Longevity Risk Management for Life and Variable Annuities

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Introduction

- **Longevity risk management:** reinsurance, derivatives, longevity bonds

- **Products:** Annuities, Deferred Annuities, Variable Annuities (+GLWB)

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
\hline
\text{Life Expectancy at 65} & \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
\text{Annuity Market} & \text{Life & Term Annuities} & \text{Indexed Annuities} & \text{Variable Annuities} \\
\hline
\text{2001} & 74.3 & 6.8 & 113.3 \\
\text{2002} & 103.3 & 11.8 & 115.0 \\
\text{2003} & 89.4 & 14.4 & 126.4 \\
\text{2004} & 87.9 & 23.1 & 129.7 \\
\text{2005} & 79.5 & 27.2 & 133.1 \\
\text{2006} & 78.3 & 25.4 & 157.3 \\
\text{2007} & 72.8 & 21.8 & 182.2 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
\text{Year} & 1950 & 1970 & 1990 & 2010 \\
\hline
\text{Life Expectancy at 65} & \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
\text{Annuity Market} & \text{Life Annuities} & \text{Term Annuities} \\
\hline
\text{2001} & 166 & 794 \\
\text{2002} & 154 & 1,096 \\
\text{2003} & 200 & 1,357 \\
\text{2004} & 281 & 2,758 \\
\text{2005} & 27 & 548 \\
\text{2006} & 29 & 530 \\
\text{2007} & 36 & 787 \\
\hline
\end{array}
\]

**Table 1:** US Annuity Sales (US$b)  
(Source: Morningstar, Inc. and LIMRA International)

**Table 2:** Australian Annuity Sales (A$m)  
(Source: Plan for Life Research)
Challenges and Opportunities

- Successful provision of longevity insurance to individuals
  - longevity uncertain, systematic improvements, risk pooling less efficient
  - capital is costly
  - cost effective hedging to offer annuities
- Financial market developments
  - JPMorgan q-Forwards
  - Longevity swaps
  - Possibilities – government longevity bonds, securitization
Paper Coverage

- Life annuities and hedging instruments
- Market and mortality models
- Hedging strategies and effectiveness using Longevity Bonds and Derivatives
- Static hedging (ALM):
  - Dynamic hedging – lack of markets and liquidity
  - ALM and Risk based capital
Hedging Instruments

- **q-Forwards**
  - Pay actual $q_{x,t}$ in exchange for fixed $q^F_{x,t}$
  - Individual ages and 5-yr Bucketed

- **Coupon Longevity Bond**
  - Payments in line with actual survival probability $S_{65}(t)$

- **Zero Coupon Longevity Bond**
  - Single payment of $S_{65}(t)$ at maturity $t$
Hedging Strategies

**Figure 10:** Sample Cash Flows for Life Annuity with No Longevity Hedging (ZCBs)

**Figure 11:** Sample Cash Flows for Life Annuity with q-Forwards

**Figure 12:** Sample Cash Flows for Life Annuity with a 20% Longevity Bond

**Figure 14:** Sample Cash Flows for Life Annuity with a Longevity Swap (ZCLBs)
Market Model

Vector Error Correction Model with Regime Switching (RS-VECM)
- Long run equilibrium, volatility regimes

\[
\begin{align*}
\ln G_t &= \text{Log Gross Domestic Product (GDP)} \\
\ln B_t &= \text{Log Bond Index (Accumulated 90-day Bank-Accepted-Bill Yields)} \\
\ln S_t &= \text{Log Stock Price Index (ASX All Ordinaries)} \\
\ln F_t &= \text{Log Inflation Index (CPI)}
\end{align*}
\]

*Figure 16: Smoothed Regime Probabilities (Regime 1)*

*Figure 20: Overall Distribution of Simulated Annual Returns*  
(Note: Red = Historical Distribution)
Mortality – Longevity Model

