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Longevity Risk Management for Life and Variable Annuities

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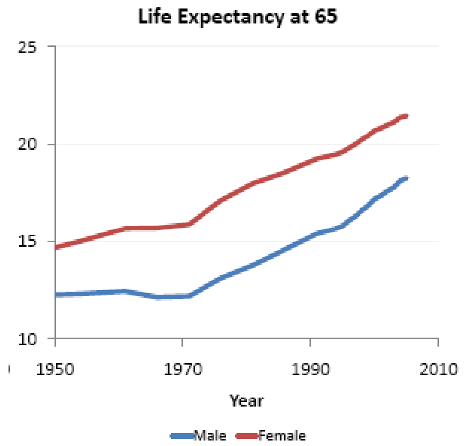
Introduction

- Longevity risk management: reinsurance, derivatives, longevity bonds



- Products: Annuities, Deferred Annuities, Variable Annuities (+GLWB)

Annuity Market	2001	2002	2003	2004	2005	2006	2007
Life & Term Annuities	74.3	103.3	89.4	87.9	79.5	78.3	72.8
Indexed Annuities	6.8	11.8	14.4	23.1	27.2	25.4	24.8
Variable Annuities	113.3	115.0	126.4	129.7	133.1	157.3	182.2

TABLE 1: US Annuity Sales (US$b)
(Source: Morningstar, Inc. and LIMRA International)

Annuity Market	2001	2002	2003	2004	2005	2006	2007
Life Annuities	166	154	200	281	27	29	36
Term Annuities	794	1,096	1,357	2,758	548	530	787

TABLE 2: Australian Annuity Sales (A$m)
(Source: Plan for Life Research)

Challenges and Opportunities

- Successful provision of longevity insurance to individuals
  - longevity uncertain, systematic improvements, risk pooling less efficient
  - capital is costly
  - cost effective hedging to offer annuities
- Financial market developments
  - JPMorgan q-Forwards
  - Longevity swaps
  - Possibilities – government longevity bonds, securitization

Paper Coverage

- Life annuities and hedging instruments
- Market and mortality models
- Hedging strategies and effectiveness using Longevity Bonds and Derivatives
- Static hedging (ALM):
  - Dynamic hedging – lack of markets and liquidity
  - ALM and Risk based capital

Hedging Instruments

- q-Forwards
  - Pay actual $q_{x,t}$ in exchange for fixed $q^F_{x,t}$
  - Individual ages and 5-yr Bucketed
- Coupon Longevity Bond
  - Payments in line with actual survival probability $S_{65}(t)$
- Zero Coupon Longevity Bond
  - Single payment of $S_{65}(t)$ at maturity t

FIGURE 3: Structure of a τ-year q-forward

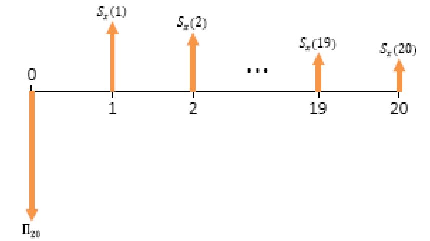


FIGURE 5: Structure of a 20-year (Immediate) Longevity Bond

FIGURE 4: Structure of a τ-year Zero Coupon Longevity Bond

Hedging Strategies

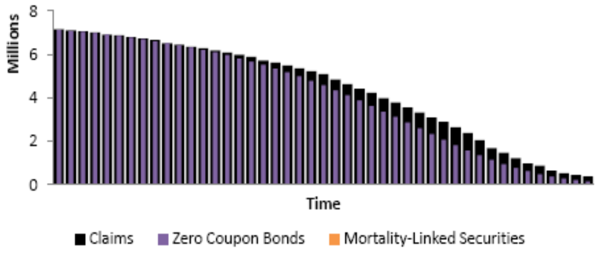


FIGURE 10: Sample Cash Flows for Life Annuity with No Longevity Hedging (ZCBs)

