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A 'Simple' Stochastic Model for Longevity Risk revisited through Bootstrap

Xu Shi
Bridget Browne

Financial Services Forum

Scoring Goals in a Changing World
5-6 May 2014 • Hilton Sydney
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Projecting Australian Mortality using the CMI Mortality Projections Model

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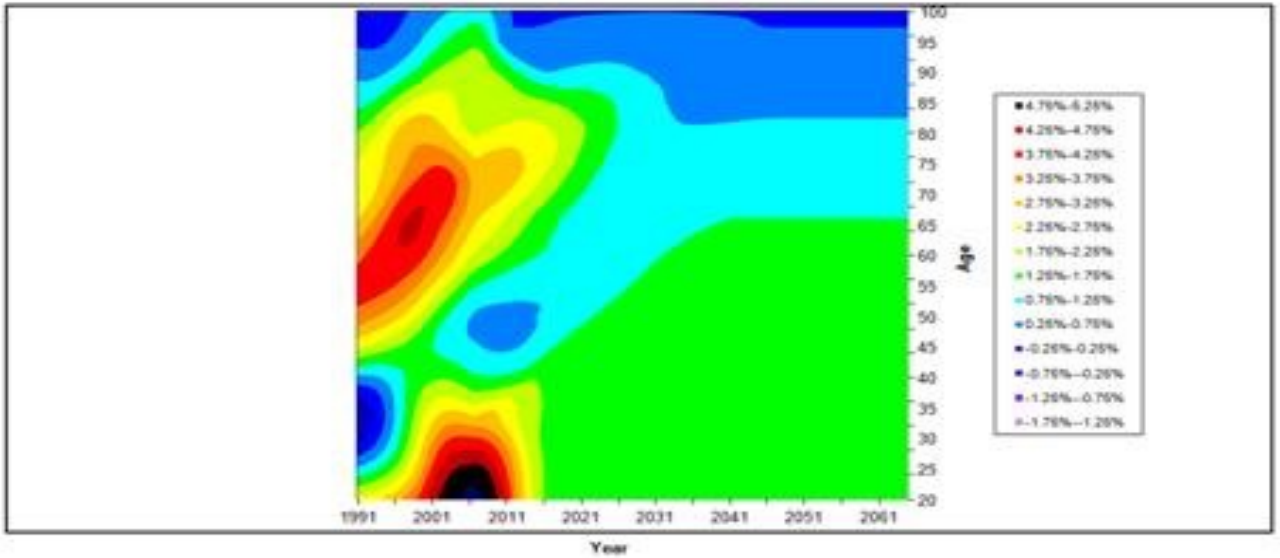
Results - Males



Future work

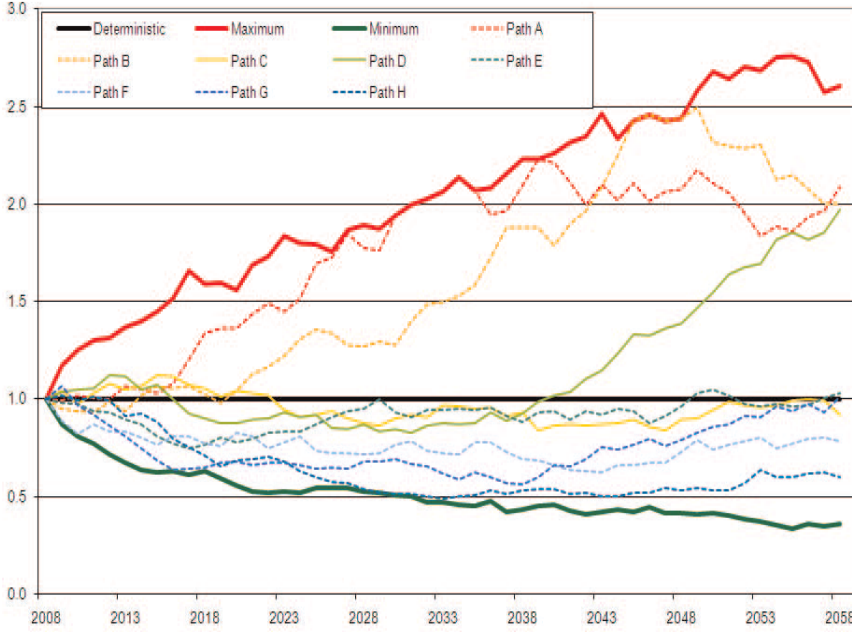
Major limitation: deterministic

Next step: range of outcomes

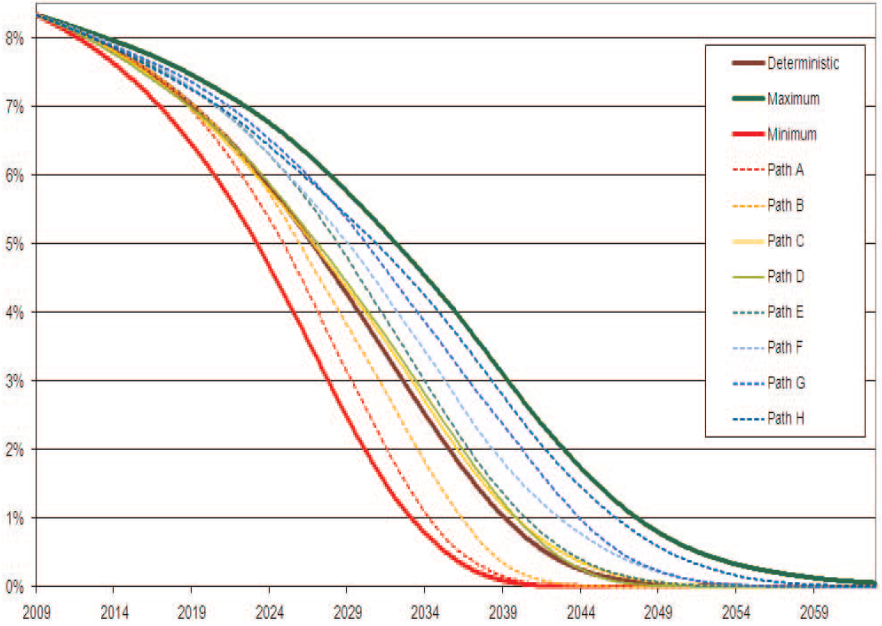
Why does the issue arise?

- UK's CMI method is deterministic
- How to determine percentiles for
  - Pricing
  - Reserving
  - Internal capital modelling
  - Of longevity risk in annuity portfolio?
- How to incorporate prudence or conservatism with quantification?

