Collateral and re-hypothecation
  - Funding liquidity risk
  - Central clearing

Multiple curve term structure models with stochastic volatility

- Stochastic evolution of 2 curves:
  - Discount rates and forward rates; or
  - Discount rates and spreads
- Price contracts linked to Libor, Euribor or equivalent
- Instantaneous or discrete forwards
- Separate stochastic evolution of rate volatility

Model Definition

Stochastic Volatility modelling in term structure models - current methods

1. Heston (mean-reverting process):

$$dz(t) = \kappa(z_0 - z(t))dt + \eta(t)\sqrt{z(t)}dZ(t)$$

κ – mean reversion speed

z_0 – mean reversion level

$\eta(t)$ or η – volatility of variance

2. SABR: (non mean reverting):

$$dz(t) = \alpha z(t)dZ(t)$$

α – volatility of variance

Regime Switching + Mean Reversion

Heston: $dz(t) = \kappa(z_0 - z(t))dt + \eta(t)\sqrt{z(t)}dZ(t)$

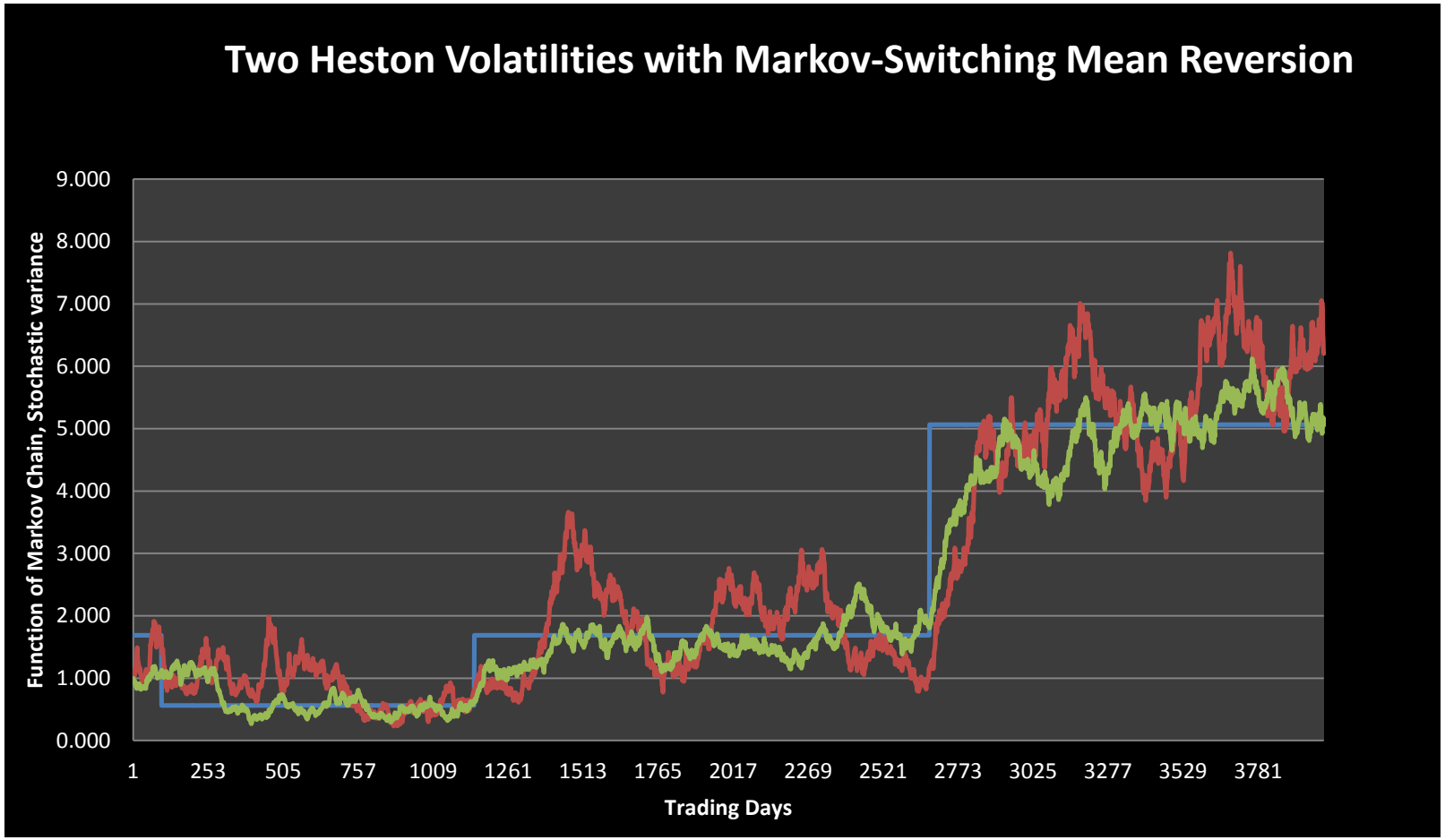
Markov-switching Heston:

$$dz(t) = \kappa\big(f(\alpha(t), t) - z(t)\big)dt + \eta(t)\sqrt{z(t)}dZ(t)$$

$f(\cdot,\cdot)$ – deterministic function of $\alpha(t)$ and t

$\alpha(t)$ - continuous time Markov chain with discrete state space

Volatility Simulation Sample Path



Interest Rate Definitions – Discrete Forward Rate Model

- Times T_0, T_1, T_2 to T_N
- Discrete tenor x spacing (e.g. $x = 3m, 6m$)

At time $t < T_n$, for period T_n to T_{n+1}:

- $L_n^x(t) = F_n^x(t) + S_n^x(t)$ - FRA rate - par swap rate against Libor over (T_n, T_{n+1}) exchanged at T_{n+1}
- $F_n^x(t)$ - forward rate on OIS (overnight indexed swap) curve (for discounting)
- $S_n^x(t)$ - forward rate of FRA-OIS spread

Example Full Model Dynamics – Full Collateralisation

OIS equation (n=1...N): $dF_n^x(t) = \sqrt{z_1(t)}\lambda_n^F(t)\varphi_n^F\big(t, F_n^x(t)\big)dY_F^n(t)$

Spread equation (n=1…N): $dS_n^x(t) = \sqrt{z_2(t)}\lambda_n^S(t)^T\varphi_n^S\big(t, S_n^x(t)\big)dY_S^n(t)$

OIS volatility: $dz_1(t) = \kappa_1\big(f(\alpha(t),t) - z_1(t)\big)dt + \eta_1(t)\sqrt{z_1(t)}dZ_1^n(t)$

Spread vol.: $dz_2(t) = \kappa_2\big(g(\alpha(t),t) - z_2(t)\big)dt + \eta_2(t)\sqrt{z_2(t)}dZ_2^n(t)$