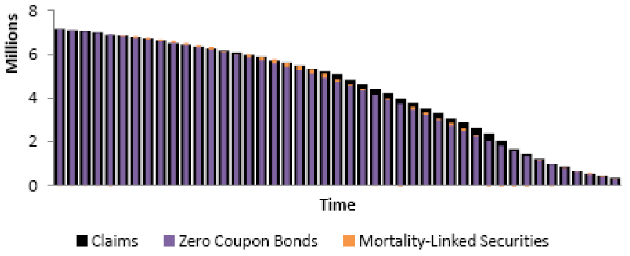


FIGURE 11: Sample Cash Flows for Life Annuity with q-Forwards

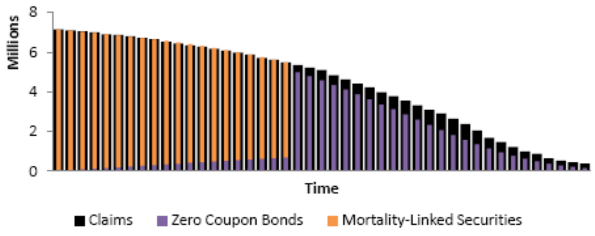


FIGURE 12: Sample Cash Flows for Life Annuity with a 20yr Longevity Bond

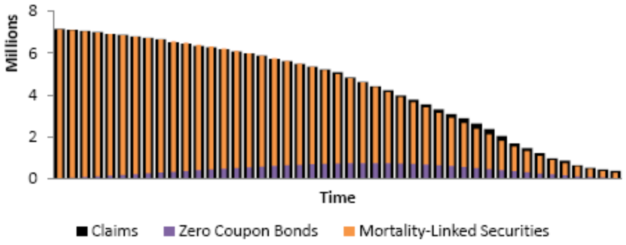


FIGURE 14: Sample Cash Flows for Life Annuity with a Longevity Swap (ZCLBs)

Market Model

Vector Error Correction Model with Regime Switching (RS-VECM)

- Long run equilibrium, volatility regimes

$$\ln G_t = \text{Log Gross Domestic Product (GDP)}$$
$$\ln B_t = \text{Log Bond Index (Accumulated 90-day Bank-Accepted-Bill Yields)}$$
$$\ln S_t = \text{Log Stock Price Index (ASX All Ordinaries)}$$
$$\ln F_t = \text{Log Inflation Index (CPI)}$$

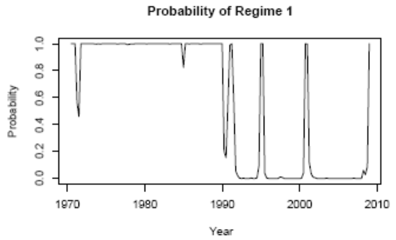


FIGURE 16: Smoothed Regime Probabilities (Regime 1)

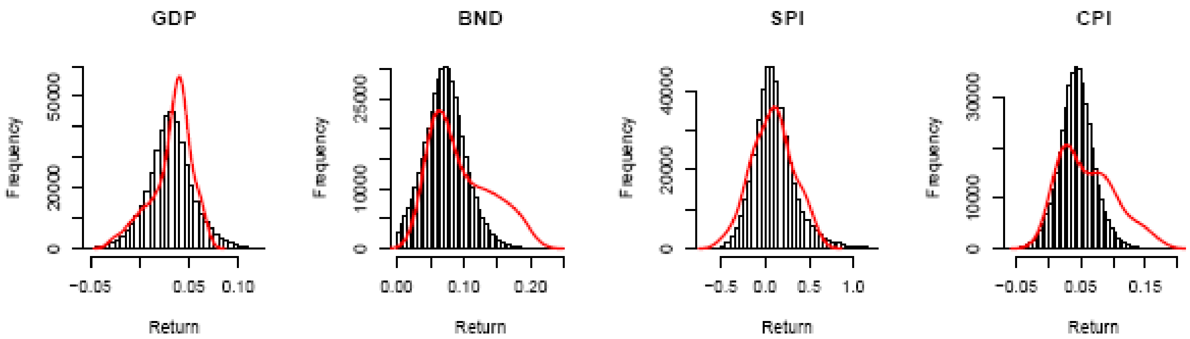


FIGURE 20: Overall Distribution of Simulated Annual Returns
(Note: Red = Historical Distribution)

Mortality – Longevity Model

Models mortality rates in cohort direction
Logit age structure for rate changes (stationary)
Age dependence using principal components

$$\Delta \operatorname{logit} \mu_{x,t} \equiv \Delta \ln \frac{\mu_{x,t}}{1-\mu_{x,t}} = a + bx + \varepsilon_{x,t}$$

$$\Delta \operatorname{logit} \mu_{x,t} = \operatorname{logit} \mu_{x,t} - \operatorname{logit} \mu_{x-1,t-1}$$

Parameter	Male	Female
a ($\times 10^{-2}$)	-0.3516	-0.3320
b ($\times 10^{-2}$)	0.1054	0.1244
σ	0.0891	0.0840

