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Longevity Risk Management for Life and Variable Annuities: Effectiveness of Static Hedging using Longevity Bonds and Derivatives

Prepared by Andrew Ngai and Michael Sherris

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The Institute of Actuaries of Australia
Level 7 Challis House 4 Martin Place
Sydney NSW Australia 2000
Telephone: +61 2 9233 3466 Facsimile: +61 2 9233 3446
Email: actuaries@actuaries.asn.au Website: www.actuaries.asn.au

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Andrew Ngai
PricewaterhouseCoopers
Sydney, Australia
andrew.ngai@au.pwc.com

Michael Sherris
Australian School of Business
University of New South Wales
m.sherris@unsw.edu.au

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Abstract

For many years the longevity risk of individuals has been underestimated as survival probabilities improved across the developed world. The uncertainty and volatility of future longevity has posed significant risk issues for both individuals and product providers of annuities and pensions. This paper investigates the effectiveness of static hedging strategies for longevity risk management using longevity bonds and derivatives (q-forwards) for the retail products: life annuity, deferred life annuity, indexed life annuity and variable annuity with guaranteed lifetime benefits. Improved market and mortality models are developed for the underlying risks in annuities. The market model is a regime switching vector error correction model for GDP, inflation, interest rates and share prices. The mortality model is a discrete time logit model for mortality rates with age dependence. Models were estimated using Australian data. Basis risk between annuitant portfolios and population mortality was based on UK experience. Results show that static hedging using q-forwards or longevity bonds reduce longevity risk substantially for life annuities, but significantly less for deferred annuities. For inflation indexed annuities static hedging of longevity is less effective because of inflation risk. Variable annuities provide limited longevity protection compared to life annuities and indexed annuities, as a result longevity risk hedging adds little value for these products.

Keywords: longevity risk, risk based capital, static hedging, q-forwards, longevity bonds, life annuities, variable annuities

JEL Classifications: G22, C50

1 Introduction

Retirement systems around the world are increasingly relying on individuals to accumulate personal saving for retirement through defined contribution accumulation schemes. On retirement these accumulated funds will need to finance an increasing and uncertain lifetime. Figure 1 shows the rapid mortality improvements in the late 20th century in Australia. Government support through social security and aged pensions will be under pressure as more individuals will be expected to finance their longevity risk with retirement products that include longevity insurance.

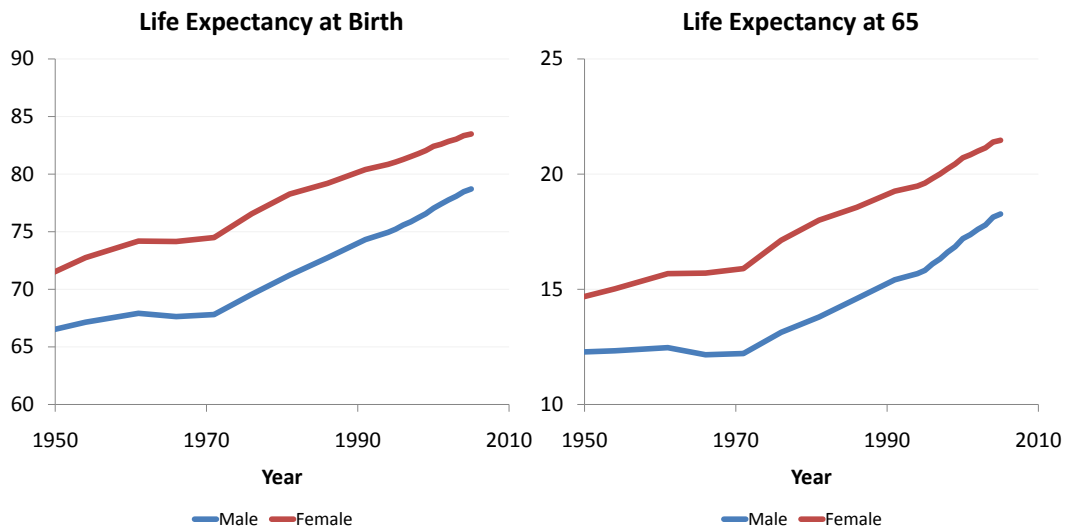


FIGURE 1: Australian Life Expectancies, 1950–2005 (Source: ABS)

The traditional longevity insurance products have included life annuities, indexed annuities and deferred annuities offered by life insurers. Annuity markets have not developed except in a few countries, typically where there has been an element of compulsion to annuitise retirement savings. A product innovation recently introduced into Australia, popular in the US, Japan, and Europe, is the variable annuity with a guaranteed lifetime withdrawal benefit (GLWB) rider. The GLWB is designed to cover insurance against longevity and market risks while allowing individuals more flexibility and liquidity as compared to life annuities (Ledlie et al, 2008 [33]). The recent global financial crisis has drawn attention to the risks for variable annuity writers resulting in insurers increasing fees, reducing benefits, or withdrawing from selling variable annuities altogether (Burns, 2009 [13]; Lankford, 2009 [32]).

The annuity markets in the US and UK are relatively well developed, with US annuity sales amounting to US\$255b in 2007 (Source: Morningstar Inc and LIMRA). Table 1 shows how the US fixed annuity market (including both term and life annuities) has been relatively stable while the variable annuity market has been growing rapidly until recently. Variable annuities have proved to be attractive, addressing consumer concerns over issues

Annuity Market	2001	2002	2003	2004	2005	2006	2007
Life & Term Annuities	74.3	103.3	89.4	87.9	79.5	78.3	72.8
Indexed Annuities	6.8	11.8	14.4	23.1	27.2	25.4	24.8
Variable Annuities	113.3	115.0	126.4	129.7	133.1	157.3	182.2

TABLE 1: US Annuity Sales (US\$b)
(Source: Morningstar, Inc. and LIMRA International)

Annuity Market	2001	2002	2003	2004	2005	2006	2007
Life Annuities	166	154	200	281	27	29	36
Term Annuities	794	1,096	1,357	2,758	548	530	787

TABLE 2: Australian Annuity Sales (A\$m)
(Source: Plan for Life Research)

such as bequests, early death, and income flexibility (HM Treasury, 2006 [26]). In the US the proportion of variable annuities sold with a GLWB has increased from 24% in 2004 to 43% in 2006, the highest election rate among all the variable annuity guarantees offered (Source: NAVA).

A range of annuities have been offered in Australia, including fixed term, fixed life, and inflation-indexed products. Table 2 shows recent sales. Annuity demand has been low and falling, with sales of A\$3.04b in 2004 decreasing to A\$0.82b by 2007. Most annuities sold in Australia are term certain annuities, with life annuities accounting for only 2% of annuity sales in 2006 (Source: IFSA). The decline in annuity demand is partly due to changes in taxation and regulation in 2004 and 2007, when the Australian Government reduced the assets test exemption for complying income streams and made superannuation benefits tax free for those aged 60 or over (Ganegoda, 2007 [22]). Both of these changes have decreased the attractiveness of life annuities, and the effect of the first change can be seen by the 90% fall in life annuity sales from 2004 to 2005.

A viable life annuity market requires that life insurers be able to effectively manage the risks of retail products without holding excessive levels of capital. Longevity risk can be managed using a variety of techniques including reinsurance and hedging in financial markets (Blake et al, 2006 [8]). As reinsurers have been unwilling to accept substantial amounts of longevity risk (Wadsworth, 2005 [46]), insurers are increasingly considering hedging and securitization. Financial market hedging requires mortality/longevity linked securities and derivatives. Pooling longevity risk and natural hedging are not effective for systematic longevity improvement. This impacts all ages and can only be hedged in financial markets or by reinsurers through diversification with relatively uncorrelated risks (Loeys et al, 2007 [36]).

Mortality-linked securities were first proposed by Blake and Burrows (2001) [7] who

proposed a survivor bond as a method of hedging longevity risk, and suggested that the government issues survivor bonds based on population mortality. Dowd (2003) [19] proposed other methods of hedging longevity risk, including survivor options, futures, and swaps. Blake et al (2006) [10] and Dowd et al (2006) [20] discuss longevity bonds and survivor swaps respectively, including construction and use in hedging longevity risk. Brown and Orszag (2006) [11] provide a detailed discussion of the advantages and disadvantages of government involvement in longevity bonds, such as the ability of the government to spread longevity risk across generations.

There have been innovations to transfer longevity risk to the capital markets including mortality-linked securities (MLS), longevity derivatives such as q-forwards (Coughlan et al, 2007 [16]) and longevity swaps. There is also a potential role for government to issue longevity bonds (Blake et al, 2009 [5]) to provide the hedging. At the same time regulatory capital requirements for European insurers under Solvency II and risk based capital requirements in other countries need to be allowed for in the successful provision of longevity insurance to individuals (Blake et al, 2009 [5]). Although securitization of extreme mortality risk has been successful, there has yet to be a successful securitization of longevity risk (Biffis and Blake, 2009 [4]). Lin and Cox (2005) [35] proposed a longevity bond with payments based on the insurer's loss experience eliminating basis risk while Wills and Sherris (2010) [49] analysed tranching in a longevity bond structure showing how age dependence is a critical factor in the design and pricing of a securitization.

In 2007 JPMorgan launched the LifeMetrics mortality index, initially based on US and UK data (Blake et al, 2008 [9]). By disclosing the data and the methodology, LifeMetrics provided a transparent mortality index on which survivor derivatives can be traded. In 2008, the first transaction based on this index took place, a q-forward transaction between JPMorgan and Lucida, a UK buyout firm. The first longevity swap in the capital markets soon followed, with JPMorgan facilitating the swap between Canada Life and a group of investors including ILS and hedge funds (Blake et al, 2009 [6]). Several further transactions between life insurers and investors have since occurred (Ribeiro and di Pietro, 2009 [42]).

There is relatively little analysis of annuities and longevity risk from an insurer's perspective. Bauer and Weber (2008) [2] assess the risk of annuities for a single premium immediate life annuity using relatively simple stochastic models to describe the market and mortality processes. Dowd et al (2006) [21] analyse the risks of various mortality-dependent positions, including an annuity book with a longevity bond hedge. The longevity bond involves coupons based on survival probabilities, and is structured to match a typical life annuity book for a given cohort. A range of risk measures such as expected shortfall and spectral measures are used to quantify the risk in the portfolios. More recently, Plat (2009) [39] assesses longevity risk and q-forward hedge effectiveness for a sample insurance portfolio with a stochastic factor model to describe portfolio specific mortality in relation to population mortality.

Most techniques available to hedge longevity risk involve illiquid or developing markets.

It is not practical to dynamically hedge longevity risk. Against this background it is important to assess the effectiveness of static hedging strategies using currently available and proposed hedging instruments for retail longevity risk management products including life annuities and variable annuities. In this paper market and mortality models are applied to assess the static hedging of mortality risk in an Australian context. A range of hedging strategies for annuity portfolios is assessed extending Dowd et al (2006) [21] and other studies. Techniques for hedging longevity risk must be assessed in a rigorous framework to support the development of retail annuity markets. The study focusses on longevity risk since there are well established markets for equity and interest rate risk. In Australia there is a limited market for inflation hedging. The effect of not being able to hedge inflation guarantees in annuity products is shown to be significant.

This paper is structured as follows. The next section provides a description of life annuities and hedging strategies using mortality-linked securities to manage longevity risk for insurers. Following that the models developed for market and longevity risk management are outlined. The results of the analysis of longevity risk for a different portfolios of annuities are then presented. The final section discusses the results and summarizes the conclusions.