Models mortality rates in cohort direction
Logit age structure for rate changes (stationary)
Age dependence using principal components

\[
\Delta \logit \mu_{x,t} \equiv \Delta \ln \frac{\mu_{x,t}}{1 - \mu_{x,t}} = a + bx + \varepsilon_{x,t}
\]

\[
\Delta \logit \mu_{x,t} = \logit \mu_{x,t} - \logit \mu_{x-1,t-1}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Male (\times 10^{-2})</th>
<th>Female (\times 10^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>-0.3516</td>
<td>-0.3320</td>
</tr>
<tr>
<td>(b)</td>
<td>0.1054</td>
<td>0.1244</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.0891</td>
<td>0.0840</td>
</tr>
</tbody>
</table>

Table 6: Parameter Estimates for Mortality Model (MLE)

<table>
<thead>
<tr>
<th>Number of Eigenvectors</th>
<th>Percentage Explained (Male)</th>
<th>Percentage Explained (Female)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>77.5</td>
<td>77.9</td>
</tr>
<tr>
<td>2</td>
<td>82.9</td>
<td>83.3</td>
</tr>
<tr>
<td>3</td>
<td>88.1</td>
<td>86.6</td>
</tr>
<tr>
<td>4</td>
<td>90.1</td>
<td>88.6</td>
</tr>
<tr>
<td>5</td>
<td>91.7</td>
<td>90.5</td>
</tr>
<tr>
<td>9</td>
<td>95.3</td>
<td>94.8</td>
</tr>
<tr>
<td>10</td>
<td>96.0</td>
<td>95.6</td>
</tr>
<tr>
<td>15</td>
<td>98.4</td>
<td>98.1</td>
</tr>
<tr>
<td>20</td>
<td>99.5</td>
<td>99.4</td>
</tr>
<tr>
<td>25</td>
<td>99.9</td>
<td>99.9</td>
</tr>
<tr>
<td>30</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 7: Proportion of Observed Variation (in Standardised Residuals) Explained

Figure 24: Projected Male Mortality Rates for Cohort 65 (with 95% CIs)
Mortality Model – Basis Risk

Portfolio of annuitants – hedging instruments based on population index

\[ \rho_{x,t} = \frac{q_{x,t}^A}{q_{x,t}} \]
\[ \rho_{x,t} = \alpha + \beta x + \nu_{x,t} \]
\[ \nu_{x,t} \sim N(0, \theta^2) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-0.1381</td>
<td>-0.1648</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.0110</td>
<td>0.0111</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.0668</td>
<td>0.0846</td>
</tr>
</tbody>
</table>

Table 9: Parameter Estimates for Annuitant-Population Ratio Model

![Graphs showing observed ratios of annuitant/pensioner to population mortality for males and females.](image)

Figure 25: Observed Ratios of Annuitant/Pensioner to Population Mortality
# Scenario Analysis

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Market Assumption</th>
<th>Mortality Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Stochastic</td>
<td>Stochastic</td>
</tr>
<tr>
<td>2</td>
<td>Average</td>
<td>Stochastic</td>
</tr>
<tr>
<td>3</td>
<td>Adverse (1930–75)</td>
<td>Stochastic</td>
</tr>
<tr>
<td>4</td>
<td>Stochastic</td>
<td>Avg + Excess Imp. (2% / yr Acc)</td>
</tr>
<tr>
<td>5</td>
<td>Stochastic</td>
<td>Avg + Excess Imp. (25% Flat)</td>
</tr>
</tbody>
</table>

