

Institute of Actuaries of Australia

Weather and Carbon Derivatives Pricing and Managing Risk in the ART Market

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Abstract:

<u>Key words</u>: Alternative risk transfer, derivatives, carbon, weather, HDD, CDD, risk, temperature, rainfall, Ornstein-Uhlenbeck, stochastic, mean-reversion, CAT Bonds, securitisation, ILS

This paper is concerned with the discussion of suitable methods for the modelling and pricing of alternative risk transfer contracts and, in particular, weather and emission based derivative contracts. An overview of the global ART market is given before an analysis of the pricing and risk management issues surrounding weather and carbon contracts is investigated with illumination provided via empirical examples.

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1: Introduction

1.1 Background

The Alternative Risk Transfer (ART) market is one that accommodates the transfer of risks that aren't generally accepted in primary, commoditised risk markets. The contracts written are usually tailored in some way to meet the specific needs of the insured's risk profile.

Initially the ART market began out of the requirements of corporations who sought to self insure particular risks associated with the operation of their business. From here the market grew to become aligned with a much wider range of products that are in someway considered alternative to more traditional forms of insurance and policy coverage.

1.1.1 What is an ART contract?

Whilst there is no strict definition of what constitutes an ART contract or an ART market participant there are several characteristic that loosely classify the instruments and investors that are involved in this market. Some of these features include:

- Multi-year coverage often for multiple lines of business;
- Multiple triggers / perils attached to the contract;
- Cross border contracts;
- Components of both insurance and financial products;
- Specifically tailored solutions.

There exists a loose division of the ART market used by practitioners (and as proposed by SwissRe [2003]), namely alternative carriers and alternative products. Alternative carriers refer to self-insurers, captives and other similar risk pooling arrangements. Alternative products refer to the products that are available to assist in addressing the transfer of alternative risk. These include contracts such as:

- CAT bonds
- Securitised Risks
- Weather Derivatives
- Insurance Linked Securities (ILS)
- Emission (Carbon) Derivatives

The range of products that are listed above will often cover a variety of risks under a single contract. This paper is focussed particularly on the latter group - alternative products - and specifically on issues relating to pricing and risk management of weather and emission related derivatives.

Generally ART markets exist to facilitate risk management vehicles that provide advantages that can't be obtained from traditional insurance and capital market products. Some of the benefits of ART solutions that attract both insurers and investors include:

- Increased underwriting capacity and capital for insurers;
- Broaden the cover offering;
- Portfolio diversification;
- Protection of existing cash-flows.

The remainder of this section seeks to outline some of the characteristics of global alternative risk transfer markets as well as to detail some of the recent developments and trends in the industry.

1.2 Alternative Risk Transfer Markets

Originally ART markets developed out of the needs of insurance companies to circumvent both the capacity constraints and accounting treatments of traditional insurance and reinsurance contracts. In particular, many of the early securitisations of insurance cash-flows were used as a means of transferring the economic risk from the insurance market, where discounting of reserves was not allowed in some jurisdictions (most notably the UK), to the broader financial markets where the practise of discounting future liabilities was permitted.

It has been noted by several authors that the first insurance securitisations took place in the US during 1988 and involved the sales of rights to emerging profits from blocks of life insurance policies and annuities (Cowley and Cummins 2005). This securitisation of cash-flows is a feature that is common to many ART market products.

1.2.1 Global ART Market

The ultimate size of the ART market is difficult to determine largely due to the hazy definitions that are used to classify these types of contracts. Captives are the most dominant end-user of ART based risk management solutions. These organisations include self-insurers and other vehicles that are specially created to assist in the transfer or mitigation of risks from their parent company.

There exists several important centres in regard to the global ART market, amongst the top being London, New York, Bermuda and the Cayman Islands. The latter two being due to the large number of captives that have historically been domiciled in jurisdictions with favourable taxation environments.

The vast majority of these transactions will in someway involve a large, multinational bank, insurer or reinsurer. Some of the key market participants across the various sub-industries are:

- Investment Banks: Goldman Sachs, JPMorgan Chase, Citibank
- Insurers: XL, AIG, Zurich
- Re-insurers: Swiss Re, Munich Re, Hannover Re.
- Brokers: AON, Willis, Marsh
- Advisory: AON-Benfield, Mercer, Watson Wyatt

The pricing and modelling of the risks that make up typical ART contracts are by their very nature technical with the methods to a large extent kept 'in-house' thus making it difficult for smaller players to obtain the intellectual property required to be an effective market participant.

1.2.2 The Australian ART Market

The Australian ART market is significantly less developed than its overseas counterparts but has achieved significant expansion in recent years. Whilst Australia does not posses

the equivalent of the US's risk retention groups there exists a range of captive insurers that facilitate local companies to self-insure a range of risks. There have been several issues of CAT bonds covering Australian risks, most notably the earthquake bonds, *Australis*, introduced by Swiss Re in 2006.

More recently superannuation funds in Australia have been turning to alternative investment classes in order to generate excess returns whilst maintaining a diversified portfolio of assets. The relative resilience of asset values for ART contracts (particularly CAT bonds) during the recent financial crisis has proved its worth as an alternative asset class and as a portfolio diversification tool.

1.3 Convergence of Capital and Insurance Markets

In recent times we have seen a significant convergence of insurance and financial markets in terms of both the investors they attract as well as the perils that they provide coverage for. The proliferation of CAT bonds following hurricane Andrew in 1992 were an early attempt to bridge the gap between traditional reinsurance markets and the broader financial markets which provided a much larger capacity to absorb such massive costs.

Some of the factors that have contributed to overall convergence of capital and insurance markets include:

- Financial institutions seeking to diversify risk profiles;
- Capacity constraints of traditional insurance markets;
- Requirement for an integrated approach to risk and ERM practices.

Indeed ART contracts are at this juncture between capital and insurance markets with the contracts traded generally containing components of both. Traditional insurance markets have much to gain from a gradual convergence of risk products between the two markets.

As previously alluded to, the broader capital markets have a much greater risk bearing capacity than the corresponding insurance and reinsurance markets and ART contracts provide much of the bridge that enables the transfer of risks between these two groups.

What are the important differences between these two approaches to risk management? One of the key requirements of an insurance contract, and that still differentiates it from general capital market instruments, is the requirement that the investor (policyholder) has an interest in the object or outcome that is being insured. This distinguishing characteristic has important implications as to how individual contracts are traded and therefore, ultimately, how prices for these contracts are established. The requirement for the insured to have an interest removes the speculation-based market participants that provide so much liquidity in the broader capital markets.

There are many examples of risk management structures that have been developed that include elements of traditional insurance along with concepts and payoffs that are normally associated with derivative (or index) based contracts and several of these will be described in section 2.

1.4 The ART Market as an Alternative Asset Classes

The properties of many ART products, like weather and emission derivatives, are attractive to portfolio managers due to their unique diversification benefits.

As the weather derivative market has demonstrated over the past decade, some of the most significant market participants are those investors who are seeking to access the particular qualities that this asset classes exhibits. Much of the rapid expansion that the market experienced during the early 2000's was due to this interest from non 'end-users' providing significant liquidity to the market. In particular they seek to add products to their investment portfolios that have low correlations to other asset classes and thus add to the diversification of the portfolio.

1.4.1 Hedge Fund Interest

Fund managers are particularly interested in investments that achieve returns with specific relationships to traditional financial variables either to exploit arbitrage structures or to reduce the risk profile of the portfolio through diversification. In particular, hedge funds and other specialist asset managers seek investments where the correlation of returns generated from the investment to the returns of equity markets are negligible to provide a diversification asset.

In recent years Hedge funds have been interested in the ART market as an alternative asset class. In the last 5 years several funds have originated solely dedicating themselves to these instruments and are sometimes referred to as 'Insurance Risk' hedge funds. Examples of investment funds dedicated to global ART markets include Nephilia, Fermat and Securis.

The following section provides a numerical background to this interest.

1.4.2 Diversification benefits

To demonstrate the diversification benefits that can be obtained between various financial indices and a typical (albeit hypothetical) weather contract an analysis of correlations of returns was undertaken between the ASX 200, the US S&P 500, a composite global bond index as well as a hypothetical weather contract over temperature recorded at Sydney airport. The weather contract is based on a monthly accumulated CDD¹ measure during the period 2002 – 2010. The deviance of the accumulated CDD's from the daily average is compounded with an initial level of 1000 similar to the method proposed by Cao et al [2006]

The table that follow outlines the results of the daily correlation analysis that was undertaken over several financial indices and the hypothetical weather contract described above.

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¹ Cumulative degree days, defined in section 2.2.1.1

Table 1.1: Summary of correlations (daily)

	S&P 500	ASX 200	Bonds	Commodities	Weather
S&P 500	1.00	0.88	0.26	-0.06	0.08
ASX 200	0.88	1.00	0.13	0.12	0.04
Bonds	0.26	0.13	1.00	0.16	-0.12
Commodities	-0.06	0.12	0.16	1.00	-0.09
Weather	0.08	0.04	-0.12	-0.09	1.00

It can be seen that the synthetic weather contract has relatively negligible correlation to the other financial variables on the basis of daily returns suggesting their suitability as a diversification tool.

1.4.3 Insurance Securitisation

Since the fall of global financial markets during 2008 the term 'securitisation' has attracted negative connotations due to the fact that much of the blame was directed at the failure of sub-prime backed securities at the beginning of the financial crisis in the US summer of 2007, and ultimately (and more generally) the broader CDO (Collateralised Debt Obligation) market during 2008.

This has seen the overall volumes of new transactions being significantly reduced in 2009 as financial institutions avoided entering into new securitisations and were forced to liquidate current holdings as the credit ratings on many of the instruments were reduced to sub-investment grades. Anecdotally it is anticipated that this decline will reverse itself during 2010 with segments of the industry expected to increase substantially on the back of the stabilisation of global financial markets.

