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Scale of Longevity Risks for Pension and Life Annuity Providers

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Outline

- Norman Cohen’s critique (motivation of the paper)
- Mathematical framework
  - Force of Longevity
  - Longevity Risk Premium
- Longevity Risk vs. Investment Risk
- Numerical Examples
  - Empirical Evidence of Force of Longevity (UK and AUS)
  - Longevity Risk for Australian Male age 60
- Discussion and Conclusion

Norman Cohen's critique (FT 9 Feb 2004)

- *"Was the actuarial profession asleep at the wheel?"*
- *"Given the scale of the crisis in the pensions industry - for it is not just on longevity that actuaries got it wrong, but on investment and funding as well - it is a wonder that the profession has not come in for the opprobrium given to, say, accountants"*
- *…actuaries have consistently underestimated human longevity.*

-- Norman Cohen 9 Feb 2004 Financial Times

Was the critique fair? What actuaries did?

- Friedland (1998) had a summary of a day-long discussion among experts titled “impact of mortality improvement on Social Security: Canada, Mexico, and the U.S.” on October 30, 1997.
- Khalaf-Allah (2002) built a model to project future mortality improvement in term of reduction factors for UK.
- Lin and Cox (2004) also discussed the securitisation of the longevity risks in life annuities.
- Australian Government Actuary (1999,2004) published Mortality Improvement Factor in Australian Life Table 1995-97 and 2000-2002.

What was missing?

- Need to add concept of longevity in actuarial textbook
- Need to compare and manage the longevity risk together with investment risk in a more systematic way
  - Scale of the measurement (This Paper)
  - Correlation to investment (Next Paper)

Mathematical Framework (Continuous)

Survival Function: $S(x) \longrightarrow S(x,t)$

Force of Mortality $\mu(x,t) = \frac{-1}{S(x,t)} \cdot \frac{\partial S(x,t)}{\partial x}$

Force of Longevity $\upsilon(x,t) = \frac{1}{S(x,t)} \cdot \frac{\partial S(x,t)}{\partial t}$

$$S(x+n,t+n)/S(x,t) = \exp\left\{\int_0^n \left[-\mu(x+\tau,t+\tau)+\upsilon(x+\tau,t+\tau)\right]\cdot d\tau\right\}$$

$$\bar{a}_{x,t_0} = \int_0^{\varpi-x} \exp\left\{-\left[\int_0^t \left[r(t_0+\tau)+\mu(x+\tau,t_0+\tau)-\upsilon(x+\tau,t_0+\tau)\right]\cdot d\tau\right]\cdot dt\right\}$$

Mathematical Framework (Discrete)

Conditional Survival Probability $P(x,t) = S(x+1,t)/S(x,t)$

Conditional Force of Longevity $\xi(x,t) = \frac{1}{P(x,t)} \cdot \frac{\partial P(x,t)}{\partial t}$

$$a_{65,2004} = \sum_{k=1}^{\varpi-65} \exp(-r_1 k) \cdot {}_k p_{65,2004} = \sum_{k=1}^{\varpi-65} \exp(-r_1 k) \cdot \left(\prod_{m=0}^{k-1} P(65+m, 2004+m) \right)$$

$$a_{65,2044} = \sum_{k=1}^{\varpi-65} \exp(-r_2 k) \cdot {}_k p_{65,2044} = \sum_{k=1}^{\varpi-65} \exp(-r_2 k) \cdot \left(\prod_{m=0}^{k-1} P(65+m, 2044+m) \right)$$

Extremely Simplified Assumption $\xi(65+m, n+m) = \overline{\xi}$

⇨ $$a_{65,2044} = \sum_{k=1}^{\varpi-65} \exp\left[-(r_2 - 40 \cdot \overline{\xi})k\right] \cdot \prod_{m=0}^{k-1} P(65+m, 2004+m)$$

⇨ $$a_{65,2044,r_2} = a_{65,2004,r_2-40\overline{\xi}}$$

Longevity Risk Premium (indexation)

Longevity vs. Investment

- Fundamental driver of the longevity risk is $\bar{\xi}$, which is a weighted average of the conditional forces of longevity over age 65 and beyond.
- The **longevity risk premium** is $40\bar{\xi}$, which implies that the longevity risk usually increases with expanding horizon.
- The longevity effect in life annuity pricing is somehow equivalent to the case of reduced rate of investment return, which implies that longevity risk could be measured in a similar way of investment risk.
- Alternatively, the concept of **longevity indexation** can be used, so that an annuity sold in 2044 would be equivalent to an indexed annuity sold in 2004, indexed by $40\bar{\xi}$ per annum.