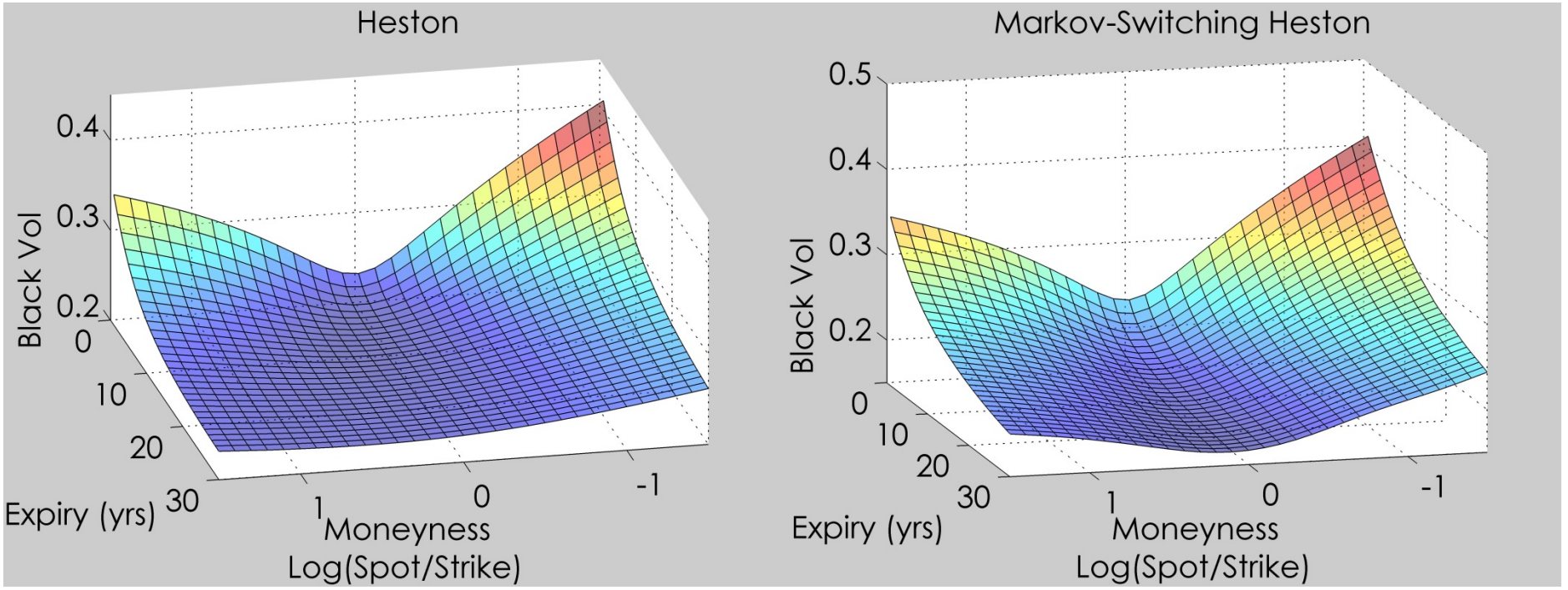
$\lambda_n^F(t)$, $\lambda_n^S(t)$ - deterministic volatility vectors defining vol term structure

$\varphi_n^F(t, x(t)) = \big(b_n(t)x(t) + \big(1 - b_n(t)\big)x(0)\big)$ - displaced lognormal skew

$Y_F^n(t)$, $Y_S^n(t)$, $Z_1^n(t)$, $Z_2^n(t)$ Independent standard Brownian motions ($Y_F^n(t)$, $Y_S^n(t)$ vectors) under OIS T_n-forward martingale measure