2 Life Annuities and Hedging Strategies

The analysis considers the range of potential retail products suited for longevity risk for individuals: life annuity, inflation indexed life annuity, deferred annuity, inflation indexed deferred annuity and variable annuity with GLWB. A portfolio of policies is considered consisting of 1,000 male policies of initial age 65, reflecting the current retirement age in Australia. The methodology used is based on Bauer and Weber (2008) [2] and Dowd et al (2006) [21]. This analysis considers only a cohort aged 65 to make the comparison simpler. The terminal age assumed is $\omega = 111$, so that the time horizon for the simulations is $T = 46$. For immediate and variable annuities, the initial payment is assumed to be \$100,000. For deferred annuities a lower initial investment of \$10,000 is assumed, to ensure a comparable level of longevity risk at the older ages. For the variable annuity, the initial lump-sum payment is invested in a portfolio of a money market account and equity index, with the individual assuming all the market risk. A GLWB rider to cover longevity risk is included. Each retail product is priced as realistically as practical reflecting available data on market prices.

To assess the longevity risk and effectiveness of static hedging strategies, the insurer's final surplus after all policyholders have died is simulated using market and longevity models calibrated to Australian data. Claims, and regular GLWB charges for the variable annuity, depend on mortality and market outcomes for inflation indexed and variable annuities. Static hedging strategies are designed to match the expected liabilities. These strategies include fixed interest (zero coupon) bonds, longevity bonds, and q-forwards. The insurer is assumed to be able to invest in risk-free bonds of all maturities, a money

market account and an equity portfolio. After receipt of the initial premium and payments for the static hedge for the liabilities any remaining initial premium is invested in a T -year risk-free bond. In each future year the surplus is invested in a money market account which accumulates at the risk-free rate. The insurer is assumed to borrow against this money market account to meet negative net cash flows. Investment of surplus in the money market account ensures consistency. Surplus at time t , U_t , is the accumulated net cash flows from premiums, claims outgoings, and net cash flows from assets purchased in the hedging strategy. The surplus, accumulating at a rate of return R_t^U , is given by

$$U_t = U_{t-1} (1 + R_t^U) + P_t - C_t + H_t \quad (1)$$

where

$$\begin{aligned} U_t &= \text{insurer's surplus at time } t \\ P_t &= \text{total premiums (fees) at time } t \\ C_t &= \text{total claims outgoing at time } t \\ H_t &= \text{net hedging cash flows at time } t \end{aligned}$$

Expenses are not included to ensure consistency between the products.

2.1 Annuity Cash Flows

For life and indexed annuities there is a single upfront premium P_0 paid at time 0 and the regular annual payments to survivors are the policy claims. Market prices were used for the immediate annuities based on Ganegoda and Bateman (2008) [23]. Deferred annuities are priced to ensure consistency with the money's worth ratio for immediate annuities. The deferred annuity prices were determined using:

$${}_{20|}a_{65}^* = \left(\frac{a_{65}^*}{a_{65}} \right) {}_{20|}a_{65} \quad (2)$$

where a^* is a deferred annuity factor implied by market prices, and a is the corresponding actuarial annuity factor determined using current life table and yield curve data. For the indexed annuity the claim payments allow for inflation by adjusting for the inflation index.

2.2 Variable Annuity Cash Flows

Cash flows, including withdrawals and fees, for the variable annuity with the GLWB are based on US products and pricing analyses for variable annuity guarantees (Bauer et al, 2006 [1]; Holz et al, 2007 [27] and Hill et al (2008) [25]). Claims for the GLWB are withdrawals in excess of the policy value. Premiums (fees) for the GLWB are charged on

a withdrawal base and deducted from the policy value. Lapse rates are included and all individuals are assumed to make the same investment and withdrawal decisions so that at each policy anniversary either the guaranteed amount is withdrawn or the entire contract is surrendered. The investment portfolio has a proportion α of the equity index and $1 - \alpha$ of the money market account with α equal to 75%, similar to the asset allocation observed in the US market.

Withdrawals are assumed to be a fixed percentage of the withdrawal base equal to 5% at age 65. The withdrawal base is assumed to automatically step up to the policy value so that it is the maximum of all previously attained policy values. The fee charged is a fixed percentage of 40bps of the withdrawal base.

Lapse rates consist of a base lapse rate (for the underlying variable annuity) multiplied by a dynamic adjustment based on in-the-moneyness (ITM) (Marcsik and Wion, 2008 [37]). Lapse rates are chosen conservatively, with the base rate a constant 5% and dynamic adjustment a step function which decreases with ITM. These rates are lower than those generally assumed in the US. The assumed lapse rate is shown in Figure 2.

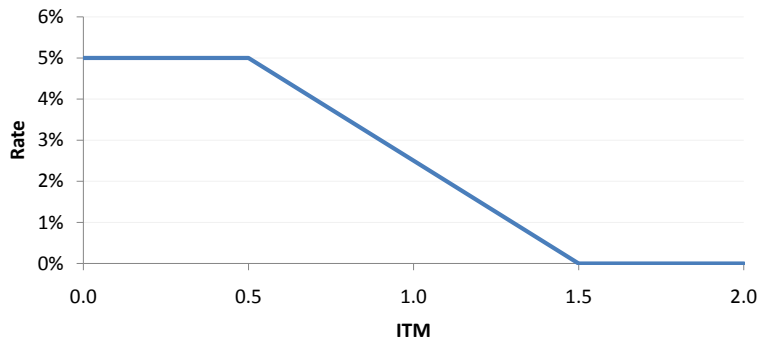


FIGURE 2: Assumed Lapse Rate Function

2.3 Hedging Instruments

The hedging instruments available are zero coupon bonds, q-forwards, and longevity bonds. q-Forwards are issued in the OTC market and customized to the insurer cash flows. At present there are no longevity bonds available to hedge annuity cash flows in Australia. It has been proposed that governments issue such bonds to promote the development of an annuity market. Interest rate risk is hedged using risk-free zero coupon matching bonds, or an equivalent matching strategy, even though the longest maturity Australian government bond has a term of 15 years, significantly less than a life insurer's longer duration liabilities and cash flows. A zero coupon bond of maturity τ pays a cash flow of \$1 at time τ , and involves no longevity risk.

2.3.1 q-Forwards

q-Forwards are forward contracts which pay an actual (floating) mortality rate $q_{x,t}$ in exchange for a fixed (forward) mortality rate $q_{x,t}^F$. The cash flows involved in a q-forward of maturity τ are shown in Figure 3.



FIGURE 3: Structure of a τ -year q-forward

q-Forwards are used to construct a longevity swap for the insurer's expected claims according to a standardised population index. Because the insurer's mortality experience will be based on annuitant mortality there will be basis risk between the hedged standardised index based on population mortality and the insurer mortality.

2.3.2 q-Forwards (Bucketed)

In practice q-forwards are often bucketed by 5 year age groups (65–69, ..., 105–109) to improve liquidity by reducing the number of contracts. The bucketed q-forwards are based on average mortality rates $q_{x,t}^B$ across the 5 year groups. The average is taken over the mortality rates for a cohort table.

2.3.3 Longevity Bonds

Payments on longevity bonds are linked to the survival experience of a cohort and based on population mortality (Dowd et al, 2006 [21]; Blake et al, 2006 [10]). Deferred longevity bonds have also been proposed (Blake et al (2006, 2009) [5, 8]). Zero coupon longevity bonds pay a single amount in proportion to the actual survival rate $S_x(\tau)$ at maturity τ . The cash flows of a zero coupon longevity bond are shown in Figure 4.

In order to assess the effectiveness of longevity bonds for hedging longevity risk, maturities of 20 and 40 years are used since these are approximately the average and maximum term of life annuities for lives aged 65. Payments are based on the actual population survival rate $S_x(t)$ at each time t . The 20-year bond is an immediate bond, whereas the 40-year bond is deferred for 20 years. These two bonds can be used to create an immediate 40-year bond by purchasing both the 20-year immediate and 40-year deferred bonds. The cash flows involved in the longevity bonds are illustrated in Figures 5 and 6.

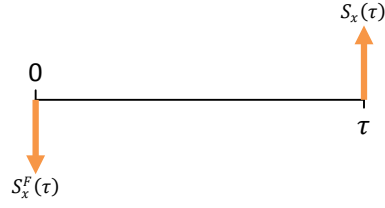


FIGURE 4: Structure of a τ -year Zero Coupon Longevity Bond

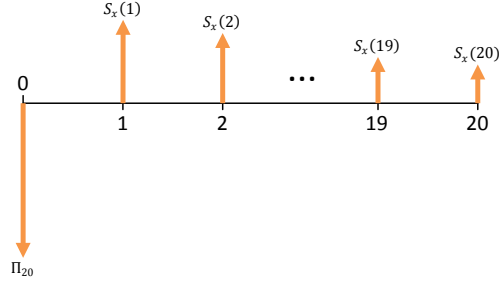


FIGURE 5: Structure of a 20-year (Immediate) Longevity Bond

Longevity bonds are equivalent to a portfolio of zero coupon longevity bonds. A zero coupon longevity bond has payoff similar to a zero coupon bond plus a portfolio of q-forwards since all mortality rates $q_{x+t,t}$ for $t < \tau$ affect the survival probability $S_x(\tau)$, and therefore the hedge for claims at time τ involves using q-forwards with payout at all times $t < \tau$.

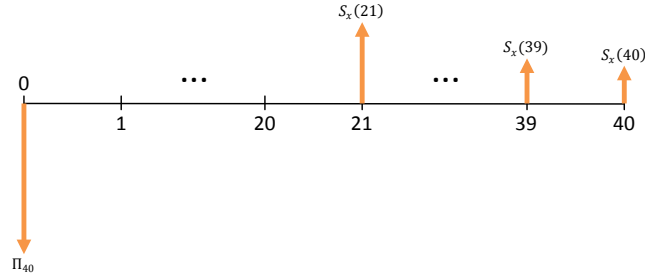


FIGURE 6: Structure of a 40-year (Deferred) Longevity Bond

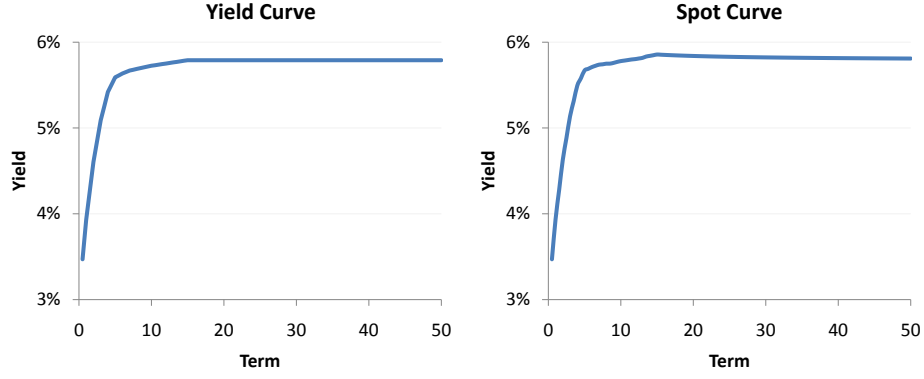


FIGURE 7: Assumed Yield and Spot Curves

2.4 Pricing of Hedging Instruments

Zero coupon bonds are priced using market yield data from the Reserve Bank of Australia shown in Figure 7.