The Original Model (Koller 2011)



Sample paths of C_t



Sample paths of annuity payments

The Original Model (Koller 2011)

$$\hat{q}_{x,t} = q_{x,t} \times C_t + \varepsilon_{x,t}$$

$$C_t = e^{X_t} \times C_{t-1}$$

$$C_0 = 1$$

The Original Model (Koller 2011)

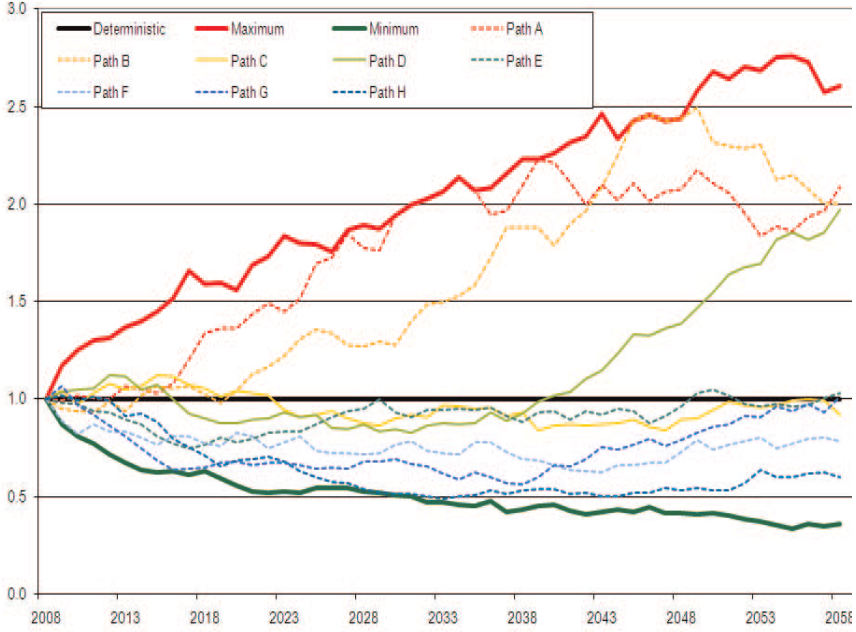
$(X_t)_{t\in\mathbb{N}_0}$, iid N(μ, σ^2)

$(e^{X_t})_{t\in\mathbb{N}_0}$, iid lognormal

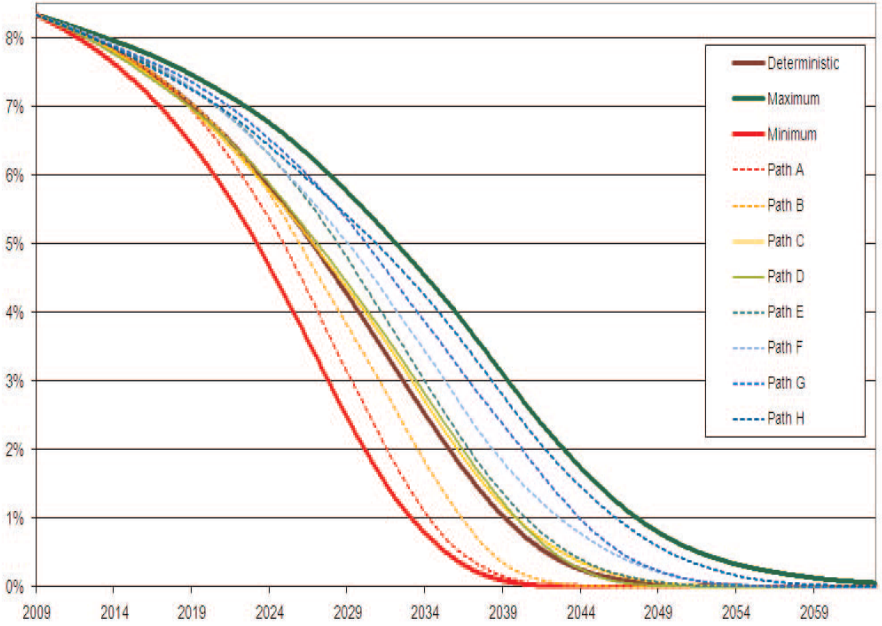
$\mathbb{E}[e^{X_t}] = 1$, is imposed, so that best estimate is followed

$\mathbb{E}[e^{X_t}] = e^{\mu+0.5\sigma^2}$, follows from above

The Original Model (Koller 2011)



Sample paths of C_t



Sample paths of annuity payments

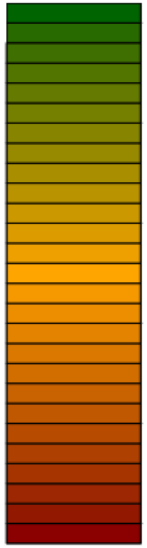
Modifications

- What if X_t is not normal?
- Transformation - rejected
- Non-parametric approach – bootstrap
- Improved model selection
- Consider X rather than X_t