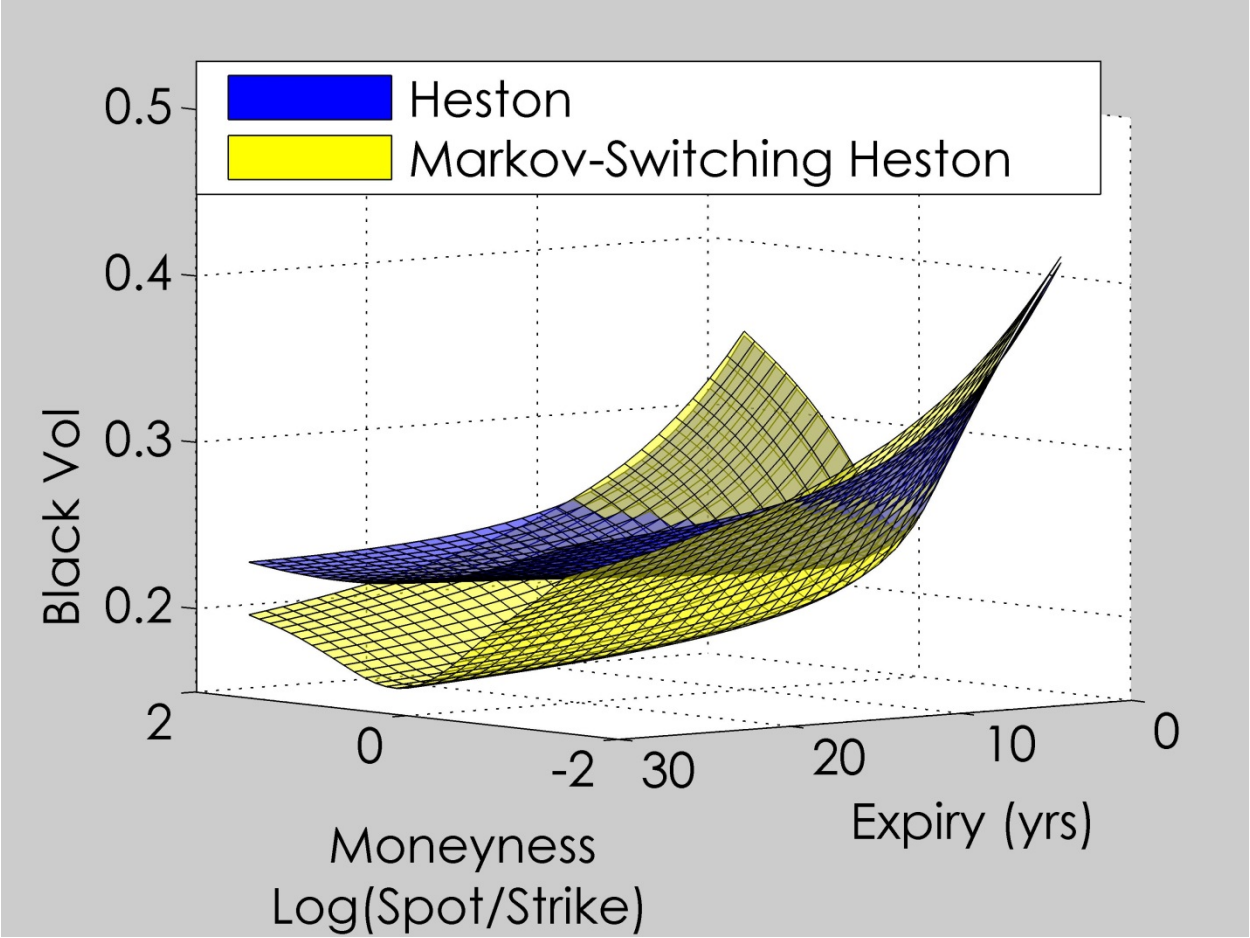
Model Statics

Model-Generated Volatility Surface

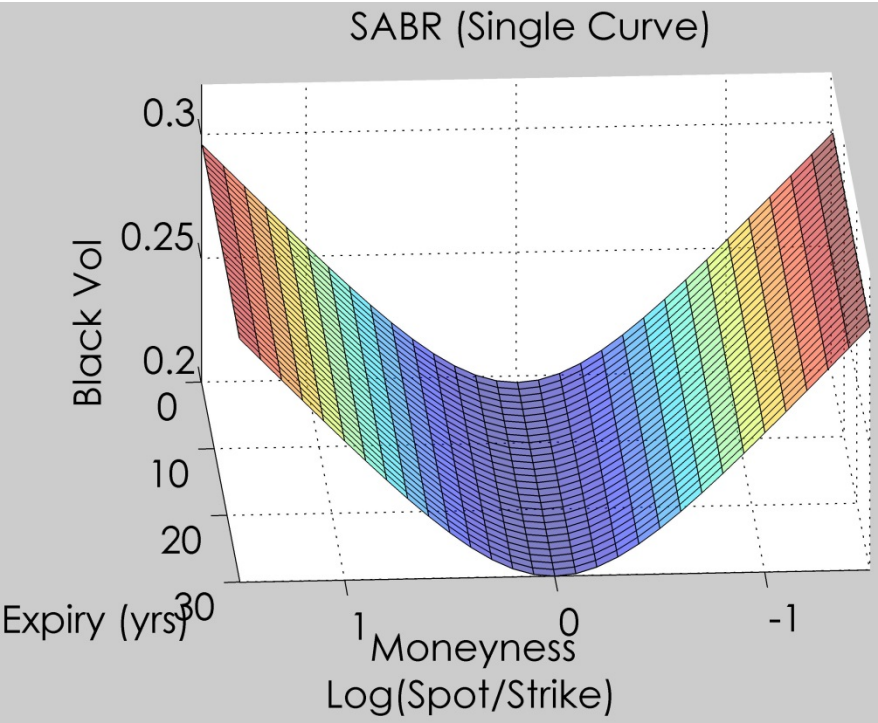


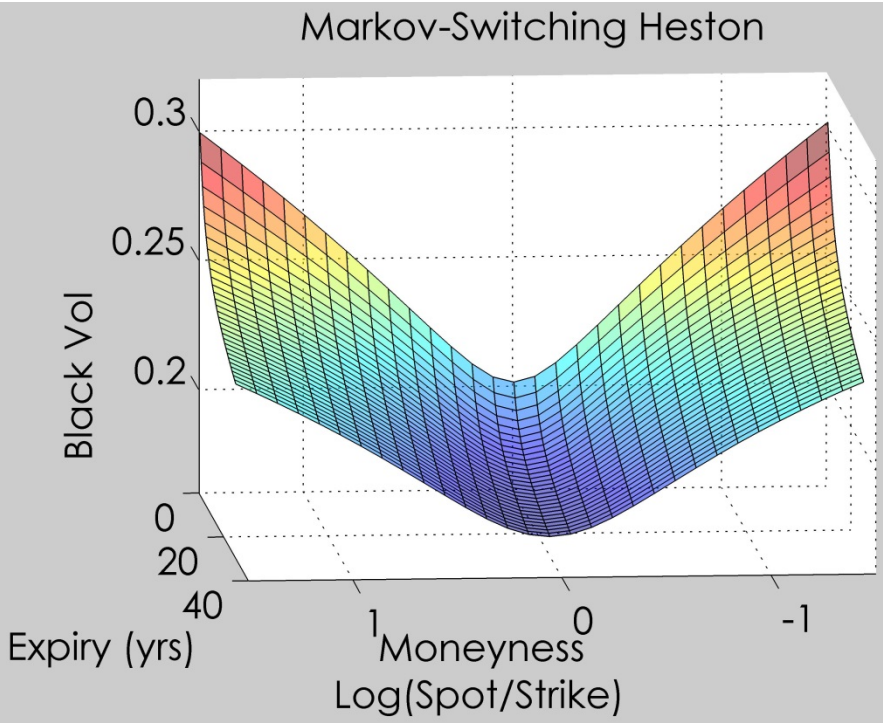
One SV, mean reversion rate = 0.25, volatility of variance = 1.5, 16 state Markov chain