The q-forward prices are determined assuming a Sharpe ratio as in Loeys et al (2007) [36]:

$$q_{x,t}^F = (1 - S_q \sigma_x t) q_{x,t}^E \quad (3)$$

where S_q is the Sharpe ratio for the q-forward, $q_{x,t}^E$ is the expected mortality rate under the real-world P measure, and σ_x is the historical (percentage) volatility of the mortality rates:

$$q_{x,t}^E = E_P(q_{x,t} | \mathcal{F}_0) \quad (4)$$

$$\sigma_x^2 = \text{Var} \left(\frac{\Delta q_{x,t}}{q_{x,t}} \right) \quad (5)$$

An estimate $\hat{\sigma}_x$ was obtained from historical data using the sample variance for graduated (smoothed) mortality rates to remove random noise. The required Sharpe ratio was chosen as 0.20, which is the implied Sharpe ratio from UK annuity markets (Loeys et al (2007) [36]). Longevity bonds are similarly priced.

2.5 Hedging Strategies

Static hedges are considered since these are the most effective for blocks of annuity business. At present, dynamic longevity hedges are difficult to implement due to illiquidity and high transaction costs. Dynamic hedges may be more effective in future as the market for mortality-linked securities develops. The effectiveness of static hedging strategies are assessed by comparing the surplus distributions along with expected shortfall risk measures. The case where there is "no longevity hedging" is used as a base case along

with the case where mortality is deterministic, the "no longevity risk" case. For the no longevity hedging case an asset liability matching strategy is implemented using zero coupon bonds and expected mortality rates for cash flows. This strategy is considered with and without longevity risk to provide benchmarks against which the longevity hedging strategies can be evaluated.

Hedging strategies assessed are:

- q-Forwards
- q-Forwards (Bucketed)
- 20yr Longevity Bond
- 40yr Longevity Bond
- Both Longevity Bonds
- Longevity Swap

The q-forwards are used along with a portfolio of zero coupon bonds to match expected claims under the fixed mortality index. The difference between expected claims and the actual claims is hedged using the q-forwards. The survival probability has to be expressed in terms of differences between actual and forward mortality rates so that the hedge can be constructed. Following Cairns et al (2008) [15], a first order approximation is used:

$$\begin{aligned}
S_x^A(t) &= \prod_{\tau=1}^t (1 - q_{x+\tau-1}^A) \\
&\approx \prod_{\tau=1}^t (1 - \hat{\rho}_{x+\tau-1} q_{x+\tau-1}) \\
&= \prod_{\tau=1}^t (1 - \hat{\rho}_{x+\tau-1} q_{x+\tau-1}^F + \hat{\rho}_{x+\tau-1} (q_{x+\tau-1}^F - q_{x+\tau-1})) \\
&\approx S_x^{FA}(t) \left(1 + \sum_{\tau=1}^t \frac{\hat{\rho}_{x+\tau-1} (q_{x+\tau-1}^F - q_{x+\tau-1})}{1 - \hat{\rho}_{x+\tau-1} q_{x+\tau-1}^F} \right) \tag{6}
\end{aligned}$$

where $(\cdot)^A$ denotes an annuitant rate, $S_x^{FA}(t) = \prod_{\tau=1}^t (1 - \hat{\rho}_{x+\tau-1} q_{x+\tau-1}^F)$ is the fixed (forward) survival probability corresponding to $S_x^A(t)$, and $\hat{\rho}_{x,t}$ is the ratio between annuitant and population mortality rates $q_{x,t}^A$ and $q_{x,t}$.

q-Forwards bucketed by age are used in an identical manner as individual-age q-forwards with the cash flows based on average mortality rates across the 5 year groups.

Longevity bonds are used in place of zero coupon bonds to better match the liabilities of the insurer. Three types of longevity bond strategies are included: purchasing a 20-year

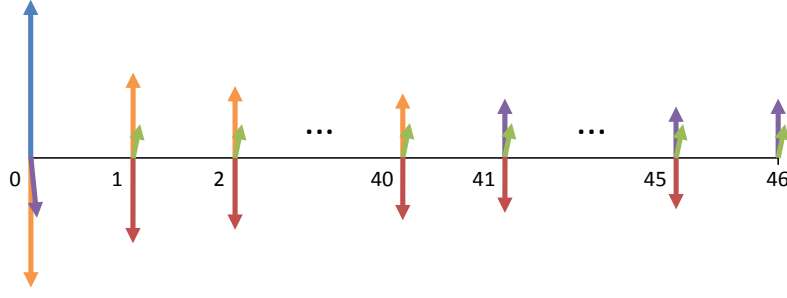


FIGURE 8: Cash Flows for Immediate Life Annuity with Both Longevity Bonds
 (Blue = Premiums, Red = Claims, Green = Investment Returns,
 Purple = Zero Coupon Bonds, Orange = Longevity Bonds)

bond, a (deferred) 40-year bond, and both. The hedging strategies involve purchasing one or both longevity bonds to match the portfolio cash flows. The cash flows from the longevity bond are proportional to population survival probabilities $S_x(t)$, whereas the insurer's claims depend on annuitant mortality $S_x^A(t)$. For indexed annuities, the insurer's claims increase with inflation whereas the longevity bond payouts will be based on fixed amounts adjusted for the survival probability. Zero coupon bonds are included to allow for expected inflation and the difference in annuitant and population mortality. An illustration of the cash flows for an immediate life annuity portfolio involving the purchase of both longevity bonds is shown in Figure 8. Only the 40yr longevity bond was included for deferred annuities and the GLWB.

The claims from a GLWB are the most challenging to construct an effective static hedge, as the claim amounts and timing both depend on volatile stock markets. In an average or strong market, there are very little longevity claims as the policy value generally does not deplete before the policyholder dies. However, in an adverse market, there may be a very large number of claims as all policyholders will have depleted their policy value early in their retirement. The GLWB can be viewed as a deferred annuity with a stochastic (market-dependent) payment amount and deferral period. Once the policy value is depleted, the product becomes a life annuity as there will be no further step-ups and lapses. The GLWB is hedged in a similar manner as a deferred life annuity by purchasing a 40-year deferred longevity bond.

The longevity swap is constructed using zero coupon longevity bonds. The longevity bonds are based on population survival probabilities $S_x(t)$, and zero coupon bonds are included to cover the difference in expected annuitant and population mortality. The cash flows for a portfolio of life or indexed annuities with a longevity swap constructed from zero coupon longevity bonds are illustrated in Figure 9.

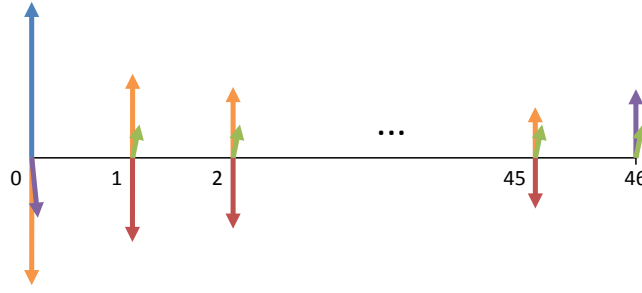


FIGURE 9: Cash Flows for Immediate Life/Indexed Annuity with Longevity Swap
(Blue = Premiums, Red = Claims, Green = Investment Returns,
Purple = Zero Coupon Bonds, Orange = Longevity Bonds)

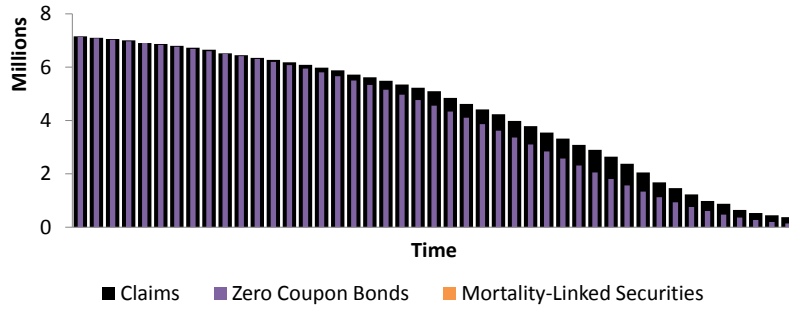


FIGURE 10: Sample Cash Flows for Life Annuity with No Longevity Hedging (ZCBs)

2.6 Comparison of Hedging Strategies

A comparison of the cash flows in a portfolio of immediate life annuities arising from the hedging strategies is shown for a market/mortality simulation in Figures 10 to 14. Initial and final cash flows (at times 0 and T) are not shown. In the sample shown, mortality improves gradually, eventually reaching an improvement of approximately 25% over the projected rates.

The final surpluses for each strategy in this sample path are summarized in Table 3. The final surplus increases from the mortality hedge and decreases due to the cost of hedging.

No L Hedging	No L Risk	q-Forwards	L Bond (20y)	L Bond (40y)	L Swap
94.69	139.37	118.58	108.28	109.23	122.91

TABLE 3: Sample of Simulated Final Surpluses for Life Annuity

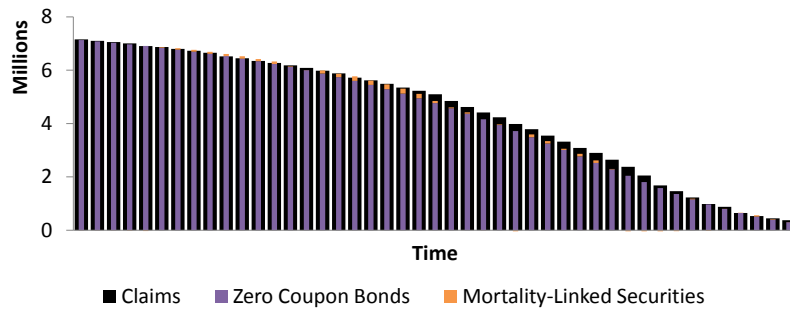


FIGURE 11: Sample Cash Flows for Life Annuity with q-Forwards

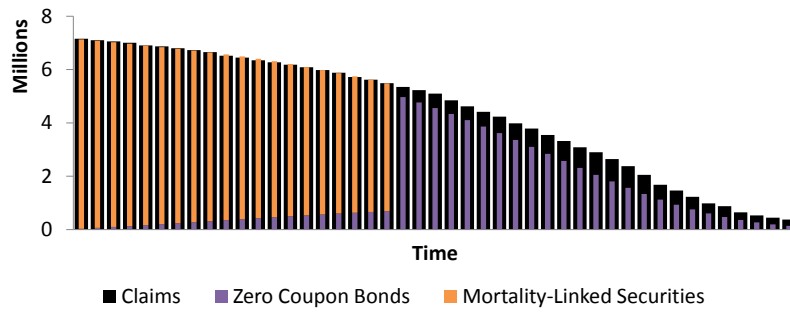


FIGURE 12: Sample Cash Flows for Life Annuity with a 20yr Longevity Bond

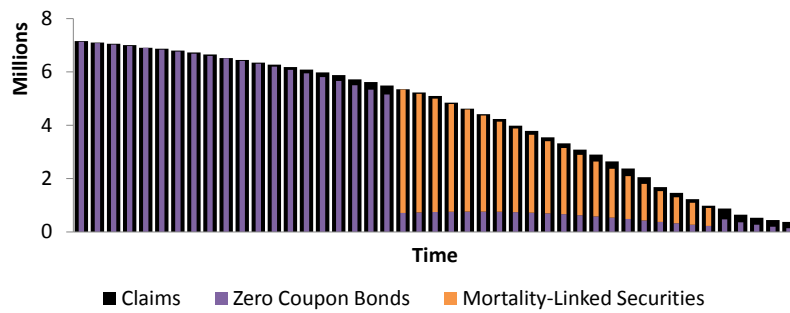


FIGURE 13: Sample Cash Flows for Life Annuity with a 40yr Longevity Bond

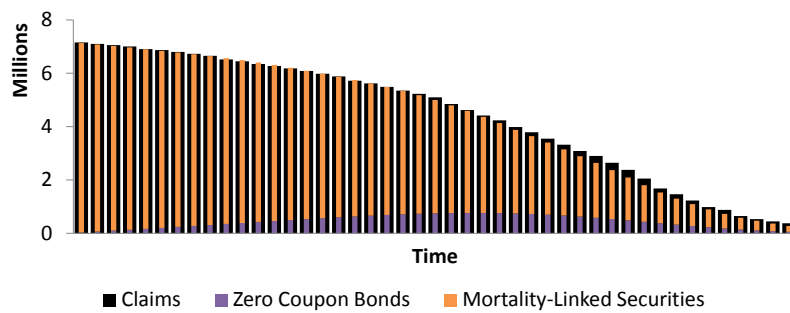


FIGURE 14: Sample Cash Flows for Life Annuity with a Longevity Swap (ZCLBs)