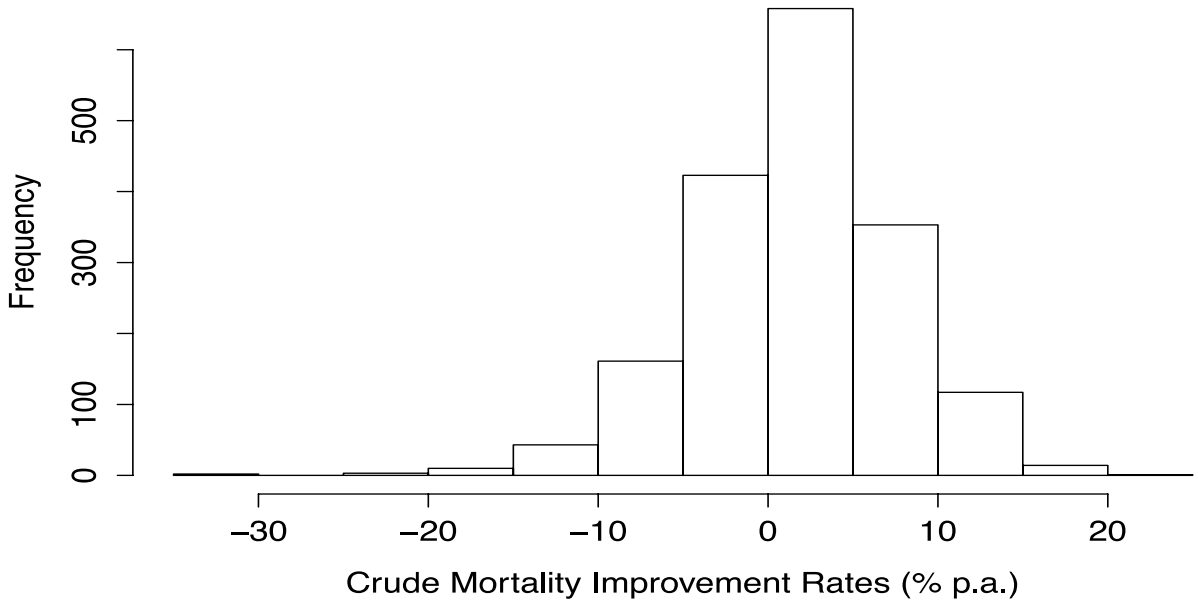
A numerical example

- Australian Females
- Human Mortality Database
- Ages 55 to 89 inclusive
- 1954 to 2008
- Surface of 35 ages and 55 cal. years

Crude Mortality Improvement Rates



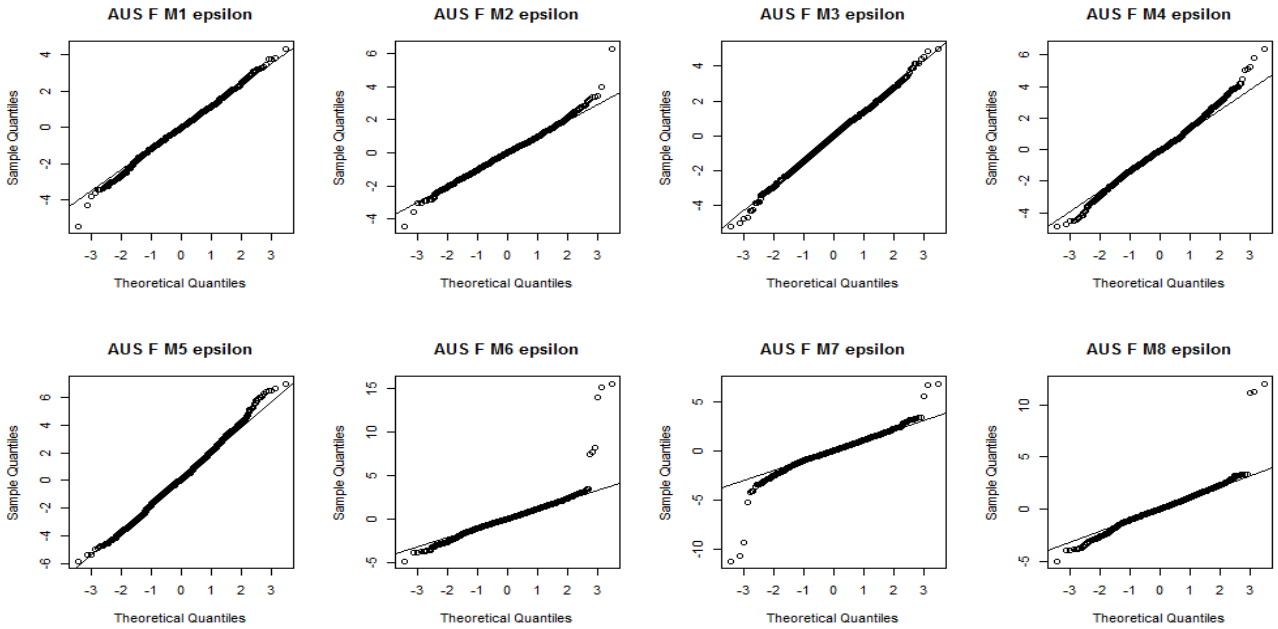
Histogram of Crude MI Rates



Model assessment

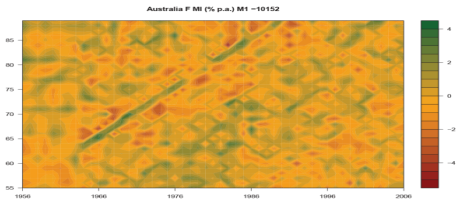
- Use smoothing models only to help determine our best estimate of future variability
- M1-M8 from LifeMetrics (JP Morgan)
  - Standardised residuals and BIC
  - Forecasting properties

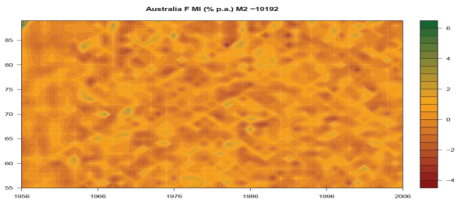
QQ Plots

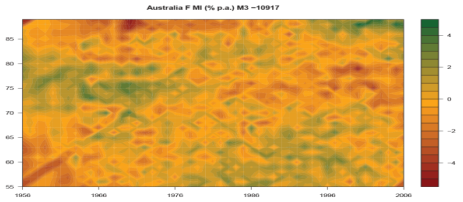

AUS F M1 epsilon
AUS F M2 epsilon
AUS F M3 epsilon
AUS F M4 epsilon
AUS F M5 epsilon
AUS F M6 epsilon
AUS F M7 epsilon
AUS F M8 epsilon
Sample Quantiles
Theoretical Quantiles

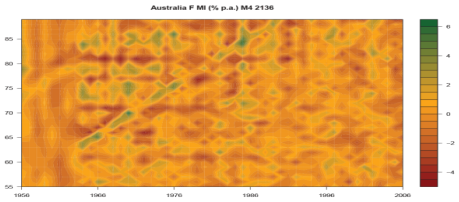
Summary statistics for epsilon and BIC

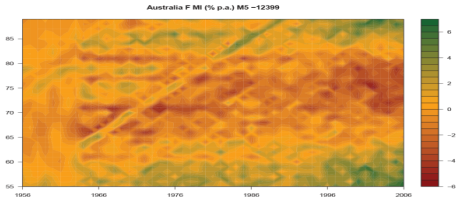
Model	Mean	Variance	Skewness	BIC
1	-0.000606	1.489723	-0.087587	-10 152
2	0.007536	1.088392	0.134849	-10 192
3	-0.003997	2.028962	-0.085894	- 10 917
4	-0.019290	2.078517	0.172425	2 136
5	0.166069	3.876925	0.139640	-12 399
6	0.034808	1.842268	2.350169	-10 309
7	-0.005507	1.496240	-1.015056	-10 397
8	0.020308	1.582045	1.035231	-10 297