Advantages exist for both insurance and financial markets in the securitisation of insurance risks. To the insurance industry it provides opportunities to access alternative cover and thus can increase the competitive pressure on reinsurance premiums. Indeed the IMF states that the securitisation of insurance risks "has the ability to reduce the premium spikes in the insurance cycle" (IMF: Global Economic Outlook 2008). We discuss insurance securitisation in more detail in section 2.3

1.5 Overview

The remainder of this paper is organised as follows. Section 2 outlines the current state of the weather risk market and discusses the recent trends experienced by particular sectors of the market. Section 3 undertakes a similar endeavour for the more immature, though rapidly expanding, emission markets.

Some of the key mathematics that is required in both the modelling and pricing of weather and carbon derivatives is outlined in section 4. Section 5 demonstrates the modelling and pricing process for by way of an empirical investigation into temperature modelling. The following section outlines some of the modelling techniques and risk management applications of carbon based contracts. Finally an overview and summary of the discussion is provided in section 7 that concludes the paper.

2: Weather Risk Markets

2.1 Background

Weather risk markets were, like the broader ART market, were developed in the search for alternative finances to overcome the capacity constraints of traditional insurance markets and to access the wider financial markets. This section describes the major components of the weather risk market and outlines some of the recent development in this section of financial markets which broadly include weather derivatives, ILS, CAT bonds amongst other types of weather securitisations.

2.2 Insurance Linked Securities (ILS)

The Insurance Linked Security (ILS) market covers those products that are essentially a securitisation of a range of insurance liabilities and is synonymous with the term 'insurance securitisation'. Index based risk transfer has become increasingly popular in recent years with a plethora of tailor made indices being created in the past decade to provide a reference trigger for a range of index-based risk management products.

Swiss Re have recently created a CAT bond index (SRCBI) that enables improved transparency in the market and that in the future should be able to be used as a reference trigger for risk-management structures and products. Late in 2009, the Caribbean Catastrophe Risk Insurance Facility (CCRIF) announced the introduction of an excess rainfall index-based weather risk product. This contract is based on a real time rainfall model that has been developed for the region and relates to the maximum peak rainfall in a given period rather than the average measured rainfall as is common amongst other rainfall derivative structure. The intention is that this will provide a trigger for a risk management structure that can cover the risks associated with flooding in various regions of the Caribbean.

The ILS market remained surprisingly resilient during the recent financial crisis with the returns from these assets proving as a bright spot for those who had diversified into such asset classes. This was despite the failure of several institutions exposed to the ILS market.

2.2.1 Cat Bonds

As has previously be highlighted, the market for CAT bonds largely grew out of the shortage of reinsurance capital following a series of catastrophes in the early 1990's. Hannover Re is recognised as having issued the first CAT-linked securitisation in 1994 with a total notional value of US \$85m. From here the market has grown substantially with the coverage broadening to a larger range of perils. More recently FIFA purchased a CAT bond to cover cancellation costs for the 2006 World Cup in Germany. The coverage extended to a number of perils including windstorm, flood and terrorism.

There is set to be a significant amount of activity in the CAT bond and ILS markets during 2010 with AONBenfield Securities estimating the total value of catastrophe bonds maturing during 2010 will approach US\$5bn. Investors will be seeking to purchase new issues to replace these maturities, which will maintain liquidity in the ILS market.

This will provide much stability to the market that came under a significant amount of pressure following the collapse of Lehman Brothers who was the backer of 4 major CAT bond issues in the market at that time. Whilst generally the collateralisation of the exposure would exist to back the contract the quality of the assets that had been used in the collateralisation account was substandard resulting in the default of several of the bonds.

CAT bond issues until that time had followed the practise of enlisting large banks to act as the total-return-swap counterparties as they were considered to be, for all intents and purposes, a near risk free counterparty for the transaction. The bonds that Lehman covered were suddenly left without a counterparty and their value drastically declined before their ultimate failure. Since then the market has evolved to develop more rigid counterparties and collateralisations. Recent transactions have used US Treasury bonds, AAA rated debt and tri-party repurchase agreements to provide greater security to the CAT bond structure.

2.3 Weather Derivatives

Weather derivative products were largely born out of the deregulation of the US energy industry in the early 1990's which meant that these corporations now had shareholders who were more concerned about the impact of weather conditions on the variability of their returns. Thus temperature based derivatives were the first to be considered based on the direct link they have to energy consumption. The market has expanded significantly from these beginnings to now incorporate a wide range of products that cover over a dozen common weather variables.

2.3.1 Contract Types

Active markets exist in the trading of options, futures and swaps over a variety of underlying weather variables. Often these derivative contracts have special, tailor-made features that are introduced in order to properly match the hedging requirements of the client. Typically these specific features are designed to limit the payout in some way and thus make the product more affordable to the consumer. They include cap's and option barriers such as 'up-and-in' barriers that can be applied to the contract in a variety of ways.

The majority of the weather derivatives market is still comprised of temperature and precipitation based contracts however other weather variables are becoming increasingly more popular in recent years as industries begin to realise the risk management benefits that these contract can bring to their businesses. These include, amongst others, snowfall, wind and sunshine based derivative contracts.

2.3.1.1 Temperature

Heating Degree Days (HDD) and Cooling Degree Days (CDD) contracts still account for more than half of the weather derivative transactions that are made both on exchanges and as OTC contracts. The primary purpose of these contracts is to allow organisations to hedge against excessively high or low temperature distributions over a pre-specified period of time.

Most contracts make reference to the arithmetic average of the maximum and minimum temperatures recorded in a 24hr period. In other words:

$$T_i = \frac{T_{\text{max}} + T_{\text{min}}}{2} \tag{2.1}$$

Naturally this average temperature is more reliable to model than either the maximum or minimum temperature are by themselves however information is thus lost and this measure would not provide a satisfactory hedge for those seeking protection from extreme daily temperatures. A HDD is simply the number of degrees the days average temperature was below some reference level, \overline{T} , generally set at 18°. Hence the HDD's for the month are given as:

$$HDD = \sum_{month} \max\{0, (\overline{T} - T_i)\}$$
 (2.2)

This terminology originated out of the fact that generally heating would be required below a particular reference level (here set at 18°) thus requiring the expenditure of energy. The converse is also true for CDD's with the consumer's general requirement for 'cooling' energy above the reference temperature. From here an option contract can then be set with a payoff and exercise price based on these measures. The payoff of a call option on the HDD index is of the form:

$$V_{t} = \max\{0, (HDD - K)\} \cdot tick \tag{2.3}$$

Where K is the exercise price of the option and the 'tick' provides the conversion from degrees to \$ (i.e. its units are \$ per degree) and generally ranges anywhere from \$1,000 to over \$1,000,000 per degree.

A CDD contract is the reverse of the HDD in terms of its payoff. A CDD is just the number of degrees a particular days average temperature was above some pre-determined reference level. As before the accumulated CDD's for a month are given as:

$$CDD = \sum_{month} \max(0, (T_i - \overline{T}))$$
 (2.4)

This is only one possible construction of a temperature derivative and other popular structures in involve the averaging of an index over a period of time (an asian-style option) or to consider the maximum or minimum temperatures over a period (a 'lookback' style option)

2.3.1.2 Rainfall

Rainfall contracts have received significantly less focus, in terms of both the literature and the amount of trading activity, when compared to temperature based derivatives. Primarily this was due to the fact that the weather derivatives market was largely created from the needs of energy and utility companies who's major exposure was to temperature related weather variations. Adding to this, rainfall has proven significantly more difficult to model accurately, particularly where geographically small areas are concerned. This is primarily due to the discrete nature of rainfall and as we shall see in Section 4, two geographically close recording stations can produce widely different precipitation readings, something that is not encountered with temperature based statistics.

These difficulties manifest as geographical 'basis' risk in rainfall derivative contracts in that the situation of the risk must be relatively close to the measuring station for an effective hedge to be possible. Not only are you required to find someone who will benefit and someone who will lose-out if rain falls but you also have to create an underlying rainfall index (i.e. weather station) that will protect both users. This can often be difficult to achieve in practise.

2.3.1.3 Other Variables

Whilst temperature and rainfall contracts account for the majority of the traded derivatives, other important weather variables are also becoming realised for their hedging opportunities. Interest in these new products is purely demand driven and is generally initiated by a client demanding protection from unfavourable weather conditions.

Wind speed contracts have been one of the major growth points for the weather derivatives market, accentuated by a significant global 'push' towards wind based power generation and other renewable sources of energy. Wind-generating power companies are at the mercy of the winds in terms of the amount of electricity that they can produce and hence require some type of protection against lower than expected wind speeds. The Merrill Lynch - Global Commodities Wind Power Indices (WPIs) are produced as a reference by which wind farmers and other producers can hedge their exposure to the variability of wind speeds.

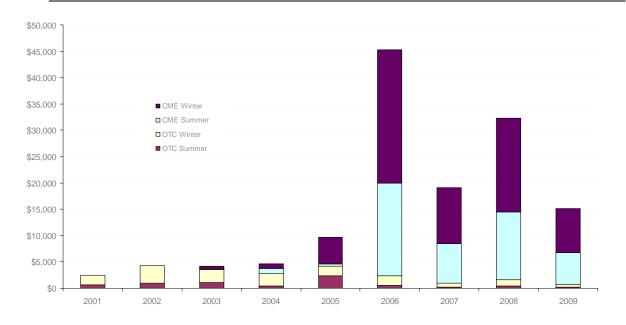
More recently, the CME cleared its first snowfall based contract on 22nd December 2009 being a seasonal maximum binary option. This followed a significant increase in the amount of similar OTC contracts executed in recent years.

2.3.2 Weather Markets

The primary markets for weather

From their beginnings covering largely temperature-based contracts the market has expanded to include a much larger variety of weather variables and along with this the contracts themselves have become increasingly standardised promoting the listing of contracts on exchanges and moving the market away from its OTC roots.