TABLE 6: Parameter Estimates for Mortality Model (MLE)

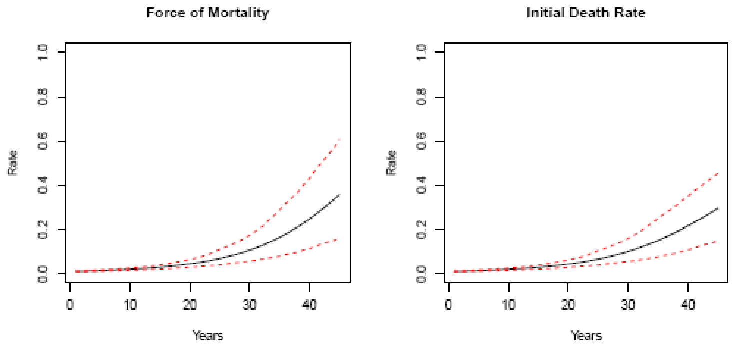


FIGURE 24: Projected Male Mortality Rates for Cohort 65 (with 95% CIs)

Number of Eigenvectors	Percentage Explained (Male)	Percentage Explained (Female)
1	77.5	77.9
2	82.9	83.3
3	88.1	86.6
4	90.1	88.6
5	91.7	90.5
9	95.3	94.8
10	96.0	95.6
15	98.4	98.1
20	99.5	99.4
25	99.9	99.9
30	100.0	100.0

TABLE 7: Proportion of Observed Variation (in Standardised Residuals) Explained

Mortality Model – Basis Risk

Portfolio of annuitants – hedging instruments based on population index

$$\rho_{x,t} = \frac{q^A_{x,t}}{q_{x,t}}$$

$$\rho_{x,t} = \alpha + \beta x + \nu_{x,t}$$
$$\nu_{x,t} \sim N(0, \theta^2)$$

Parameter	Male	Female
α	-0.1381	-0.1648
β	0.0110	0.0111
θ	0.0668	0.0846

Table 9: Parameter Estimates for Annuitant-Population Ratio Model

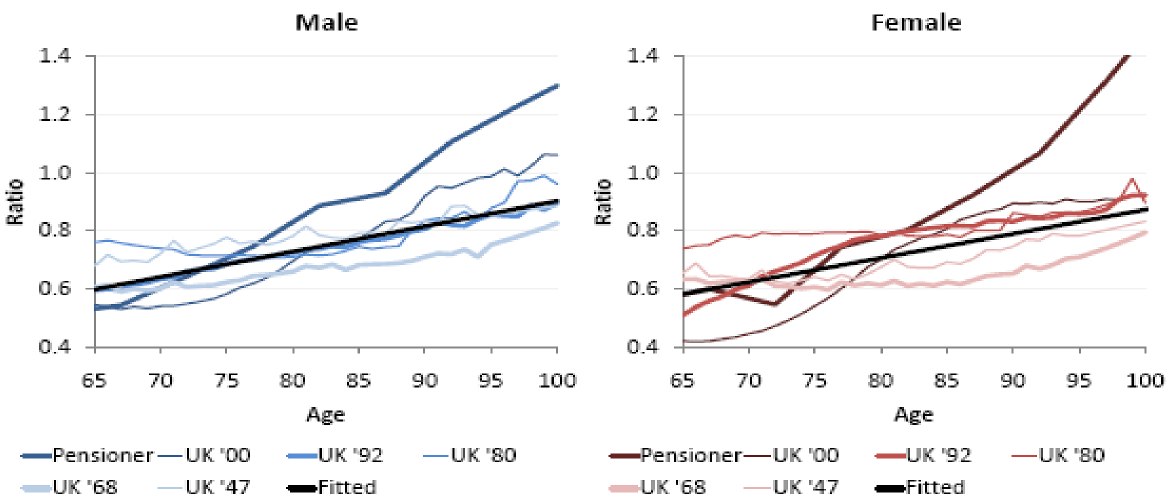


Figure 25: Observed Ratios of Annuitant/Pensioner to Population Mortality

Scenario Analysis

Scenario	Market Assumption	Mortality Assumption
1	Stochastic	Stochastic
2	Average	Stochastic
3	Adverse (1930–75)	Stochastic
4	Stochastic	Avg + Excess Imp. (2%/yr Acc)
5	Stochastic	Avg + Excess Imp. (25% Flat)