2.7 Risk Measures

The hedging strategies are compared using the distribution of the final surplus U_T for 100,000 simulations. The expected shortfall (ES) is used since it includes both the probability as well as the severity of adverse losses in the surplus distribution. The α VaR for a random variable X is the α quantile of the distribution:

$$\text{VaR}_\alpha(X) = \inf \{X : F_X(\text{VaR}_\alpha) \geq \alpha\} \quad (7)$$

The corresponding expected shortfall is given by:

$$\text{ES}_\alpha(X) = E(X|X < \text{VaR}_\alpha) \quad (8)$$

2.8 Scenario Analysis

The hedge strategies were assessed for a range of scenarios, including stress tests. The scenarios used were:

Scenario	Market Assumption	Mortality Assumption
1	Stochastic	Stochastic
2	Average	Stochastic
3	Adverse	Stochastic
4	Stochastic	Average + Excess Imp. (2%/yr Accumulating)
5	Stochastic	Average + Excess Imp. (25% Flat)

Scenario 1 simulates paths using the stochastic market and mortality models. Scenario 2 uses a deterministic market (average case from simulated paths) but with stochastic mortality. Scenario 3 uses a deterministic market based on the historical period of 1930–1975, beginning just prior to the Great Depression. This scenario is primarily for the purpose of assessing the GLWB, which was found to insure mainly against (extreme) market risk. Scenario 4 stress tests the longevity hedging strategies by applying a deterministic mortality but with 2% annual excess improvement above the projected rates. This reflects future mortality trends, where the effects of health improvements such as a cure for cancer, gradually impact on the mortality rates beyond the expected level. Finally, Scenario 5 stress tests the strategies with a 25% flat shock across mortality rates of all ages and times. This is the scenario that is proposed in determining the longevity risk capital charge under Solvency II.

3 Models for Market and Longevity Risk

3.1 Market Model

The market model used is a cointegrating vector error correction model with regime switching (RS-VECM). This allows for both short-term fluctuations, through the autoregressive structure, and long-run equilibria using a cointegration relationship (Johansen, 1995 [29]), along with heteroskedasticity through the use of regime switching. The RS-VECM was found to have the best fit to Australian data under various model selection criteria. Harris (1997) [24] fits a regime switching VAR model using Australian data to four key economic variables, showing that regime switching is a significant improvement over ARCH and GARCH processes in accounting for volatility. Sherris and Zhang (2009) [44] also fit a VAR model to Australian data with simultaneous regime switching across many economic and financial series.

A general RS-VECM with lag of p is expressed as:

$$\Delta \mathbf{y}_t = \boldsymbol{\mu} + \sum_{i=1}^{p-1} \mathbf{A}_i \Delta \mathbf{y}_{t-i} + \mathbf{B} \mathbf{C} \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t(\omega_t) \quad (9)$$

where \mathbf{y}_t is a d -dimensional vector of observations, $\Delta \mathbf{y}_t = \mathbf{y}_t - \mathbf{y}_{t-1}$ is the first differenced series, $\boldsymbol{\mu}$ is the mean vector, \mathbf{A} is a $d \times d$ parameter matrix of coefficients, \mathbf{B} and \mathbf{C}' are $d \times r$ matrices of rank r describing the cointegration (equilibrium) relationship between the variables, and $\boldsymbol{\varepsilon}_t(\omega_t)$ is a vector of regime-dependent multivariate normal random errors with covariances $\boldsymbol{\Sigma}_{\omega_t}$:

$$\boldsymbol{\varepsilon}_t(\omega_t) \sim \mathcal{N}_d(\mathbf{0}, \boldsymbol{\Sigma}_{\omega_t}) \quad (10)$$

Two regimes are used in the model, representing a normal state and high-volatility state respectively:

$$\omega_t = \begin{cases} 0 & \text{normal regime} \\ 1 & \text{high-volatility regime} \end{cases} \quad (11)$$

The probabilities of switching between regimes are described in a Markov chain with transition matrix \mathbf{P} :

$$\mathbf{P} = \begin{pmatrix} p_0 & 1 - p_0 \\ 1 - p_1 & p_1 \end{pmatrix} \quad (12)$$

where the probabilities p_0 and p_1 are constant over time.

The model was fitted using a two-stage procedure detailed by Krolzig (1997) [30]. The first step in fitting a RS-VECM is to determine the cointegration rank r and matrix \mathbf{C} using the Johansen (1988, 1995) [28, 29] methodology. The second stage of the estimation procedure involves the use of an Expectation-Maximisation algorithm to estimate the

remaining parameters. The cointegration analysis and estimation was performed in R using the packages ‘urca’ and ‘vars’ written by Pfaff (2008) [38]. The EM algorithm was performed using code written in R, and was checked by replicating the results in Section 9.3 of Krolzig (1998) [31] which involves an identical EM algorithm procedure (except with 3 regimes).

A lag of $p = 2$ was used based on model selection criteria, AICc and SBC, and for parsimony. The AICc is a corrected version of the AIC which is appropriate for small samples as it avoids selecting models with excessive parameters and ensures parsimony (Burnham and Anderson, 2004 [12]). For a multivariate model of dimension d , the model selection criteria are given by:

$$\text{AICc} = \ln L - k \left(\frac{n}{n - (j + 1 + d)} \right) \quad (13)$$

$$\text{SBC} = \ln L - k \left(\frac{1}{2} \ln n \right) \quad (14)$$

where L is the log-likelihood, k is the number of parameters, n is the number of observations (time points), and j is the number of structural parameters (per dimension). For comparison with the log-likelihood, the criteria have been defined so that a higher value is preferred, and can be interpreted as a goodness of fit measure (log-likelihood) with a penalty for complexity.

The financial and economic series included in the model are:

$\ln G_t$ = Log Gross Domestic Product (GDP)

$\ln B_t$ = Log Bond Index (Accumulated 90-day Bank-Accepted-Bill Yields)

$\ln S_t$ = Log Stock Price Index (ASX All Ordinaries)

$\ln F_t$ = Log Inflation Index (CPI)

Each market variable is a component of the time series vector \mathbf{y}_t .

The bank bill yield is representative of the risk-free interest rate, and is used in place of a short-term Treasury bill yield due to data availability. The stock index and interest rates are required for the investments and claims, and inflation is required for inflation-linked cash flows of the indexed annuities. GDP is an important macroeconomic variable which interacts with the other market variables.

Market data was obtained from the Reserve Bank of Australia (RBA). The data consists of observations over the period 1970 to 2009, the maximum period over which data was available for all 4 variables. Quarterly data was used as GDP and CPI observations are not available at a higher frequency. The bond index was obtained by accumulating the 90-day bank-accepted-bill yield. A statistical summary of the data (log quarterly returns) is shown in Table 4. Plots of the data are shown in Figure 15

Unit root tests indicated that all the series were $I(1)$ with the exception of $\ln B_t$, which is

Statistic	$\Delta \ln G_t$	$\Delta \ln B_t$	$\Delta \ln S_t$	$\Delta \ln F_t$
Mean	0.0078	0.0227	0.0137	0.0146
Standard Deviation	0.0101	0.0101	0.1010	0.0115
Skewness	-0.2010	0.7783	-1.8734	0.9470
Excess Kurtosis	1.4224	-0.5803	7.6267	1.1543
1st Quartile	0.0021	0.0140	-0.0262	0.0060
Median	0.0082	0.0191	0.0258	0.0125
3rd Quartile	0.0129	0.0296	0.0692	0.0216
Minimum	-0.0299	0.0107	-0.5719	-0.0046
Maximum	0.0367	0.0483	0.2298	0.0566

TABLE 4: Summary Statistics for Market Log Quarterly Returns

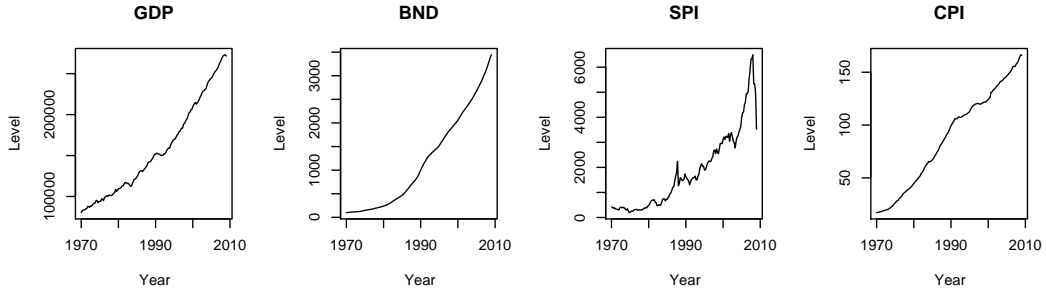


FIGURE 15: Observed Paths of the Market Variables (1970–2009)

$I(2)$. However a model based on the levels $\ln B_t$ for simulated scenarios was found to be more reasonable and reflective of history. The model based on $\Delta \ln B_t$ simulated a large number of negative returns due to the downward trend in interest rates over the period 1970–2009 which also causes the model to fit a downward trend, resulting in negative future simulated returns.

The cointegration matrix \mathbf{C} was estimated as:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & -0.272 & -0.274 \\ 0 & 1 & -0.463 & -0.921 \end{pmatrix}$$

The long run equilibrium is then given by $\mathbf{C}\mathbf{y}_t = \mathbf{0}$, which can be written as:

$$\ln G_t = 0.272 \ln S_t + 0.274 \ln F_t \quad (15)$$

$$\ln B_t = 0.463 \ln S_t + 0.921 \ln F_t \quad (16)$$

Production (G_t) is positively related to equities (S_t), which represent the value of real assets in firms, and inflation (F_t) which increases the dollar value of production. Nominal

returns should reflect productivity and price inflation as shown by the second cointegration equation, where the bond index B_t , comprised of accumulated nominal returns, is positively related to equities S_t , the value of real assets, and the price level F_t .