Volatility surface – Markov Chain Impact

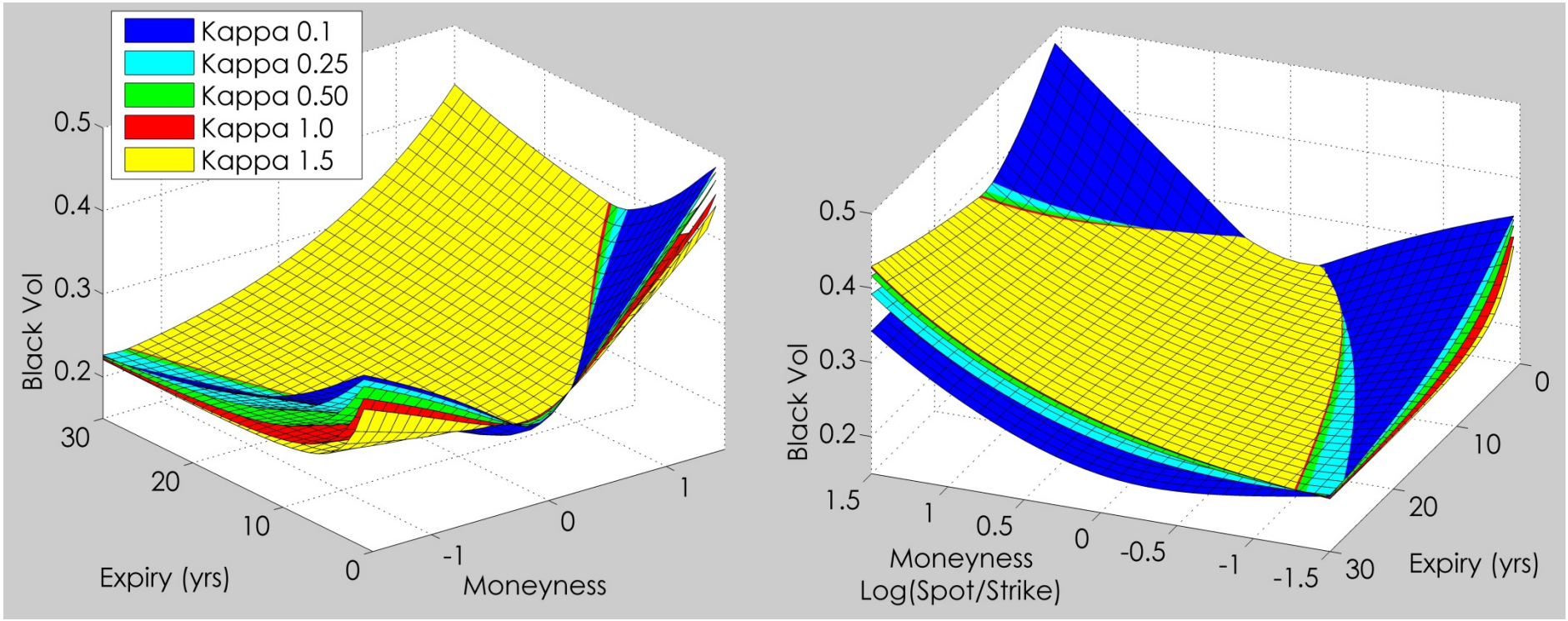


SABR vs Markov-Switching Heston



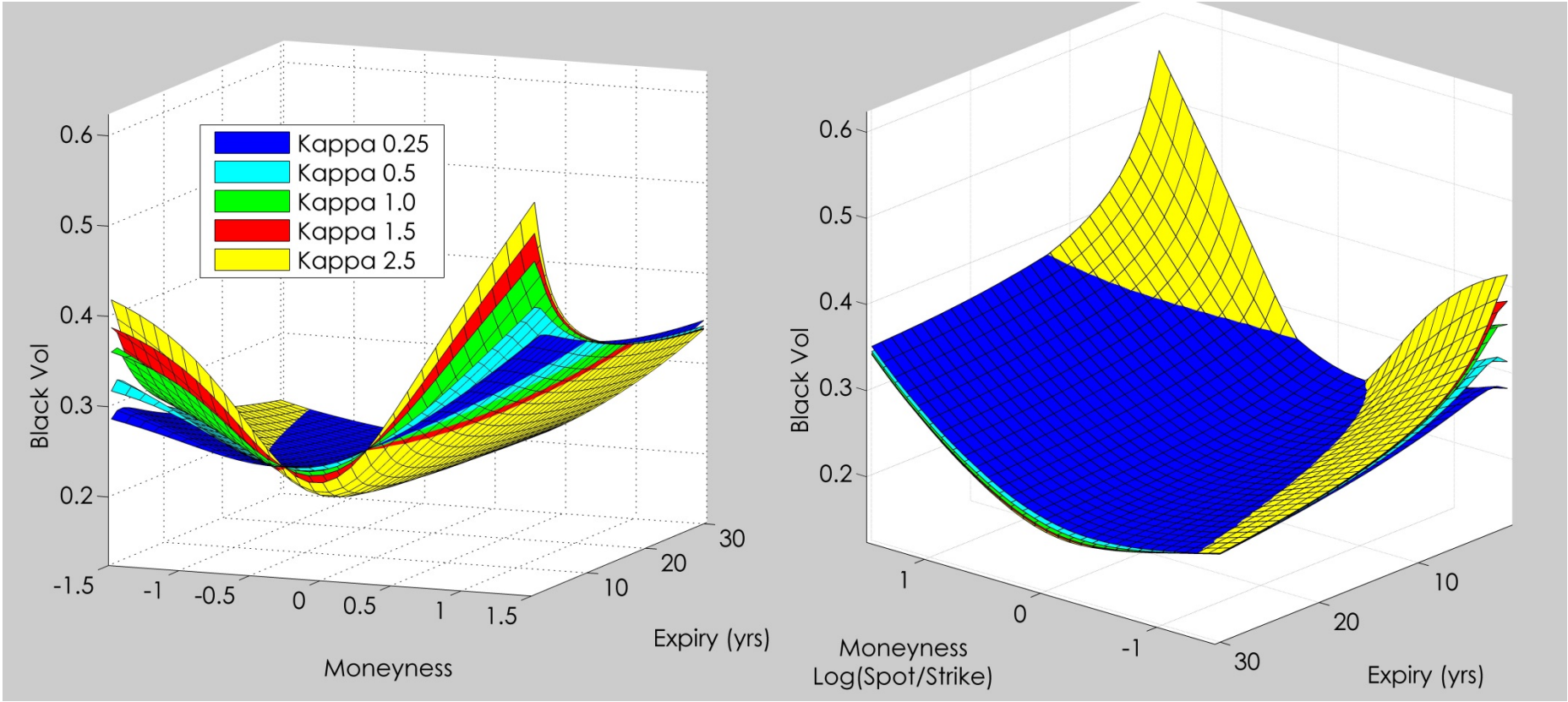


Traditionally Mean Reversion Kills the Smile



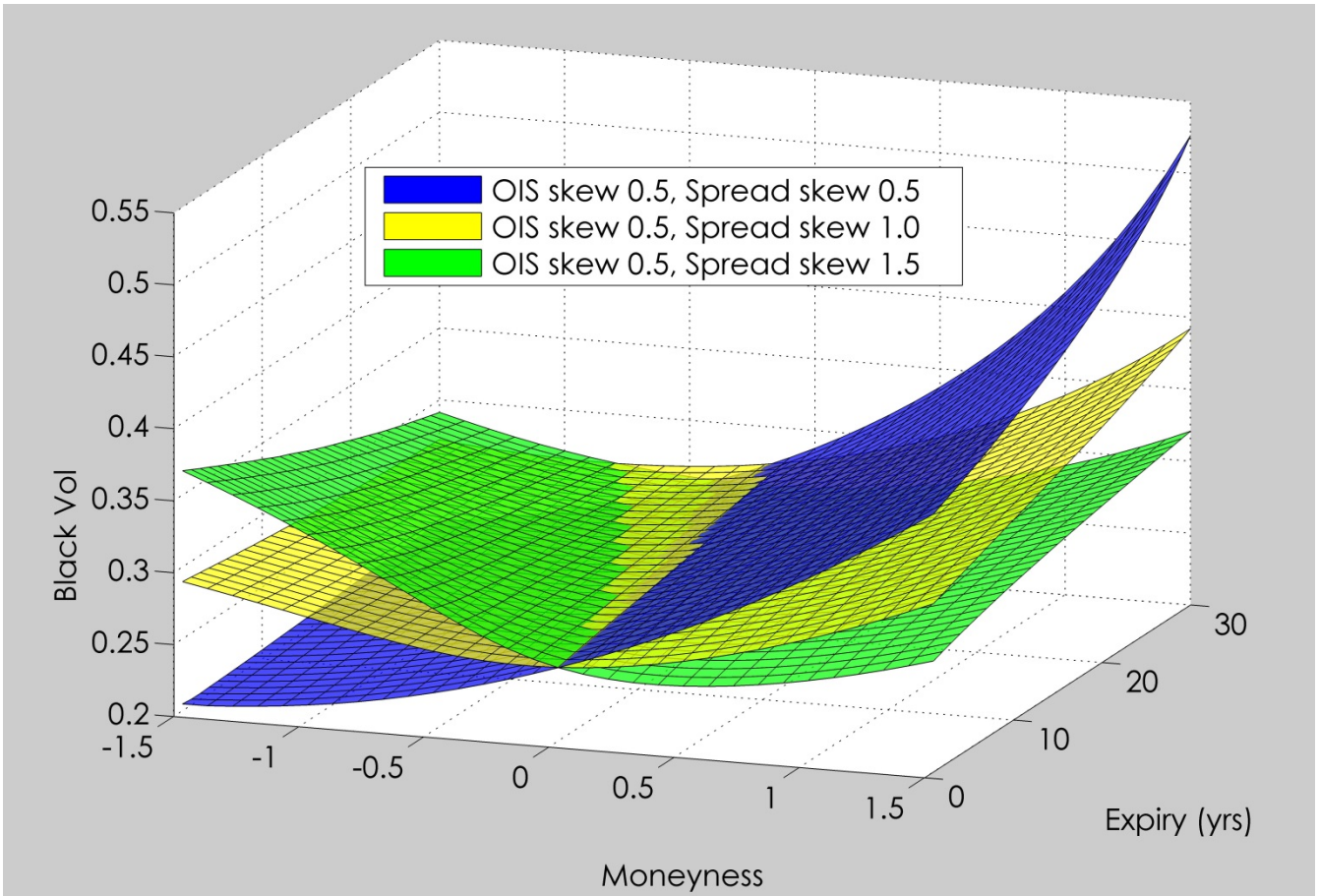
Displaced lognormal Heston, volatility of variance = 1.5, OIS and Spread Skew 0.6