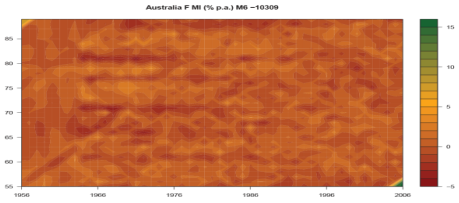

Australia F MI (% p.a.) M1 −10152

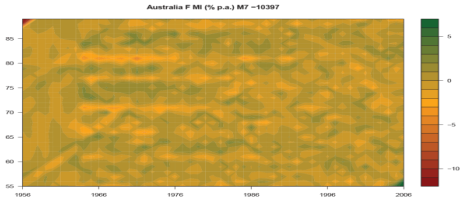

Australia F MI (% p.a.) M2 −10192

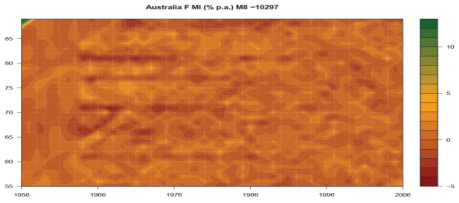

Australia F MI (% p.a.) M3 −10917


Australia F MI (% p.a.) M4 2136


Australia F MI (% p.a.) M5 −12399


Australia F MI (% p.a.) M6 −10309

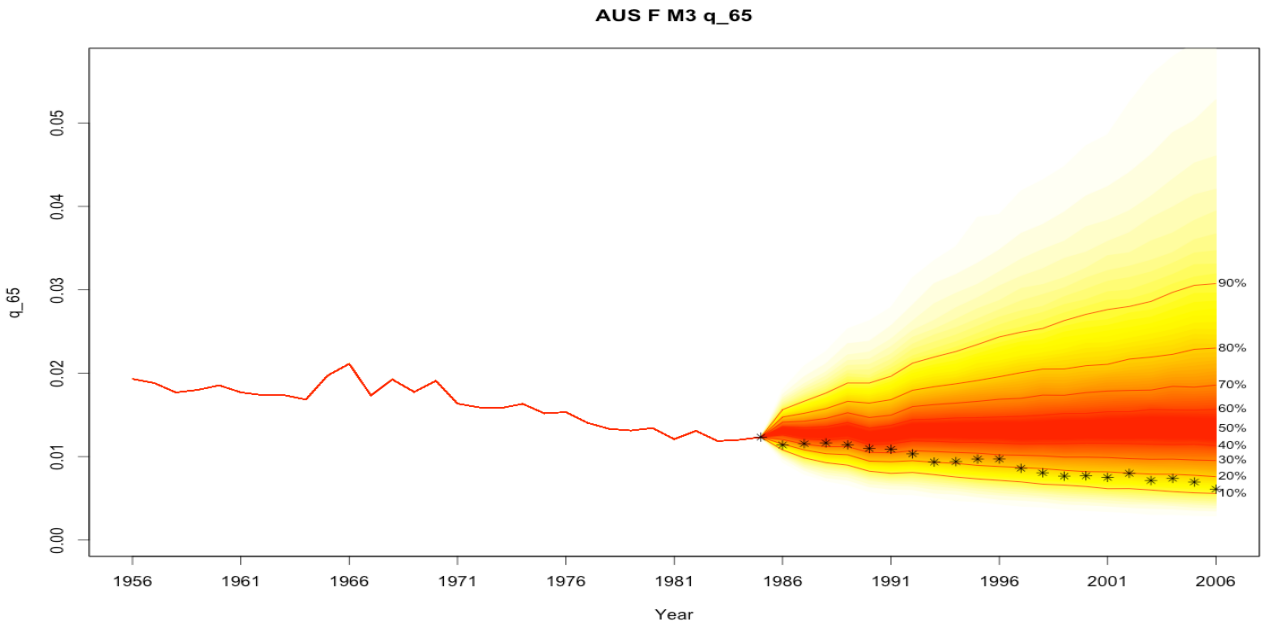

Australia F MI (% p.a.) M7 −10397


Australia F MI (% p.a.) M8 −10297

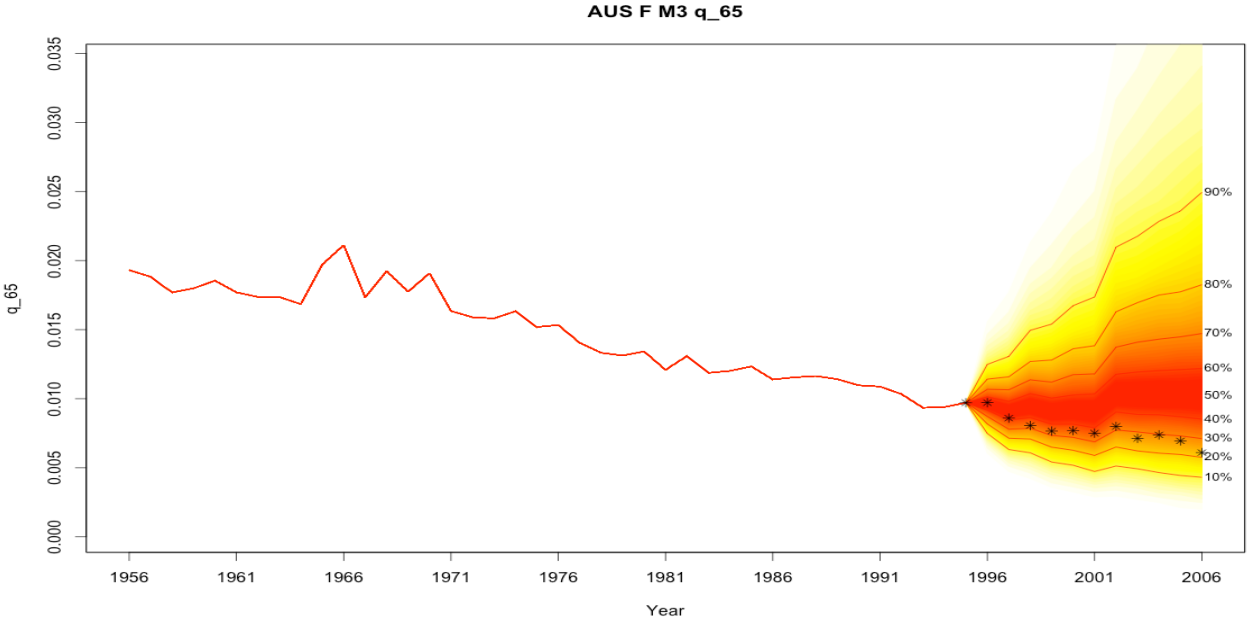
Forecasting properties

- Now choosing between M2 and M3
