Reserving methods: future trends

Greg Taylor

Taylor Fry Consulting Actuaries
University of Melbourne
University of New South Wales

Overview

- Discussion focuses on stochastic reserving models
- Some comments on current stochastic reserving practices
- Discussion of some of the more advanced models currently available
- Examination of some extensions of these that are within reach in the near future

Stochastic models

Opening observations

General framework

- Data vector Y
- Model $Y=f(\beta)+\varepsilon$

Parameter vector

Stochastic error vector

- Estimate $\hat{\beta}$ of β
- Future observation vector $Z=g(\beta)+\eta$
- Forecast $Z^*=g(\hat{\beta})$ of Z
- Prediction error $Z-Z^*=[g(\beta) - g(\hat{\beta})] + \eta$

Estimating prediction error

- Prediction error $Z\text{-}Z^*=[g(\beta) - g(\hat{\beta})] + \eta$

Parameter error

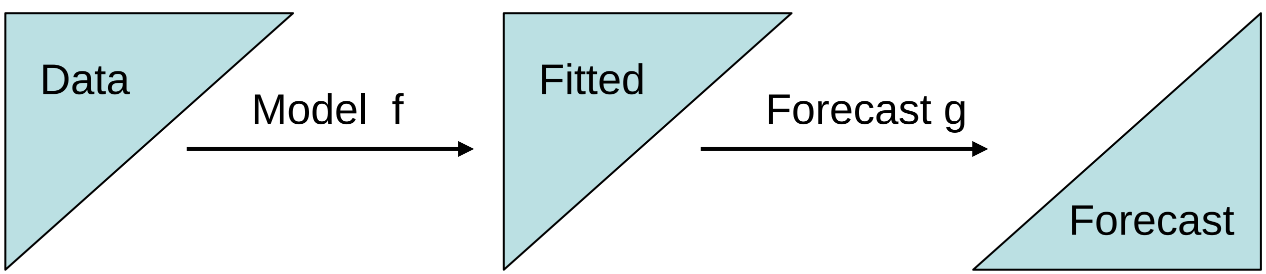
Process error

- Both errors estimated in terms of residuals of data with respect to model

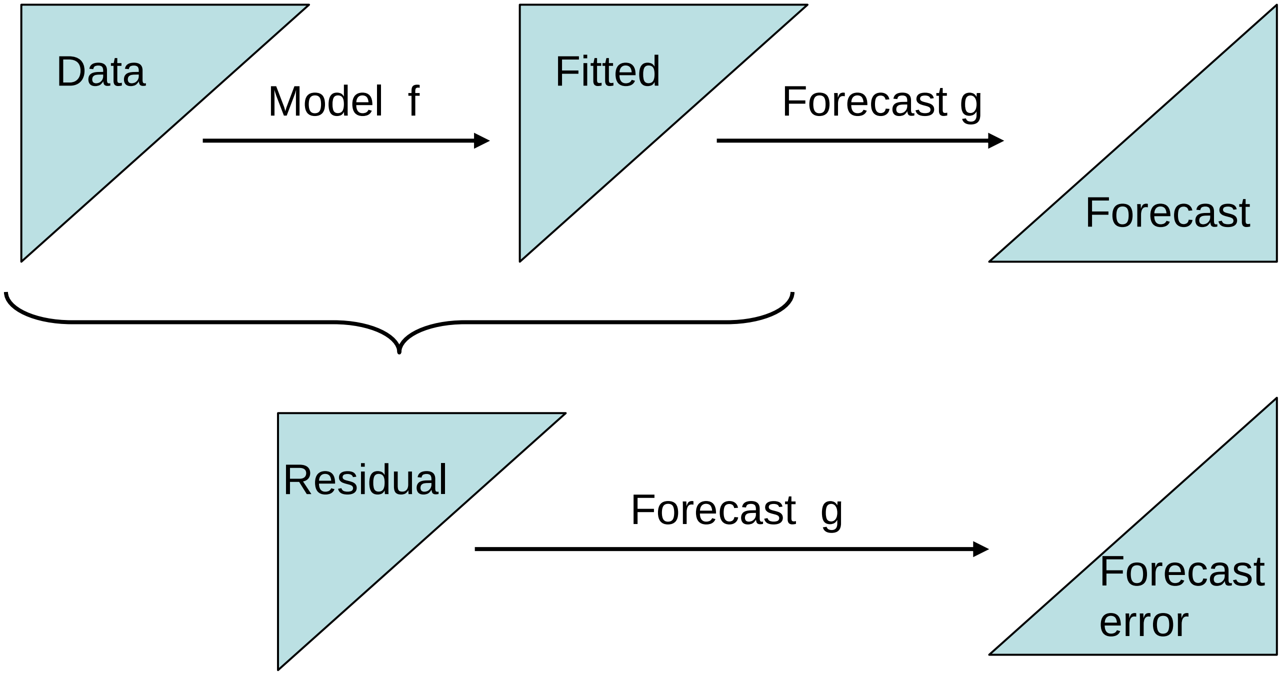
$$R=Y-\hat{Y}=Y-f(\hat{\beta})$$

- Ultimately distributional properties of Z^* depend on R(f) and g

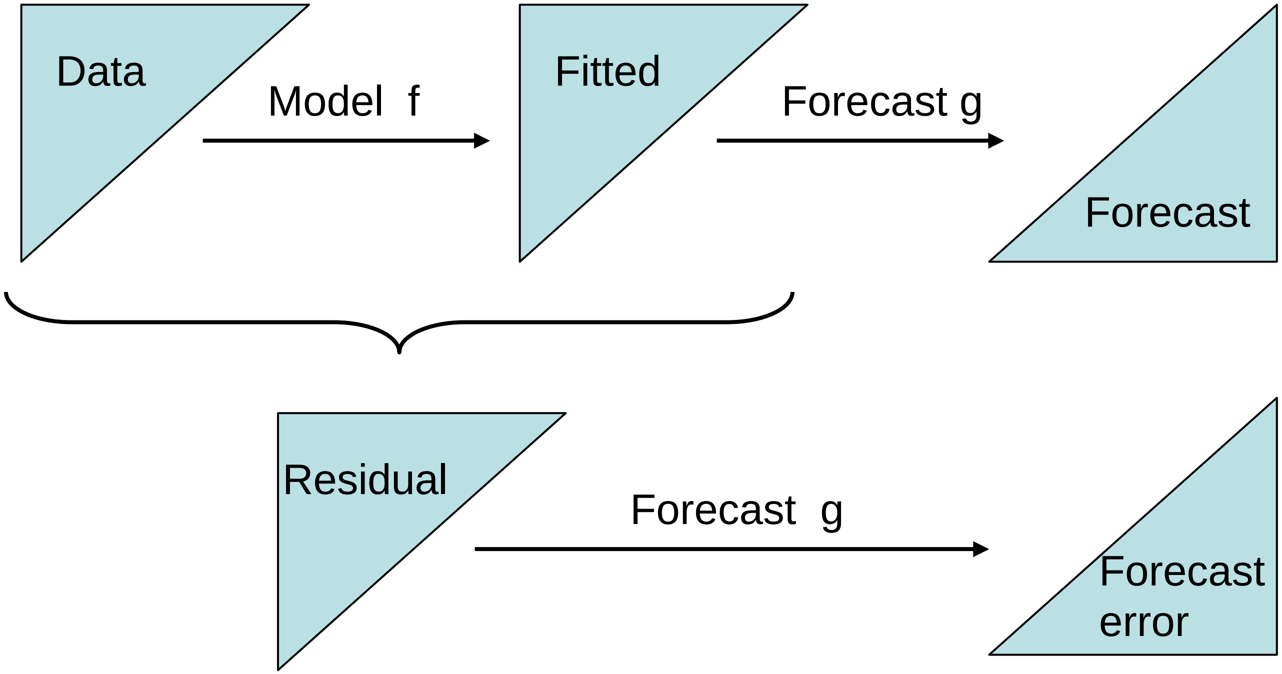
Pictorially



Pictorially

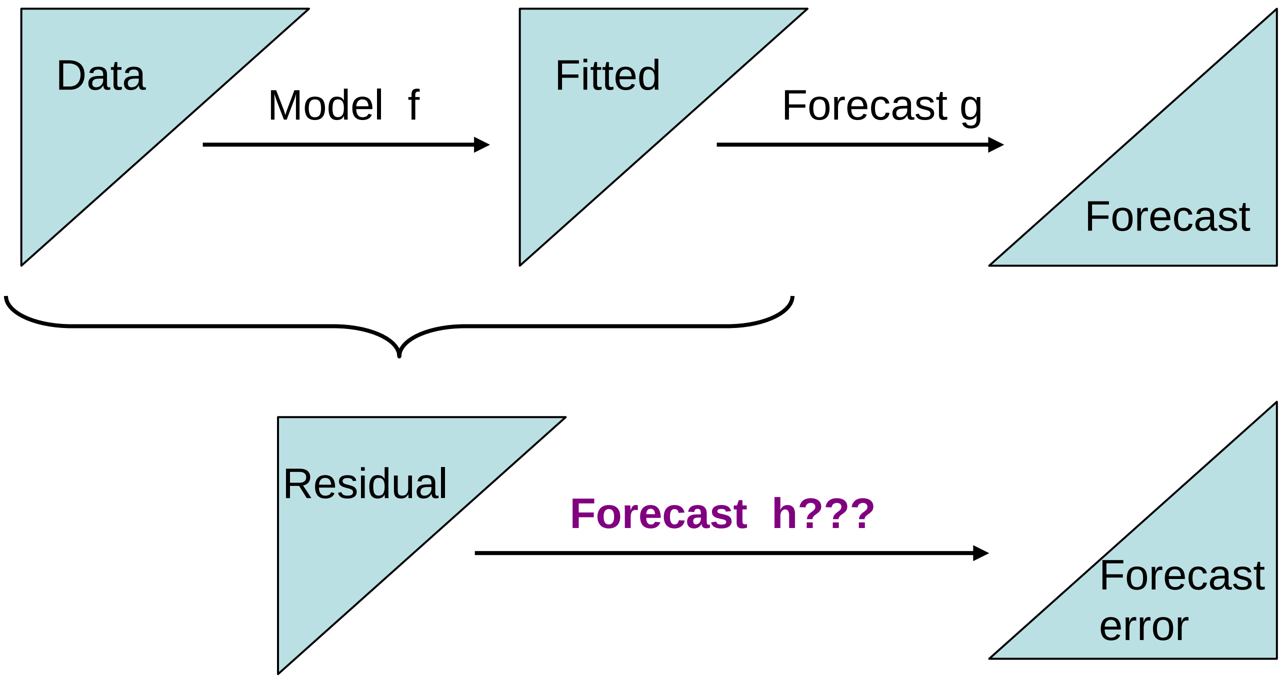


Pictorially

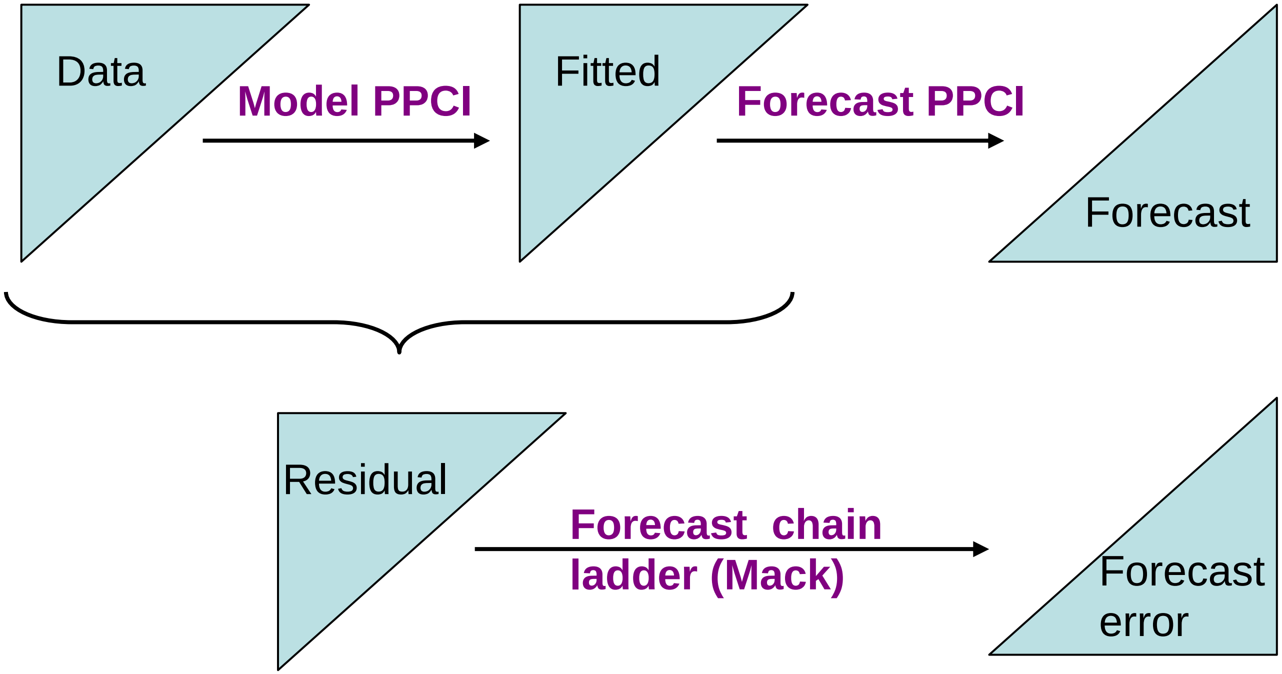