Kappa can now increase short end smile



Markov-switching Heston, Volatility of variance = 0.5, OIS and Spread Skew 0.6

Smiles Without SV – Sum of 2 Displaced Lognormal Processes



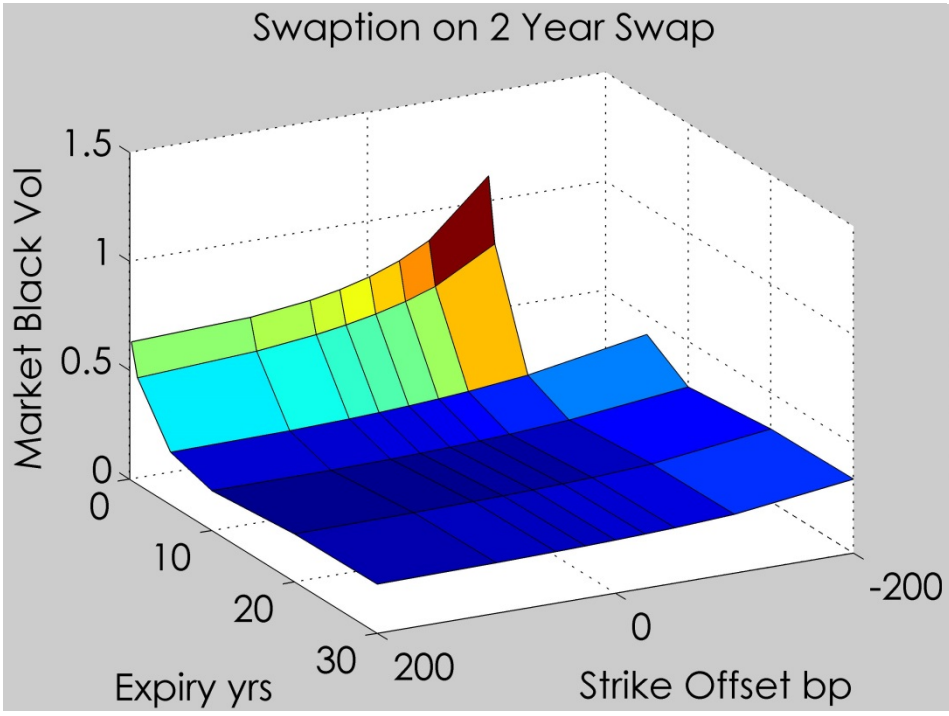
Empirics –

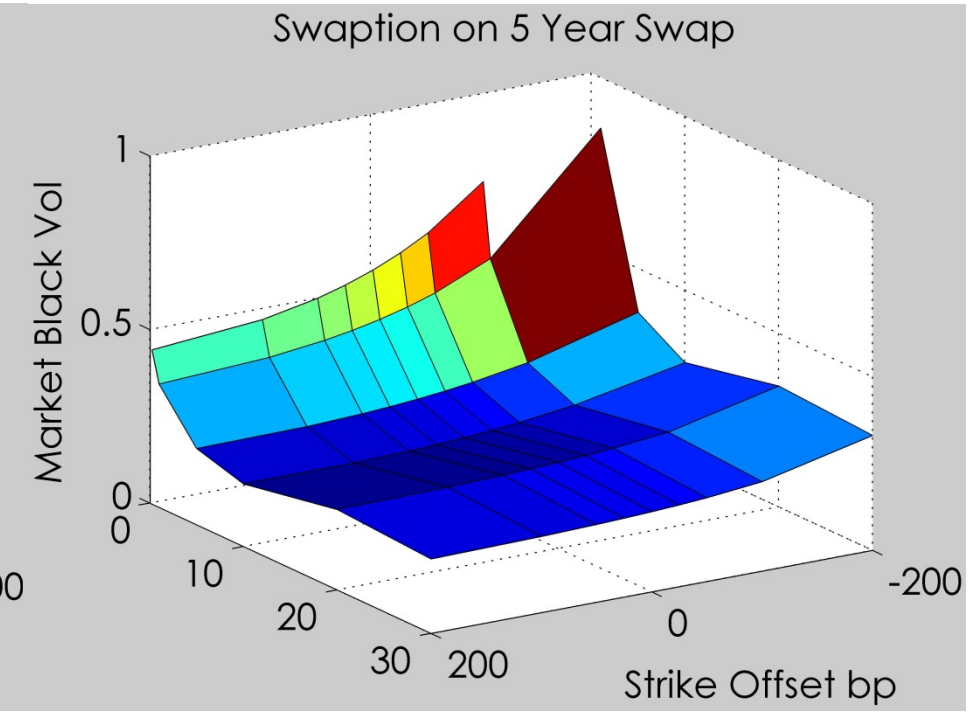
Term Parameter Calibration to Swaption Cube