- M2 unstable
- Backtesting for M3

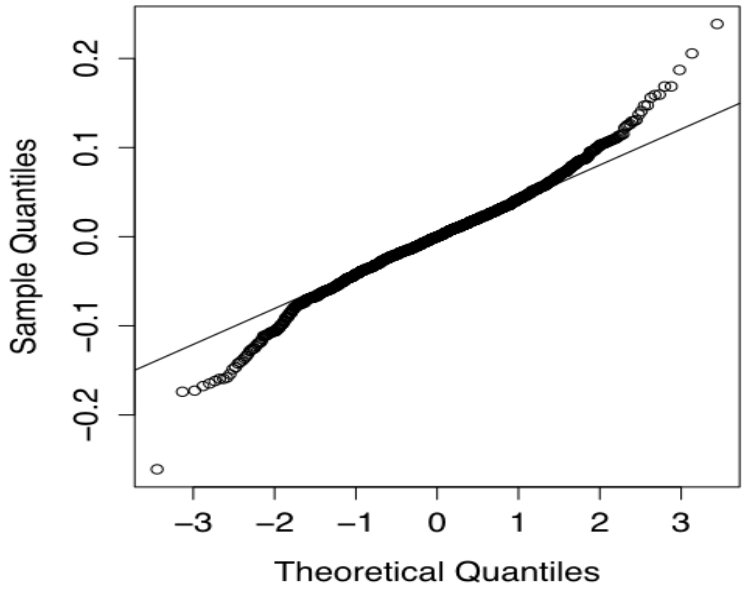
Prediction Intervals I



Prediction Intervals II



QQ Plot of X_t for M3 Australian Females

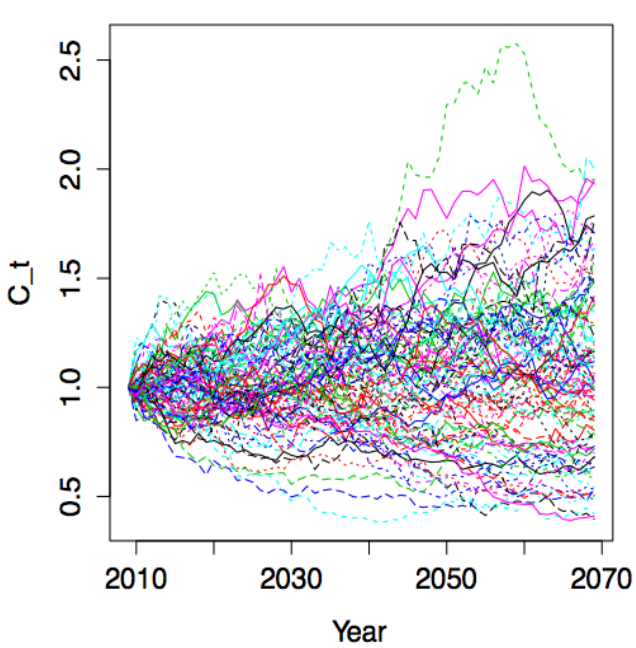


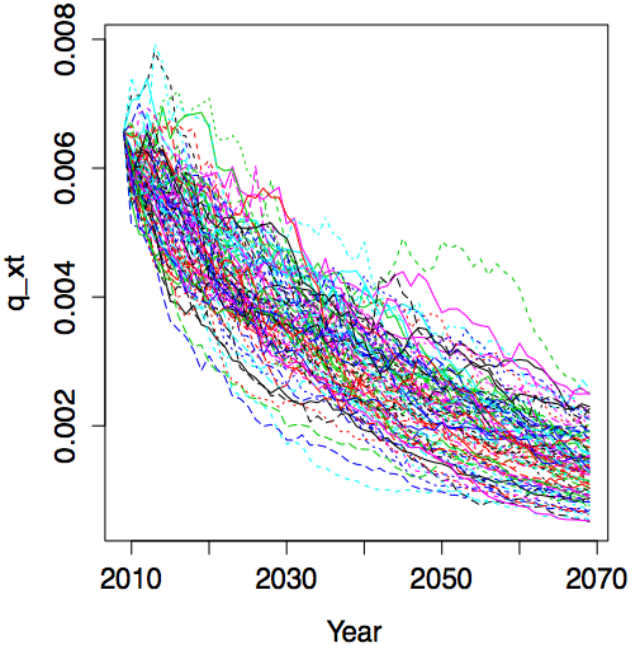
Age dependence or independence?

- P-values of two-variable regression of X against age (x) and year (t)

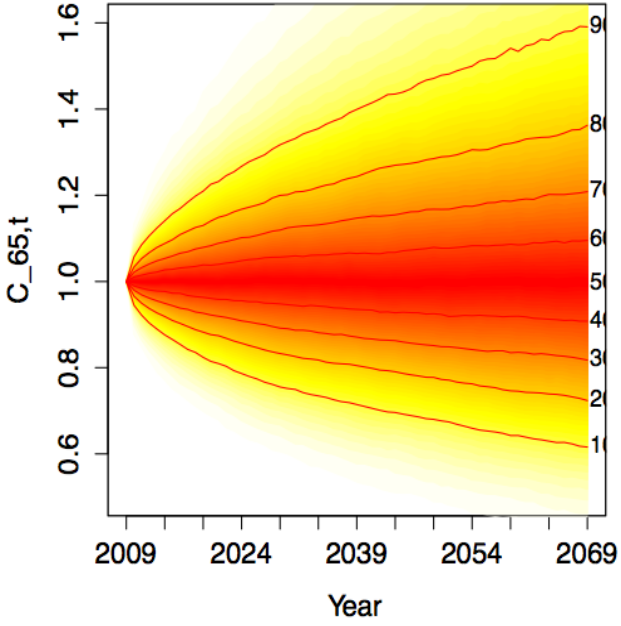
Model Type	Variable	ANOVA p-value
Continuous Case	Year	0.1079
	Age	0.8350
Discrete Case	Year	0.9965
	Age	0.9999