Call this case of forecast error estimation **coherent**

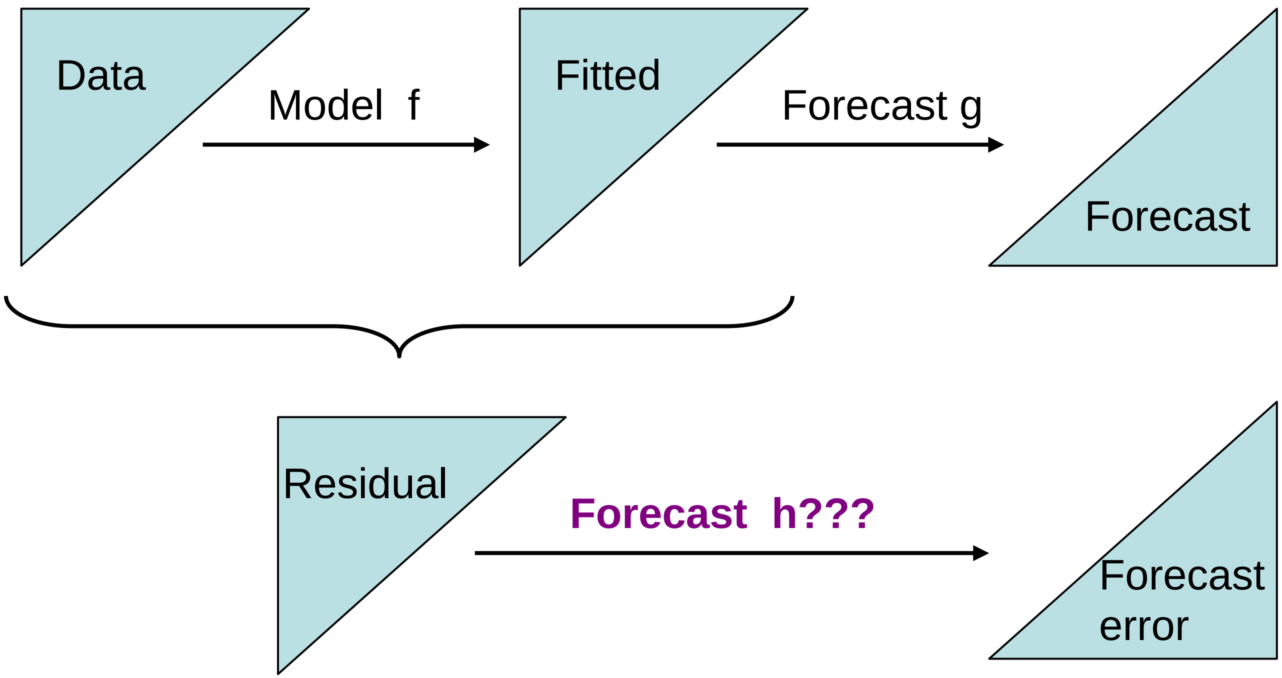
Pictorially



For example

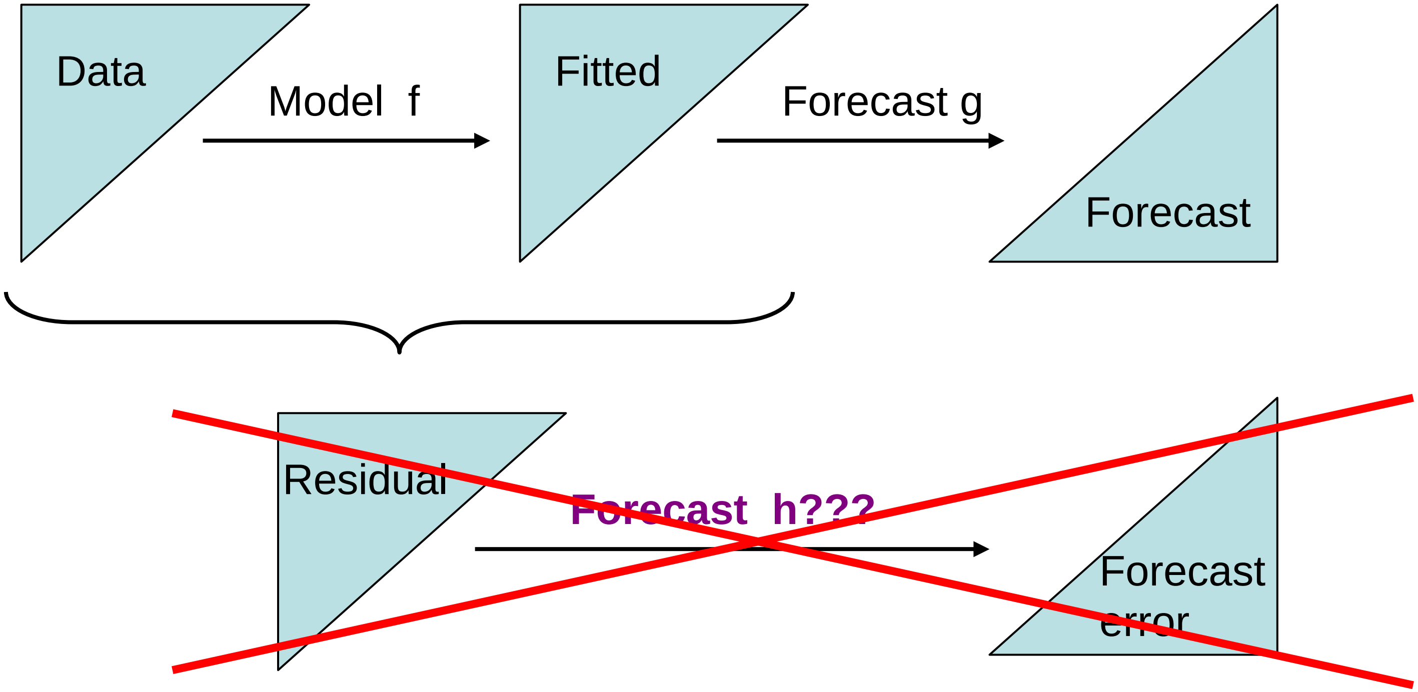

Data
Model PPCI
Fitted
Forecast PPCI
Forecast
Residual
Forecast chain ladder (Mack)
Forecast error

Pictorially



Call this case of forecast error estimation **incoherent**

Pictorially



Conclusion 1

- Any incoherent estimation of stochastic properties of a loss reserve is meaningless

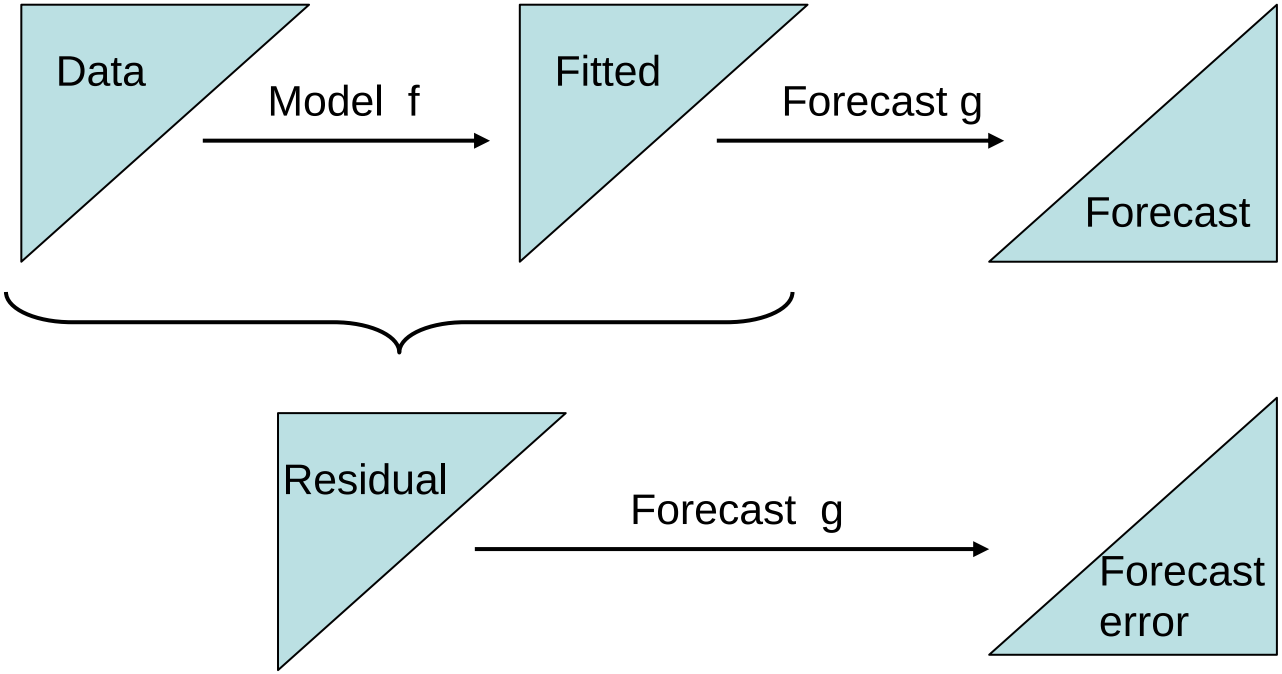
Available options for forecast error estimation

- Only two
  - **Internal estimation**
    - Based on measured error between data and model (such as just illustrated)
    - Good for capturing features inherent in the model
      - Parameter error
      - Process error
  - **External estimation**
    - Based on
      - Identification of specific components of forecast error (see O'Dowd, Smith & Hardy, 2005) e.g.
        - » Future changes in superimposed inflation
        - » Generally systemic changes that are not well represented in past data
      - Judgmental assessment of their contributions

Conclusion 2

- Ideally, forecast error should be composed of
  - Internal estimates
    - Parameter error
    - Process error
  - External estimates
    - Model specification error
    - Errors due to other systemic effects