Figure 2.1: Weather Derivative Markets



Source: PwC Annual WRMA survey

2.3.2.1 CME

The Chicago Mercantile Exchange has the largest weather derivatives market in the world. The exchange offers both futures and option contracts over a range of US and European cities. In 2009 the CME traded 14.50 billion of notional value on its exchange which accounts for almost 85% of the total traded volume worldwide.

For example the CME recently cleared its first snowfall based contract which covered the period January – March 2010 and was executed on 22nd December 2009.

2.3.2.2 Europe

The acceptance of weather derivatives has been slower in Europe than in the US market. One of the main reasons for this is the lack of a standardised weather-recording framework that exists across the Atlantic. This is as a result of the many individual countries that make up Europe who have vastly different levels of development, which has meant that it has historically been very difficult to obtain consistent, reliable data. The recent expansion of the European Union (EU) and the creation of a single European currency should greatly facilitate the propagation of unified procedures for data capture and analysis.

2.3.2.3 Asia

More recently Asia, and in particular Japan, have become a source of weather derivative demand as their energy markets gradually became deregulated. The first official transaction in Japan was during 1999 between Mitsui Marine and a local sporting goods manufacturer that consisted of an option over the snow depth recorded for the following winter season. Natsource Japan, a large energy-based broker, is one of the major promoters of weather derivatives in Japan and has created a measure, the Japan Weather Derivatives Index (JWDI), around which market participants can design weather risk management products. As well as this the company has created an interbank electronic exchange, the Japanese Weather Exchange (JWX), on which the large

financial organisations can transfer weather risk both within Japan as well as to the rest of the world.

2.3.2.4 Australia

In Australia, the first reported transaction of a weather derivative occurred in March of 1998 between United Energy Marketing and Utilicorp², a US based energy utility. The contract structure was for a payout if the temperature rose above 35°C in Melbourne or 33°C in Sydney for 5 days or more during the summer months. As it turns out Sydney reached this level on 5 days and Melbourne on 6 days and hence the contract exercised at roughly 8 times the initial one-off premium paid.

The CME has recently begun offering weather derivative products over a range of Australian locations. From early 2009 contracts covering both monthly and seasonal cooling and heating indices for 3 locations:

- Bankstown, Sydney
- Brisbane Airport
- Melbourne

These contracts have only been thinly traded and are largely supported by the exchange however, along with several recent listings for European cities, this demonstrates CME's commitment to the expansion of the weather derivatives market.

2.3.3 Market Participants

In the early years of the weather derivative market the major transactions were generally between large energy companies and large financial institutions. The overall complexities of the risk inherent in these products required a significant amount of research in order to properly price and was beyond the expertise of most of the participants. The financial impact of weather was so great on these energy and utility companies that is was economical to spend large amounts of money endeavouring to reduce this uncertainty.

To achieve this they utilised temperature-based derivatives in an attempt to 'smooth' the uncertainty in their financial performance that was attributable to temperature variations. As an example, consider a company who supplies gas to consumers for heating during the winter months. Clearly the company will see reduced profits if temperatures are higher than usual during the winter period in question and, conversely, will experience higher profits if the region experiences lower than average temperatures. Hence this business might seek to reduce the overall variability of its expected profits buy purchasing a put option on the HDD index for the particular months in question

2.3.4 Weather Derivatives and Insurance

There are important distinctions between a derivative contract and traditional insurance policies in terms of the coverage offered and the payouts received by the purchaser of the protection. Some of the points of difference include:

 Identifiable Loss: Insurance contracts are more often designed for protection from the extreme weather events such as a storm or flood where an identifiable loss has occurred. This loss is required as evidence that a claim can be made

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² Energy and Power Risk Management – June, 1998

against the insurer. The payout of a weather derivative on the other hand is determined by reference to an index whose composition is transparent, such as temperature and rainfall. In this sense the purchaser of the protection does not require proof and significant savings can be made on legal fees required to defend appropriate payments.

Moral Risk: The moral risk that is inherent in all insurance contracts can be nearly
entirely removed as the reference is made to an index that is (hopefully) out of the
control of both the counterparties. This will again act to reduce the cost of
covering the associated risk and hence make weather derivatives a more
affordable option to primary producers.

The insurance industry has contributed to much of the growth in recent years. Weather derivative contracts have allowed many insurers and reinsurers to offer a wider range of products to their clients as well as being able to assist in managing the significant weather related risk that a general insurer will often retain. These now include non-catastrophic weather insurance contracts that can protect crops and their associated income streams from what are relatively predictable weather extremes.

Large reinsurers such as Swiss Re (New Markets division) and Hannover Re are currently using weather derivatives as part of their broader risk management programs and more recently in Asia companies such as Westpac Bank and Element Re have entered the market. Whilst these large reinsurers are well suited due to the generally large size of weather derivative contracts, theoretically they should be more appealing to geographically more concentrated insurers who are less easily able to obtain 'natural' diversification. Brokers such as Willis, AON and Marsh are actively promoting weather derivative products to their clients however these contracts tend to be highly tailored to meet the specific needs of the consumer and hence generally require specific pricing approaches.

2.3.5 Other Users

As was outlined in section 1, there are other, more technical, users of weather derivatives Who are seeking to invest in these securities in order to access their unique diversification profiles. More and more financial institutions are promoting these derivative products to their clients and, as will be shown throughout this paper, the breadth of risk management applications appear to have no bounds. These include:

- Construction companies hedging against temperature, rainfall, snowfall;
- Drink manufactures temperature;
- Farmers all weather variables;
- Event Organisers mainly precipitation;
- Tour Operators all weather variables.

In later sections we will see some recent practical examples of how weather based contracts can assist businesses across a wide range of industries.

3: Emission Markets

3.1 Counting Carbon

During 1997 in Kyoto, Japan the Kyoto Protocol was adopted by the UN member countries in response to the growing evidence suggesting that human activities are contributing to the global warming phenomenon. The protocol divides the word into two; Annex 1 countries and non-Annex 1 countries. Annex 1 countries are considered the developed nations as well as those that are in transition. Non-Annex 1 countries implicitly refer to developing nations.

Currently 186 countries have both signed and ratified the Kyoto protocol, most recently being Brunei in August 2009. The only notable exception, in terms of their contribution to global GHG emissions, is the US who has signed the protocol but is currently not intending to ratify the agreement. The US represents approximately 36.1% of the Annex 1 emissions and thus will be a crucial stakeholder at the table for any true global emission reduction framework.

The Kyoto protocol provides three mechanisms (known as 'flexibility mechanisms') that can be used by Annex I members to assist them in managing their emission targets as specified by the agreement. These are:

- International Emission Trading (IET) This is the means by which emission units
 can be traded between Annex I countries. The Kyoto protocol states that the trading
 of carbon emissions is to be one of the primary mechanisms used to achieve the
 targeted reductions in GHG's on a global basis under the IET flexibility mechanism.
 This clause has led to the development of emission markets, most notably the EU
 ETS.
- Joint Implementation (JI) This allows Annex 1 countries to offset their emissions by investing in emission reduction projects in other Annex 1 countries. These include bio-sequestration and geo-sequestration projects, many of which have been developed in Annex 1 countries in recent years.
- Clean Development Mechanism (CDM) emission reduction projects in non-Annex
 1 countries that produce Certified Emission Reductions (CER's) that can be used to
 meet emission targets in Annex 1 countries. This mechanism is described in more
 detail when we discuss risk management issues surrounding the carbon market in
 section 6.

Under the protocol, Annex 1 countries are required to report annually that details all anthropogenic GHG emissions in that country as well as GHG reductions from 'sinks' in accordance with the protocol.

3.1.1 The need for a pollution charge

There is little debate amongst policy-makers as to the overall need for the adoption of a pollution charge (in some form) that will incentivise the gradual reduction in the consumption of products and services that cause damage to our atmosphere. The distribution of this charge amongst the business population will require instruments to be

available that can securitize the profits that can be generated from progressing towards a 'greener' economy. The most popular framework proposed by signatories to the protocol is what is known as a 'cap-and-trade' system where allocations of 'capped' emission targets are specified for each country with the ability of surplus's to be traded between other countries to assist in meeting their targets under the scheme.

This structure can then be replicated at subsequent points below this international level with individual 'cap-and-trade' systems applying to organisations within that country. This leads to the requirement for carbon markets to exist within each jurisdiction to facilitate the transfer of these emission certificates between organisations. The level of development of these markets varies significantly amongst those nations that have signed and/or ratified the Kyoto protocol.

3.2 Carbon Markets

Carbon markets have seen rapid expansion in recent years, particularly in Europe where the adoption of the Kyoto protocol has been taken most literally. These markets generally list the emission certificates that are specific to their region and for a particular compliance date. Derivatives based on these contracts are also generally available with futures and options being the most popular structures.

3.2.1 Europe

The European carbon market is the most liquid carbon market in the world and is based around the EU Emission Trading Scheme (EUTS) that came into effect at the beginning of 2005. This represented the first full scale implementation of the framework specified in the Kyoto protocol. The EU ETS covers approximately 50% of the EU carbon emissions and approximately 40% of the GHG emissions.

The EU scheme provides the mechanism through which European Union Emission Allowances (EUA's) can be traded between corporations to assist them in meeting their emission targets under the EU ETS. The primary exchanges based in Europe that list carbon contracts include:

- French Bluenext
- Nordic Nord Pool Group
- EEX the German European Energy Exchange.
- ECX the British European Energy Exchange.

Other security exchanges such as the NYSE and EUREX have cooperation policies to enable the through processing of carbon based contracts. These markets had average monthly volumes of EUA's traded of approximately 370Mt of CO₂ during 2009. CER's have seen a significant increase in trading activity with volumes more than doubling in March this year to 83 Mt.

3.2.2 US

There have been several attempts by US parliaments in recent years to introduce legislation that seeks to set a framework for the US's response to the reduction of its GHG emissions. Whilst not intending to ratify the Kyoto protocol the US has proposed An alternative framework be drawn up for a post Kyoto world and has obtained agreement from several of worlds large emitters un the 'Washington Declaration'

including the governments of France, UK, Canada, Germany, Brazil, China, Japan, Russia amongst others. The system that is visualized in the declaration is a global capand-trade system that covers both developed and developing nations. This initiative has yet to be taken further by the current administration.