Swaption cube

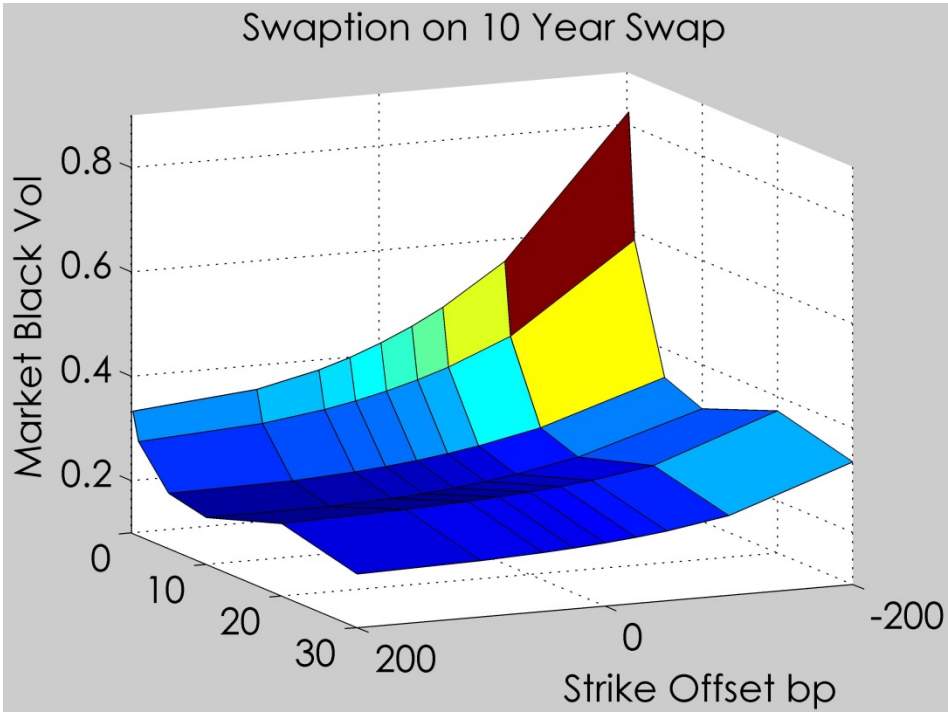
- Swaption – option to enter interest rate swap at a fixed rate (strike) instead of the prevailing future par forward swap rate
- Swaption Cube – Implied volatility (price) by 3 factors – option expiry, underlying swap maturity, strike
- Bloomberg swaption cube implied volatilities available for
  - Option expiries 3M, 1Y, 5Y, 10Y, 20Y and 30Y
  - Swap maturities for each expiry 2Y, 5Y, 10Y, 20Y, 30Y
  - 9 Strikes for each expiry/maturity pair – At The Money and +/- 25,50,100,200 basis points
  - 30 “smiles” each with 9 strikes
- Euro area money market : 6 month tenor