Internal estimation of forecast error

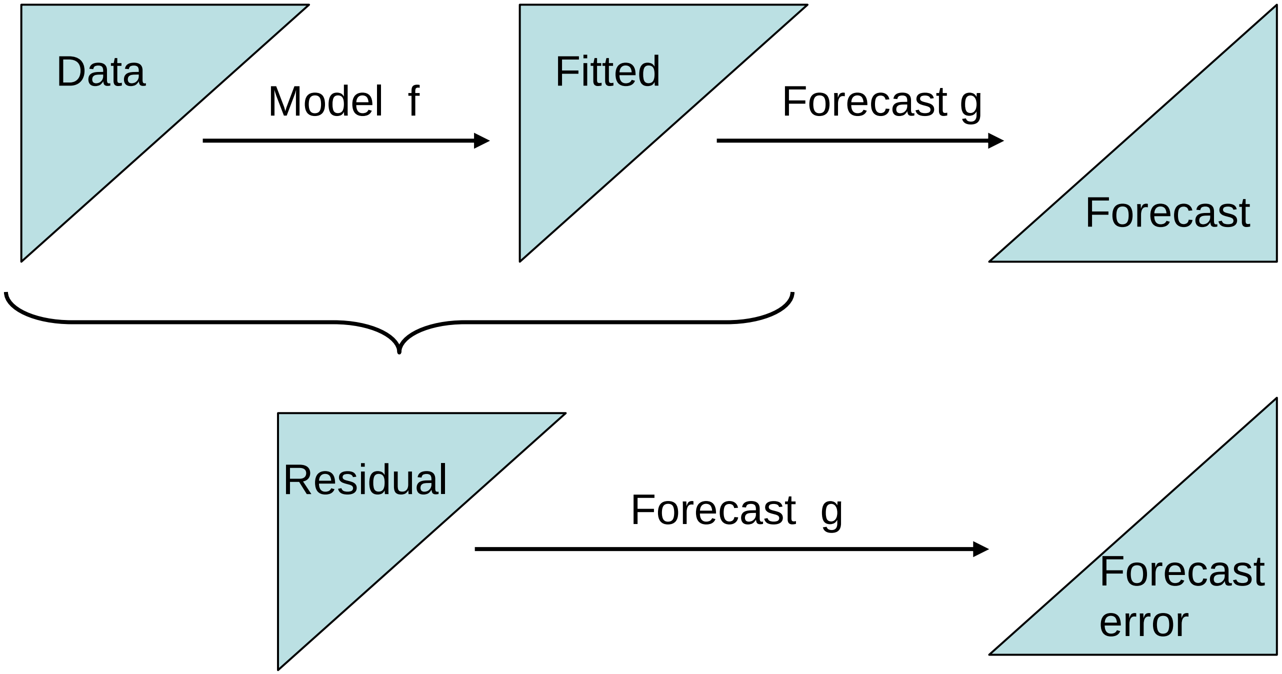


Large
residuals imply
large forecast
error

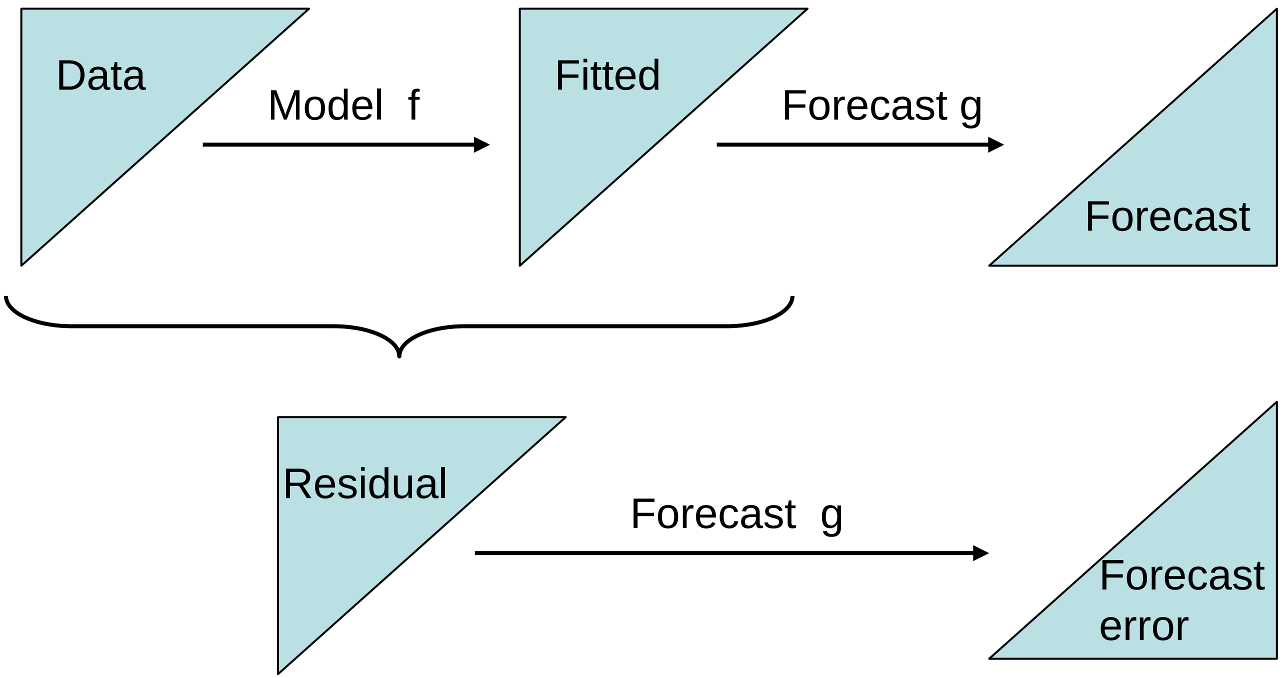
Conclusion 3

- Good models produce low forecast error (CoV)
  - Economic in use of capital
- Poor models produce high forecast error
  - Uneconomic in use of capital

Internal estimation of forecast error



Internal estimation of forecast error

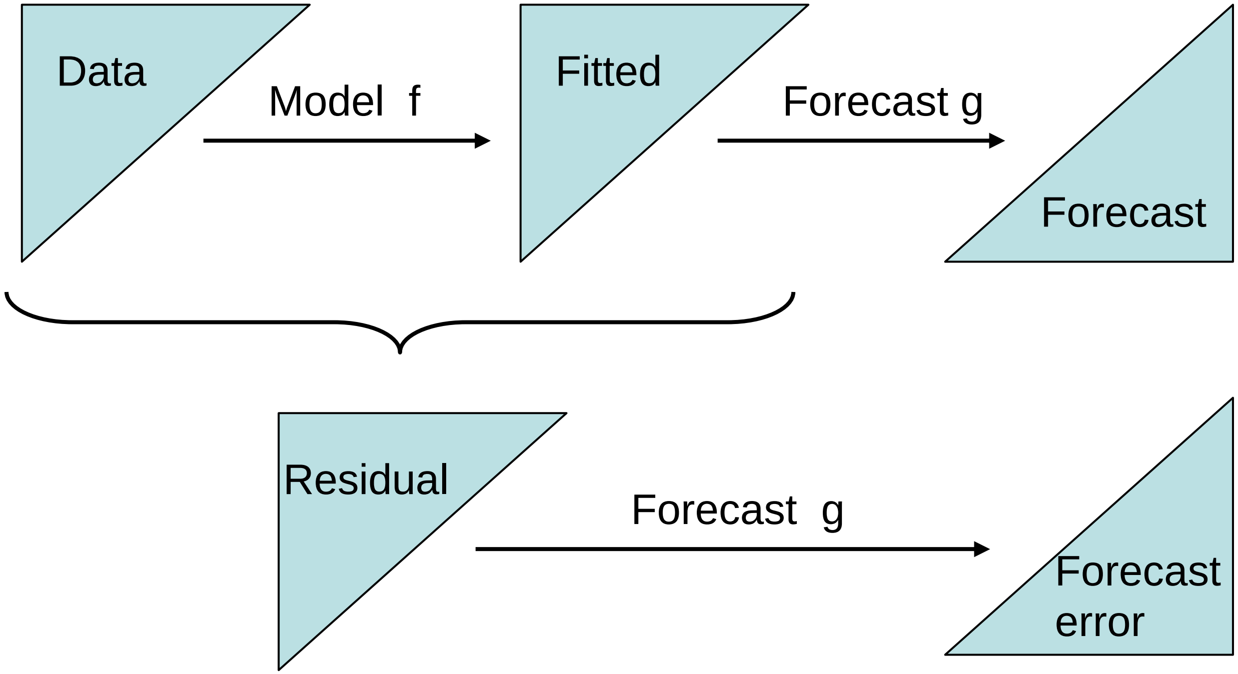


This is what the bootstrap does

Bootstrapping

- One internal form of forecast error estimation
- Are there others?
- Very rarely
  - Due to intractable mathematical complexity in mapping residuals to forecast error
- So need to make the bootstrap work

Bootstrap


Data
Model f
Fitted
Forecast g
Forecast
Residual
Forecast g
Forecast
error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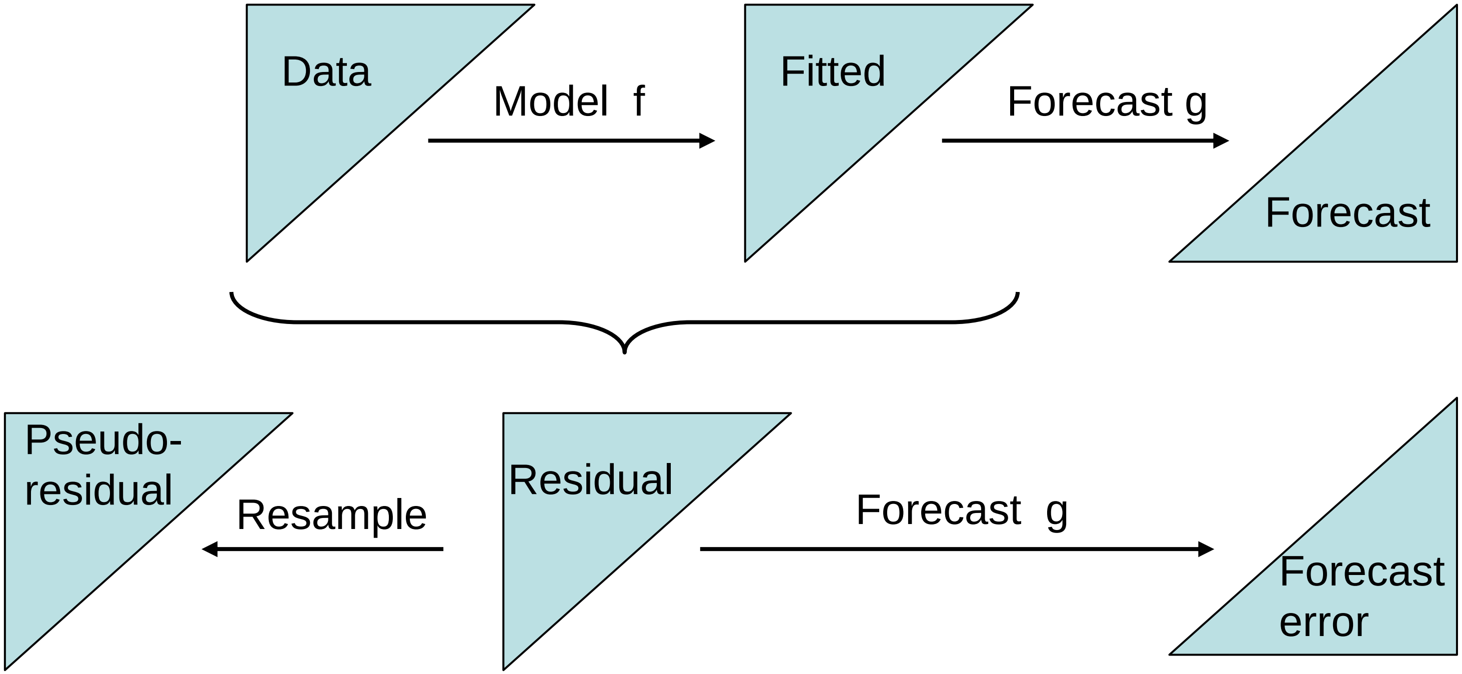
Bootstrap