The American Clean Energy and Security Act of 2009 (ACES) was approved by the House of Representatives on June 26, 2009, by a vote of 219-212, but has not yet been approved by the Senate. This is commonly referred to as the Waxman-Markey bill in recognition of its authors. Underlying the proposals in the bill is the establishment of a national 'cap-and-trade' system that is similar in form to that proposed by other nations.

3.2.3 Australia

One of the primary purposes in establishing an ETS is to create a market for carbonbased products so that businesses can plan for the future around the current expected price of carbon. Recent volatility in the price of carbon demonstrates the difficulty in funding future investments when the future price of carbon is so uncertain.

Domestically, the current political situation facing the CPRS legislation and the ETS is precarious with the proposed CPRS legislation (actually consisting of 11 individual bills) being twice rejected by the senate and recently having been postponed by the government until at least 2013. Although the implementation of any trading system within Australia now seems several years away, the reporting requirements in relation to GHG emissions has been enacted in the form of the National Greenhouse and Energy Reporting Act 2007 (NGER). This places requirements on the nations top emitting companies to report GHG inventories on an annual basis to the Greenhouse and Energy Data Officer.

3.2.4 Recent Challenges

Unfortunately the carbon markets in Europe have experienced several recent setbacks that have reduced the confidence in the market at a time in its development when it still needs to prove itself as an efficient mechanism for the abatement of greenhouse gases.

Two recent incidents that have undermined confidence in the carbon market are"

- Phishing scam February 2010 An estimated 250,000 permits were stolen from 6
 German organisations who inadvertently handed over company details that enabled
 third parties to steal their emission permits. These permits were then spruiked to
 hundreds of companies across the globe. Several exchanges were suspended for
 short periods on February 2nd in response to the scandal.
- Recycled CER scandal March 2010 The Hungarian government unintentionally sold around 2 million recycled CER's onto the market. These certificates had already been used to meet compliance targets by a range of Hungarian companies which means that they cannot be resold as CER certificates. Trading of contracts on most European exchanges were halted as a result of the discovery with trading resuming on all exchanges soon afterwards.

3.3 Carbon Insurance

The various mechanisms that exist under the protocol to assist in reducing the global level of emissions require projects to be established that depend on foreign capital in order for the emission certificates to be realised. These projects attract a range of risks over their lifetime and investors require solutions to help them manage these risks up until the point that the certificates generated are on-sold to the wider emission markets. Carbon insurance contracts are now available in many developed countries where carbon sequestration projects are being developed or that have been planned. These contracts seek to address the multitude of risks that are encountered when seeking to establish projects through which carbon credits are expected to be generated. These contracts will be discussed in more detail in section 6 in relation to the development of CDM projects.

4: Pricing Fundamentals

This section seeks to lay down some of the key building blocks required to model and price contracts based on alternative asset class such as weather and carbon derivatives.

Many of the modelling and pricing techniques are common to both of these asset classes primarily due to the fact that the markets are generally incomplete and thus require a different approach to pricing derivatives in more developed, liquid markets. The pricing methods required for both weather and carbon derivative contracts have many similarities particularly in their dependence on numerical methods for their solution.

4.1 Generalised Hyperbolic Distributions

Generalised hyperbolic distributions have been suggested as being more appropriate to the modelling of returns as experience by many financial assets. The use of Hyperbolic distributions in modelling financial variables has most notably been put forward by Eberlein and Keller [1995] and Hu and Kercheval [2008]. These distribution have been found to provide a better fit to the returns of many financial variables particularly those with fat-tails.

Specifically the probability density function representing a hyperbolic distribution is of the form:

$$f_{x} = \frac{\chi^{-\lambda} \left(\sqrt{\chi \delta}\right)^{\lambda}}{2K_{\lambda} \left(\sqrt{\chi \delta}\right)} x^{\lambda - 1} e^{\left[-\frac{1}{2}(\chi x^{-1} + \delta x)\right]}$$

Where x , δ >0 and K_{λ} is a modified Bessel function of the third kind.

This is the general form of the distribution. Some specific examples of generalised hyperbolic distributions are:

- Hyperbolic where $\lambda = 1$
- Normal Inverse Gaussian where $\lambda = -1/2$
- Skewed t-distribution where $\lambda = -\upsilon/2$ and $\chi = \upsilon$

We will see an example of the NIG being used later in section 5.

4.2 Geometric Brownian Motion

The theory in terms of weather and carbon derivative pricing is still extremely sparse with no widely satisfactory formula that can be used by the growing number of practitioners in the marketplace. The current pricing methodologies can be broadly put into two groups; analytical solutions or numerical solutions. Whilst standard equity options have the famous Black-Scholes equation to provide practitioners with a reliable pricing basis, their weather-based counterparts have no agreeable equivalent to the BS framework and generally require a numerical approach.

A quick revision of standard option pricing theory is helpful when investigating its extensions later. Arithmetic Brownian motion is commonly represented by the following stochastic differential equation:

$$dX_{t} = \mu . dt + \sigma . dW_{t} \tag{2.1}$$

Whilst this might be suitable for modelling biological processes, Geometric Brownian motion (gBm) is the process that is generally used to model financial variables such as stock and commodity prices. Its necessity arises out of the fact that a log function does not permit negative values, essential when modelling asset prices. gBm is described by the following stochastic differential equation:

$$\frac{\partial X_t}{X_t} = \mu . dt + \sigma . dW_t \tag{2.2}$$

This can be reduced to arithmetic Brownian Motion via the substitution into the above equation of $y = F(X_t) = \log X_t$ and the use of Ito's formula, to give:

$$dF = (\mu \cdot -\frac{1}{2}\sigma^2)dt + \sigma \cdot dW_t$$

which with an initial condition, X_0 , has a solution given by:

$$F(X_t) = \log x_0 + (\mu - \frac{1}{2}\sigma^2)(t - t_0) + \sigma W_{t - t_0}$$
 (2.3)

A solution for the process X_t is then found by reversing the substitution (exponentiating), Hence we arrive at our distribution for the initial gBm process:

$$X_{t} = X_{0} \cdot e^{\left[\left(\mu - \frac{1}{2}\sigma^{2}\right)(t - t_{0}) + \sigma W_{t - t_{0}}\right]}$$
(2.4)

From here Black and Scholes use their now famous hedging method to derive a partial differential equation (i.e. no longer a stochastic equation) for the dynamics of the option price that is based on the Brownian motion of equation (2.2). The Black-Scholes p.d.e is:

$$\frac{\partial V_t}{\partial t} = rV - ry. \frac{\partial V}{\partial y} - \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 V}{\partial y^2}$$
 (2.5)

Where V_t represents the payoff of the option contract. This differential equation can then be solved to obtain the explicit Black-Scholes formula. Alternatively a martingale approach can be adopted via equation (2.4), i.e. seeking a solution to the equation:

$$V_{t} = \mathbf{E}_{Q} \left[X_{0} \cdot e^{\left[(\mu - \frac{1}{2} \sigma^{2})(t - t_{0}) + \sigma W_{t - t_{0}} \right]} \mid F_{t} \right]$$
 (2.6)

Where Q represents the risk-neutral martingale measure.

4.3 Mean Reversion

The key difference to modelling weather-based variables when compared with a general financial variable, such as a stock price, is that most weather components exhibit some degree of mean reversion. A mean reverting process is one in which the drift component of the stochastic differential equation (equation 2.1) always acts in a direction that

opposes the current displacement from the mean process, in much the same way as a spring acts on a weight.

This concept is utilised frequently in the modelling of interest rates, which like many weather variables are at least partially mean reverting in that do not rise or fall without bound. The Vasicek(1979) model of forward interest rates is based on this mean reverting approach as well as other well known interest rate model such as the Hull-White model and the Cox-Ignersoll-Ross model.

The mean reversion component is deterministic and is an extension of the drift term, μ .

$$\frac{dX_{t}}{dt} \propto -(X_{t} - \overline{X}) \tag{2.7}$$

where X represents the mean process. In this way the drift will always act in a way so as to bring the process closer to its mean. A proportionality constant is now required, called the mean-reversion parameter, which is a measure of the restoration force acting on the process dynamics. It is akin to the spring constant k, for all those who can still recall high school physics classes.

$$\frac{dX_t}{dt} = -\gamma \cdot (X_t - \overline{X}) \tag{2.8}$$

Substituting this into the standard Brownian motion dynamics, equation (2.1), we get:

$$dX_{t} = \gamma (\overline{X} - X_{t}).dt + \sigma.dW_{t}$$
(2.9)

Note that this process will no longer be gBm and that negative values are permitted by equation (2.9). The probability that negative values will occur depends on the mean process level as well as the strength of the mean reversion. For a strongly reversionary process who's mean is significantly above zero it is highly unlikely that the process would ever go negative. The Ornstein-Uhlenbeck process is the name given to this modified Brownian motion. It has since been shown in Dornier and Queruel [2002] that the process given in equation (2.8) does not actually revert to its mean when temperature is used as the variable. This is due to the fact that the mean process that the equation is reverting to, \overline{X} , is not constant. To overcome this an extra term (the time derivative of the mean process) is required to be added to the drift component of the stochastic equation above. Hence we get:

$$dX_{t} = \left[\gamma(\overline{X} - X) + \frac{d\overline{X_{t}}}{dt} \right] dt + \sigma . dW_{t}$$
 (2.10)

This representation is now mean reverting in the long-run, in other words: $\mathbf{E}[X_{_t}] = X$. Another advantage of including this term is that equation (2.10) can now be solved by the traditional integrating factor method. Multiplying through by \mathbf{e}^{ys} we obtain:

$$e^{\gamma s} dX_s - e^{\gamma s} \gamma (\overline{X} - X) dt + e^{\gamma s} . d\overline{X}_s = e^{\gamma s} \sigma . dW_s$$
 (2.11)

Now the left hand side of the above expression is just the differential of a product, i.e:

$$e^{\gamma s} dX_s - e^{\gamma s} \gamma (\overline{X} - X) dt + e^{\gamma s} . d\overline{X_s} = d[e^{\gamma s} . (X_s - \overline{X}_s)]$$
 (2.12)

$$d[e^{\gamma s}.(X_{s} - \overline{X}_{s})] = \sigma.\int_{0}^{t} e^{\gamma s} dW_{s}$$
 (2.13)

Note that this would not have been possible without the extra term being added to equation (2.10) to make it properly mean reverting. After rearranging this expression we obtain the solution to the stochastic process:

$$X_{t} = \overline{X}_{t} + (X_{0} - \overline{X}_{o}) e^{-\gamma \Delta t} + \int_{0}^{t} e^{-\gamma \Delta t} .\sigma_{\tau} dW_{\tau}$$
 (2.14)

This equation will then become the basis of the Monte Carlo simulations that are undertaken in section 3.