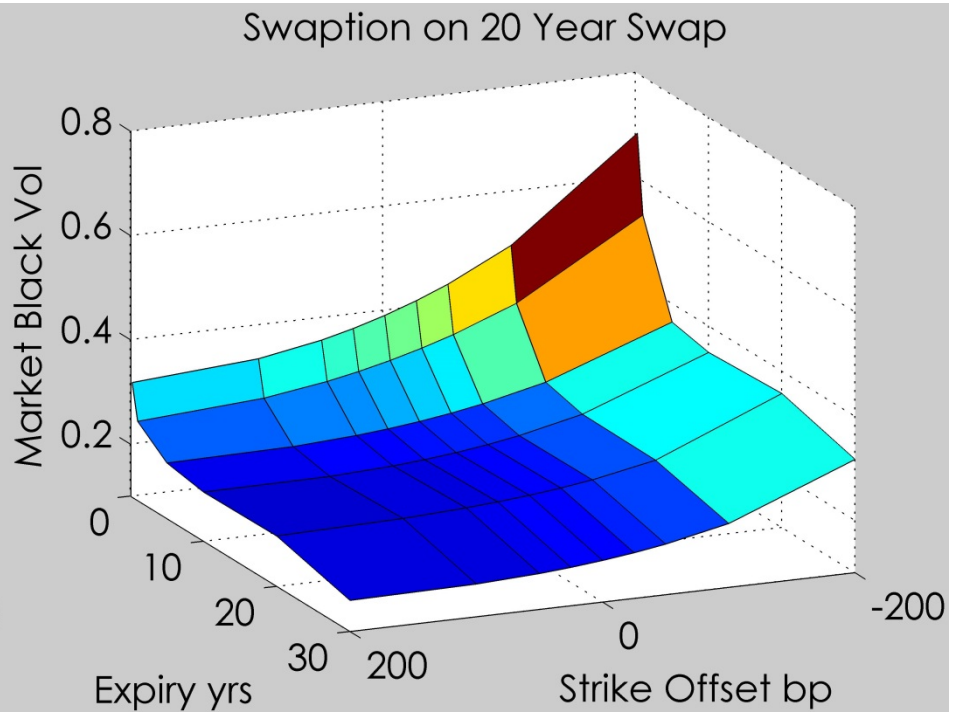
Swaption Cube 2yr/5yr – Euro 1/9/11



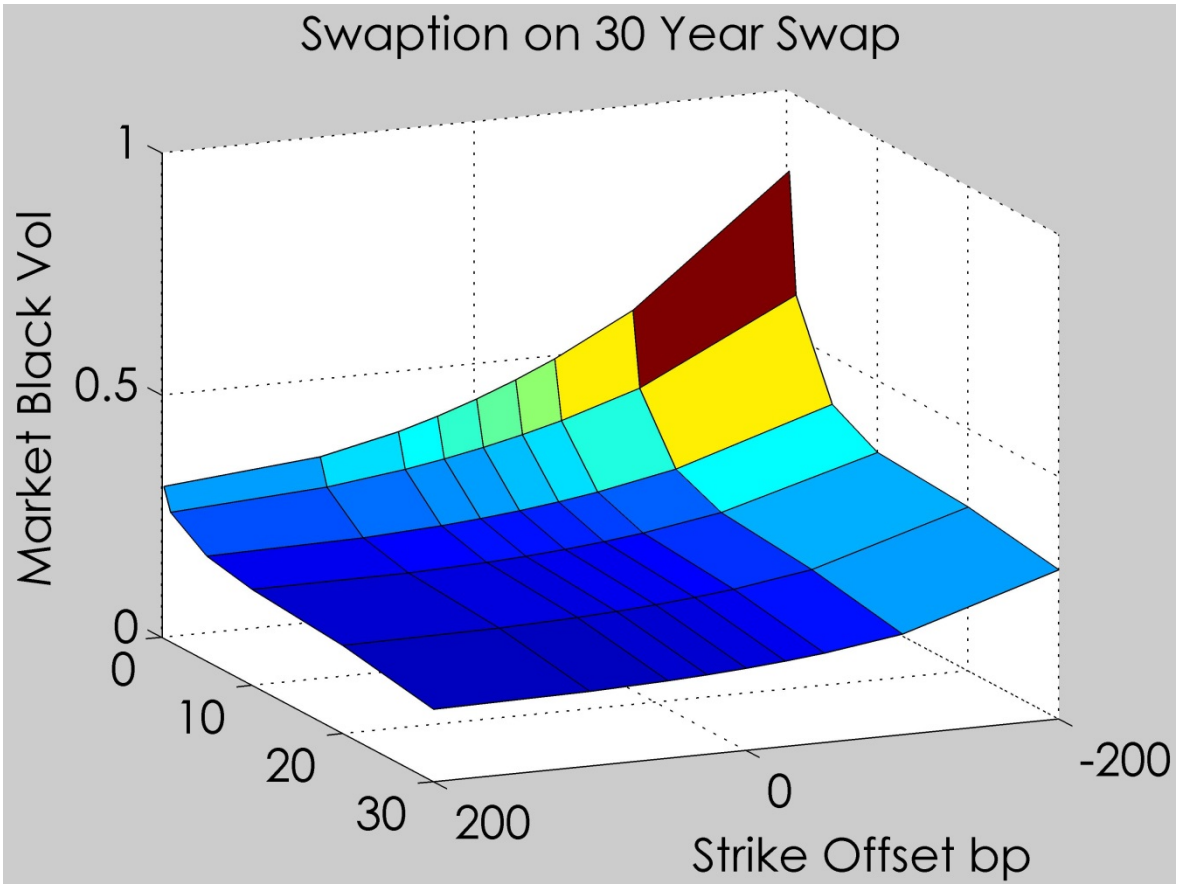


Swaption Cube 10yr/20yr – Euro 1/9/11





Swaption Cube 30yr – Euro 1/9/11



Term Parameter Calibration

- Cube data as parameters of a set of models
- One model per smile
- Constant parameters
  - Per smile parameters
  - Global parameters
- Example for each swaption (expiry n, maturity m) :

OIS : $$dF^x_{n,m}(t) = \sqrt{z^F_{n,m}(t)}\lambda^F_{n,m}\left(b^F_{n,m}F^x_{n,m}(t) + \left(1 - b^F_{n,m}\right)F^x_{n,m}(t)\right)dY_F^{n,m}(t)$$

Spread: $$dS^x_{n,m}(t) = \lambda^S\left(b^S S^x_{n,m}(t) + (1 - b^S)S^x_{n,m}(t)\right)dY_S^{n,m}(t)$$

OIS volatility: $$dz^F_{n,m}(t) = \kappa^F\left(f\left(\alpha(t)\right) - z^F_{n,m}(t)\right)dt + \eta^F_{n,m}\sqrt{z^F_{n,m}(t)}dZ_F^{n,m}(t)$$

Fitted Term Parameters: Euro 1/9/2011

OIS Per Smile Parameters

	Volatility $\lambda^{F}_{n,m}$					Skew $b^{F}_{n,m}$				
Maturity (yrs)	2	5	10	20	30	2	5	10	20	30
Expiry										
3 Months	1.20	0.70	0.49	0.43	0.47	0.27	0.06	-0.15	-0.18	-0.50
1 year	0.83	0.50	0.38	0.34	0.36	0.26	0.21	0.03	-0.14	-0.11
5 Year	0.31	0.28	0.26	0.26	0.27	0.22	0.16	0.12	0.10	0.11
10 Year	0.22	0.22	0.23	0.24	0.24	0.13	0.09	0.10	0.14	0.21
20 Year	0.26	0.28	0.29	0.24	0.22	0.60	0.63	0.67	0.43	0.32
30 Year	0.27	0.26	0.24	0.20	0.19	0.75	0.67	0.52	0.14	0.05

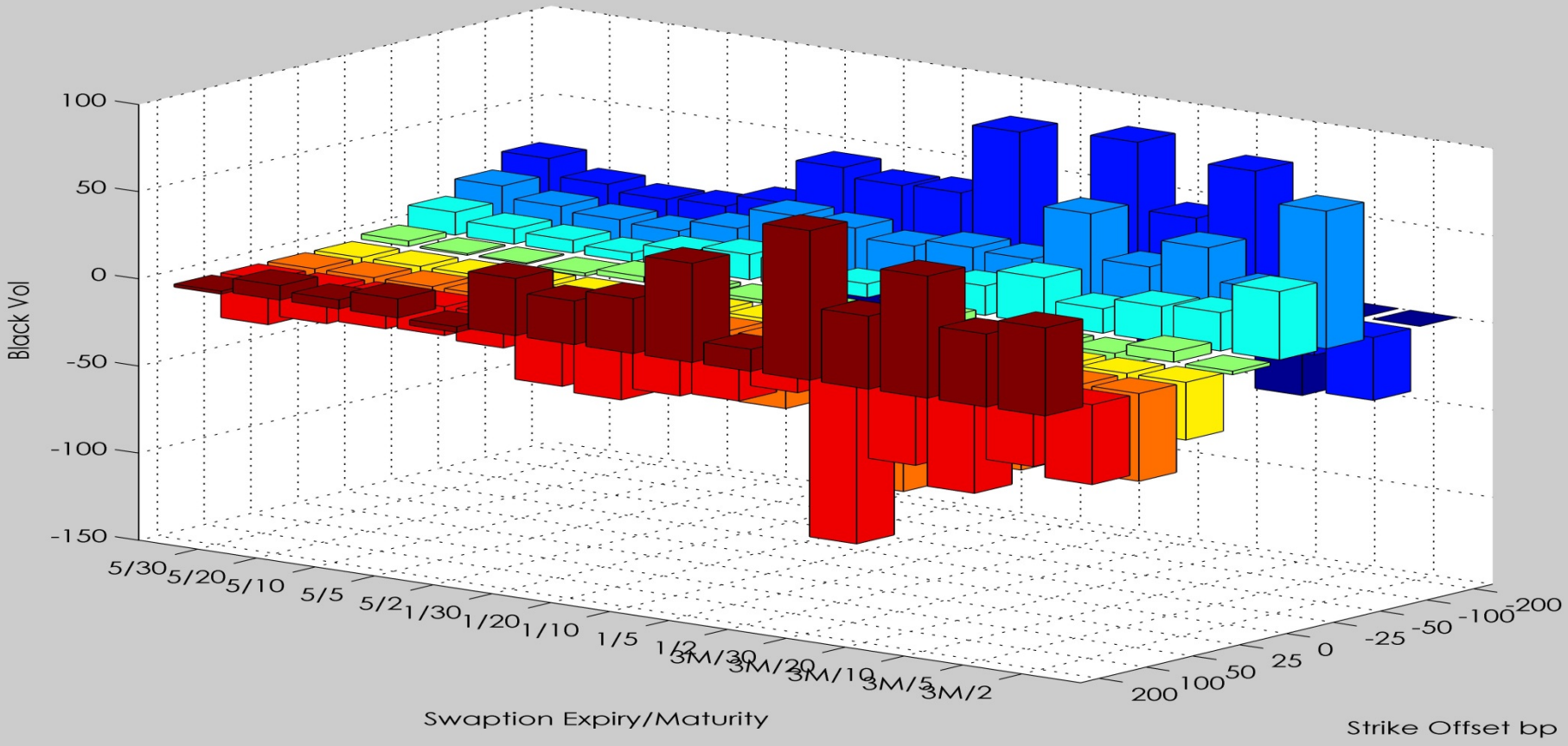
	Volatility of Variance $\eta^{F}_{n,m}$				
Maturity (yrs)	2	5	10	20	30
Expiry					
3 Months	1.02	1.05	0.99	1.03	1.15
1 year	0.65	0.54	0.74	0.79	0.72
5 Year	0.95	0.99	1.01	1.02	0.98
10 Year	1.05	1.07	1.07	1.06	1.05
20 Year	0.98	0.98	0.96	0.99	1.01
30 Year	0.97	0.98	1.00	1.02	1.03