Data
Model f
Fitted
Forecast g
Forecast
Pseudo-residual
Resample
Residual
Forecast g
Forecast error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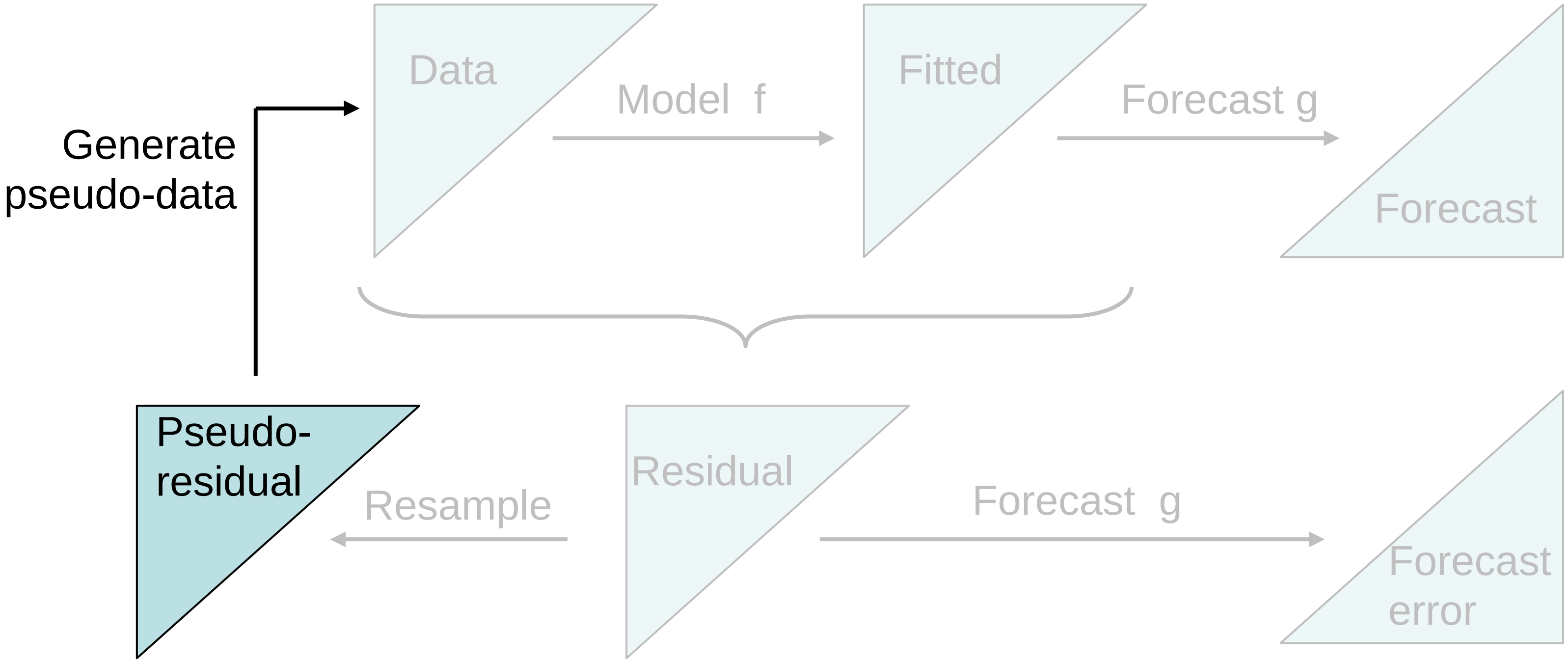
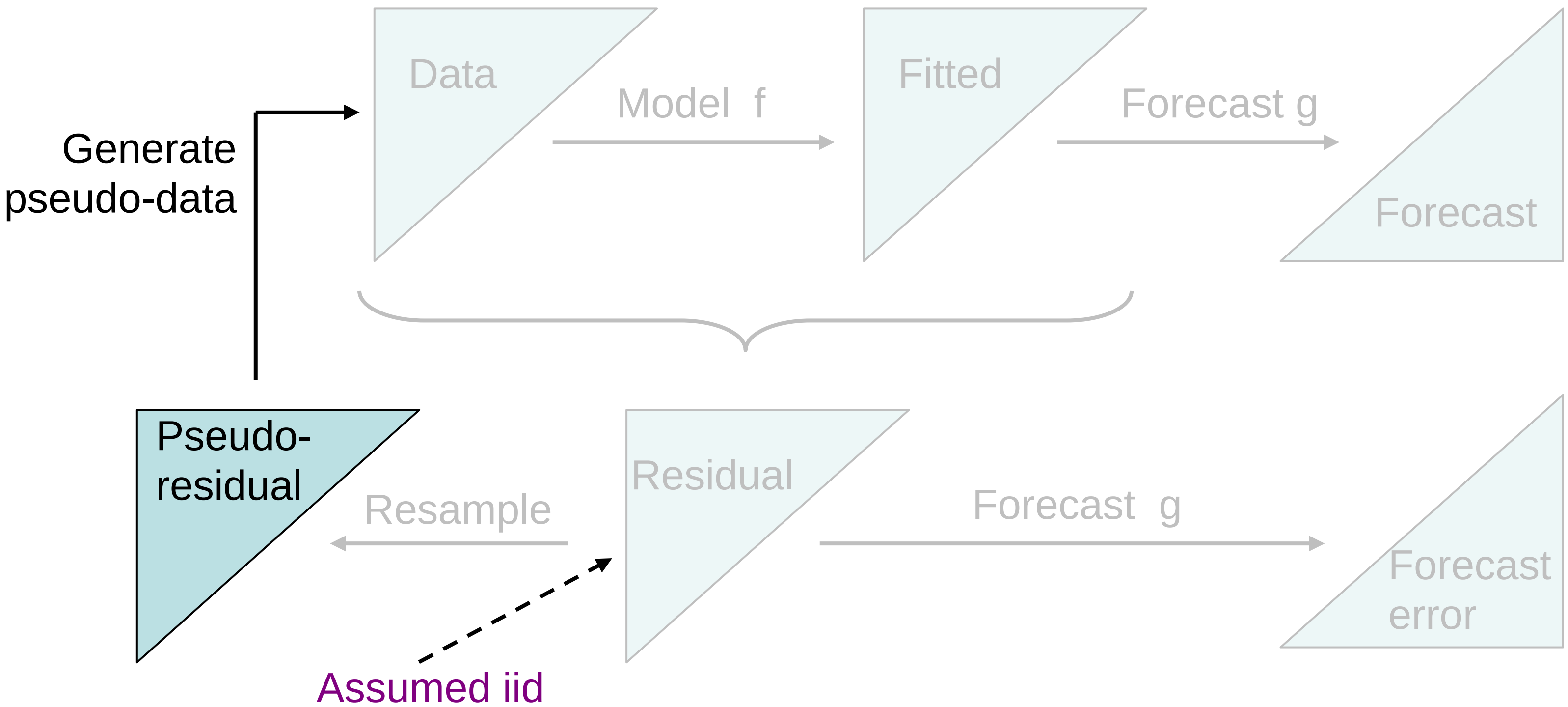

Data
Model f
Fitted
Forecast g
Forecast
Generate
pseudo-data
Pseudo-
residual
Resample
Residual
Forecast g
Forecast
error

Bootstrap


Generate
pseudo-data
Data
Model f
Fitted
Forecast g
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residual
Resample
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Forecast
error
Assumed iid


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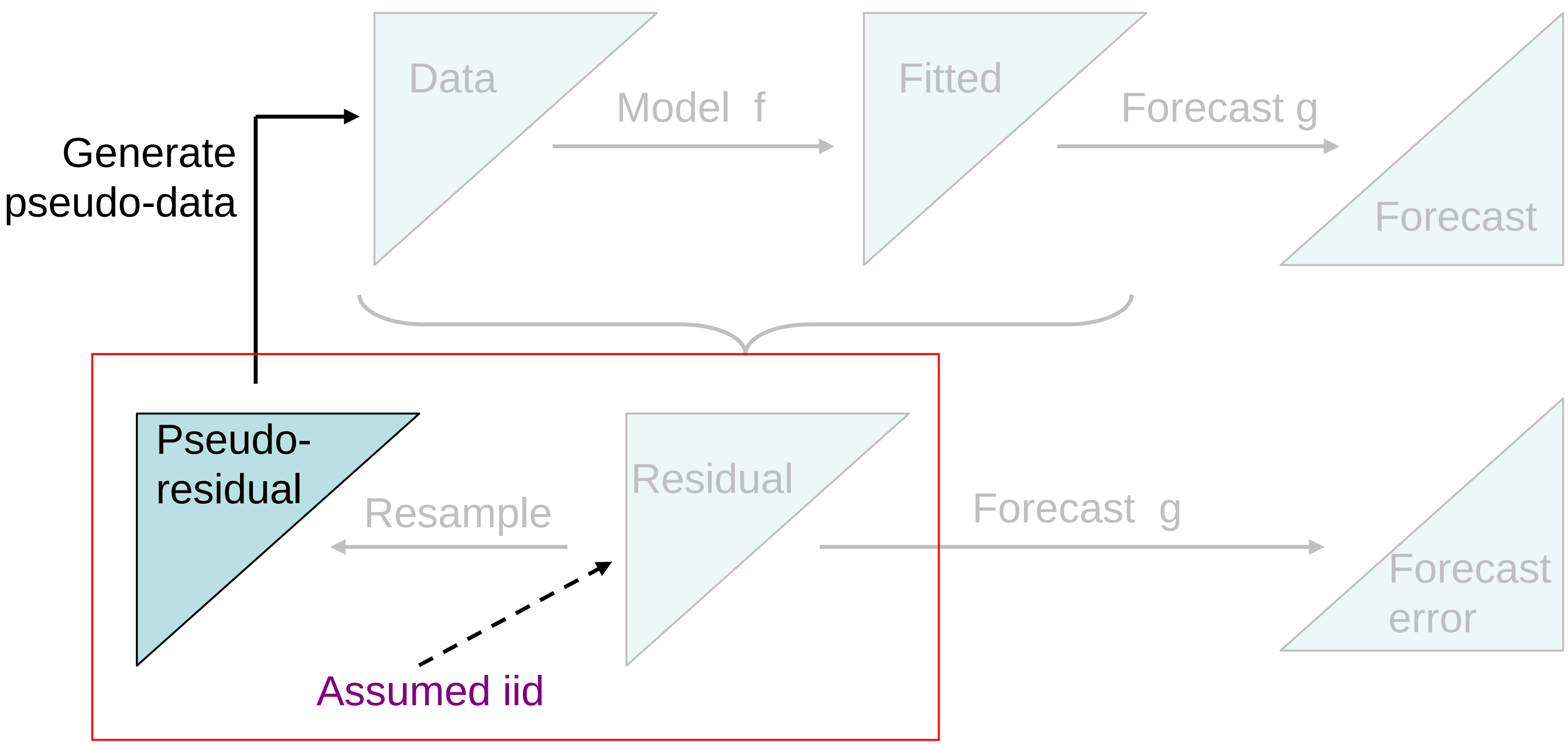
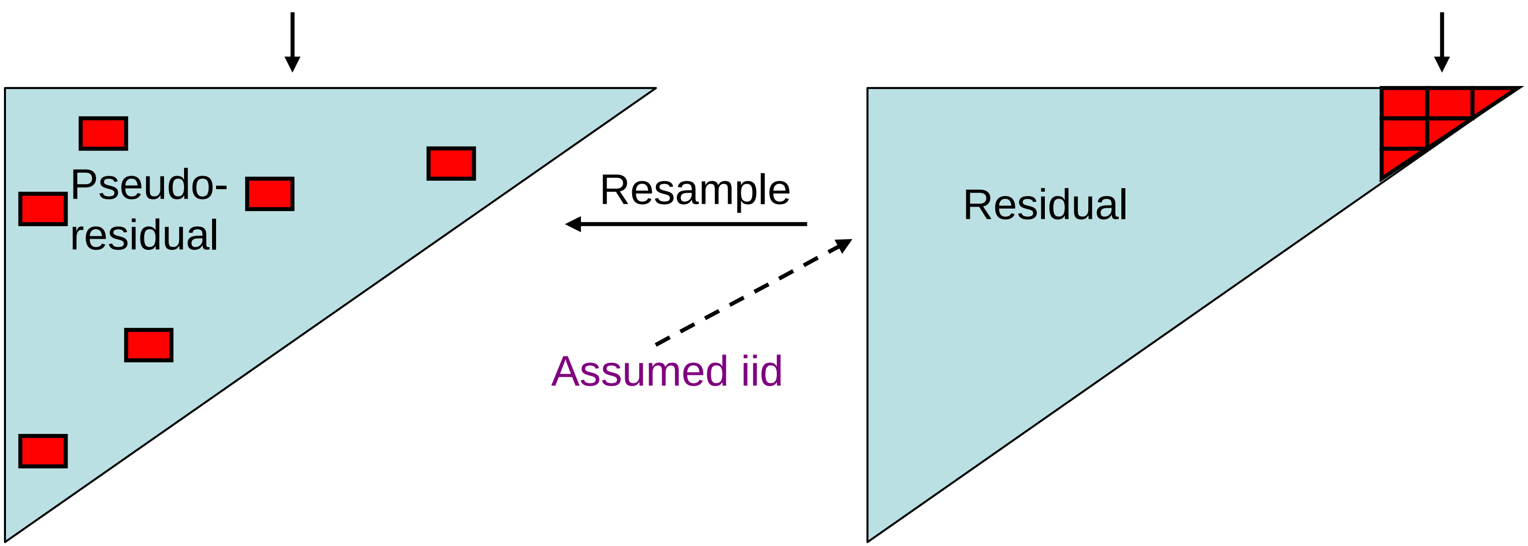
Bootstrap


