Risk Based Capital and Capital Allocation in Insurance

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Introduction – What Is Capital?

• Financial capital
  – Funding of productive assets (real capital), expected claims cost in insurance

• Risk based capital
  – Providing financial solvency and managing volatility in business outcomes

• Focus on Risk Based Capital
  – Most significant for financial intermediaries
  – Prudential/solvency regulation (Basel II, Solvency II)
  – Economic capital (based on risk) used for pricing and financial management
Capital Allocation

- Capital allocation used for many purposes:
  - Determining actual or expected return on capital by line of business
  - Assessing value of acquiring and divesting businesses/assets
  - Determining line management compensation based on return on capital
  - Pricing allowing for costs of capital
  - Risk quantification using risk based and economic capital
Current Practice

• Economic capital
  – Risk measure used to allocate capital to lines of business
    • VaR in banking, TailVaR in insurance
  – Capital is aggregated (allowing for diversification) to determine enterprise wide risk based capital
    • Diversification, dependence (copula, conditional dependent factor models)
• Pricing in multiline/multiproduct insurer or bank
  – Capital allocation to risk or line of business
  – Expected return on capital (RAROC).
Current Practice

• Fair rate of return for regulated lines of insurance business
  – Allocation of capital to line of business
  – “Fair” rate of return on capital - Enterprise wide or varying by-line
  – Frictional costs (tax, agency, financial distress)

• Regulatory solvency requirements
  – Based on risks such as market, credit, insurance, operational and aggregated for enterprise wide solvency
  – Risk models varying for different risks (multivariate normal, frequency and severity, extreme value)
Current Practice

• Many different risk measures
  – VaR, ruin probability, TailVaR,
  – Expected Policyholder Deficit, Insolvency Default Put Option.
• Which measure makes most economic sense?
• Many different approaches to allocating capital to line of business
  – proportional to risk measure, proportional to liabilities, marginal allocations,
    equal expected returns to capital, covariance of losses.
• How to determine an economically sensible measure?
Current Practice

• Capital allocation generally considers lines of business or risks on an individual basis (which may be assets or liabilities)
  – no direct allowance for dependence between risks or business lines
  – Diversification benefit considered later at the aggregated level
• Yet, capital is available to support all lines of business.
  – How to allow for this in allocating capital to line of business?
Current Practice

Surplus Allocation with Different Risk Measures - Normal Assumption
Capital Allocation Irrelevance

• Famous Corporate Finance Theory on Irrelevance of Capital Structure (Modigliani and Miller)
  – Perfect market assumptions, no frictional costs of capital
• Under these assumptions similar result (almost) holds for capital allocation to line of business/division
  – Different capital allocations are consistent with different expected returns on capital by line and an infinite number of alternatives are possible
  – No value maximising optimum (without market imperfections)
• Qualification – risk based insolvency put must be allocated based on payoffs by line (contribution to insolvency risk has financial impact)
Approaches To Capital Allocation

• Allocate capital to each line based on a risk measure and derive an expected return on capital
  – Choice of risk measure VaR, TailVaR?
  – Expected return on capital – same for all lines? adjusted for risk?
  – No agreement. Is there an optimal approach?

• Use an ERM value maximising (cost minimising) objective and derive implied capital allocations consistent with value maximisation
  – Costs of capital (tax, agency and financial distress) minimisation produces an enterprise VaR target for capital
  – Price elasticity of demand produce value maximising optimum
  – Equity or debtholder perspective? (shareholder or policyholder in insurance)
Capital Allocation and Pricing In Multi-line Businesses

• Risk based capital should be determined at an enterprise level
  – Minimisation of (expected) tax, agency and financial distress costs
  – Maximisation of shareholder value added (allowing for price elasticity of demand)

• Pricing of lines of business
  – Cost based on risk-adjusted discounted expected values of cash flows (income minus expenses)
  – No need to allocate capital (except for by-line contribution to insolvency risk)
  – Market price reflects market factors such as price elasticity, profit margins on sales
Pricing and Insolvency

• Framework for fair pricing, capital allocation and insolvency put option value
  – Sherris, M., (2006), Solvency, Capital Allocation and Fair Rate of Return in Insurance, Journal of Risk and Insurance, Vol 73, No 1, (March 2006), 71-96. (this paper awarded the Casualty Actuarial Society (CAS) 2007 prize for the most valuable contribution to casualty actuarial science published in American Risk and Insurance Association (ARIA) literature in the preceding year)

• Practical model based on dependent log-normal risks
Simple Insurance Example

• Single period example
  – emphasise main concepts
  – Two lines of business - Liability 1 and Liability 2
  – 4 outcomes or states

• Which liability line is the most risky?
• VaR or economic capital compared with the insolvency put option value
• Is capital allocation required for pricing?
# Simple Insurance Example

## Table 1
Probabilities and Payoffs for Example Insurer

<table>
<thead>
<tr>
<th>State</th>
<th>P-probs</th>
<th>Q-probs</th>
<th>Risky Asset</th>
<th>Risk-Free Asset</th>
<th>Liability 1</th>
<th>Liability 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.6</td>
<td>1.05</td>
<td>200</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>0.4</td>
<td>1.1</td>
<td>1.05</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.4</td>
<td>1.0</td>
<td>1.05</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.1</td>
<td>1.5</td>
<td>1.05</td>
<td>0</td>
<td>310</td>
</tr>
<tr>
<td>Time 0 value</td>
<td>1.0</td>
<td>1.0</td>
<td>21.3333</td>
<td>38.6667</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 2
Insurer Balance Sheet Payoffs