Paths of Ct, q65,t for Australian Females

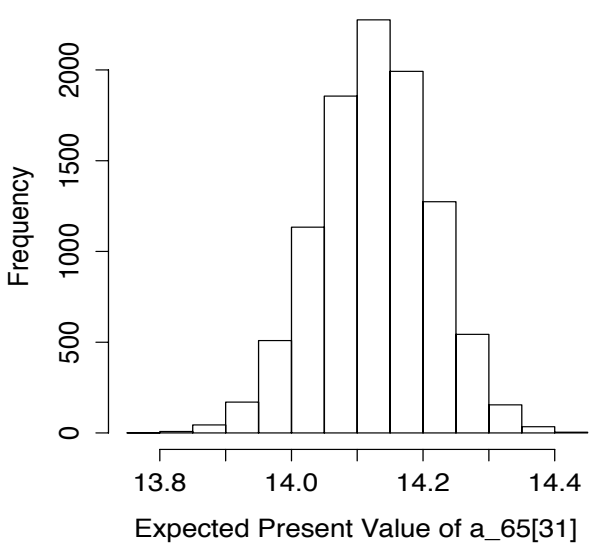


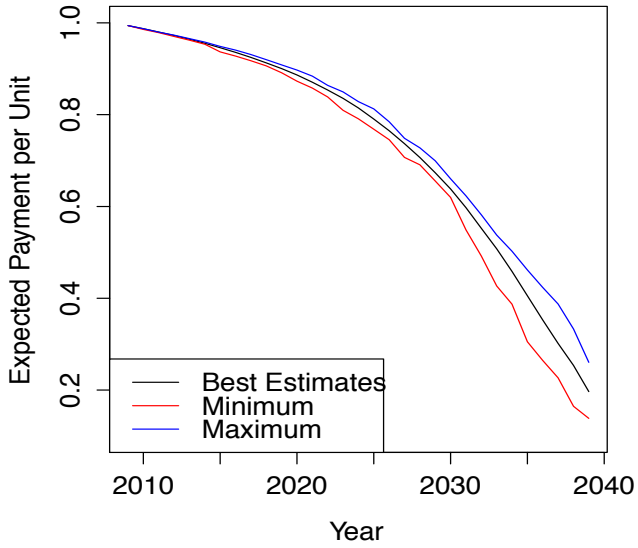


Fan plots of Ct, q65,t for Australian Females



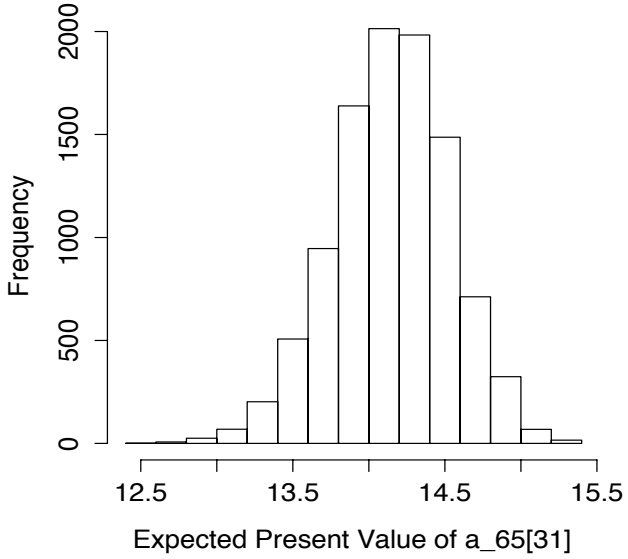
Aust Females (Age Independent Case, X)

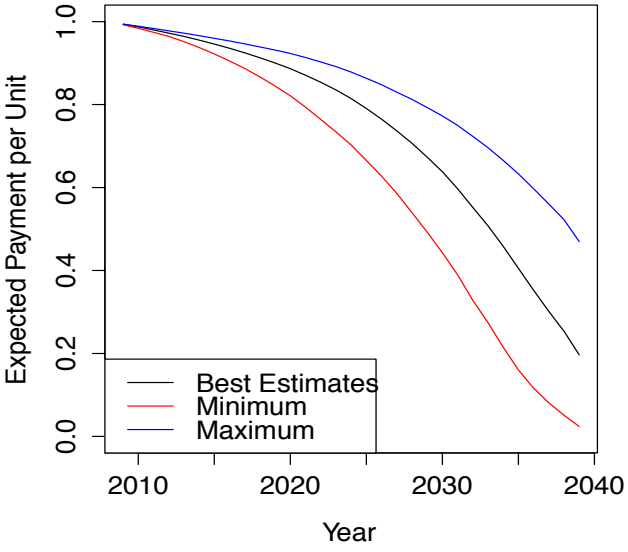




Histogram (LHS) of $a_{65:[31]}$ (at 4% pa) and cohort survival curves(RHS)

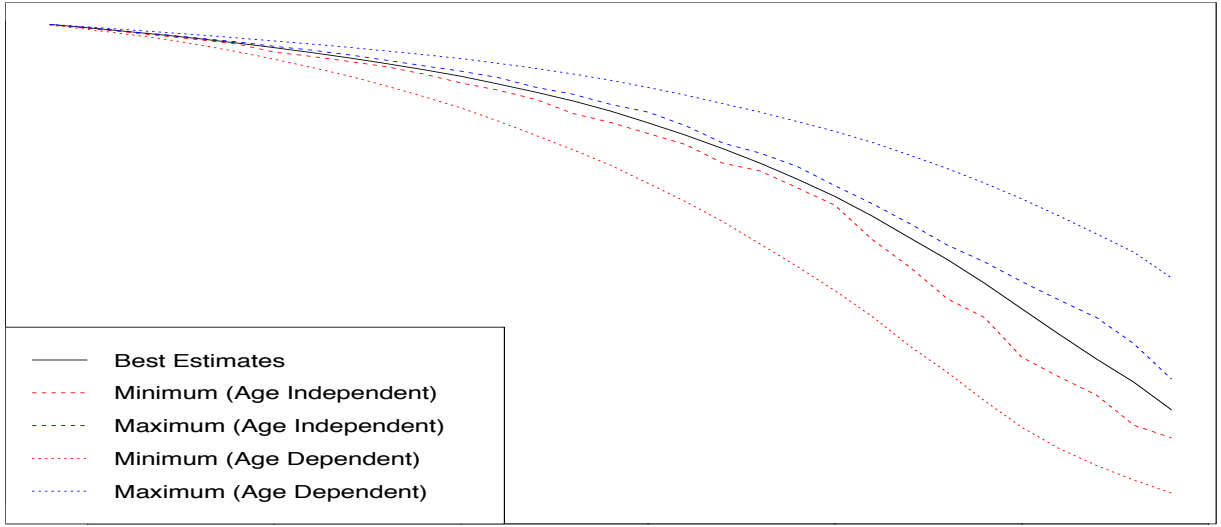
Aust Females (Age Dependent Case, X_t)