Data
Model f
Fitted
Forecast g
Forecast
Generate
pseudo-data
Pseudo-
residual
Resample
Residual
Forecast g
Forecast
error
Assumed iid

What happens when residuals not iid? - example

Residuals assume more influential positions – can distort model and forecast

Actually large residuals

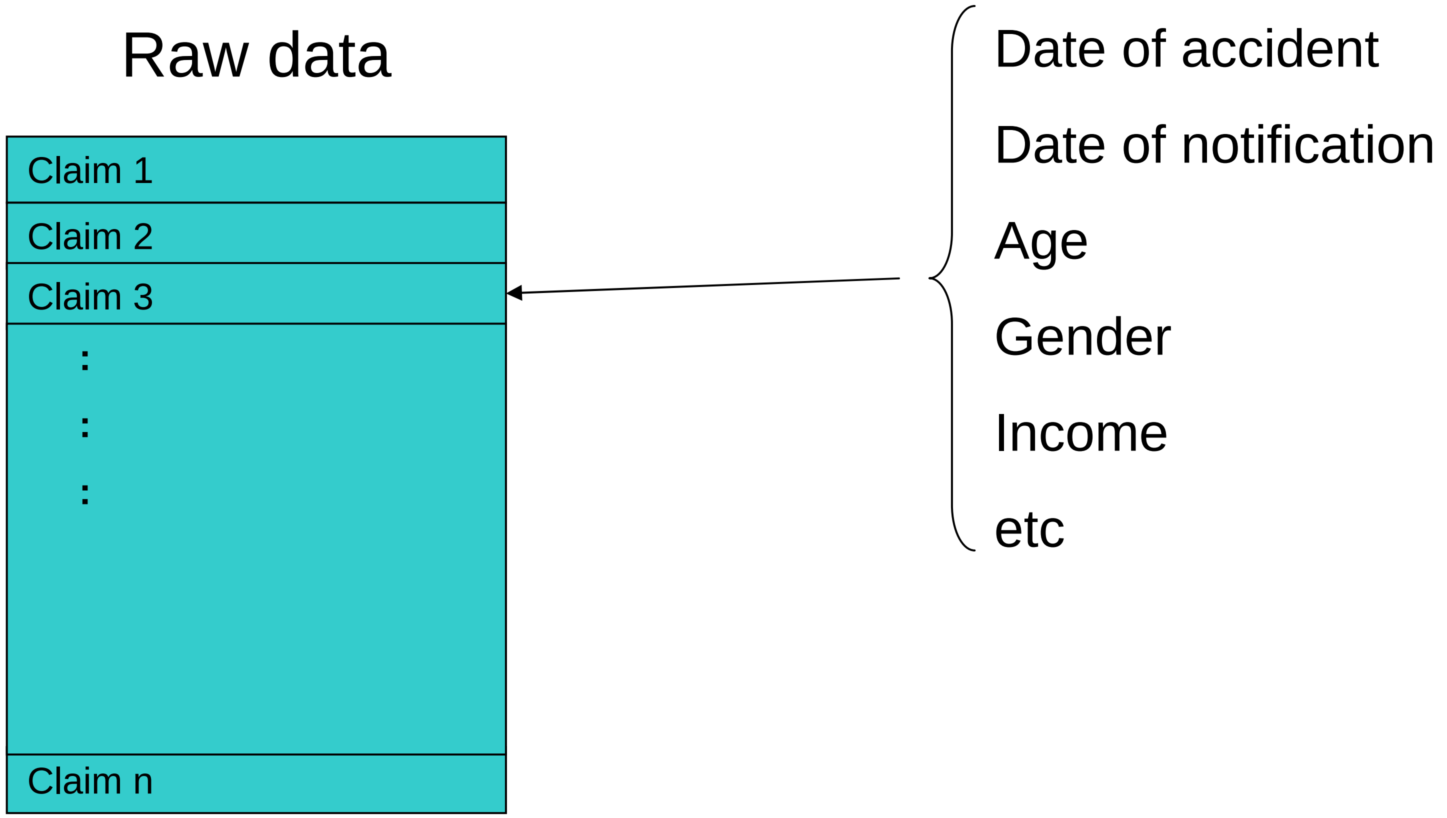


Conclusion 4

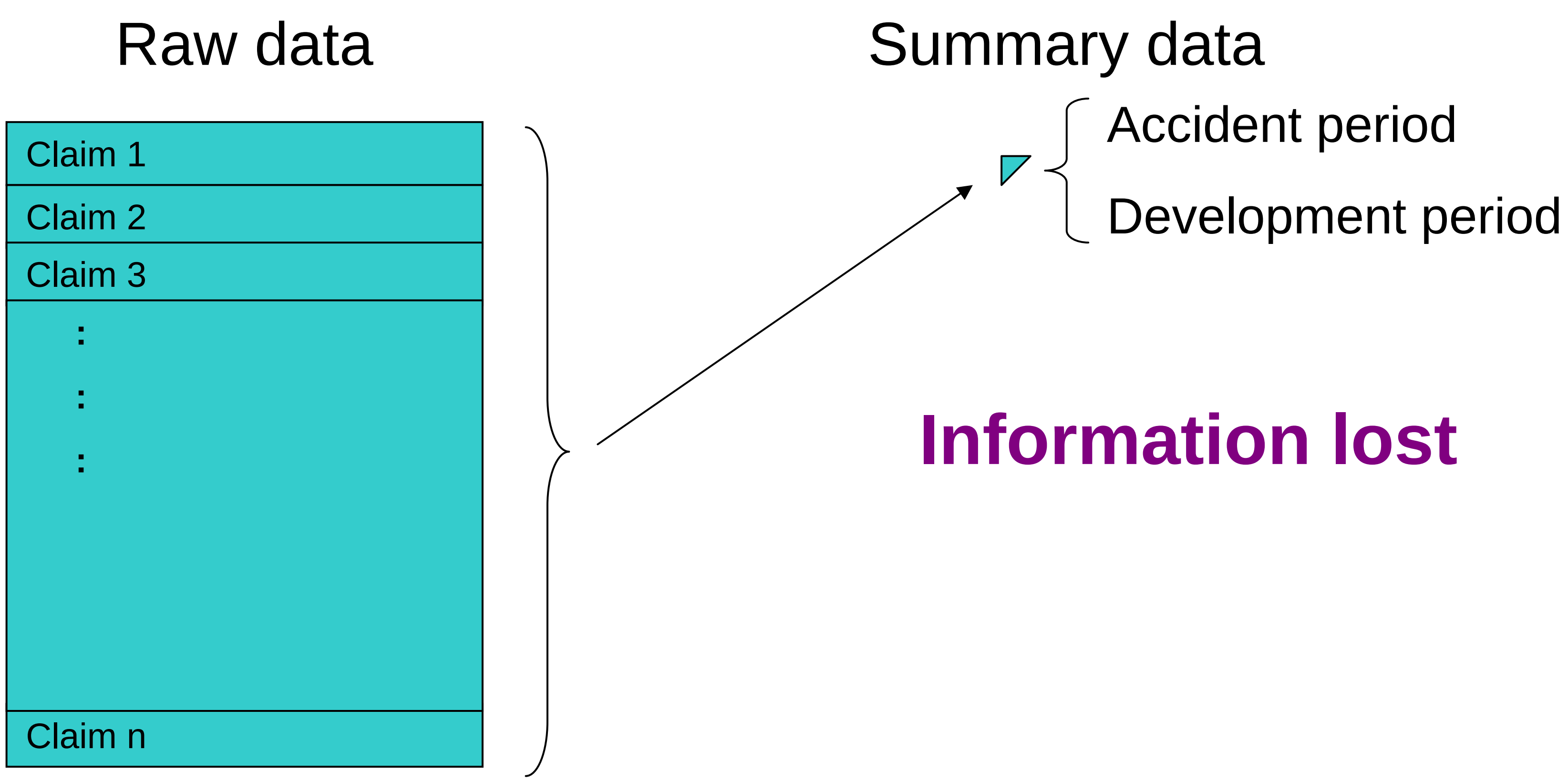
- Particular care is needed to ensure that model residuals are consistent with iid assumption if ludicrous bootstrap results are to be avoided

Individual claim reserving
and
Statistical case estimation

Reserving data treatment

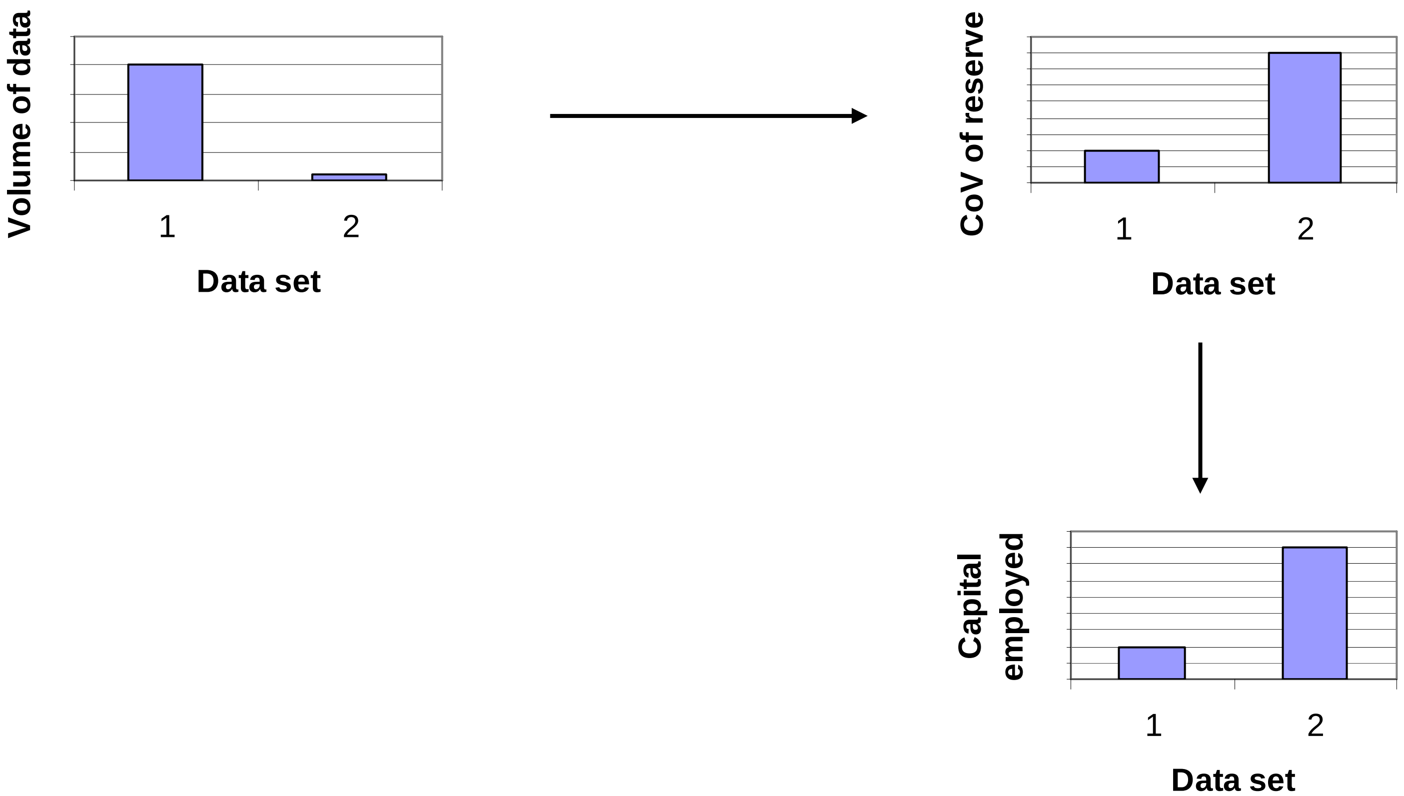


Reserving data treatment



Information lost

Why does quantity of data matter?