4.4 Black-Scholes formulation

The seminal paper of Black and Scholes [1977] provided an analytical framework for the pricing of contingent claims and in particular options, however the Black Scholes (BS) formula relies on some fairly stringent assumptions. Most importantly it is assumed that the underlying process is driven by gBm as given above by equation (2.1). Most empirical studies show that in fact asset returns are strongly leptokurtic (More concentrated in the middle and 'fat-tailed'), such as Fama (1965).

As well as this, with respect to weather derivatives, there is generally no underlying process that is actively traded (i.e. people don't trade degreesyet) and as such the BS framework has no method of hedging the derivative in order to derive an analytical solution. For these reasons a standard BS approach is not applicable and other more indirect methods must be pursued.

4.4.1 Asian Options

Many common weather derivative contracts have as their underlying index, an average of some statistic over a period of time. For example, many temperature-based derivatives will have a payoff that is determined by reference to the average temperature over a week or a month. These types of options are generally referred to as 'Asian' options and it is only by coincidence that the majority of 'Asian' weather derivative options are actually traded in Asia. The averaging can be done on a variety of time frames however it is generally the daily temperature that is averaged. For example a call option on the monthly average temperature would have a payoff of the form:

$$V_{t} = \max\{0, (\frac{\sum_{i=1}^{n} T_{i}}{n} - K)\} \cdot tick$$
 (2.15)

where there are n days in the month. Analogies to the BS partial differential equation can be derived for the varieties of 'Asian' options that exist³ however we still have no underlying asset with which to perform the required hedge.

³ These include fixed strike and floating strike amongst others. See Buchen [2002] for derivations.

4.4.2 Black '76 Model

The alternative model that was devised by Black and known as the Black '76 model has important implications for both the pricing of both weather and carbon derivative products.

Several authors have decided to proceed with the BS framework ignore the problems of the underlying assumptions or have attempted to alter the BS approach to accommodate these deficiencies. The most serious assumption that must be relaxed is that there is no underlying asset to base the derivative price around.

In regard to the pricing of weather derivative contracts, Jewson et. al [2003] has suggested that an alternative BS formula can be derived akin to the derivation of an option on a futures contract. To overcome the fact that there does not exist an underlying, traded asset, Jewson creates a hypothetical forward weather index that is used as the underlying asset which the BS framework can use as a hedge to enable the pricing of the contingent claim.

The theory is based on the Black (76) model where a futures contract is used as the hedge when deriving the partial differential equation that governs its motion. To illuminate, let us assume that the futures price process is governed by the 'cash-and-carry' relationship:

$$Y_t = X_t \cdot e^{r(T-t)}$$

By using Ito's formula when substituting this relationship into the standard gBm form equation (2.1) we get the altered stochastic differential equation:

$$dY_{t} = y[(\mu - r)dt + \sigma dW_{t}]$$
(2.16)

Following the same hedging procedure as used in deriving the Black Scholes partial differential equation we arrive at the following relationship for the dynamics of options on a futures contract:

$$\frac{dV_{t}}{dt} = rV - \frac{1}{2}\sigma^{2}y^{2}\frac{d^{2}V}{dy^{2}}$$
(2.17)

If we compare this relation with equation (2.5) we can see that the term, $-ry.\frac{\partial V}{\partial y}$ is

removed from the right hand side of the equation. This is the same equation one would get if calculating the price of an option on a dividend paying stock where the dividend yield was equal to the risk free rate. Hence by using notation similar to Buchen [2002] we can write the option price over a futures contract as:

$$V(y,t) = BS(ye^{-r\tau}, t, r, \sigma)$$

$$= e^{-r\tau}.BS(y,t,0,\sigma)$$
(2.18)

where $BS(x,t,r,\sigma)$ represents the 'standard' Black Scholes pricing formula.

4.5 'Burn' Analysis

This is a typical actuarial approach adopted to price a contingency where no assumptions are required to be made as to the nature of the process on which the contingency relies. A typical 'burn' analysis seeks to answer the question: "What would be the return from the contract had I purchased it each year for the last x years?" Generally, an arithmetic average is then taken of the results.

For example, to price a February HDD call with exercise of 100, simply find what the financial return would have been for each of the February months in the historical data set, with the appropriate indexing of the exercise value in order to standardise the temperatures over time⁴.

4.6 Monte Carlo Simulations

Monte Carlo simulations differ from the 'burn' analysis approach above in that they require assumptions to be made as to the dynamics of the underlying variable. Essentially they involve running a series of simulations based on a statistically derived model and then calculating what the expected return is from all of these simulations.

Mathematically speaking we find a solution to:

$$\mathbf{E}[f(X_t)] = \frac{1}{N} \cdot \sum_{i=1}^{N} f(\overline{X}(t, \psi_i))$$
 (2.19)

i.e. the arithmetic average of the simulation outcomes. Here t represents time and ψ_i is the series of calculation points.

The concept of mean reversion discussed earlier has important consequences for the selection of an appropriate starting point for the simulation process. If the time period of interest is in the short-term then it is necessary to begin the simulations from the present day values as any current deviation from the mean will have an impact on the process in the short-term. Consider for example a one-day option over the average daily temperature that was being valued at both 1 day and 1 month before the contract period.

For the valuation close to the contract period it is important to take into account the current displacement from the mean as this value will have an impact on the temperature on the following days. Hence the simulation should be started on the actual raw value. If however we are 1 month from the contract period then we can begin the simulation on the mean temperature process as the actual displacement from the mean will not have an appreciable influence on the temperatures in a month's time. An example of pricing via a Monte Carlo simulation is shown in section 5 when the valuing of temperature based derivatives are investigated

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⁴ If using data sets over 100 years then a quadratic parameterisation of the long-term trend will be required to properly estimate the effective strikes. See section 3.3.1 for more discussion.

5: Modelling and Pricing Temperature

In this section we will outline some of the practical consideration required when modelling weather variable, in particular temperature and precipitation based derivative contracts.

The analysis will be undertaken for weather stations located both in Sydney and Melbourne to apply some context to the results obtained. Much of this analysis is an updated version of analysis undertaken in Tindall [2006].

5.1 Temperature

Compared with rainfall, the analysis of temperature has received significant attention from the literature in recent years. Primarily this is due to the fact that the majority of traded contracts in the weather derivatives market are temperature based, primarily related to energy supply or demand. More recently the search for a reliable statistical model for temperature dynamics has been intensified by the need to provide sound evidence for the impacts of human interaction on the planet.

Much of the effort required in modeling the average temperature is in reducing the observed process to a stationary process by removing the deterministic trends. The temperature model that is developed in the next sections follows a combination of approaches used by Benth et al [2002], Alaton [2002] and has been applied to the weather stations outlined above

The genera outline of the process of modelling and pricing temperature based contracts is:

- 1. Clean the data
- 2. De-trend the data.
- 3. De-seasonalise;
- Estimate mean-reversion;
- 5. Model and investigate the residuals;
- 6. Simulation of the process;
- 7. Pricing the derivative.

The remainder of this section seeks to illuminate this process by undertaking an empirical investigation on temperatures recorded at Sydney airport station for the period 1948 – 2010.

5.1.1 Data

In order to highlight much of the discussion made herein, reference will be made to an analysis undertaken using temperature data from the Sydney region for the period 1856-2010. The data was obtained from the Australian Bureau of Meteorology (BOM). The following subsections of the data were used in this investigation:

Sydney Airport Jan 1940 – Mar 2010
 Melbourne Jan 1971 - Mar 2010

The majority of the analysis in this paper is in respect of the Sydney Airport weather station which consists of 66 complete years of minimum and maximum temperature as

well as precipitation readings. Please refer to the appendix for a discussion of the treatment of missing values in these data sets.

5.1.2 Temperature Distributions

The following histogram shows the distribution of temperatures at Sydney Airport for the period Jan 1940 through to March 2010

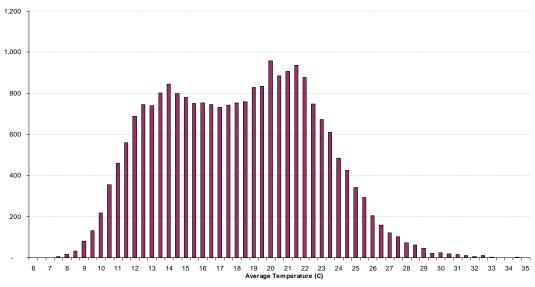


Figure 5.1: Temperature Histogram – Sydney Airport

As figure 3.1 shows, the distribution is bimodal, reflecting peaks for both the summer and winter months. This feature is common amongst most temperature distributions throughout the world and Appendix B shows a range of these. It is interesting to note that southern hemisphere locations tend to have a negatively skewed distribution where as northern hemisphere locations are usually positively skewed. If one were to restrict attention to just summer months or just winter months then the following patterns are revealed:

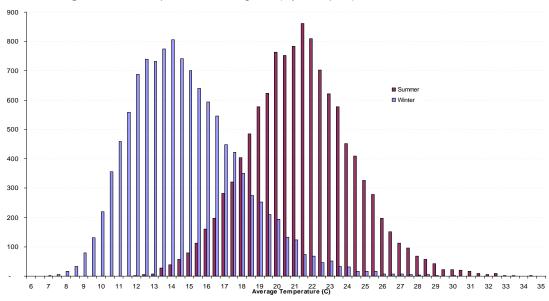


Figure 5.2: Temperature Histogram (Syd. Airport) – 'Summer' vs 'Winter'

The following chart depicts the average temperature distribution for Melbourne airport over the period 1971 – 2010.