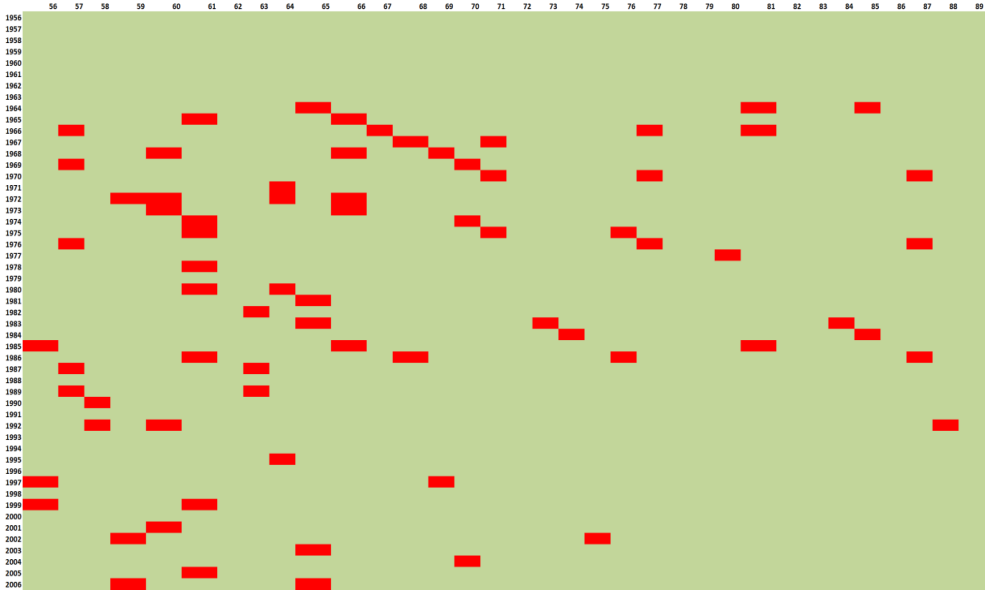


Histogram (LHS) of $a_{65:[31]}$ (at 4% pa) and cohort survival curves(RHS)

Paths of cohort survival curves for $a_{65:[31]}$ for Australian Females (Two Cases)

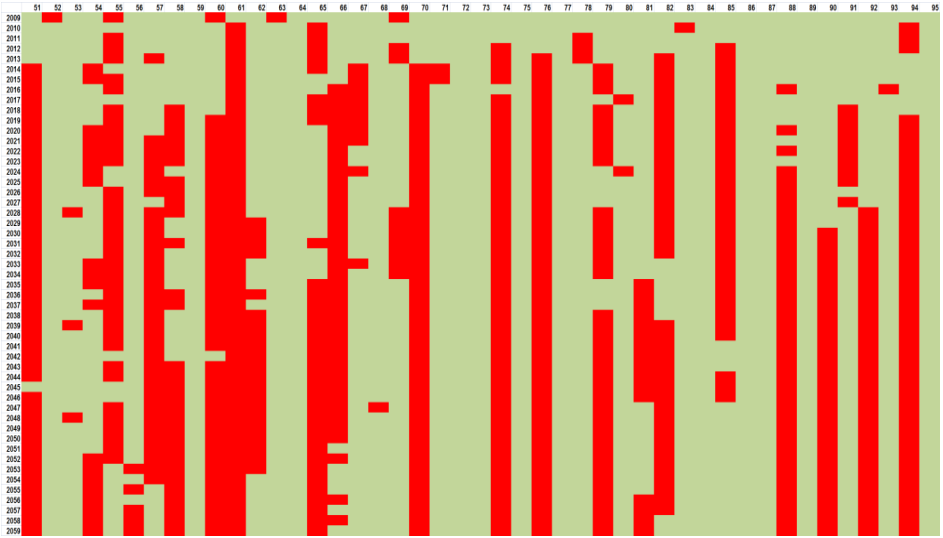


Age independence => coherence?



- Example of crossover in Crude Mortality Rates for Australian Females (red cell indicates mortality in year t is greater for age x than for age x+1)

Age independence => coherence?