Individual claim modelling

- Data vector Y
- Model $Y=f(\beta)+\varepsilon$
- Let components Y_i of Y relate to individual claims
  - Y_i denotes some outcome for the i-th claim, e.g. finalised size, paid to date, etc.
- Call this model an **individual claim model**
- Call a reserve based on such a model an **individual claim reserve**

Example

Y_i = finalised individual claim size

Y_i ~ Gamma

$E[Y_i]$

= exp {function of operational time

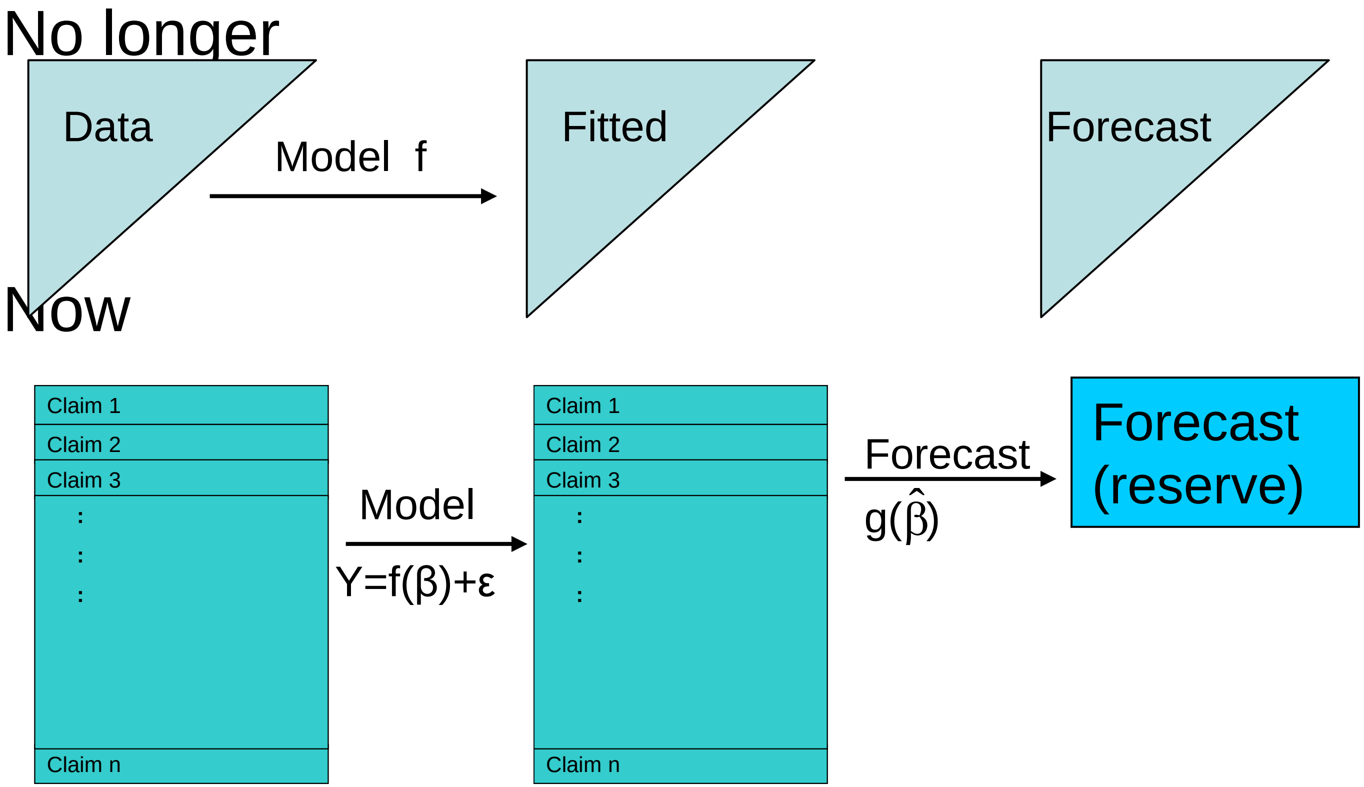
+ function of accident period (legislative change)

+ function of finalisation period (superimposed inflation

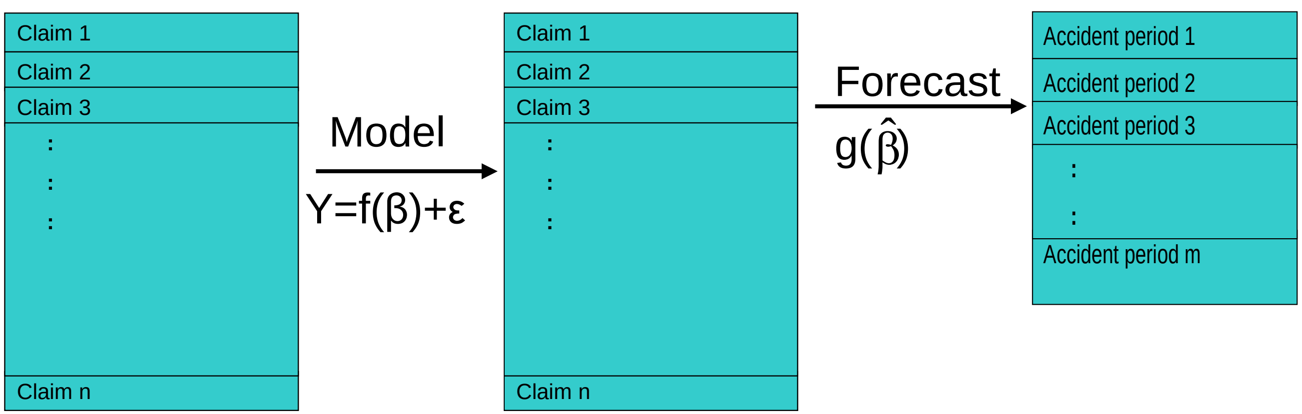
+ joint function (interaction)of operational time & accident period (change in payment pattern attributable to legislative change)}

Discussion in Taylor & McGuire (2004)

Pictorially

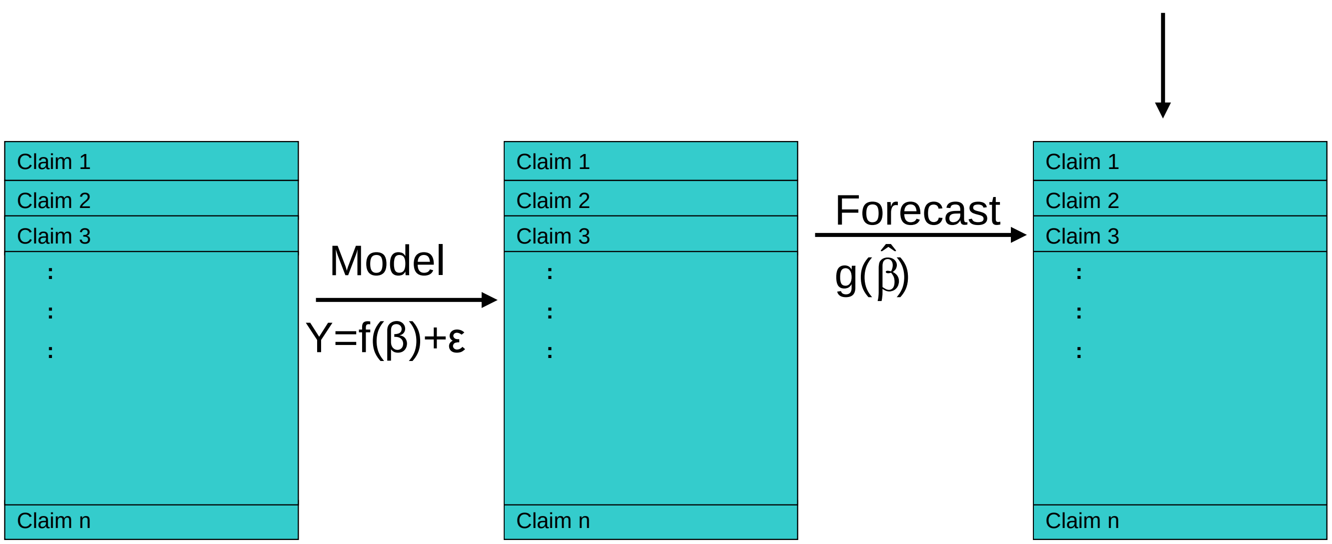


For example


Claim 1
Claim 2
Claim 3
:
:
:
Claim n
Model
Y=f(β)+ε
Claim 1
Claim 2
Claim 3
:
:
:
Claim n
Forecast
g($\hat{\beta}$)
Accident period 1
Accident period 2
Accident period 3
:
:
Accident period m

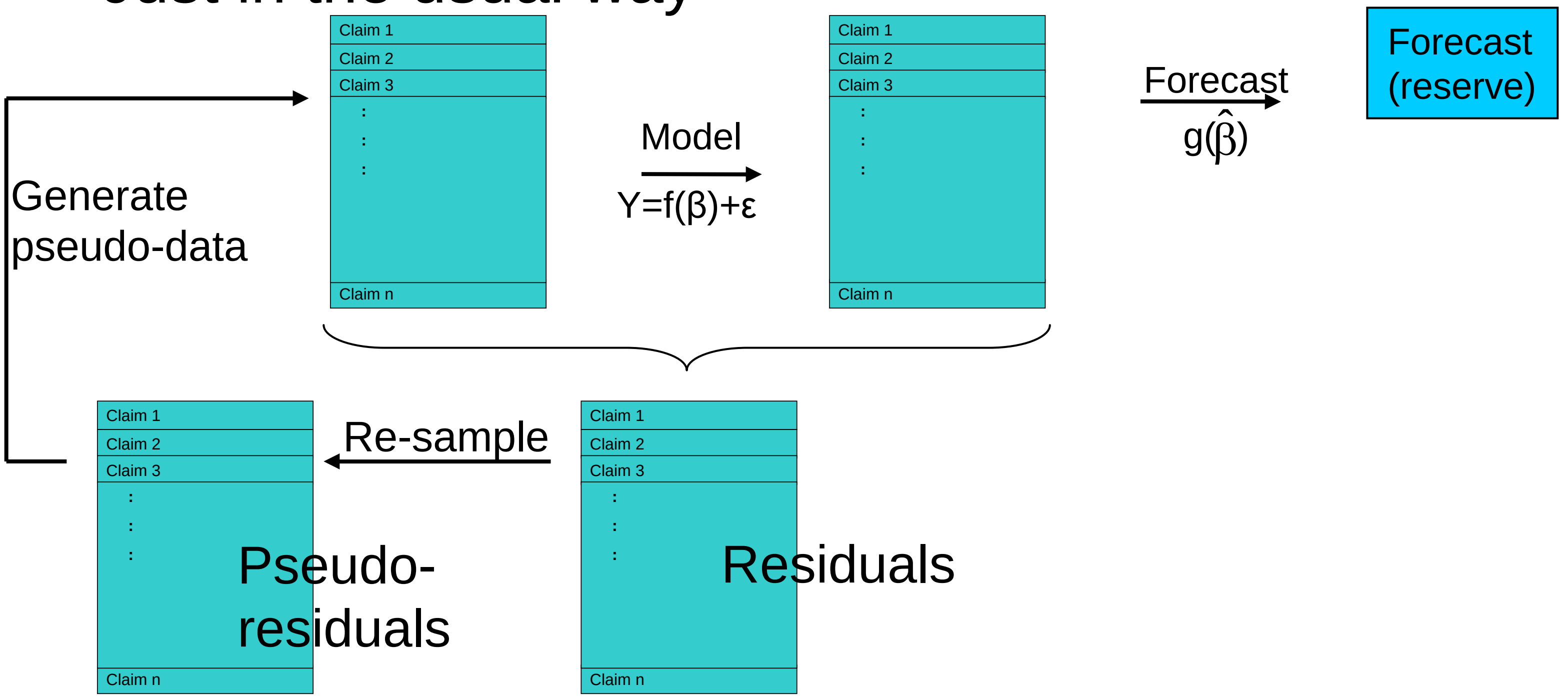
Alternatively

Special case of individual claim reserving – **statistical case estimation**



Can bootstrap individual claim reserve

Just in the usual way



Adaptive reserving

Static and dynamic models

- Return for a while to models based on aggregate (not individual claim) data
- Model form is still $Y=f(\beta)+\varepsilon$
- Example
  - i = accident quarter
  - j = development quarter
  - $E[Y_{ij}] = a\, j^b \exp(-cj) = \exp\,[\alpha+\beta \ln j - \gamma j]$
    - (Hoerl curve PPCI for each accident period)

Static and dynamic models (cont'd)