<table>
<thead>
<tr>
<th>State</th>
<th>Assets</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>Total L</th>
<th>$\max (L - A, 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>200</td>
<td>40</td>
<td>240</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>4</td>
<td>10</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>0</td>
<td>310</td>
<td>310</td>
<td>10</td>
</tr>
<tr>
<td>Time 0 value</td>
<td>200</td>
<td>21.3333</td>
<td>38.6667</td>
<td>60</td>
<td>12.381</td>
</tr>
</tbody>
</table>

Payoffs By Line
### Default Put Risk Measure

**Table 3**
Liability Shortfalls in the Event of Insolvency

<table>
<thead>
<tr>
<th>State</th>
<th>( D_1 = L_1 \max(1 - \frac{A}{L}, 0) )</th>
<th>( D_2 = L_2 \max(1 - \frac{A}{L}, 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Time 0 value</td>
<td>9.5238</td>
<td>2.8571</td>
</tr>
</tbody>
</table>
Equity Payoffs

Table 4
Insurer Equity Payoffs

<table>
<thead>
<tr>
<th>State</th>
<th>P-probs</th>
<th>Assets</th>
<th>Total L</th>
<th>Equity = max (A - L, 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>120</td>
<td>240</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>220</td>
<td>14</td>
<td>206</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>200</td>
<td>6</td>
<td>194</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>300</td>
<td>310</td>
<td>0</td>
</tr>
<tr>
<td>Time 0 value</td>
<td>200</td>
<td>60</td>
<td></td>
<td>152.3810</td>
</tr>
</tbody>
</table>
Risk Based Capital – Frictional Costs

• Australian Prudential Regulations and Risk Based Capital using an Internal Model

• Minimising frictional costs of insurer capital produces an optimal capital level based on VaR at much lower levels than observed
Risk Based Capital

VaR Probability for Insurer Liability

- Probability (L>A+C)
- Frictional costs of capital (%)
- Financial distress costs (%)

Risk

0.50%
1.25%
2.00%
2.75%
3.50%
4.25%
5.00%
11%
12.00%
13.00%
14.00%
15.00%
Value Maximisation with Frictional Costs and Demand Elasticity

- Value maximising approach compared to economic capital and VaR approaches incorporating frictional costs of capital and price elasticity

- Value maximising approach to risk based capital and by-line pricing
Maximizing Enterprise Value Added (EVA)

\[
\max_{R_0, p_{i,0}} \{ EV A_0 \} = \max_{R_0, p_{i,0}} \left\{ \sum_{i=1}^{N} \left[ p_{i,0} - c_{i,0}' - (1 - d_0) e^{-r} \mu_{i,1} \right] q_{i,0} - \delta R_0 \right\}
\]

- Optimize prices \( p_{i,0} \) and capital \( R_0 \).
- \( q_{i,0} = q(p_{i,0}, d_0, f) \) = quantity of business sold in the \( i \)th line.
- \( d_0 = e^{-r} E^Q \left[ \max \left( 1 - \frac{V_1}{L_1}, 0 \right) \right] \).
- \( c_{i,0}' \) = marginal expenses.
- \( \mu_{i,1} \) = expected loss at time 1.
- \( r \) = risk-free rate.
- \( \delta = \frac{(1-e^{-r})\tau_1 + e^{-r}\tau_2}{1-\tau_1} \), where \( \tau_1 \) = taxes and \( \tau_2 \) = agency costs of capital.
Lower optimal capital for higher agency costs
Higher optimal capital for increasing bankruptcy costs
## Pricing Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Approach</th>
<th>Method of Allocation</th>
<th>Assumed Cost of Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maximize EVA</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>VaR at 99.5%</td>
<td>Proportional</td>
<td>15%</td>
</tr>
<tr>
<td>3</td>
<td>VaR at 99.5%</td>
<td>Proportional</td>
<td>20%</td>
</tr>
<tr>
<td>4</td>
<td>VaR at 99.5%</td>
<td>Proportional</td>
<td>15% (Commercial) and 25% Personal/Compulsory</td>
</tr>
<tr>
<td>5</td>
<td>Maximize Firm Value</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

Table 8: Strategies for insurer pricing and capitalization.
Economic Profit Margins

Based on assumed price elasticity (not actual market data)
The science of capital allocation has made significant advances in our understanding of allocation and use of risk based capital, yet:

- limited theoretical guidance on which risk measure is consistent with value maximisation and no well developed economic theory underlying the risk measures
- different firms use different risk measures
- no agreement on the appropriate risk measure
- risk measures are applied inconsistently for different risks, different lines of business, products and divisions
- for insurer pricing the price of risk should vary with the type of risk under consideration yet most risk based capital approaches implicitly use a common price of risk based on a firm wide expected cost of capital for pricing.

Conclusions
Conclusions

- Recent developments in capital allocation for risk capital for solvency and by-line pricing indicate a new direction is required
  - Importance of risk measure – insolvency default option value
  - Allocation by line and fair pricing
  - Frictional costs and market imperfections
  - Value maximising and demand elasticity