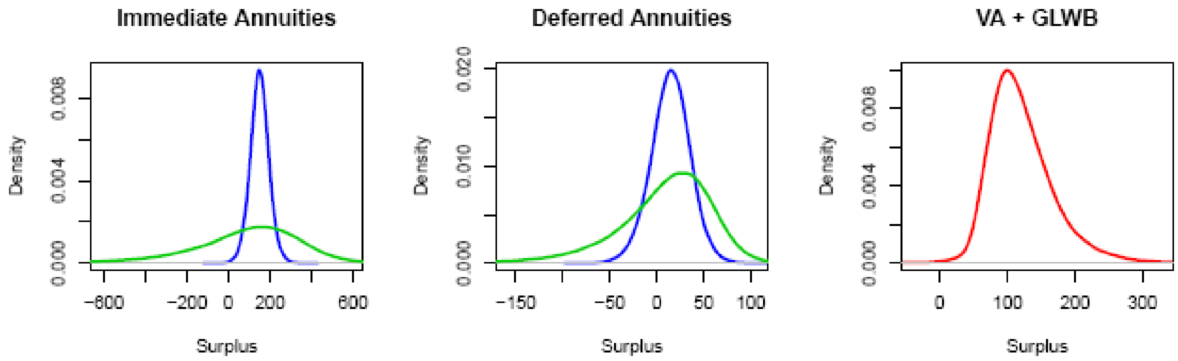


FIGURE 27: Simulated Distribution of Final Surplus U_T
(Note: Blue = Life Annuity, Green = Indexed Annuity)

Risk Measures for Longevity

Annuity	Mean (No Hedging)	$ES_{0.05}$ (No Hedging)	$ES_{0.05}$ (No L Risk)
Life	150.3	54.6	116.2
Indexed Life	81.5	-642.3	-587.9
Deferred	14.7	-30.6	7.7
Def. Indexed	7.5	-133.5	-106.2
VA + GLWB	121.43	40.80	41.27

TABLE 10: 5% Expected Shortfall (No Hedging vs No Longevity Risk)

Annuity	$\mu - ES_{0.05}$ (Stochastic)	$\mu - ES_{0.05}$ (Average)	% Difference
Life	95.7	88.5	8%
Indexed Life	723.8	173.8	76%
Deferred	45.3	42.8	6%
Indexed Def.	141.0	54.4	61%
VA + GLWB	80.6	5.3	93%

TABLE 11: Change in 5% Expected Shortfall (Stochastic vs Average Market)

Risk Measures for Longevity Risk

Indexed annuities have substantial shortfall risk - inflation

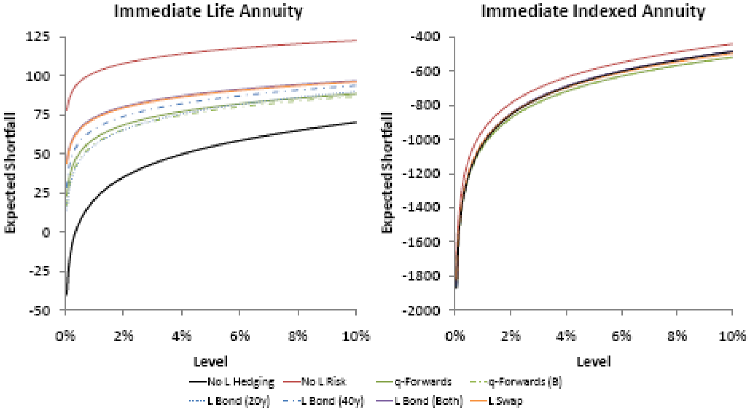


FIGURE 28: Effect of Hedging Strategies on ES (Immediate Annuities)