**Figure 27:** Simulated Distribution of Final Surplus $U_T$

(Note: Blue = Life Annuity, Green = Indexed Annuity)
Risk Measures for Longevity

<table>
<thead>
<tr>
<th>Annuity</th>
<th>Mean (No Hedging)</th>
<th>ES$_{0.05}$ (No Hedging)</th>
<th>ES$_{0.05}$ (No L Risk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life</td>
<td>150.3</td>
<td>54.6</td>
<td>116.2</td>
</tr>
<tr>
<td>Indexed Life</td>
<td>81.5</td>
<td>-642.3</td>
<td>-587.9</td>
</tr>
<tr>
<td>Deferred</td>
<td>14.7</td>
<td>-30.6</td>
<td>7.7</td>
</tr>
<tr>
<td>Def. Indexed</td>
<td>7.5</td>
<td>-133.5</td>
<td>-106.2</td>
</tr>
<tr>
<td>VA + GLWB</td>
<td>121.43</td>
<td>40.80</td>
<td>41.27</td>
</tr>
</tbody>
</table>

Table 10: 5% Expected Shortfall (No Hedging vs No Longevity Risk)

<table>
<thead>
<tr>
<th>Annuity</th>
<th>$\mu - ES_{0.05}$ (Stochastic)</th>
<th>$\mu - ES_{0.05}$ (Average)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life</td>
<td>95.7</td>
<td>88.5</td>
<td>8%</td>
</tr>
<tr>
<td>Indexed Life</td>
<td>723.8</td>
<td>173.8</td>
<td>76%</td>
</tr>
<tr>
<td>Deferred</td>
<td>45.3</td>
<td>42.8</td>
<td>6%</td>
</tr>
<tr>
<td>Indexed Def.</td>
<td>141.0</td>
<td>54.4</td>
<td>61%</td>
</tr>
<tr>
<td>VA + GLWB</td>
<td>80.6</td>
<td>5.3</td>
<td>93%</td>
</tr>
</tbody>
</table>

Table 11: Change in 5% Expected Shortfall (Stochastic vs Average Market)
Risk Measures for Longevity Risk

Indexed annuities have substantial shortfall risk - inflation

VA + GLWB provides limited longevity protection – hedging has little impact
Results – Annuities (Life/Indexed)

Hedging less effective because of inflation indexation

Figure 30: Avg Reduction in Expected Shortfall (Immediate Annuities)
Results – Deferred Annuities/GLWB

Limited longevity protection except in exceptionally adverse markets

Figure 31: Avg Reduction in Expected Shortfall (Deferred Annuities and GLWB)
Results – Basis Risk

Basis risk not a major concern

Figure 32: Reduction in Expected Shortfall: Effect of Basis Risk
Results – Hedging Cost

Cost of hedging matters – longevity risk

**Figure 33:** Reduction in Expected Shortfall: Sensitivity to Sharpe Ratio
Conclusions

• Static hedging (ALM) strategies reduce longevity risk particularly for life annuities (immediate and deferred)
• Much less effective for inflation indexed annuities (inflation risk predominates)
• VA with GLWB provide limited longevity protection and longevity hedging is of little value
• q-Forwards have additional basis risk over longevity bonds (mortality rates vs survival probabilities)
• Basis risk (annuitant vs population, bucketing) not critical for hedge effectiveness
• Cost of hedging (price of longevity risk) is an important factor for hedge effectiveness (both derivatives and longevity bonds)
Discussion and Q&A

• **Acknowledgements:** Ngai acknowledges the support of the Brian Gray Scholarship from the Australian Prudential Regulatory Authority and the Australian School of Business Supporters Circle Honours Scholarship. Sherris acknowledges the support of ARC Linkage Grant Project LP0883398 Managing Risk with Insurance and Superannuation as Individuals Age with industry partners PwC and APRA and Financial support from the Institute of Actuaries of Australia UNSW Actuarial Foundation.

• **Longevity 6: 6th International Longevity Risk and Capital Markets Solutions Conference** hosted by Australian Institute of Population Ageing Research, UNSW, 9-10 September 2010, Swiss Grande Bondi Beach, Sydney. This is the major international conference bringing together leading international industry and academic minds as well as policy makers to meet and discuss the assessment of longevity risk, the market and government developments and responses needed by pension funds and insurance companies to manage this risk. Key themes are “Reinsurance and Financial Markets Solutions” and “Government Role, Public and Private Market Solutions”.