Equivalent Longevity Term Structure (ELTS)

- Now move on to a more flexible example, where neither term structure nor longevity improvement is flat any more. Two Alternative Assumptions:

$$P(65+m, 2004+m) = \exp\left[\mu(65+m, 2004+m)\right]$$

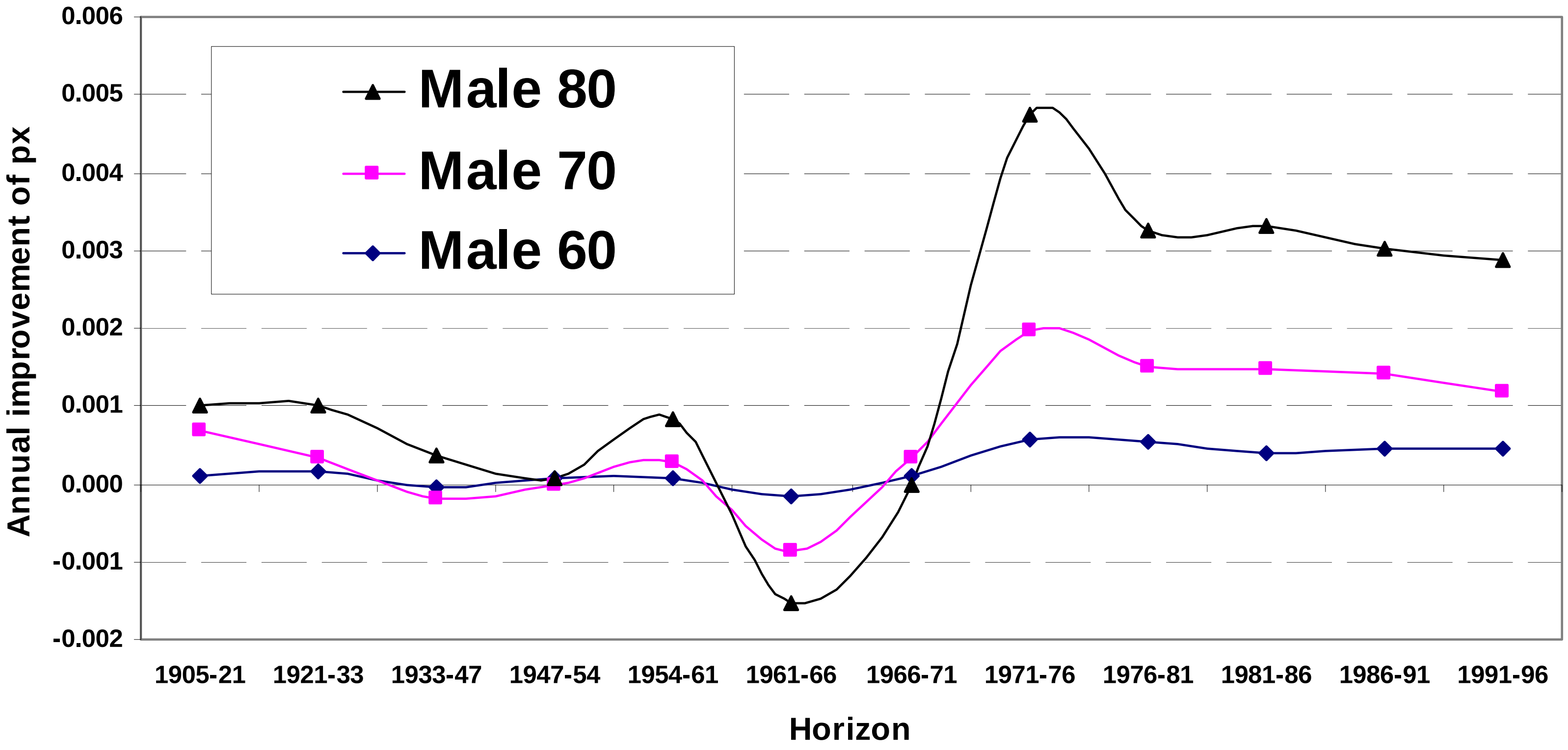
$$\xi(65+m, n+m) = \xi(65+m)$$

$$a_{65,2044} = \sum_{k=1}^{\varpi-65} \exp\left\{-\sum_{m=0}^{k-1}\left[\left(R_F(2044+m) - 40\times\xi(65+m)\right) + \mu(65+m, 2004+m)\right]\right\}$$