The estimated parameters for the market model were:

$$\begin{aligned}\boldsymbol{\mu} &= \begin{pmatrix} 0.180 \\ -0.034 \\ -0.465 \\ -0.322 \end{pmatrix} \\ \mathbf{A} &= \begin{pmatrix} -0.104 & -0.499 & 0.015 & 0.079 \\ 0.017 & 0.809 & 0.003 & 0.047 \\ 0.759 & 1.722 & 0.045 & -0.523 \\ 0.016 & 0.258 & -0.007 & 0.222 \end{pmatrix} \\ \mathbf{B} &= \begin{pmatrix} -0.020 & -0.023 \\ 0.004 & -0.009 \\ 0.062 & 0.162 \\ 0.036 & -0.015 \end{pmatrix} \\ \boldsymbol{\Sigma}_0 &= 10^{-3} \begin{pmatrix} 0.262 & 0.009 & 0.324 & -0.009 \\ 0.009 & 0.005 & -0.017 & 0.002 \\ 0.324 & -0.017 & 27.796 & -0.264 \\ -0.009 & 0.002 & -0.264 & 0.132 \end{pmatrix} \\ \boldsymbol{\Sigma}_1 &= 10^{-3} \begin{pmatrix} 1.436 & -0.087 & 1.404 & 0.041 \\ -0.087 & 0.183 & -0.405 & 0.067 \\ 1.404 & -0.405 & 157.706 & -1.110 \\ 0.041 & 0.067 & -1.110 & 0.922 \end{pmatrix} \\ \mathbf{P} &= \begin{pmatrix} 0.931 & 0.069 \\ 0.054 & 0.946 \end{pmatrix}\end{aligned}$$

There is strong autoregressive dependence of ΔS_t on ΔG_{t-1} with $A_{31} = 0.759$ so that high growth and productivity is associated with a stronger stock market. Also, the strong negative dependence of ΔG_t on ΔB_{t-1} with $A_{12} = -0.499$ reflects a contractionary effect of rising interest rates. The strong dependence of ΔS_t on changes in the other market variables as opposed to ΔS_{t-1} shows that the stock market is strongly influenced by macroeconomic factors in the short run.

The volatilities in $\boldsymbol{\Sigma}_1$ are also significantly larger than in $\boldsymbol{\Sigma}_0$, with the variances (diagonal terms) in $\boldsymbol{\Sigma}_1$ being at least 5 times greater than those in $\boldsymbol{\Sigma}_0$, showing that regime 1 represents a high-volatility state.

In order to confirm the model selection for the market simulations a number of alternative models specifications were compared. These included Vector Autoregressive Model (VAR), Vector Error Correction Model (VECM), Regime-Switching Vector Autoregressive

Model Type	p	r	k	RS Terms	$\ln L$	AICc	SBC
VAR	1	-	30	-	1815.92	1780.15	1740.27
VAR	2	-	46	-	1838.19	1775.50	1722.34
VECM	2	1	37	-	1841.95	1795.32	1748.64
VECM	2	2	42	-	1856.19	1801.02	1750.28
VECM	2	3	45	-	1863.43	1802.78	1749.95
VECM	3	2	58	-	1868.66	1780.22	1722.58
RS-VAR	1	-	42	Σ	1928.87	1878.80	1822.96
RS-VAR	2	-	58	Σ	1951.82	1872.77	1805.75
RS-VECM	2	1	49	Σ	1959.65	1897.90	1836.08
RS-VECM	2	2	54	Σ	1977.98	1907.05	1841.81
RS-VECM	2	3	57	Σ	1983.17	1906.34	1839.43
RS-VECM	2	2	58	μ, Σ	1982.31	1902.04	1836.05
RS-VECM	2	2	70	A, Σ	1986.75	1878.25	1810.23

TABLE 5: Model Selection Criteria for Market Models

Model (RS-VAR), and Regime-Switching Vector Error Correction Model (RS-VECM). Table 5, shows the model comparisons where p denotes the autoregressive order, r denotes the cointegration rank, and k denotes the number of parameters. Based on the model selection criteria, the RS-VECM with a switching covariance matrix and a cointegration rank of 2 was confirmed as the best model. Autoregressive orders larger than 2 were found to be too complex, as shown by the significant reduction in the criteria for the VECMs when moving from $p = 2$ to $p = 3$. The use of a cointegrating relationship was found to improve the fit as all the VECMs and RS-VECMs obtained higher criteria values than their VAR and RS-VAR counterparts, despite the additional parameters involved. Likewise, the use of regime switching was beneficial as the RS-VAR and RS-VECM models all performed significantly better than their VAR and VECM counterparts.

Figure 16 shows the estimated (smoothed) regime probabilities (or expected state) through time. The graph confirms that regime 1 is a high-volatility state as it includes the high volatility of the 1970s and 1980s and the financial crises of 1991, 2001 and 2008.

3.1.1 Simulation

The RS-VECM produces simulations of the economic series into the future that are comparable with historical data. The model occasionally simulated negative bond returns, in around 1% of simulations, and in these cases, the bond return was set to 0%. Figures 17 to 18 show the average simulation paths and returns along with 95% confidence intervals. Figure 19 shows the simulated distribution of the GDP, bond index, stock index, and CPI after 45 years. Figure 20 shows the overall distribution of returns for each variable obtained from these simulations and provides a comparison with the observed (historical)

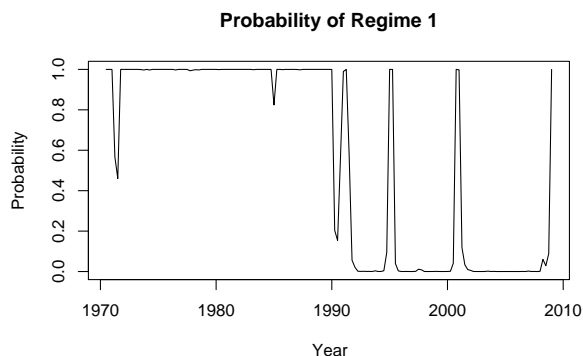


FIGURE 16: Smoothed Regime Probabilities (Regime 1)

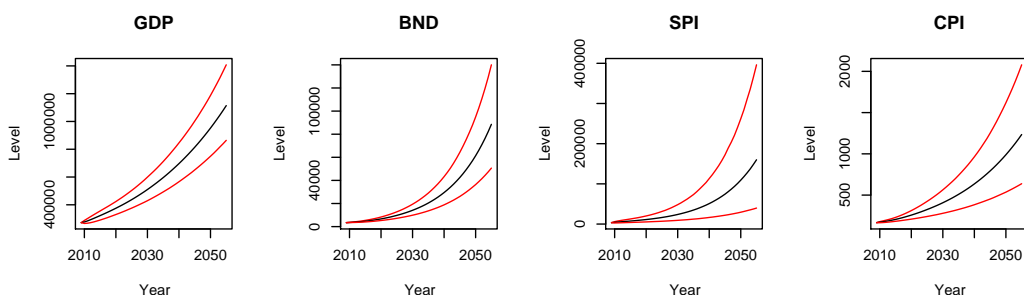


FIGURE 17: Average Simulation Path (and 95% CI) of the Market Variables

distribution.

3.2 Mortality Models

Lee and Carter (1992) [34] developed an early discrete-time model which has been strongly influential on the literature of stochastic mortality modelling with central rate of mortality given by:

$$\ln m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t} \quad (17)$$

where a_x is an age-specific constant, b_x describes the age-specific influence of the stochastic ARIMA process k_t , and $\varepsilon_{x,t}$ is a normal error term describing short-term fluctuations.

The Lee-Carter model has many shortcomings including the use of a single factor k_t , resulting in perfectly correlated mortality improvements throughout all ages. The standardised residuals were also observed to be clustered by cohort for UK data (Willets, 2004 [47]). Further developments were proposed by Renshaw and Haberman (2003, 2006) [40, 41], who introduced a multifactor model and then extended it to incorporate a cohort effect.

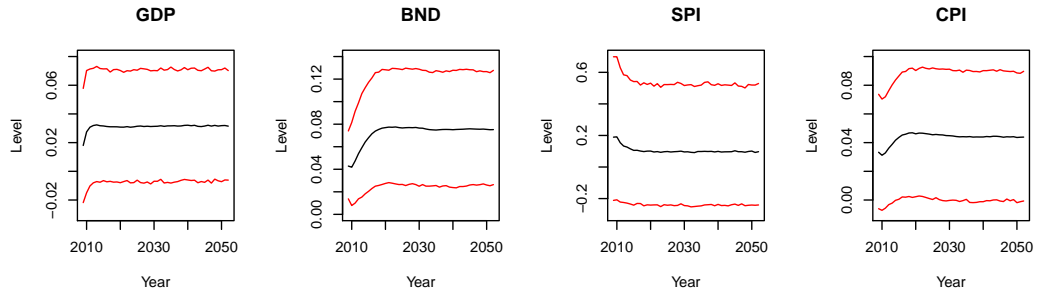


FIGURE 18: Average Simulation Returns (and 95% CI) of the Market Variables

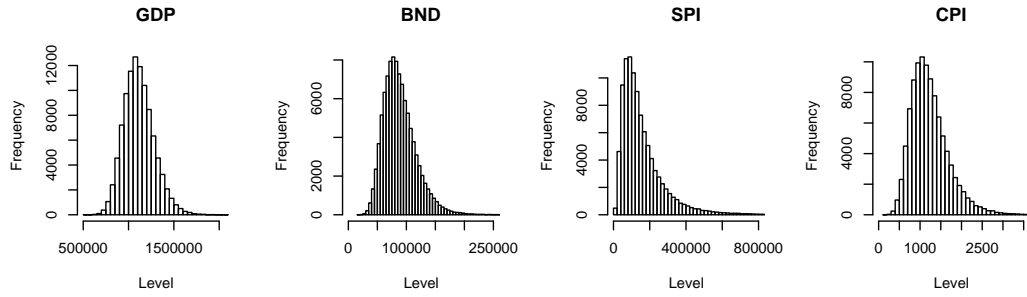


FIGURE 19: Final Distribution of Market Indices (after 45 years)

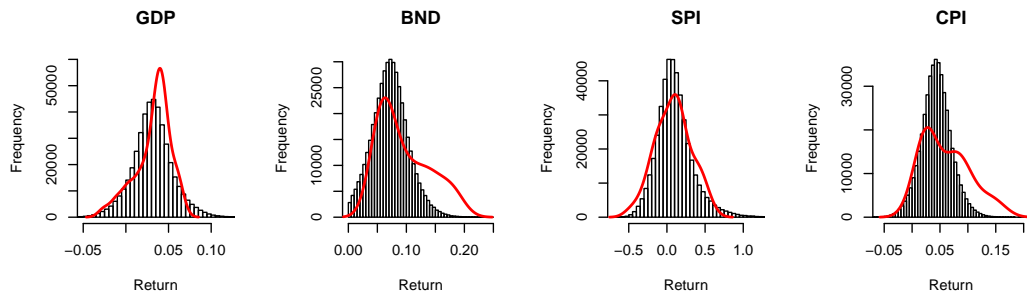


FIGURE 20: Overall Distribution of Simulated Annual Returns
(Note: **Red** = Historical Distribution)

Dahl (2004) [17] introduced a class of models, known as affine models, which have been popular due to their tractability. Affine mortality models were further developed by Biffis (2005) [3], who introduces the use of jump-diffusion, and Dahl and Møller (2006) [18], who extend the model to include a mortality improvement process using the Cox-Ingersoll-Ross model. Schrager (2006) [43] constructs a multiple factor model which is extended and applied to Australian data by Wills and Sherris (2008) [48]. The force of mortality for a life currently aged x at time t is described using a multivariate normal model which was found to fit closely to the data:

$$d \ln \mu_{x,t} = (a + bx)dt + \sigma d\mathbf{W}_{x,t} \quad (18)$$

The dependence between ages is captured by the covariance matrix of $\mathbf{W}_{x,t}$.