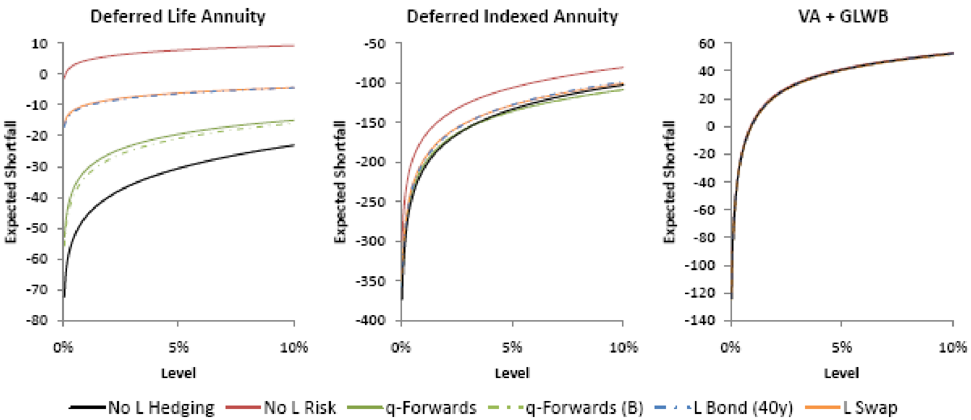


FIGURE 29: Effect of Hedging Strategies on ES (Deferred Annuities and GLWB)

VA + GLWB provides limited longevity protection – hedging has little impact

Results –Annuities (Life/Indexed)

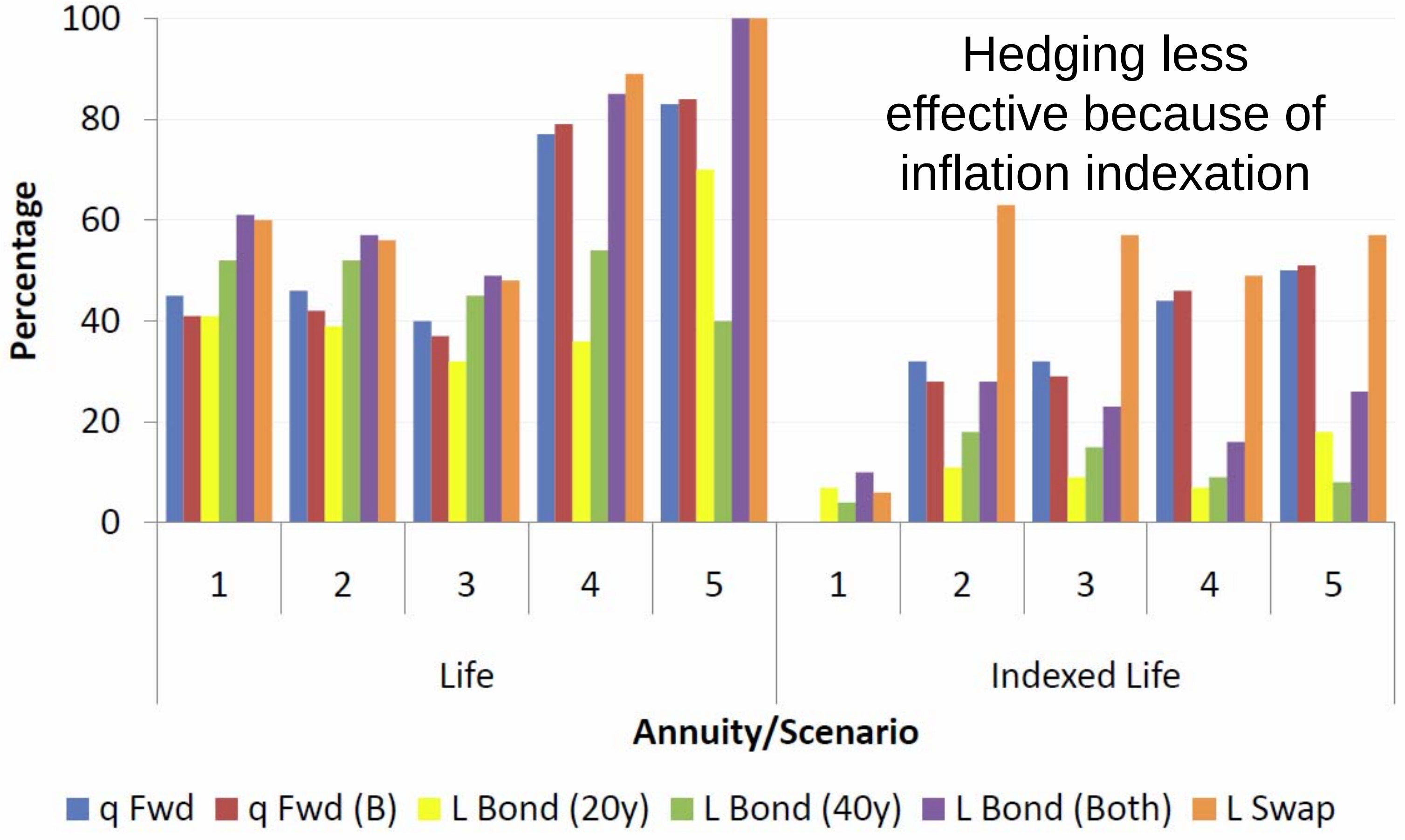


FIGURE 30: Avg Reduction in Expected Shortfall (Immediate Annuities)