- Example

  $E[Y_{ij}] = a\ j^b \exp(-cj) = \exp\ [\alpha+\beta \ln j - \gamma j]$

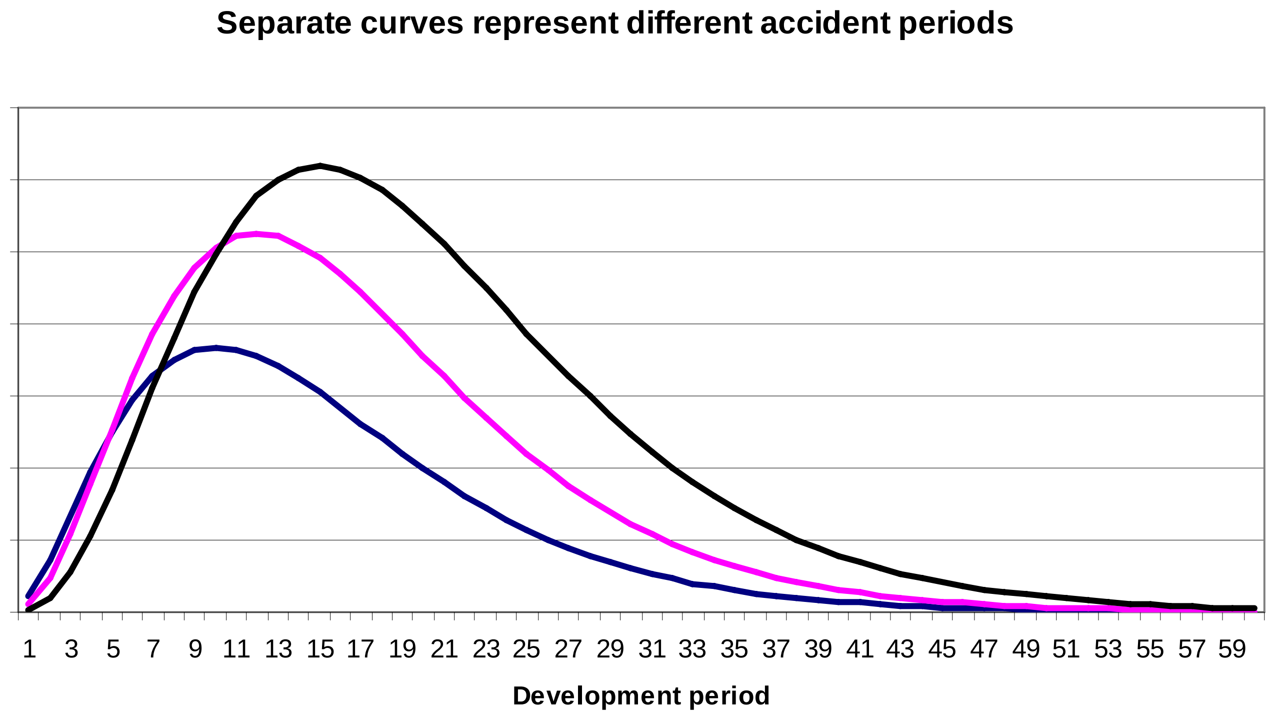
  - Parameters are fixed
  - This is a **static model**

But parameters α,β,γ may vary (evolve) over time, e.g. with accident period

Then

  - $E[Y_{ij}] = \exp\ [\alpha(i) + \beta(i) \ln j - \gamma(i)\ j]$
  - This is a **dynamic model**, or **adaptive model**

Illustrative example of evolving parameters



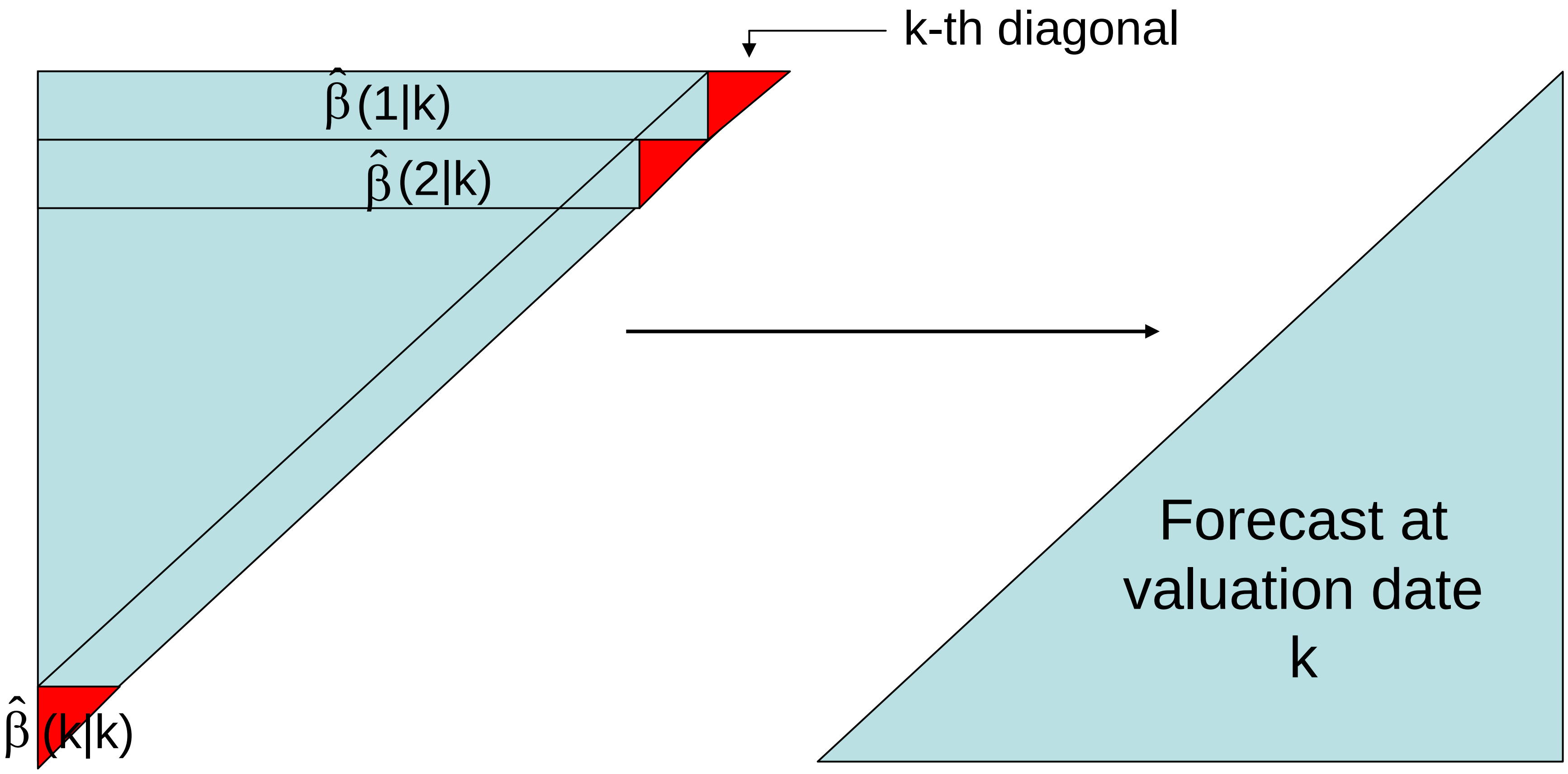
Formal statement of dynamic model

- Suppose parameter evolution takes place over accident periods
- $Y(i)=f(\beta(i)) + \varepsilon(i)$ [observation equation]
- $\beta(i) = u(\beta(i-1)) + \xi(i)$ [system equation]

Some function Centred stochastic perturbation

- Let $\hat{\beta}(i|s)$ denote an estimate of $\beta(i)$ based on only information up to time s

Adaptive reserving



Adaptive reserving (cont'd)

- Reserving by means of an adaptive model is **adaptive reserving**
- Parameter estimates evolve over time
- Fitted model evolves over time
- The objective here is "robotic reserving" in which the fitted model changes to match changes in the data
  - This would replace the famous actuarial "judgmental selection" of model

Special case of dynamic model: DGLM

- $Y(i)=f(\beta(i)) + \varepsilon(i)$ [observation equation]
- $\beta(i) = u(\beta(i-1)) + \xi(i)$ [system equation]
- Special case:
  - $f(\beta(i)) = h^{-1}(X(i)\, \beta(i))$ for matrix $X(i)$
  - $\varepsilon(i)$ has a distribution from the exponential dispersion family
- Each observation equation denotes a GLM
  - Link function h
  - Design matrix $X(i)$
- Whole system called a **Dynamic Generalised Linear Model** (DGLM)

Special case of DGLM: Kalman filter

- $Y(i)=f(\beta(i)) + \varepsilon(i)$ [observation equation]
- $\beta(i) = u(\beta(i-1)) + \xi(i)$ [system equation]
- Special case:
  - $f(\beta(i)) = h^{-1}(X(i)\ \beta(i))$ for matrix $X(i)$
  - $\varepsilon(i)$ has a distribution from the exponential dispersion family
- Further specialised
  - h(.) = identity function
    - So f(.) is linear
  - u(.) is linear
  - $\varepsilon(i), \xi(i) \sim N(0,.)$
  - This is the model underlying the **Kalman filter** (see De Jong & Zehnwirth, 1983)

Form of Kalman filter

- Let $\hat{Y}(i|s)$ be a fitted value, or forecast, of Y(i) on the basis of data to time s
- Take $\hat{Y}(i|s) = X(i) \; \hat{\beta}(i|s)$
- Kalman filter estimates

$$\hat{\beta}(i|i) = \hat{\beta}(i|i-1) + K(i) \, [Y(i) - X(i) \; \hat{\beta}(i|i-1)]$$

Kalman gain
(credibility) matrix

Implementation of DGLMs

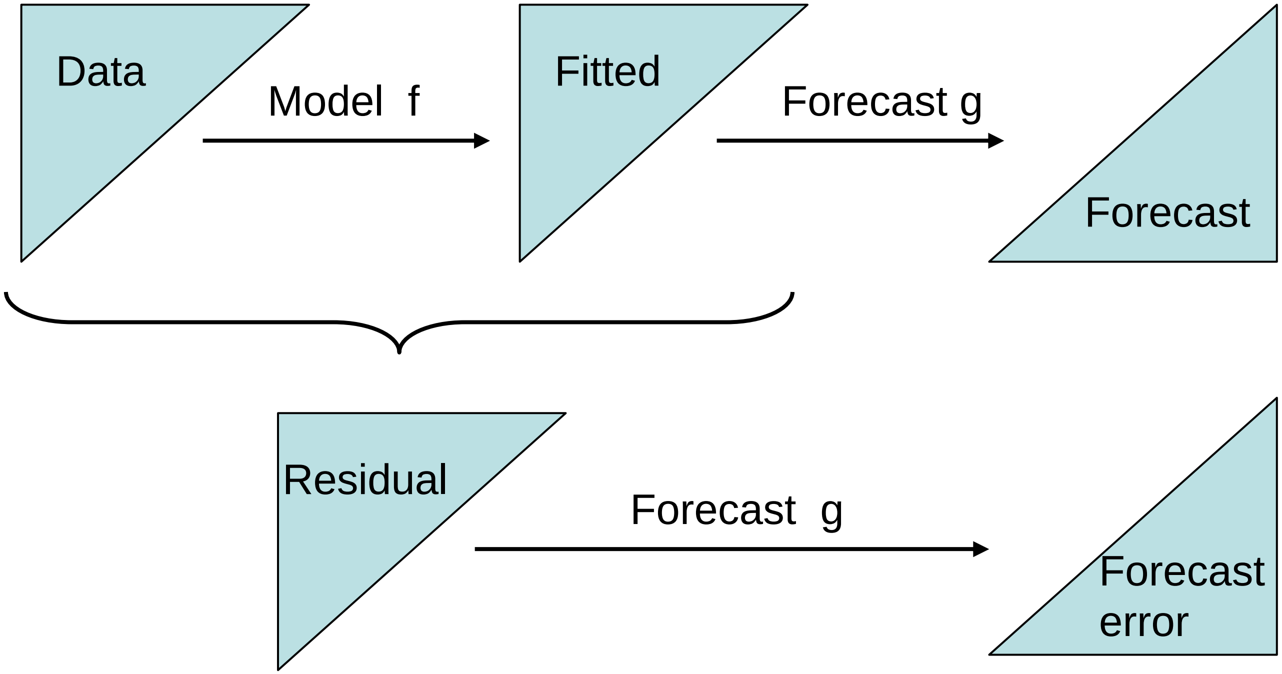
- The restrictions of the Kalman filter may not always be convenient
  - Linear relation between response variate and covariates
  - Normal distribution of claim observations
- Implementation of a more general DGLM is more difficult
- Can be done using an MCMC (Markov Chain Monte Carlo) approach
- Would be useful to have a simple updating formula similar to that of the Kalman filter (a GLM filter)
  - See Taylor, 2005