The mortality simulation model is largely based on the Wills and Sherris (2008) [48] model. A further improvement is made by modelling the relationship between annuitant and population mortality. This is important when analysing a portfolio of annuitants, and also for an analysis of basis risk in hedging strategies.

The model for the force of mortality for a life currently aged x at time t , denoted $\mu_{x,t}$, is a linear model with an age factor x :

$$\Delta \text{logit } \mu_{x,t} \equiv \Delta \ln \frac{\mu_{x,t}}{1 - \mu_{x,t}} = a + bx + \varepsilon_{x,t} \quad (19)$$

where a and b are constant parameters and $\varepsilon_{x,t}$ is an error term. As in Wills and Sherris (2008) [48], the difference operator is applied in the cohort direction:

$$\Delta \text{logit } \mu_{x,t} = \text{logit } \mu_{x,t} - \text{logit } \mu_{x-1,t-1} \quad (20)$$

The vector of errors at each time t are multivariate normal with zero mean, and can be written as:

$$\boldsymbol{\varepsilon}_t = (\varepsilon_{x_1,t}, \dots, \varepsilon_{x_N,t})' = \sigma \mathbf{W}_t \quad (21)$$

where σ is a deterministic volatility parameter, and $\mathbf{W}_t \sim \mathcal{N}_N(\mathbf{0}, \mathbf{D})$. The matrix \mathbf{D} captures the dependence structure between ages.

The logit transform was chosen based on Cairns et al (2006) [14] and observation of the data. Figure 21 shows the historical logit mortality rates.

The parameters were estimated in a two-stage procedure as in Wills and Sherris (2008) [48]. Firstly, maximum likelihood is used to estimate the parameters a, b and σ assuming independent observations of $\Delta \text{logit } \mu_{x,t}$. The covariance matrix \mathbf{D} is then determined using principal components analysis of the standardised residuals. The mortality model is estimated from population data as Australian annuitant data is not available. Mortality data was obtained from the Human Mortality Database (HMD) and the Australian Bureau of Statistics (ABS). The data contains annual central rates of mortality, $m_{x,t}$, for ages 65 to 109 and years 1970 to 2007. The minimum age of 65 is chosen to reflect the retirement

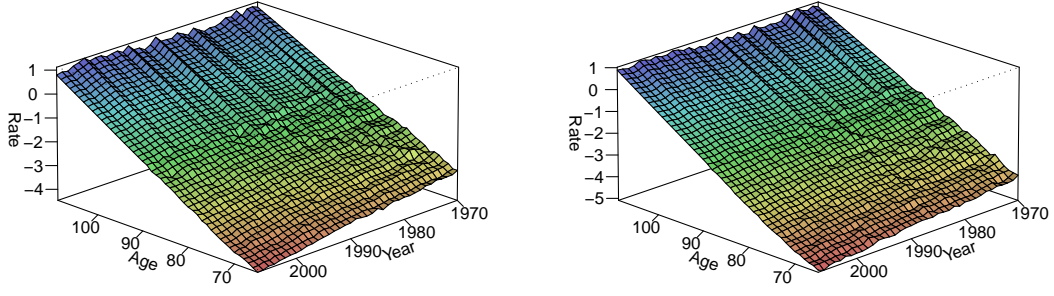


FIGURE 21: Observed Logit Mortality Rates for Males (Left) and Females (Right)

age in Australia. The data at higher ages does not consist of raw mortality rates, as the rates have been smoothed as described in the HMD Methods Protocol (Wilmoth et al, 2007 [50]) due to a lack of observed deaths at higher ages. Table 6 shows the parameter estimates for male and female data.

Parameter	Male	Female
$a (\times 10^{-2})$	-0.3516	-0.3320
$b (\times 10^{-2})$	0.1054	0.1244
σ	0.0891	0.0840

TABLE 6: Parameter Estimates for Mortality Model (MLE)

Table 7 shows the PCA results. 9 eigenvectors were sufficient to account for 95% of the observed variation in the standardised residuals.

The model provides a good fit to the data, with the residuals $r_{x,t}^*$ exhibiting no time or age trend as shown in Figure 22.

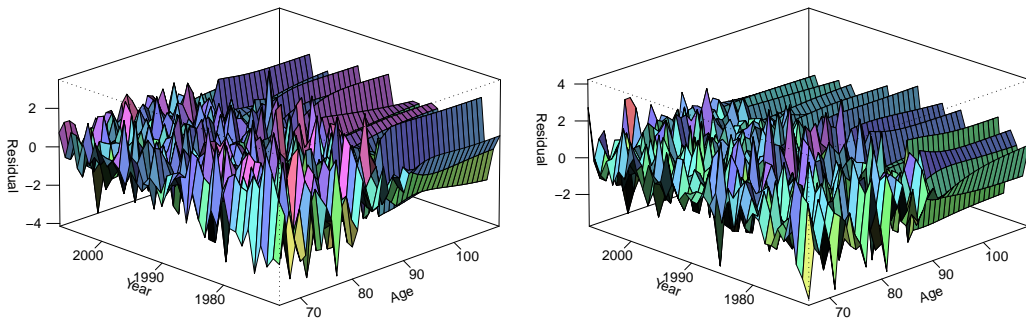


FIGURE 22: Re-Standardised Residuals $r_{x,t}^*$ for Males (Left) and Females (Right)

The unusual residuals for the higher ages result from the smoothing process of the original data. The results in Table 8 show that the model provides a good fit for male data.

Number of Eigenvectors	Percentage Explained (Male)	Percentage Explained (Female)
1	77.5	77.9
2	82.9	83.3
3	88.1	86.6
4	90.1	88.6
5	91.7	90.5
9	95.3	94.8
10	96.0	95.6
15	98.4	98.1
20	99.5	99.4
25	99.9	99.9
30	100.0	100.0

TABLE 7: Proportion of Observed Variation (in Standardised Residuals) Explained

Model	Q	r	p-value	Reject H_0 ?
Male	485.64	491	0.5598	No
Female	562.40	491	0.0140	Yes

TABLE 8: Goodness of Fit of Mortality Model

Wills and Sherris (2008) [48] noted the ‘ridges’ effect in their model which arose from the use of unsmoothed mortality rates, $\mu_{x,T}$. To prevent this, the observed logit mortality rates for the last year in the data (2007) were smoothed using cubic splines before simulation.

Realisations of the random vector \mathbf{W}_t were simulated from the multivariate normal distribution, $\mathcal{N}_N(\mathbf{0}, \mathbf{D})$. The average mortality rates for all ages 65–109 are shown in Figure 23.

The average (projected) mortality rates and 95% confidence intervals for the cohort aged 65 in 2007 are shown in Figure 24.

To allow for annuitant mortality, simulated mortality rates are adjusted based on the observed historical relationship between population and annuitant mortality. As there is no Australian annuitant data available, an investigation of Australian annuitant mortality was not possible. UK annuitant data was available from the Continuous Mortality Investigation (CMI) for years 1947, 1968, 1980, 1992, and 2000. This data was compared to UK population mortality available from the Human Mortality Database (HMD) and the Office for National Statistics (ONS). The relationship between annuitant (or pensioner) mortality and population mortality was modelled using the ratio:

$$\rho_{x,t} = \frac{q_{x,t}^A}{q_{x,t}} \quad (22)$$

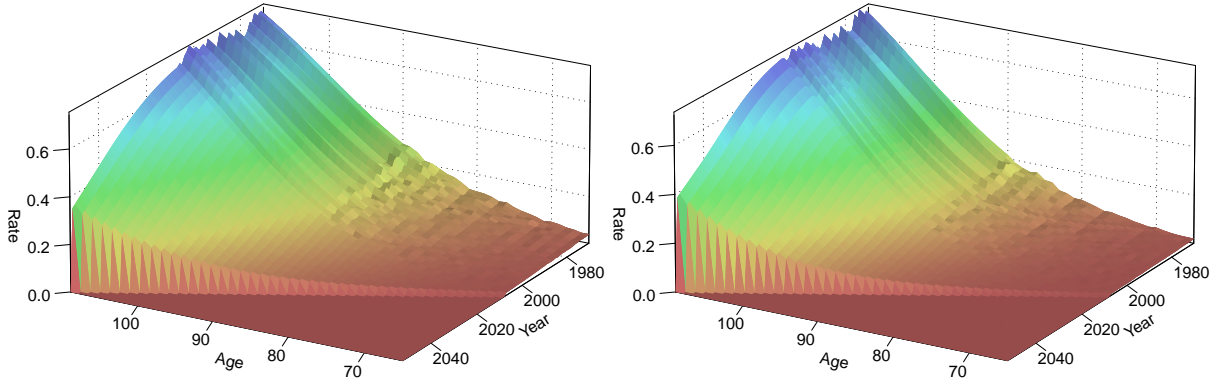


FIGURE 23: Observed and Projected Mortality for Males (Left) and Females (Right)

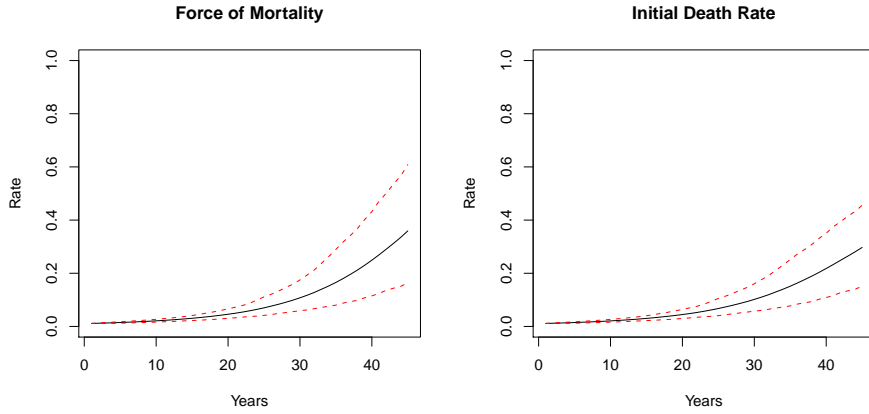


FIGURE 24: Projected Male Mortality Rates for Cohort 65 (with 95% CIs)

where $q_{x,t}^A$ and $q_{x,t}$ are the annuitant and population mortality rates for age x at time t .

A simple linear model was used to model the ratio:

$$\rho_{x,t} = \alpha + \beta x + \nu_{x,t} \quad (23)$$

$$\nu_{x,t} \sim N(0, \theta^2) \quad (24)$$

The fitted parametrs are shown in Table 9.

Stevenson and Wilson (2008) [45] investigated the relationship between Australian pensioner and population mortality for the period 2005-07, showing an approximate linear increasing relationship between age and the ratio. The relationship between UK annuitant and population mortality is also approximately linear and increasing. The variation in the ratio from year to year due to the error terms is the basis risk between population and annuitant mortality. The fitted ratios are superimposed in the graphs of observed ratios in Figure 25 for comparison and extrapolated to age 110.