Results – Deferred Annuities/GLWB

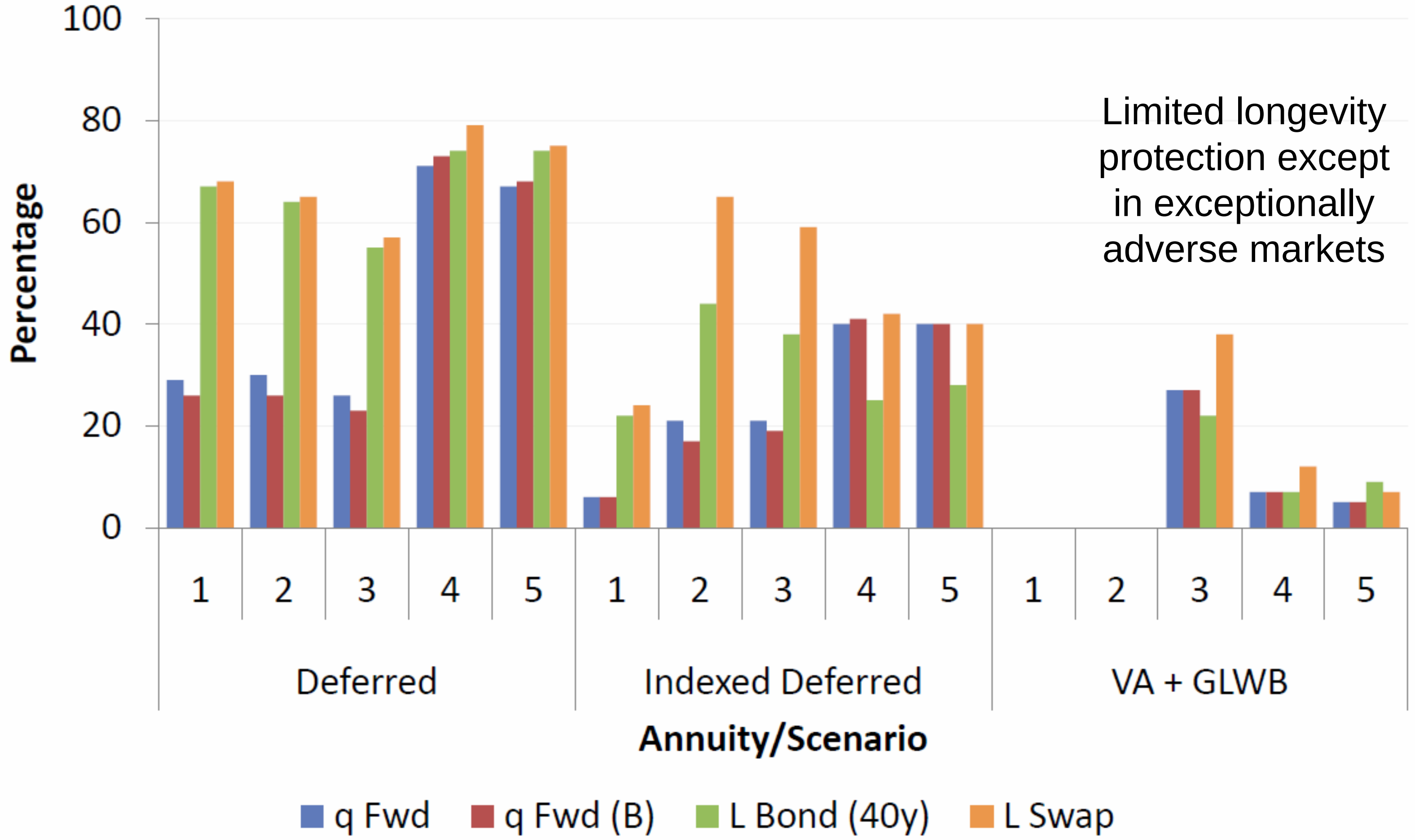


FIGURE 31: Avg Reduction in Expected Shortfall (Deferred Annuities and GLWB)