Bootstrapping DGLMs

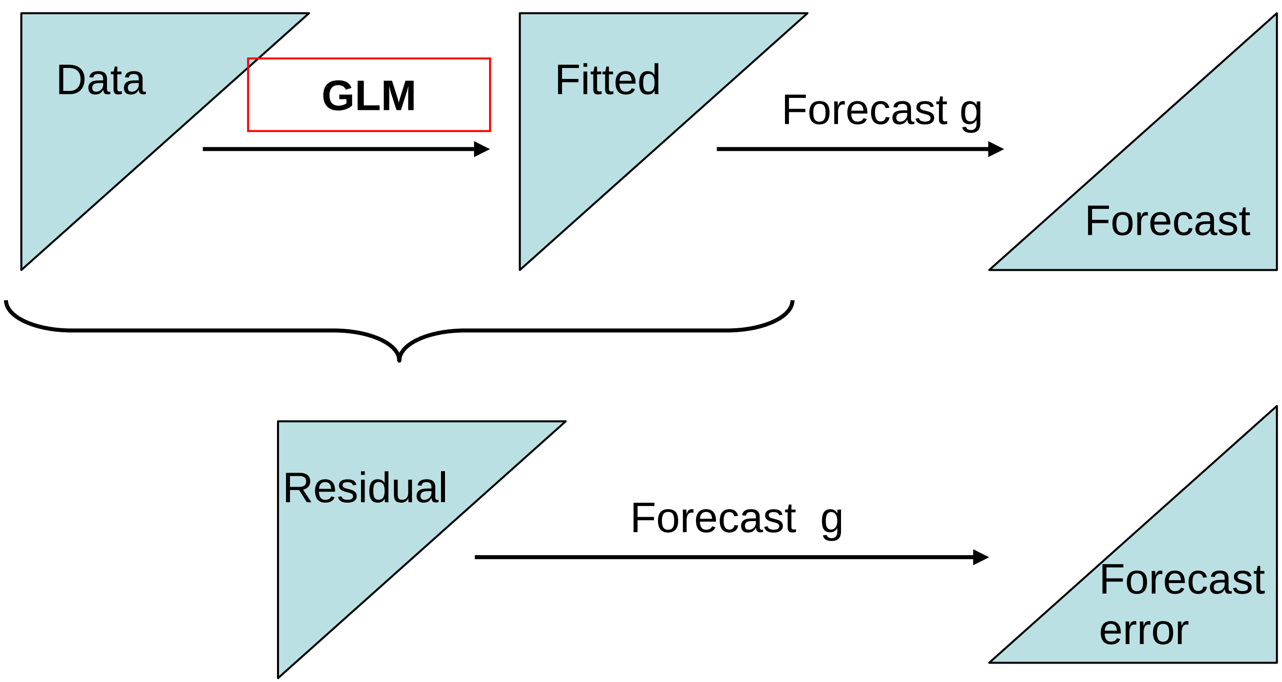
- Recursive nature of the GLM filter creates correlations between residuals
- So conventional bootstrapping is wrong
  - It assumes independence between residuals
- Necessary to modify the bootstrap to take account of the correlations
  - Say how
  - See Stoffer & Wall (1991)

Adaptive individual claim reserving

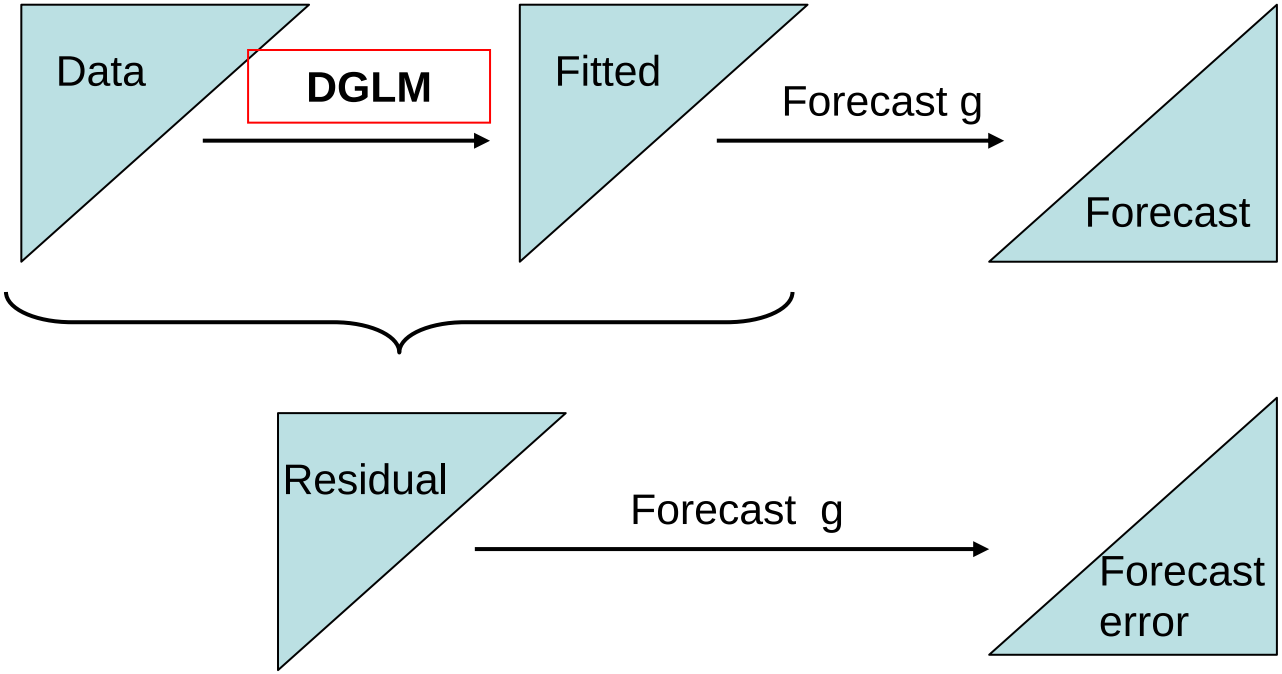
We began with…



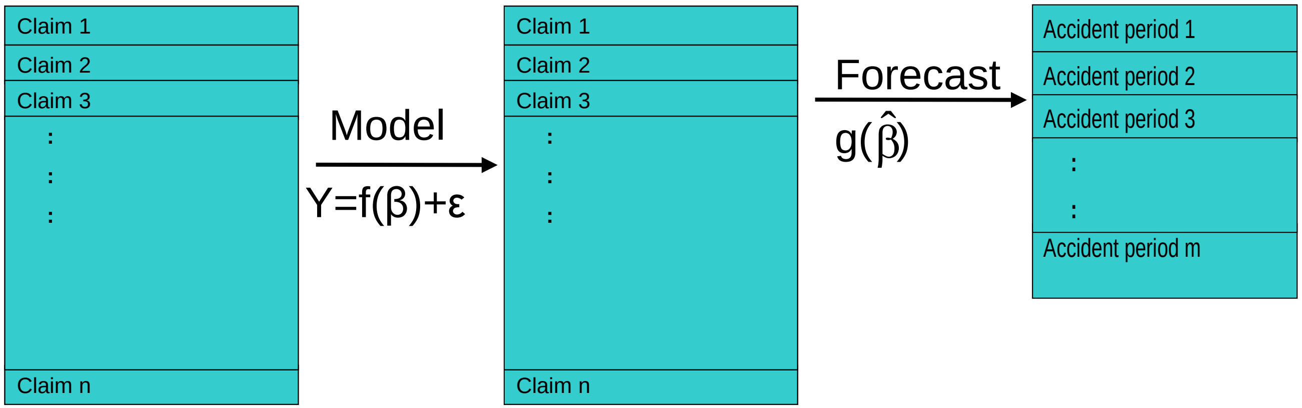
moved to GLM modelling…



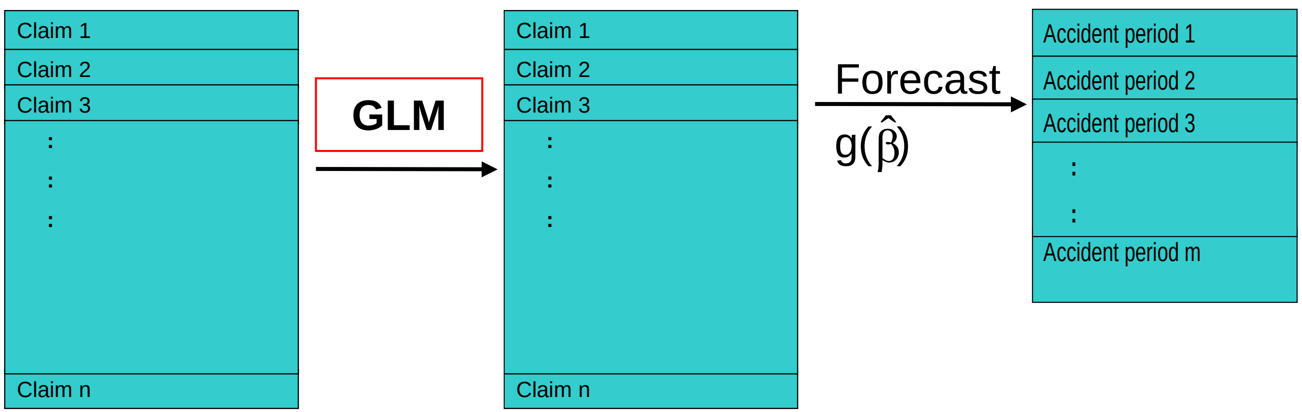
changed to adaptive GLM modelling…



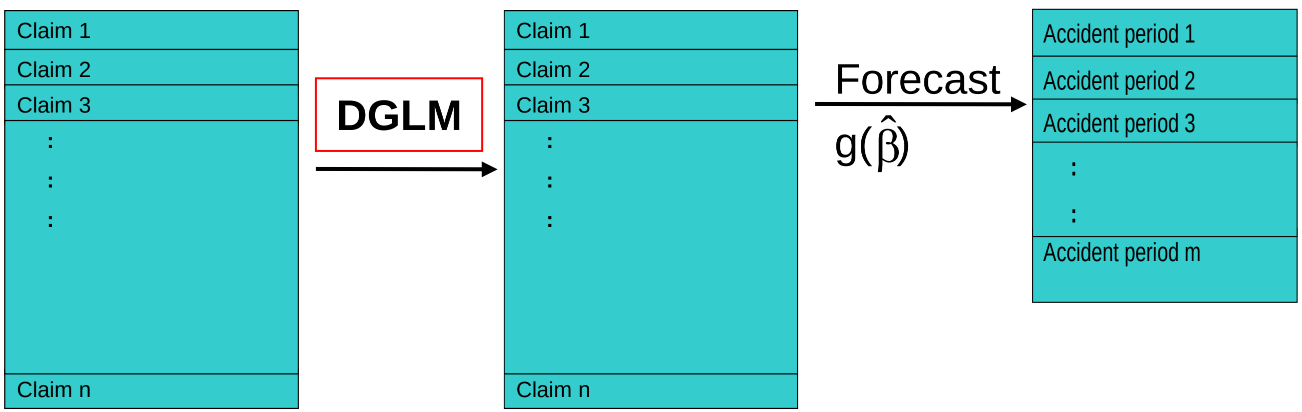
also considered individual claim modelling…



which can be individual claim GLM modelling…



and could be adaptive individual claim GLM modelling…



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- O'Dowd C, Smith A & Hardy P (2005). A framework for estimating incertainty in insurance claims cost. **XVth General Insurance Seminar,** Institute of Actuaries of Australia.
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- Taylor G (2005). Second order Bayesian revision of a generalised linear model. Available at www.economics.unimelb.edu.au/actwww/wps2005/No125.pdf
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