Parameter	Male	Female
α	-0.1381	-0.1648
β	0.0110	0.0111
θ	0.0668	0.0846

TABLE 9: Parameter Estimates for Annuitant-Population Ratio Model

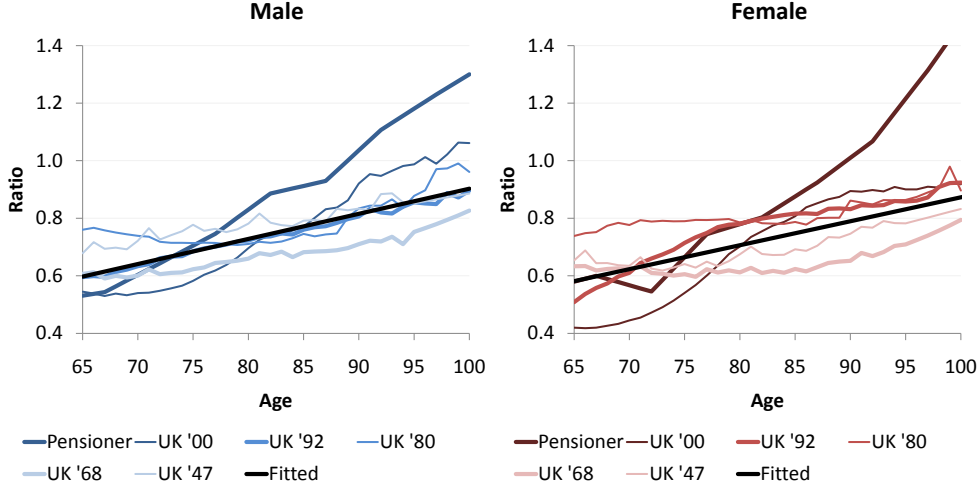


FIGURE 25: Observed Ratios of Annuitant/Pensioner to Population Mortality

4 Static Hedging Results for Annuity Portfolios

Longevity hedging strategies involving mortality-linked securities are assessed by simulating the market and mortality models. The effect of basis risk, the key disadvantage of mortality-linked securities based on population indices, is quantified. The strategies compare q-forwards and longevity bonds since these currently have the most potential for hedging longevity risk (eg. Coughlan et al, 2007 [16]; Dowd et al, 2006 [21]). q-Forwards have already been used by insurers to hedge longevity risk, whereas longevity bonds may be issued by governments in the future (Blake et al, 2009 [5]).

For the simulations of the annuity products, Figure 26 shows the expected total claims and Figure 27 shows the simulated distribution of the surplus U_T .

The difference in claims between life and indexed annuities reflects the impact of expected inflation.

Table 10 shows the impact of longevity risk on the expected shortfall for the various annuities.

With no hedging the expected shortfall for the inflation indexed annuities is very substantial arising from the inflation risk in these products. When mortality is assumed to

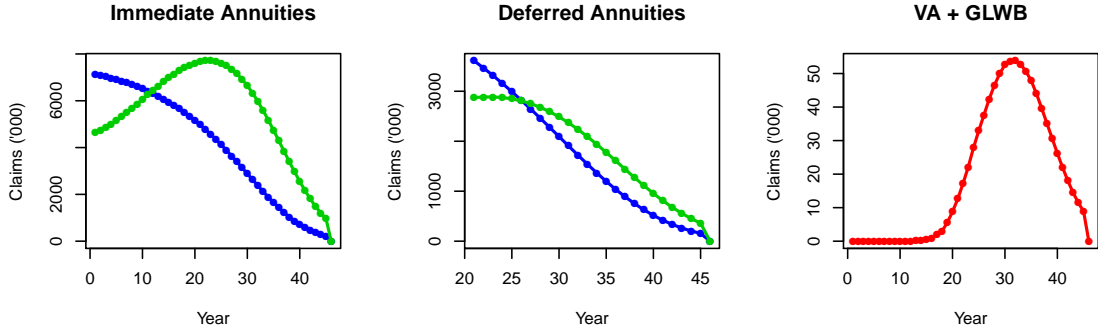


FIGURE 26: Expected Total Claims of Portfolio
(Note: Blue = Life Annuity, Green = Indexed Annuity)

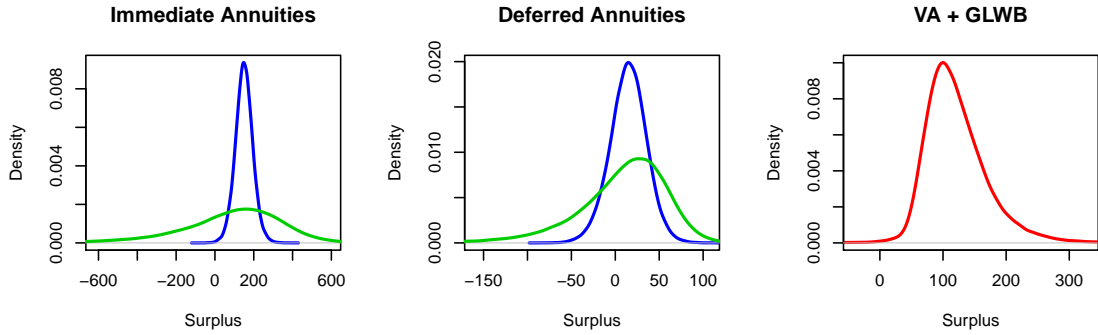


FIGURE 27: Simulated Distribution of Final Surplus U_T
(Note: Blue = Life Annuity, Green = Indexed Annuity)

be deterministic, removing the longevity risk, all products except the variable annuity with the GLWB, show improved expected shortfalls. For the inflation indexed annuities, the relative effect of longevity risk is less because of the significant inflation risk. The variable annuity with GLWB provides limited longevity protection compared with the other annuity products. Table 11 demonstrates the significance of inflation risk for indexed annuities and market risk for variable annuities.

Figures 28 to 29 show the expected shortfalls for a range of probability levels for the hedging strategies. An effective hedging strategy will shift the expected shortfall curve from the ‘no longevity hedging’ case upwards towards the ‘no longevity risk’ case. The percentage reduction in expected shortfall is quantified using:

$$\frac{ES - ES_{NH}}{ES_{NR} - ES_{NH}} \quad (25)$$

where ES_{NH} and ES_{NR} are the corresponding risk measures under ‘no longevity hedging’ and ‘no longevity risk’ respectively. Figures 30 to 31 show the average reduction in

Annuity	Mean (No Hedging)	ES _{0.05} (No Hedging)	ES _{0.05} (No L Risk)
Life	150.3	54.6	116.2
Indexed Life	81.5	-642.3	-587.9
Deferred	14.7	-30.6	7.7
Def. Indexed	7.5	-133.5	-106.2
VA + GLWB	121.43	40.80	41.27

TABLE 10: 5% Expected Shortfall (No Hedging vs No Longevity Risk)

Annuity	$\mu - \text{ES}_{0.05}$ (Stochastic)	$\mu - \text{ES}_{0.05}$ (Average)	% Difference
Life	95.7	88.5	8%
Indexed Life	723.8	173.8	76%
Deferred	45.3	42.8	6%
Indexed Def.	141.0	54.4	61%
VA + GLWB	80.6	5.3	93%

TABLE 11: Change in 5% Expected Shortfall (Stochastic vs Average Market)

expected shortfall (1% and 5% levels) for each of the 5 scenarios for each hedging strategy.

Under the stress test scenarios (4 & 5), the most effective strategies are the longevity swap (all products), q-forwards (all products), both longevity bonds (immediate life annuity), and the 40-year longevity bond (deferred life annuity). These are the strategies which have mortality-linked payments that more closely match all the cash flows.

To assess the impact of basis risk on the hedge higher and lower levels of uncertainty in the ratio of annuitant and population mortality were compared. The levels used are shown in 12. The effect of basis risk on hedge effectiveness is shown in Figure 32. Changes in the basis risk arising from uncertainty in the annuitant-population ratio does not significantly reduce hedge effectiveness.

A sensitivity analysis was performed to analyse the effect of using a higher/lower price of risk (Sharpe ratio) when pricing the mortality-linked securities. Sharpe ratios of 0.15 and 0.25 were considered for comparison with the base case of 0.20. The results in Figure 33 show that the hedge effectiveness is sensitive to the price, although the longevity bond and swap strategies for the life annuities (both immediate and deferred) change less when the Sharpe ratio is increased to 0.25. The effectiveness of q-forwards is highly sensitive

No Basis Risk	Basis Risk	150% Basis Risk
0%	6.68%	10.02%

TABLE 12: Standard Deviation of Error Term in Annuitant-Population Ratio Model

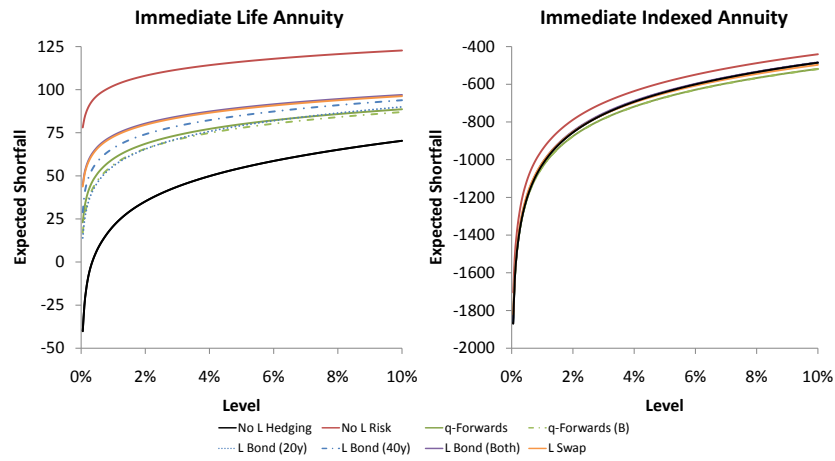


FIGURE 28: Effect of Hedging Strategies on ES (Immediate Annuities)

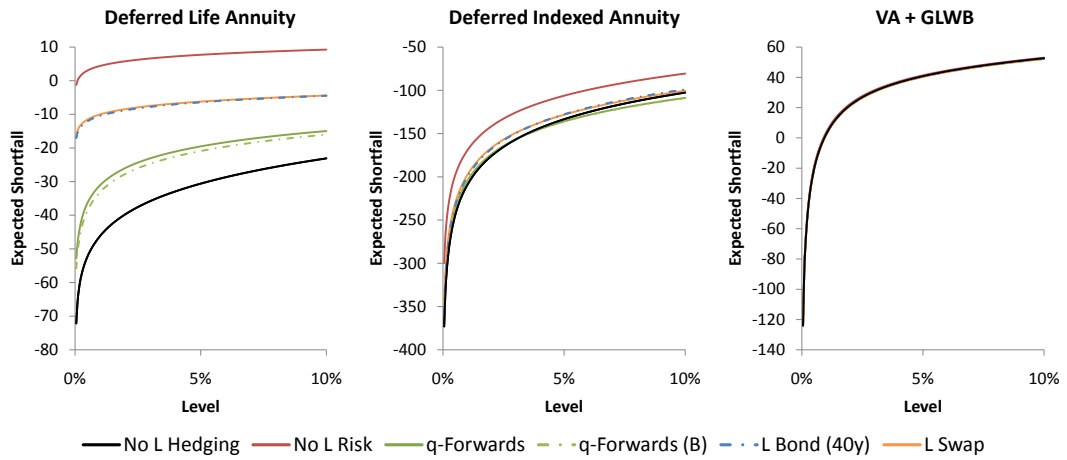


FIGURE 29: Effect of Hedging Strategies on ES (Deferred Annuities and GLWB)