Results – Basis Risk

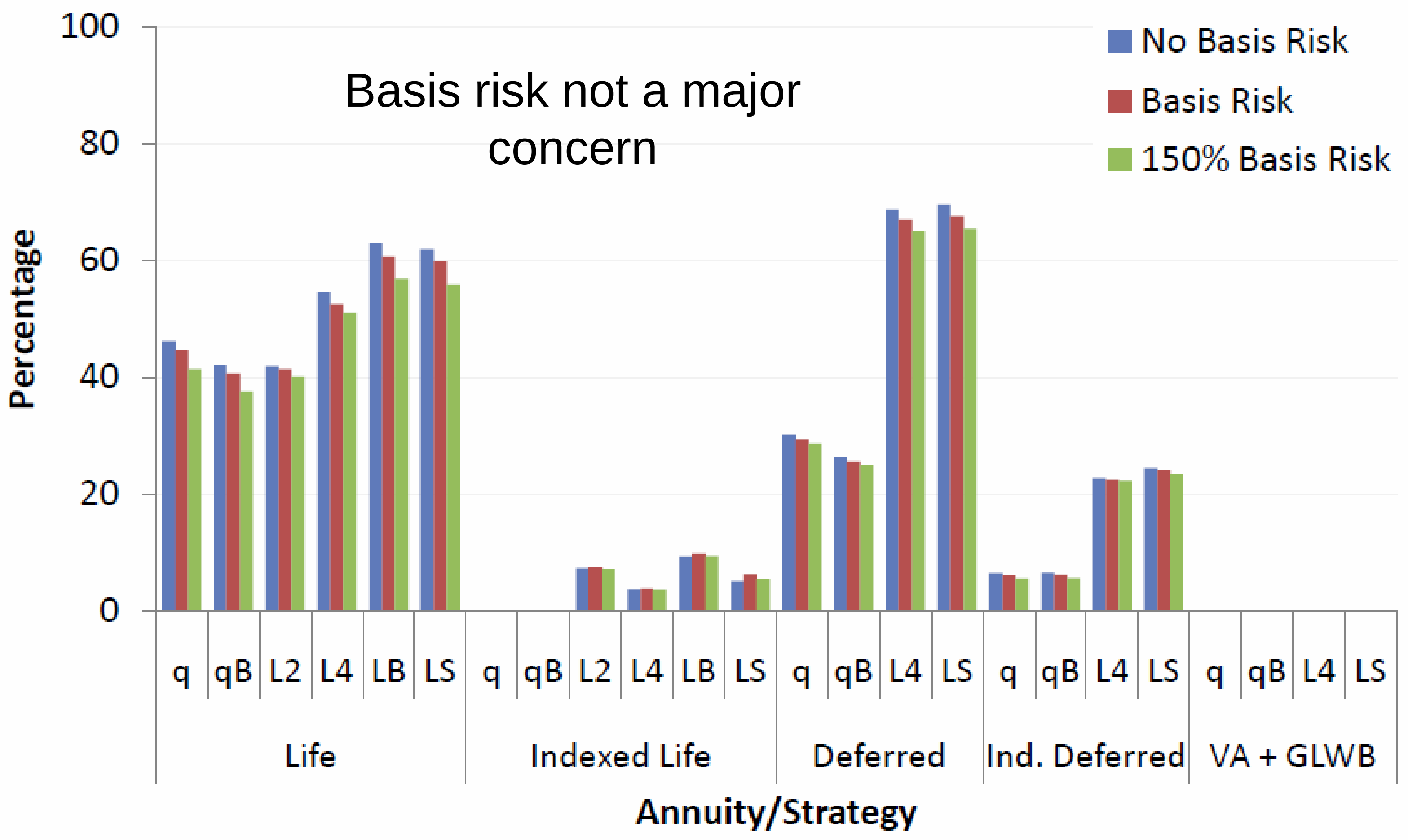


FIGURE 32: Reduction in Expected Shortfall: Effect of Basis Risk

Results – Hedging Cost

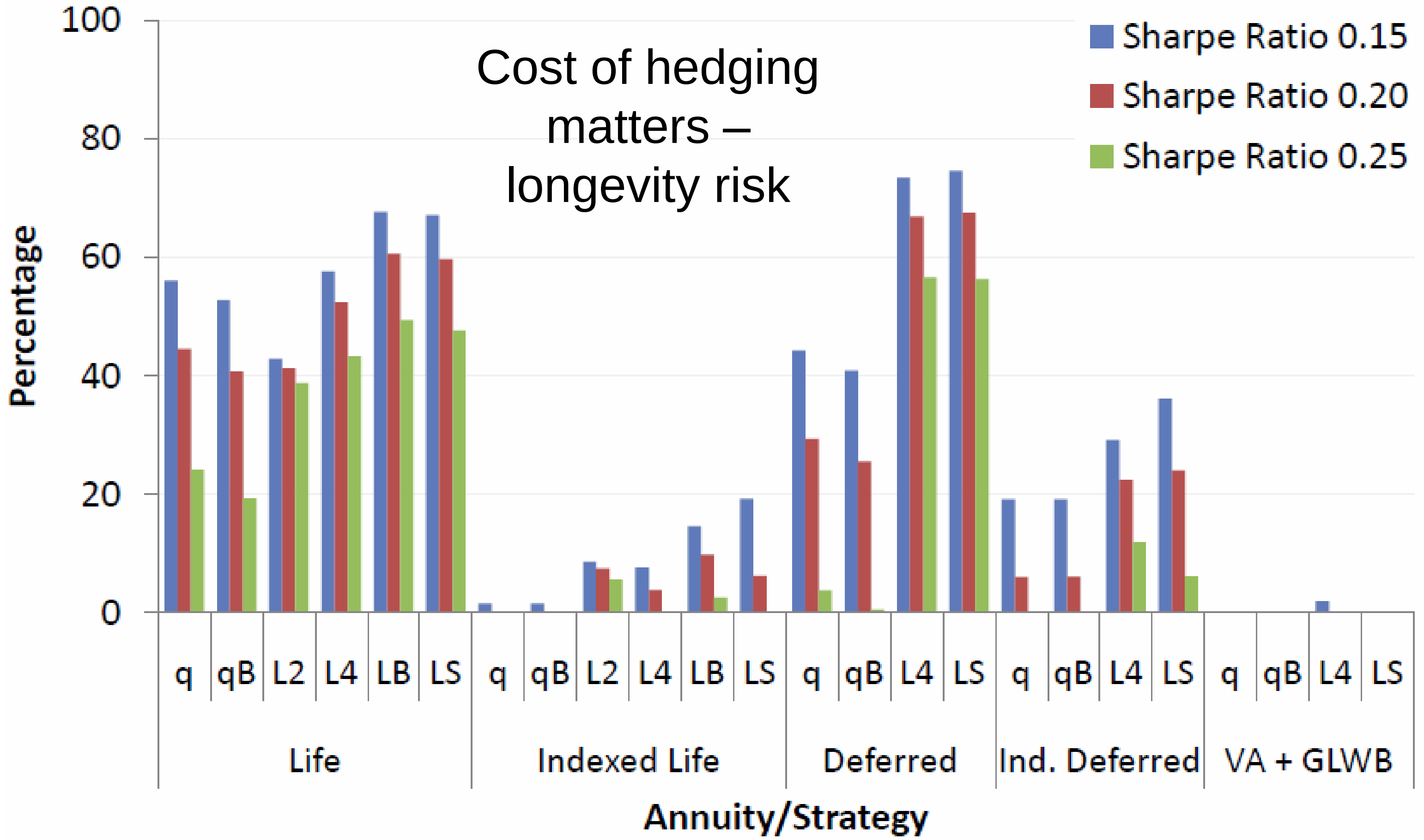


Figure 33: Reduction in Expected Shortfall: Sensitivity to Sharpe Ratio

Conclusions

- Static hedging (ALM) strategies reduce longevity risk particularly for life annuities (immediate and deferred)
- Much less effective for inflation indexed annuities (inflation risk predominates)
- VA with GLWB provide limited longevity protection and longevity hedging is of little value
- q-Forwards have additional basis risk over longevity bonds (mortality rates vs survival probabilities)
- Basis risk (annuitant vs population, bucketing) not critical for hedge effectiveness
- Cost of hedging (price of longevity risk) is an important factor for hedge effectiveness (both derivatives and longevity bonds)

Discussion and Q&A

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- **Longevity 6: 6th International Longevity Risk and Capital Markets Solutions Conference hosted by Australian Institute of Population Ageing Research, UNSW, 9-10 September 2010, Swiss Grande Bondi Beach, Sydney.** This is the major international conference bringing together leading international industry and academic minds as well as policy makers to meet and discuss the assessment of longevity risk, the market and government developments and responses needed by pension funds and insurance companies to manage this risk. Key themes are “Reinsurance and Financial Markets Solutions” and “Government Role, Public and Private Market Solutions”.