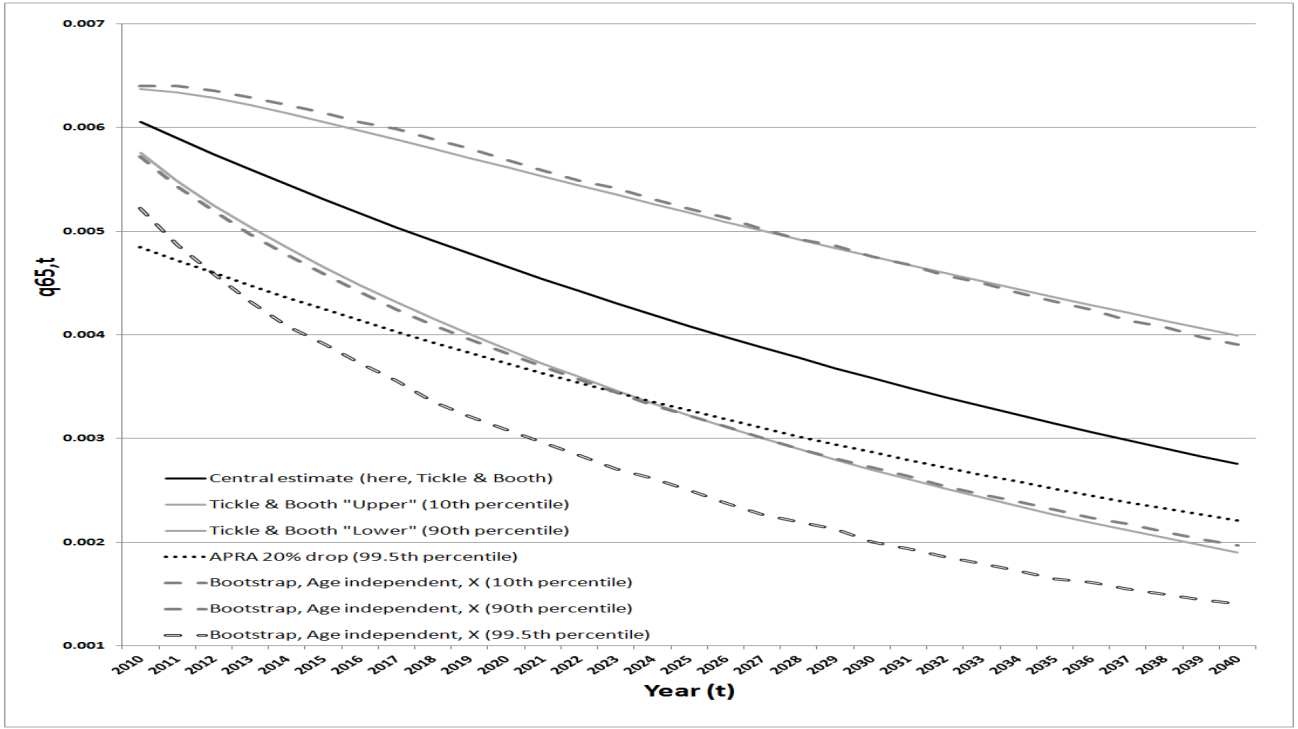


- Example of crossover in one simulation for Australian Females
(red cell indicates mortality in year t is greater for age x than for age x+1)

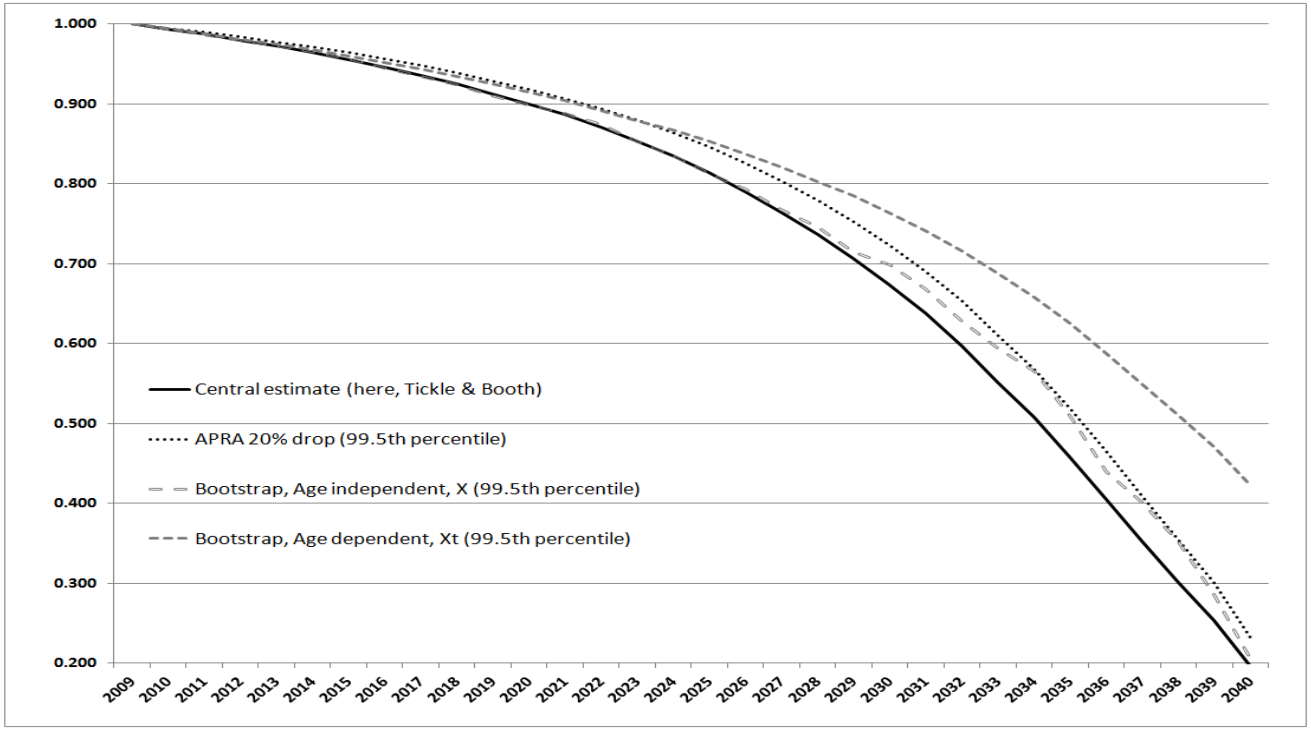
Comparison with other methods

- Tickle and Booth (2014) evaluated and updated a forecast of mortality for Australian seniors using the Booth-Maindonald-Smith variant of Lee-Carter (generously shared underlying work)
  - Central estimate and 80% PI for $q_{x,t}$
- LPS 115 Section 38 (APRA, 2013, p7)
  - 20% drop in $q_{x,t}$
  - < 0.5% probability actual claims exceed

Forecast mortality rates for Australian Females aged 65, central estimate & selected percentiles



Paths of cohort survival curves (ie cashflows) for Australian Females, central estimate, APRA basis and 99.5th percentiles for both cases



Comparison of Cohort Life Expectancy & Annuity Values (at 4% pa) for Aust. Females aged 65 in 2010, central estimate & selected percentiles

	e65[31],2010	% diff	a65[31],2010	% diff
Central estimate (here, Tickle & Booth)	22.668		14.166	
APRA 20% drop (99.5th percentile)	23.655	4.35%	14.610	3.14%
Bootstrap, Age independent, X (10th percentile)	22.287	-1.68%	14.019	-1.03%
Bootstrap, Age independent, X (90th percentile)	22.845	0.78%	14.254	0.62%
Bootstrap, Age independent, X (99.5th percentile)	23.102	1.91%	14.325	1.12%
Bootstrap, Age dependent, Xt (10th percentile)	21.415	-5.53%	13.636	-3.74%
Bootstrap, Age dependent, Xt (90th percentile)	23.817	5.07%	14.645	3.39%
Bootstrap, Age dependent, Xt (99.5th percentile)	24.912	9.90%	15.038	6.16%

Conclusion

- CMI method increasingly widely applied - UK, USA, Canada, Australia and China at least
  (SOA, 2014, Office of the Chief Actuary, 2014, Huang and Browne, 2014, CMI, 2009a, CMI, 2009b)
- Some national statistics bodies also deterministic
  (ABS, 2013, ONS, 2013)
- This method allows the user to pragmatically add stochastic variation to a deterministic model
- Bootstrapping improves tail modelling
- Help to inform understanding of longevity risk

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