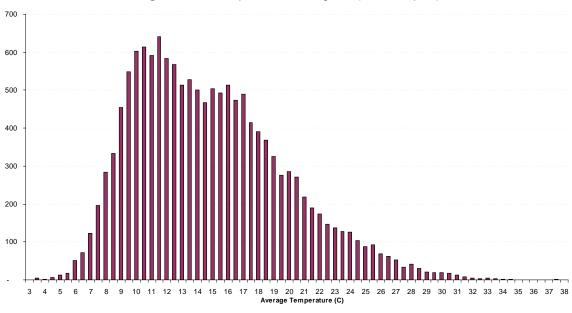


Figure 5.3: Temperature Histogram (Melb. Airport)

As can be seen the distribution is significantly more skewed than that recorded at Sydney Airport with the bi-modal nature of the distribution being less obvious. Once again it is useful to split the distribution into approximate summer and winter components. This shows:

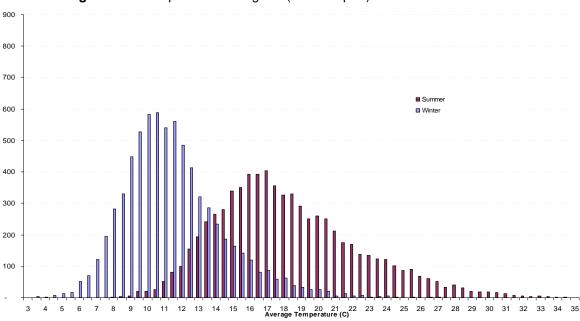


Figure 5.4: Temperature Histogram (Melb. Airport) – 'Summer' vs 'Winter'

The most notable difference when compared with the temperature distribution at Sydney Airport is the significantly higher volatility that occurs during the summer months.

Whilst we will look more specifically at the long term trends in temperature (section 5.3.1) it proves insightful to split the above histogram as follows:

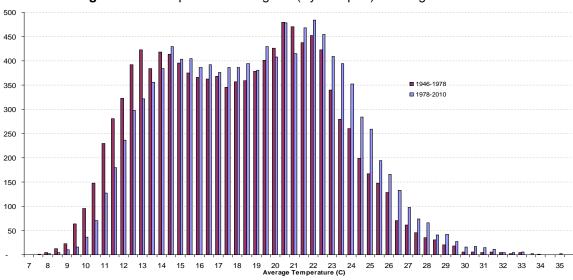


Figure 5.5: Temperature Histogram (Syd. Airport) – changes over time

This clearly shows the rising average temperatures that have been recorded at Sydney Airport over the past 80 years and justifies the use of linear and quadratic trends when we come to model the temperature data in section 5.1.3.1. A similar shift in average temperatures

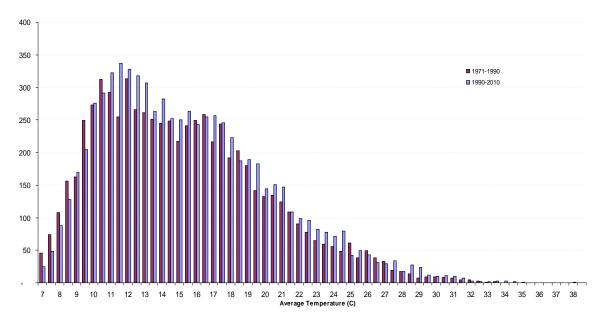


Figure 5.6: Temperature Histogram (Melb. Airport) - changes over time

Whilst the shift appears modest in terms of the entire distribution it is useful to focus in on the tail of the distribution which is heavily populated by temperatures recorded in the past 20 years:

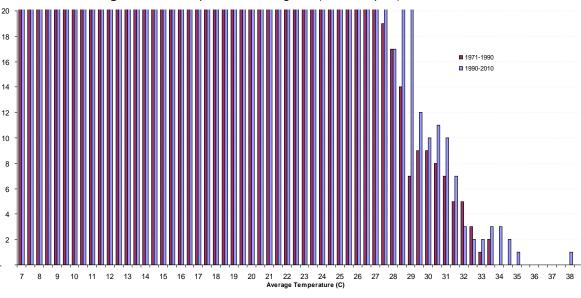


Figure 5.7: Temperature Histogram (Melb. Airport) – in the tail

Indeed the outlier recorded at 38.5 occurred in 2009 and with the next 10 highest recorded average temperatures also occurring in the second half of the study period.

The initial part of the modelling process is centred on removing both the long-term and seasonal trends from the data (the deterministic part) and from there a stochastic model is fitted to the residuals. The first part of this process is referred to as 'detrending' and several authors have proposed methods for achieving this, for example Benth [2005] and Alaton [2002].

5.1.3 De-trending

The primary aim of the de-trending process is to reduce the time series to a stationary process so that the residuals can be modelled according to a particular distribution.

The decision to de-trend the data can be time consuming and costly so there have been attempts made to quantify the necessity to carry out the procedure. Jewson [2004] provides a decision rule for the introduction of linear trend but finds that it does not significantly beat a rule to always de-trend or to always not de-trend. He found that it beat a no trend rule when the time series was relatively long and conversely beat a linear trend when the sample was short.

5.1.3.1 Long-term Trends

Analysis of most long-term temperature data reveals a slight positive trend that represents the gradual warming of the globe that has been occurring since the last ice age. Study of longer time periods shows that the trend is more significant in latter years and suggests that a quadratic term should be used to model these long-term environmental changes.

$$T_{Linear} = a + b.t + c.t^2$$

However, for shorter time periods (<80 years) it is reasonable to assume a linear form.

$$T_{linear} = a + b.t (3.1)$$

This is generally acceptable given that the majority of the weather derivative contracts that are entered into have periods of a year or less. This means that to make projections over time periods where the quadratic term becomes significant are rare in practice and for the purposes of this investigation a linear trend will be assumed.

5.1.3.2 Seasonal Variation

Seasonal variation is by far the most dominant term in the overall temperature variation. The periodic nature of temperature that results from the seasonal variation can generally be described by a sinusoidal function and for the purposes of this analysis we will assume it to be represented via a truncated Fourier series of the form:

$$T_{Seasonal} = \varepsilon \alpha_0 + \sum_i \alpha_i . Sin(\gamma t + \phi) + \sum_i \beta_i Cos(\lambda t + \theta)$$
 (3.2)

Here the coefficient α_0 is not required as this effect would be captured in the linear trend already discussed. The remaining parameters are to be estimated from the data. Combining these two effects, equations (3.1) and (3.2), we now have an equation for the mean temperature process given by:

$$\overline{T} = T_{long} + T_{seasonal}$$

hence,

$$\overline{T} = a + b.t + \sum_{i} \alpha_{i}.Sin(\gamma t + \phi) + \sum_{i} \beta_{i}Cos(\lambda t + \theta)$$
(3.3)

For the purposes of this investigation a first order Fourier series will be used so that only one α and one β need to be estimated from the data. This will entail a total of 6 parameters in total to be estimated in equation (3.3) above.

5.1.4 Parameter Estimation

The parameters contained in equation 3.3 are estimated via least-squares. In relation to the 'speeds' of the seasonal processes (γ and λ), there are two possible approaches to determining suitable parameter values. Most authors choose to constrain the two values

to $\frac{2\pi}{365}$ so that a Doppler effect is not encountered for projections over longer time

periods. Alternatively the 'speeds' could be allowed to vary, independently of each other, so that a better fit to the raw data is achieved. When the two methods were undertaken on the Sydney Airport data series the following deviances were obtained:

Fixed γ and λ : 147,614 Variable γ and λ : 143,255

Whilst this shows that the removal of the constraint improves the model fit (as would be expected) the magnitude of the improvement does not warrant the loss of tractability that

the model suffers as can be seen in the following graph that shows the resultant parameter values when the wave speeds are allowed to vary.

The clear 'kink' in the graph at around the new year is hard to justify in terms of common experience. For this reason we will constrain the individual wave speeds to the value

 $\frac{2\pi}{365}$ so that we have the superposition of waves with equal frequencies. The following

table summarises the results of the parameter estimation process for the three weather stations involved:

Table 5.1: Parameter estimation results.

	Syd. Airport	Melbourne
а	16.925	16.338
b	6.30*10 ⁻⁵	5.16*10 ⁻⁵
α	5.14	4.91
β	0.69	-0.20
φ	1.097	1.25
θ	0.97	1.10

5.1.5 Autocorrelation

In order to estimate the mean-reversion parameter a measure of the autocorrelation must be established. To achieve this, today's temperature is regressed against yesterday's temperature (i.e. an AR(1) model) and then the following relationship yields the parameter:

$$\gamma = e^{-\kappa} \tag{3.4}$$

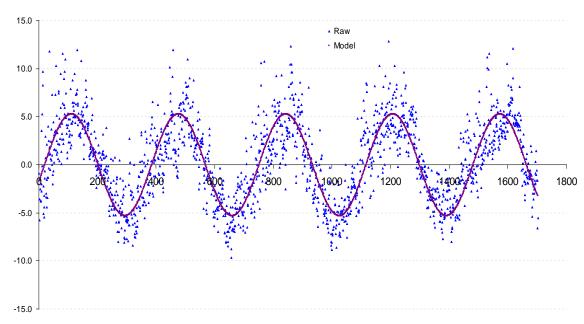
where κ is the regression parameter. This was undertaken for each of the weather stations yielding the following parameters:

Table 5.2: Autocorrelation estimation.