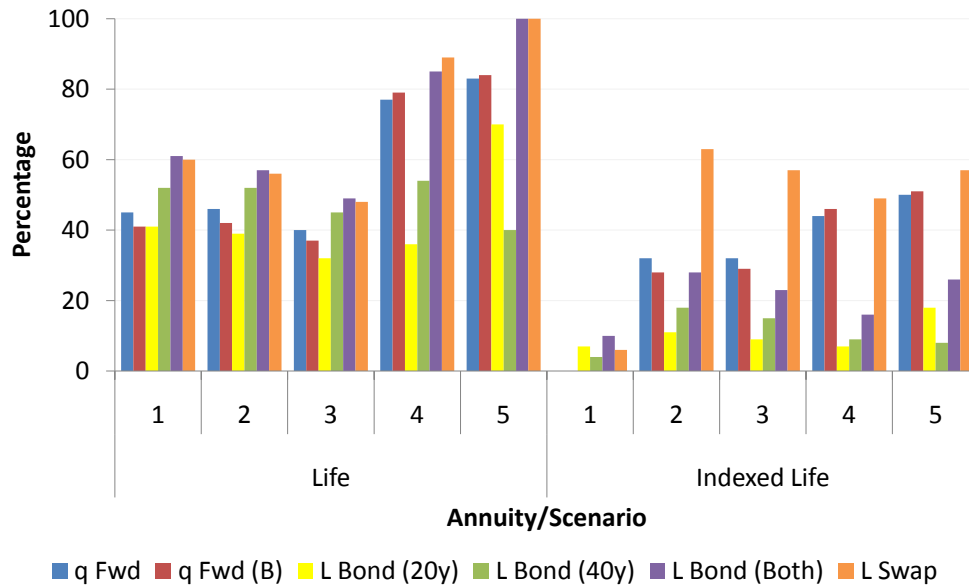


FIGURE 30: Avg Reduction in Expected Shortfall (Immediate Annuities)

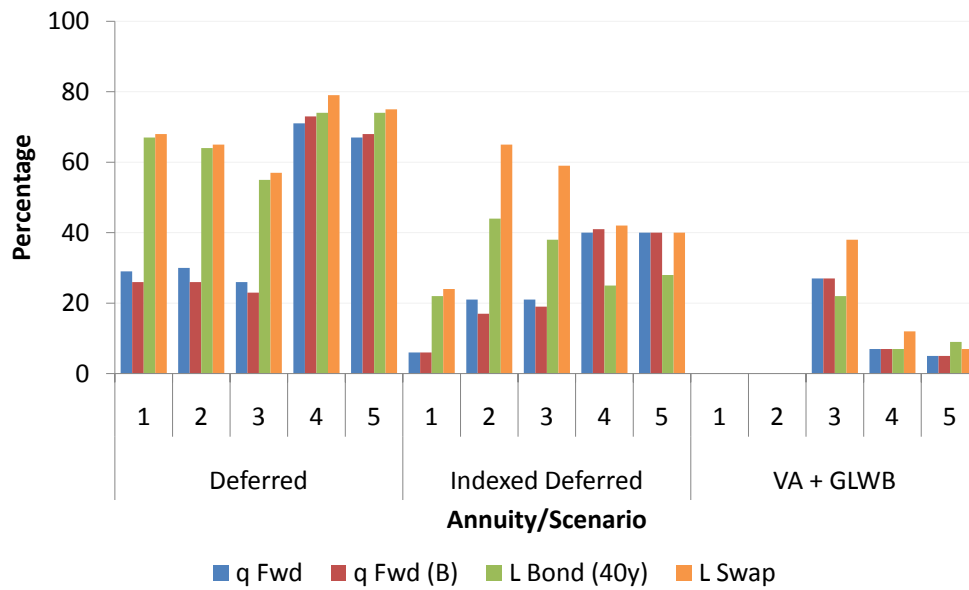


FIGURE 31: Avg Reduction in Expected Shortfall (Deferred Annuities and GLWB)

to the price of risk, with the effectiveness falling significantly when the Sharpe ratio is increased. The price of risk has a greater impact on the q-forward strategies than the longevity bonds or swap.

The GLWB provides limited longevity risk insurance, since it is only in very adverse markets that the longevity guarantee has value. A sensitivity analysis was performed with respect to the assumed lapse rates and withdrawal percentages including assuming lapse rates were 0% and/or when withdrawal percentages are increased to 6%. The reduction in lapse rates results in a larger number of policies remaining inforce, whereas an increase in withdrawal percentages increases both the amount and duration of payments. Both these situations should increase the longevity risk insurance of the GLWB. These assumption changes had virtually no impact on the hedging results.

5 Discussion and Conclusions

This paper has assessed longevity risk hedging using static cash flow matching with longevity derivatives and bonds for a wide range of annuities, including inflation indexed annuities and variable annuities with a GLWB. Improved market and mortality models were developed and estimated using Australian data to model the underlying risks. The dependence between market variables was modeled using a cointegrating VECM, which allows for both short-run dependence and a long-run equilibrium. Regime switching was also used to model the volatility of market variables. Mortality was modeled with a multiple factor model and basis risk between annuitant and population mortality was included. A wide range of static hedging strategies using q-forwards and longevity bonds were assessed and compared.

Life annuities provide protection against longevity risk and benefit the most from hedging longevity risk. Indexed annuities include inflation risk and, although longevity risk is significant, the effectiveness of hedging this risk is significantly reduced if inflation risk cannot be hedged. Variable annuities include substantial market risk and the longevity risk protection from the GLWB is extremely limited except in the most adverse market outcomes. Longevity risk is significant for deferred annuities compared to immediate annuities, as they provide longevity insurance for the higher ages where mortality improvements are more likely to differ from expected.

Longevity bonds were found to be effective in hedging the longevity risk in life annuities. Zero coupon longevity bonds allow more flexibility in constructing a hedge strategy, and are able to better hedge the longevity risk in indexed annuities. q-Forwards are also effective in hedging longevity risk, but contain a significant amount of additional basis risk compared to longevity bonds. This basis risk reflects the use of mortality rates as the underlying in q-forwards rather than survival probabilities used for longevity bonds. Constructing a static hedge using q-forward is more complex than using longevity bonds. Basis risk was found to have a minimal impact on hedge effectiveness. Annuitant mortality

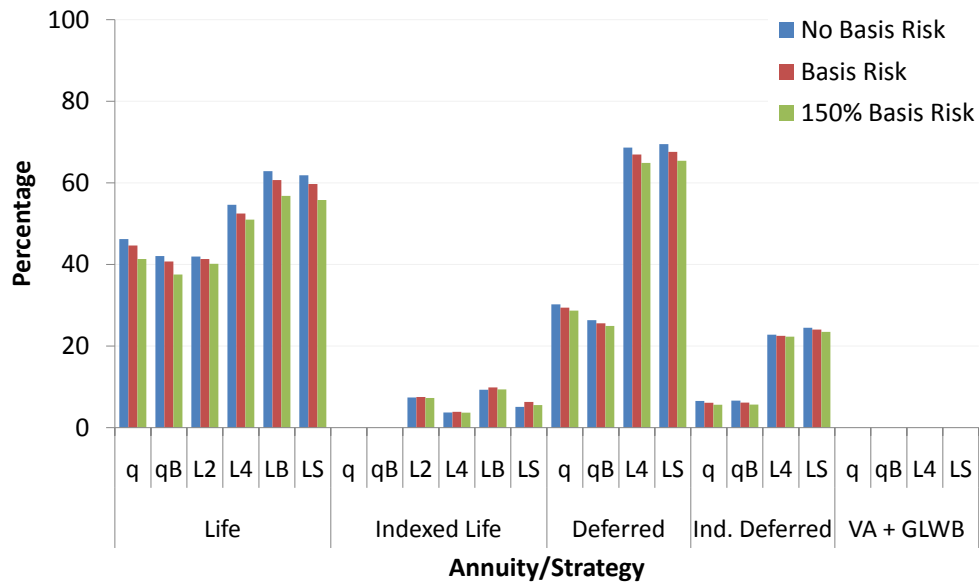


FIGURE 32: Reduction in Expected Shortfall: Effect of Basis Risk

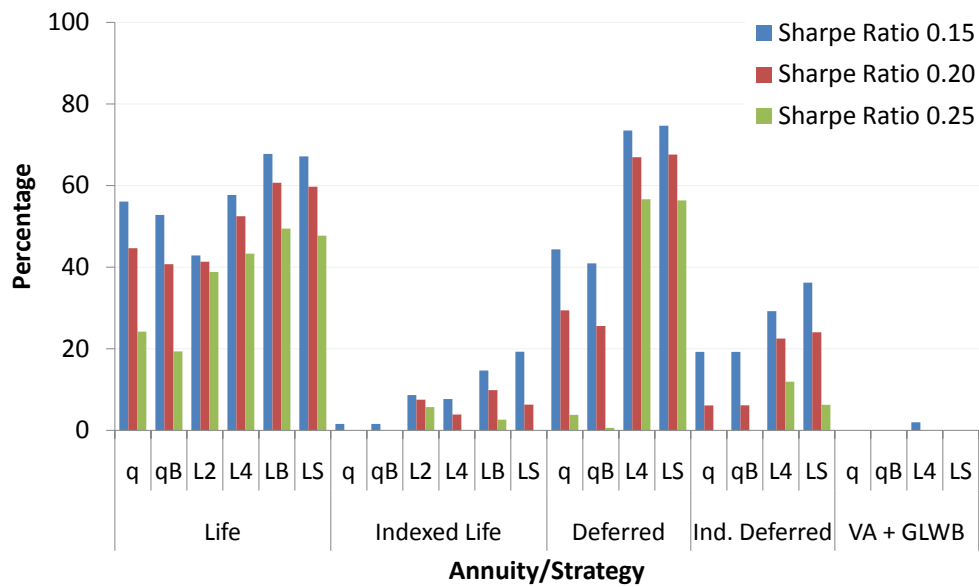


FIGURE 33: Reduction in Expected Shortfall: Sensitivity to Sharpe Ratio

can be effectively hedged using derivatives and bonds based on population mortality. Bucketing of q-forwards by 5-year age groups is also effective.

The cost of hedging, as reflected in the price of longevity risk, was found to have a reasonably significant effect on the effectiveness of hedging strategies. The longevity bond-based strategies were significantly less sensitive than the q-forward strategies to this price of risk.

There are currently no longevity derivatives or longevity bonds available in the Australian market. This paper has shown that government issuance of longevity bonds would allow insurers to manage longevity risk. If the government, through the Government Actuary or the regulator APRA, were to develop a projected mortality reference index for Australia this would allow the development of a mortality-linked securities market. An analysis of available Australian annuitant mortality based on industry experience in comparison to population mortality would be required to confirm the analysis in this paper based on UK data. The paper has also shown that in order to provide inflation indexed life annuities, there would need to be an active market for hedging inflation in Australia. This would require the government to issue inflation indexed securities.

This paper has considered static hedging because of its simplicity, lower cost, and practicality. As the market for mortality-linked securities develops further, dynamic hedging should become possible. Dynamic hedging is required for hedging market risk for variable annuities and the GLWB. However, static strategies are effective for longevity risk in the standard annuity products. The hedging strategies do not entirely eliminate longevity risk and require risk based capital to manage the residual risk. This is more significant for the inflation indexed and deferred life annuities.

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