Fitted Term Parameters: Euro 1/9/2011

Global Parameters	
OIS stochastic volatility mean reversion rate κ^F	0.66
Spread deterministic volatility λ^S	0.47
Spread Skew b^S	1.00

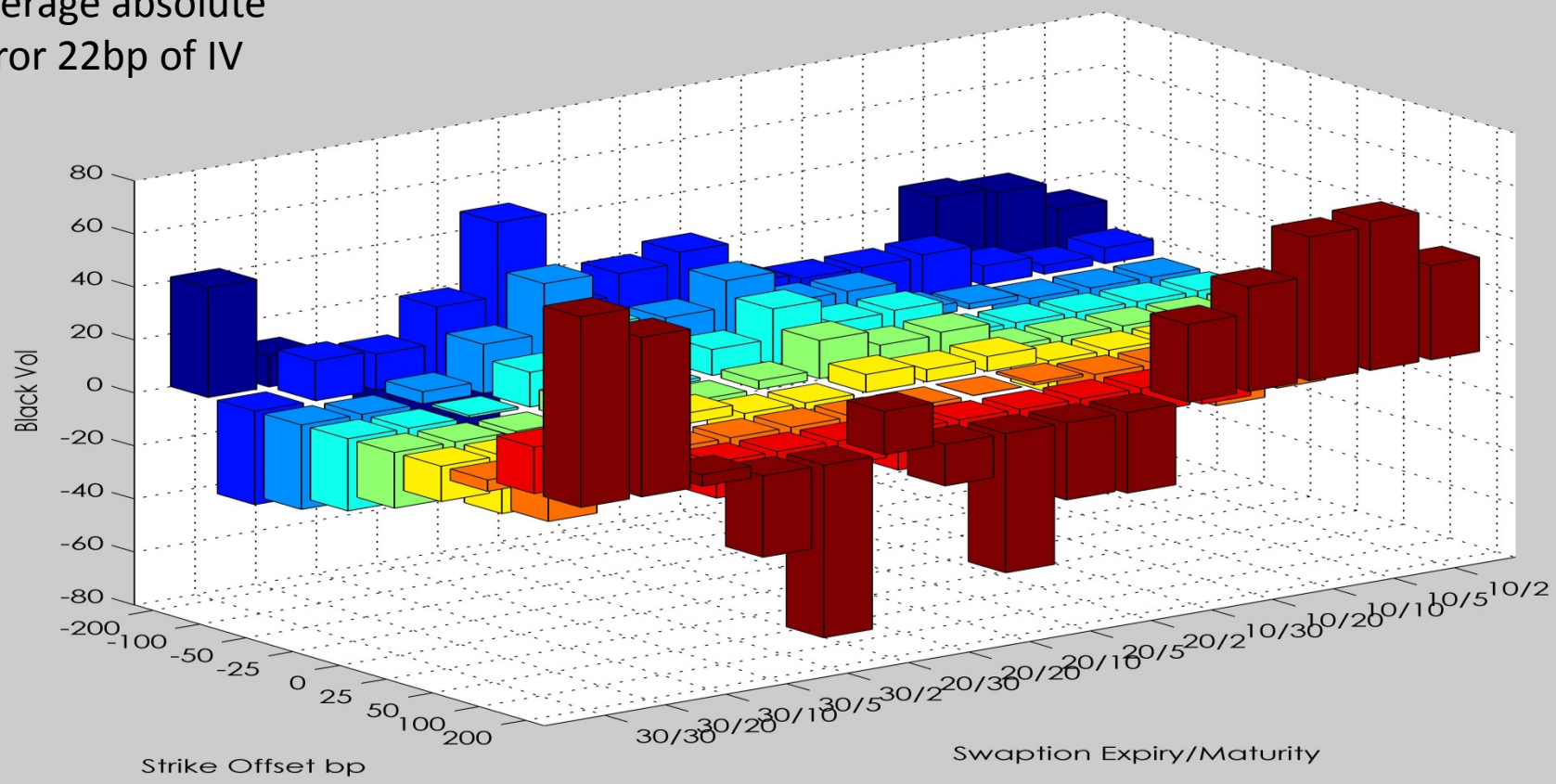
Markov Chain Summary	
$f(\alpha(t))$	Occupation Time
0.094	6.25%
0.245	25.00%
0.641	37.50%
1.675	25.00%
4.378	6.25%

Pricing error short end – IV basis points



Pricing error long end – IV basis points

Average absolute error 22bp of IV



Summary

- ✓ New SV TS model from new volatility dynamics
- ✓ First additive OIS+Spread Heston-based TS model
- ✓ First 2 curve TS with Markov switching Heston
- ✓ Smile persistence between Heston and SABR
- ✓ Heston-level mathematical rigour and tractability
- ✓ Solve Heston moment explosion
- ✓ Excellent fit to swaption cube
- × FRA dynamics not conditionally lognormal

Further research

- Collateralisation, default and funding liquidity adjustments
- Impact on exotics prices
- Same volatility different TS models
- Jumps in Spread connected with volatility regime switches
- Times series of calibrations
- Greeks