	Syd. Airport	Melbourne
γ	0.555	0.5345

Now that all of the parameter values have been estimated it is then possible to simulate typical paths that represent the dynamics of average temperature throughout the year. The following graph shows the actual and modelled temperatures for the last 10 years.

Figure 5.4: Modelled Seasonal Temperature – Sydney Airport



The increased volatility of temperatures in the summer months is evident from the graph but what is less obvious is how volatility varies throughout the rest of the year.

5.2 Patterns of Volatility

Most pricing models for weather derivative contracts are derived on the basis of a constant volatility over the term of the contingency. On a closer inspection of figure 5.5 it is evident that the volatility is not uniform over the year and there are in fact distinct seasonal patterns of volatility. Note the following pattern for the seasonal variation of the volatility of temperature at Sydney Airport:

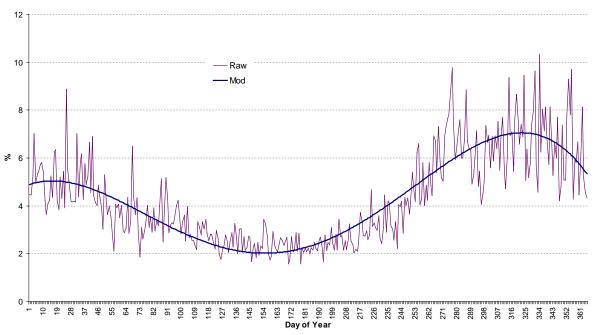


Figure 5.5: Seasonal Volatility Pattern – Sydney Airport

A degree-4 polynomial has been fitted to the data and is shown in figure 3.4 above. Other authors suggest modelling volatility as a Fourier serries (Benth [2005]), as a separate Ornstein-Uhlenbeck process (Bhowan[2003]) or as a piecewise construction of constant monthly volatilities (Alaton [2002]).

Once all of the aforementioned factors are removed from the raw data we are left with the final residuals which if the model assumptions are correct should approximate a normal distribution. The following chart shows the distribution of the residuals:

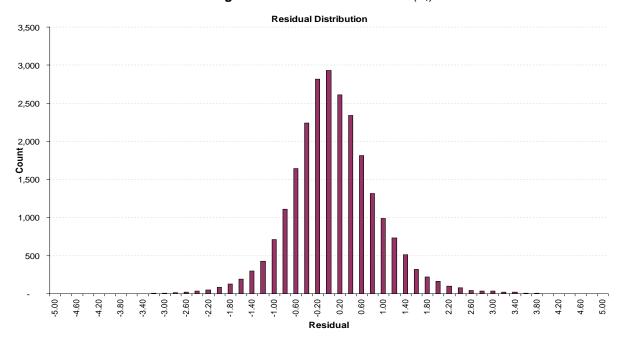


Figure 5.6: Residual Distribution (Z_i)

As can be seen from figure 3.4, the residuals are a good approximation to a normal distribution at a high level although they appear to be leptokurtic and slightly right-skewed.

5.3 Stochastic representation

Recall from equation (2.8) that the stochastic differential equation that we are using to model temperatures is:

$$dT_{t} = \left[\gamma (\overline{T} - T_{t}) + \frac{d\overline{T}}{dt} \right] dt + \sigma . dW_{t}$$
 (5.5)

This has a solution for an initial conditions of, T_0 & $\overline{T_0}$, that is given by:

$$T_{t} = \overline{T}_{t} + (T_{0} - \overline{T}_{o}).e^{-\gamma \Delta t} + \int_{\tau}^{t} e^{-\gamma \Delta t}.\sigma_{\tau} dW_{\tau}$$
 (5.6)

In order to undertake a simulation of the stochastic equation (3.3) we need to obtain a discrete approximation to the dynamics so that daily readings can be predicted. A Euler approximation to equation (3.5) is:

$$T_{t+1} - T_t = \gamma \left(\overline{T} - T_y \right) + \frac{d\overline{T}_t}{dt} + \sigma.Z$$
 (5.7)

Where $\mathbf{Z} \sim \mathbf{N}(0,1)$.

This equation can be used to simulate sample trajectories of the stochastic process and will enable the undertaking of Monte Carlo type analysis.

5.4 Pricing Option Contracts

Section 4 outlined the common approaches that are currently employed in the pricing of weather derivative contracts. Later in this section a numerical comparison of these methods will be undertaken with reference to the weather stations outlined in section (3.1). Firstly, we will look at an approximation that can be used to price both HDD and CDD option contracts.

5.4.1 Normal Approximation

Alaton [2002] proposes an approach for deriving a closed form approximation to the price of an option over CDD or HDD contracts. The catch is that generally the CDD price will only work in summer months and the HDD in winter months.

The basics behind the approach are to assume that the max() function has no effect on the resulting distribution of HDD's or CDD's. If for example the 18 degree limit was used as the basis of a CDD contract in Sydney during January it would be a rare day that has an average temperature below this value. For the record, out of the 2045 January days since 1940 only 11 have recorded an average temperature below 18 degrees (in today's terms), hence this may not be as restrictive an assumption as it may first appear.

Now consider the measurement of CDD's as given earlier by equation (1.3) but with the reference level set at 18°C, i.e.:

$$CDD_n = \sum_{i} \max\{0, (T_i - 18^\circ)\}$$
 (5.8)

If we make the assumption that the average daily teperature will always be greater than or equal to the reference temperature (18 °C) then we can rewrite this relation as:

$$CDD_n = \sum_{i} T_i - 18n \tag{3.9}$$

Now we have removed the complication of the max() function and we are free to extend the assumption of normally distributed CDD's. The relationship in equation (3.9) above is a linear combination of a Gaussian process which itself will be Gaussian and a normal approximation can therefore be legitimised for relatively large values of n.

We can then calculate the moments of this via:

$$\mathbf{E}[CDD/F_{t}] = \mathbf{E}\left[\sum_{i=1}^{n} T_{i} - 18n\right]$$
$$= \sum_{i=1}^{n} \mathbf{E}[T_{i}] - 18n$$

and

$$\mathbf{V}[CDD \mid F_t] = \sum \mathbf{V}[T] + 2\sum \sum Cov[T_{i,T_j}]$$

The value of a call option over this CDD distribution can be given as:

$$c(t) = e^{-r(t_n - t)} \mathbf{E}[\max(CDD_n - K, 0) | F_t]$$
(3.10)

where K is the strike price of the call option and $t_n > t$. Proceeding under the assumption of normality, Alaton shows that:

$$c(t) = e^{-r(t_n - t)} \int_{K}^{\infty} (x - K) . f_{CDD_n}(x) . dx$$

which on evaluation of the integral gives:

$$c(t) = e^{-r(t_n - t)} [(\mu_n - K)\mathbf{\Phi}(-a_n) + \frac{\sigma_n}{\sqrt{2\pi}} e^{\frac{-a^2_n}{2}}]$$
(3.11)

Similarly for a put option over the CDD index we can write:

$$p(t) = e^{-r(t_n - t)} \mathbf{E}[\max(K - CDD_n, 0) | F_t]$$

$$p(t) = e^{-r(t_n - t)} \int_{0}^{K} (K - x) . f_{CDD_n}(x) . dx$$
(3.12)

which as before yields:

$$p(t) = e^{-r(t_n - t)} [(K - \mu_n)[\Phi(a_n) - \Phi(\frac{\mu_n}{\sigma_n})] + \frac{\sigma_n}{\sqrt{2\pi}} (e^{\frac{-a^2_n}{2}} - e^{\frac{-\mu_n^2}{2\sigma_n^2}})]$$
(3.13)

where Φ represents the cumulative (standard) normal distribution and r is the risk-free investment rate. This is a powerful approximate method for the pricing of weather based option contracts particularly in geographic areas where the effect of the max() function is minimal.

5.4.2 An Example

To highlight much of the discussion so far we will price a CDD option for the month of January in Sydney. The CDD option is chosen as it would be popular to power generating organisations seeking to smooth their returns due to the variability of electricity demand during summer months. We will use 3 methods to calculate the option price: Normal approximation, 'Burn' analysis and Mote Carlo simulations.

The specifics of the options to be priced are:

Period:	January

Measure:	Cumulative CDD
Exercise Prices:	170 / 180 / 190 / 200 CDD's
Tick::	\$100,000 /CDD
Location:	Sydney Airport (Kingsford Smith)

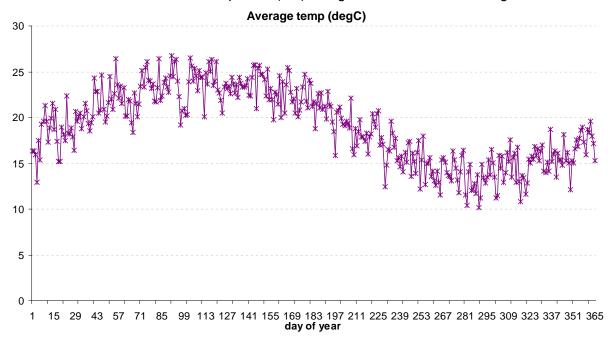
The pricing is undertaken for the three weather stations investigated throughout this section.

5.4.3 Burn Analysis

A burn analysis is undertaken using 80 years of data from the Sydney Airport weather station. To begin with, it is necessary to calculate the present value of the temperatures by adding to the raw temperatures the linear and quadratic trends that were calculated in section 3.2.1. As the linear trends will not affect the arithmetic average it is only necessary to inflate the daily average temperature rather than the raw maximum and minimum values. These present value temperatures are then used to calculate the expected payoff for this contract had it been purchased every January for the past 66 years.

5.4.4 Monte Carlo Simulation

The following chart shows a typical sample simulation for the month of January at Sydney Airport that is used to determine the expected payoff from the CDD option. The simulation was achieved via the use of equation (5.7) along with a random number generator.



6: Carbon Risk Management

6.1 Carbon Price Dynamics

The science of carbon price modelling is much less developed than the science surrounding the weather variables we have been discussing until now. The short period of time that liquid markets have existed coupled with the sometimes uneven flow of information make proper time-series analysis of the evolution of prices difficult.

6.1.1 What price Carbon?

There are many factors that influence and complicate the dynamics of carbon markets particularly due to the fact that the market is both immature (having its roots in the implementation of the EU ETS in 2005) and synthetic meaning that political decisions often have dramatic effects on the evolution of the spot price as well as the expectation of future prices.

From a theoretical point of view, the price of an emission certificate is equal the marginal cost of abatement. Fundamentally this means that if the price of an emission certificate is less than the marginal cost of abatement for a company then the company will be incentivised to purchase a certificate. If not then the company will be inclined to reduce their emissions in order to maximise their profits.

Under this philosophy the prevailing price of carbon will be influenced by the relative prices of various fuel types as coal, gas and electricity.



Figure 7.1: EUA and CER Historical Prices

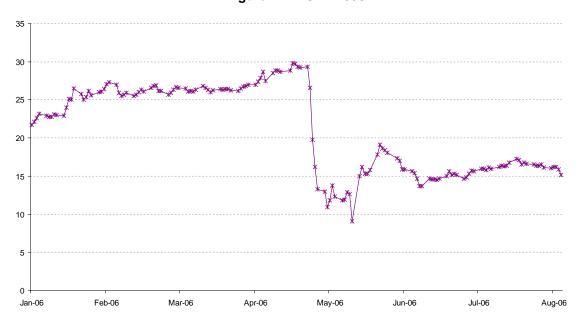


Figure 7.2: EUA - 2006

The sharp correction that occurred in the EUR price in April of 2006 was as a result of the release of market information that showed that the market was 'long' overall. Prior to this time there had been little information available indicating the open interest in the emission contracts traded during the 1st phase of the EU ETS implementation.

These informational inefficiencies are a feature that is common to many immature markets and pose significant issues when attempting to derive statistical models from the available data. Emission markets are particularly susceptible to these inefficiencies on account of the fact that the implementation of carbon markets worldwide is a deeply political issue with no single approach to implementation being adopted.

6.2 Modelling Carbon Prices

The general approach to pricing carbon derivative contracts is to model the futures price process and then apply the Black '76 approach, as outlined in section 4.4.2, to derive the fair value of a contingent claim on this futures price process. Whilst the derivation of this fair value is relatively straightforward the modelling of the emission futures price process poses significant challenges.

6.2.1 Modelling Futures Prices

The price for delivery of an emission certificate at some point of time in the future Δt is modelled via the familiar relationship:

$$F_{t} = S_{t} e^{(r-\mu).\Delta t}$$

Where μ represents the convenience yield. The convenience yield is commonly introduced into the pricing of commodities due to the fact that they are consumed and

that organisations that require these commodities pay a premium to have these inputs available.

Several authors (Benz, Benz and Truck [2007]) have suggested modelling the futures process via a GARCH(1,1) model in line with the seminal paper by Bollerslev [1986]. This requires that the variance is modelled as:

$$\sigma_n^2 = \alpha_0 \Sigma + \alpha_1 y_{n-1}^2 + \alpha_2 \sigma_{n-1}^2$$
 (6.1)

Where

$$\alpha_0 + \alpha_1 + \alpha_2 = 1$$

and Σ represents the long term average variance. Hull [2008] shows that for the process to remain stable and non-negative then:

$$\alpha_1 + \alpha_2 < 1$$

These parameters are then generally estimated via least squares from the data before the mean volatility process is calculated via equation 6.1 above.

6.2.2 Abatement Strategies

As defined earlier, the theoretical price of an emission certificate is equal to the marginal cost of abatement for a particular industrial process. Fehr and Hinz [2007] develop a theoretical model of emission certificates based on economic decision to change the fuel used as input into a commercial operation. They proceed to model the wealth of a particular trading strategy in the future, V_i , via the recursive relationship

$$V_{t+1} = V_t + \theta_t (A_{t+1} - A_t)$$

Where θ_t is the number of futures contracts traded at time t and A_t is the carbon futures process. When we consider the financial penalty (denoted by π) that exists for each ton of carbon equivalent that is not covered by valid certificates (denoted by Γ) for that particular application we get the terminal value of this process to be:

$$V_T = V_t - \theta_T A_T - \pi (\Gamma - \theta_T)$$

It is under this framework that the long-term abatement strategies are evaluated.

6.2.3 Pricing Carbon Contracts

As has previously been outlined, the pricing of contingent claims follows from the assumption that it is a fair predictor of the underlying spot prices and that the drift rate of the futures process is 0. With this assumption derivatives based on emission futures can be priced via the Black'76 model that is outlined in section 4.4.2.

6.3 Managing Carbon Risk

Carbon based financial contracts have been born out of the necessity of impending legislation that seeks to address to human contributions to climate change.

Once an ETS has been established, Australian corporations will not only have obligations under the legislation but will also have financial incentives to better manage their carbon emissions and their exposure to carbon-based risks. The significant volatility that has been experienced by the EUA and CER carbon prices highlights the need for risk management tools in order to smooth out the variability of organisations' profitability affected by the spot price of carbon.

6.3.1 Carbon Insurance

Carbon Insurance in Australia has been slow to develop primarily due to the as yet undetermined legislative situation. Other markets, such as those in Europe, have seen the development of a variety of new insurance products that seek to address the risks that corporations face under an ETS.

The price volatility outlined in section 7 represents only one of the risks associated with carbon products. Some of the other risks associated with the future procurement of carbon offsets and related products include:

- Political: Often the most significant source of risk depending on the host country of the carbon related project. Includes regulatory risk, war and civil unrest and Kyoto adoption:
- Credit: Credit worthiness of the provider/underwriter;
- Physical Delivery: Is the technology adequate? Will offsets be generated?
- Price: Uncertain price, which is also significantly correlated with the other risks mentioned previously.

Marsh estimates that the price discounting is in the order of 25%, reflecting the combined discount for all the risks associated with the future delivery of carbon offsets outlined above (*Insurance Instruments for GHG Projects*, April 2007)

Carbon Credit Delivery Insurance (CCDI) products were one of the first offerings to address the uncertainty around the future delivery of carbon offsets and are already on the market throughout Europe with companies such as AIG, Swiss Re, Zurich and Allianz offering some form of carbon capture or delivery insurance coverage. Many aspects are covered by extensions to already existing lines of business such as Political Risk Insurance (PRI) and traditional business interruption policies.

In the US, the American Clean Energy and Security Act 2009 (ACES) was passed by the House of Representatives on June 26. Importantly, the ACES mandates that carbon sequestration projects in the US must guarantee against risks facing the future delivery of carbon offsets. The legislation states that a mechanism must be provided to ensure that if a carbon credit is issued it results in a permanent carbon offset, listing insurance as an allowable mechanism.

6.3.2 Carbon Derivatives

Carbon-based derivative contracts are set to become one of the fastest growing derivative markets over the coming decade as more and more countries implement carbon pollution reduction schemes, of one form or another, with an ETS as a primary, underlying component. The evidence from Europe suggests that Australia will experience significant demand for carbon based derivative products to assist corporations in managing their carbon exposures and these products, in-themselves, will grow to form a desirable asset class based on their unique diversification benefits.

Carbon derivatives will be required to play an important role in the development of the global carbon market where by the variability in the spot carbon price can be hedged thus assisting the provision of capital to projects where the profit is generated from the sale of carbon credits at some point in the future, often many years after the provision of capital to the project.

6.3.3 The Clean Development Mechanism

The Clean Development Mechanism (CDM) is one of the cornerstones of the Kyoto protocol. This is the mechanism by which registered projects in developing nations, that reduce global emissions, can obtain 'carbon credits' that can be sold to Annex 1 nations (roughly corresponding to developed nations). The credits generated are referred to as Certified Emission Reduction units (CER's).

These projects are initially capital intensive and the profits generated from them via the issuing of certificates can be delayed by many years. This means that long term finance is generally required and that capital providers will seek to properly manage the risks associated with their investment.

The price risk that they are exposed to can effectively be managed through the use of carbon derivatives however, as section 6.3.1 suggests, there are a range of other risks that the capital provider can be exposed to that can currently only be covered by more traditional insurance covers.

7: Looking Forward

7.1 Weather Markets

The future for weather derivative markets appears positive after enduring significant turbulence during the aftermath of the global financial crisis. Volumes, whilst having reduced significantly from their boom in 2006, have now stabilised above the 2005 levels and thus continuing the upward trend experienced by the market since its inception in the second half of the 1990's.

The CAT bond market (and more generally the ILS market) is showing significant signs of recovery in the early part of 2010 after suffering due to the collapse of Lehman Brothers and the general undermining of the securitisation markets as a result of the global financial crisis. Once issues surrounding the proper collateralisation of liabilities associated with ILS's are addressed the securitisation market should be able to move forward with increased confidence in the risk management abilities of these contracts.

Going forward, the securitisation of insurance risks will continue to play an important role in providing alternative funding capabilities to global risk markets, increasing competition and thus reducing pricing spikes that are evident in many insurance and reinsurance markets.

7.2 Global Emission Legislation

The implementation of climate change legislation across the globe since the adoption of Kyoto has been sporadic with the EU clearly leading the world in establishing a functional framework following the principles laid out in the protocol. In Australia, the development of a sustainable emission market has stalled with the passage of legislation enabling an emission trading scheme now having been postponed beyond the end of the 2nd phase of the Kyoto protocol implementation period. This uncertainty will stunt the development of emission markets in the region and will leave the Australian voluntary market as the only market driven emission mechanism in the Pacific.

The end of the second compliance period is approaching us in 2012 and several countries, most notably the US, are proposing alternative frameworks with the capacity to extend an international agreement significantly beyond this date. The uncertainty that currently exists, even in well established markets as exist in the EU, hampers the development of the projects that are intending to put a price on carbon emissions that will promote the more efficient use of inputs that contribute to these emissions.

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