ELTS

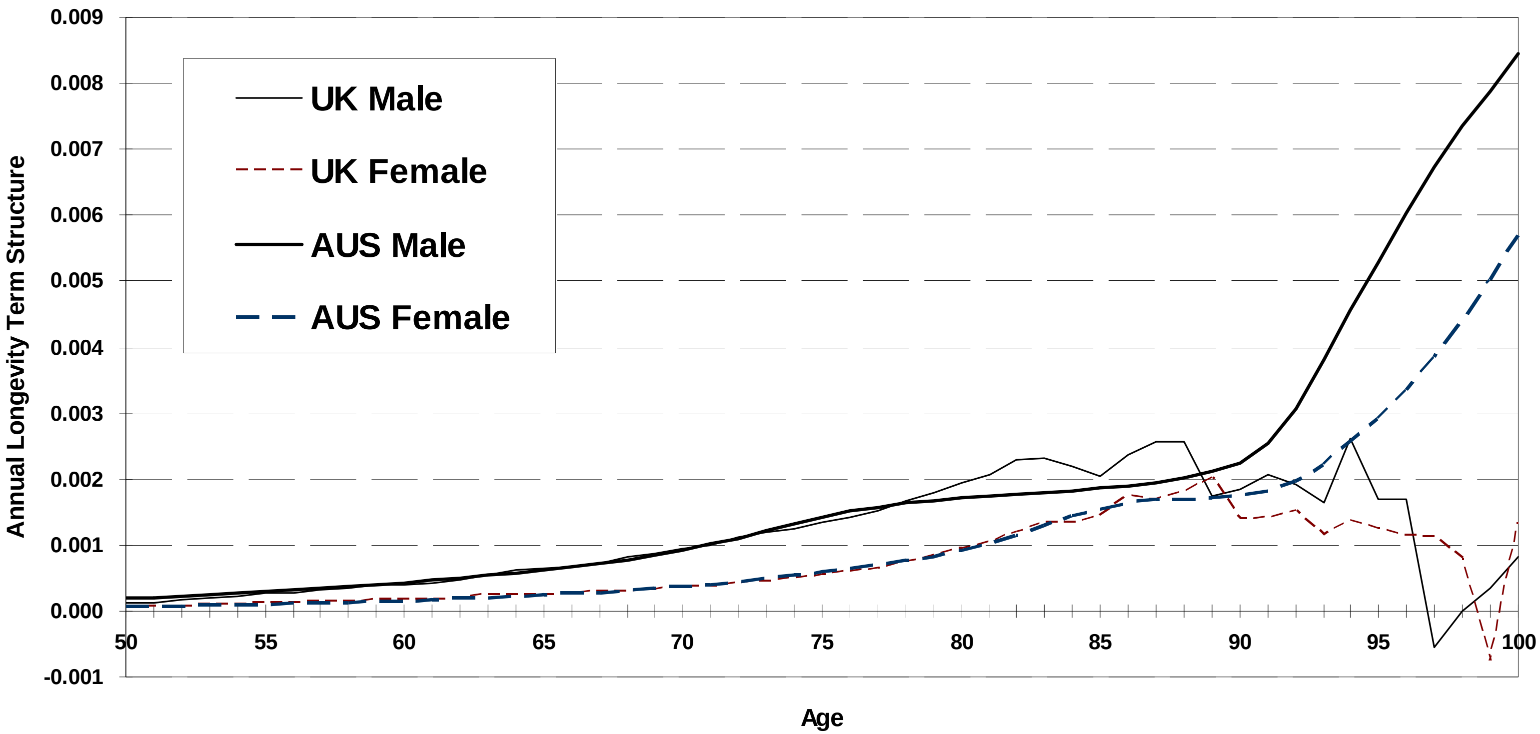
- Then the longevity adjustment in annuity pricing would be against forward rate term structure.

Empirical Longevity Evidence in Australia



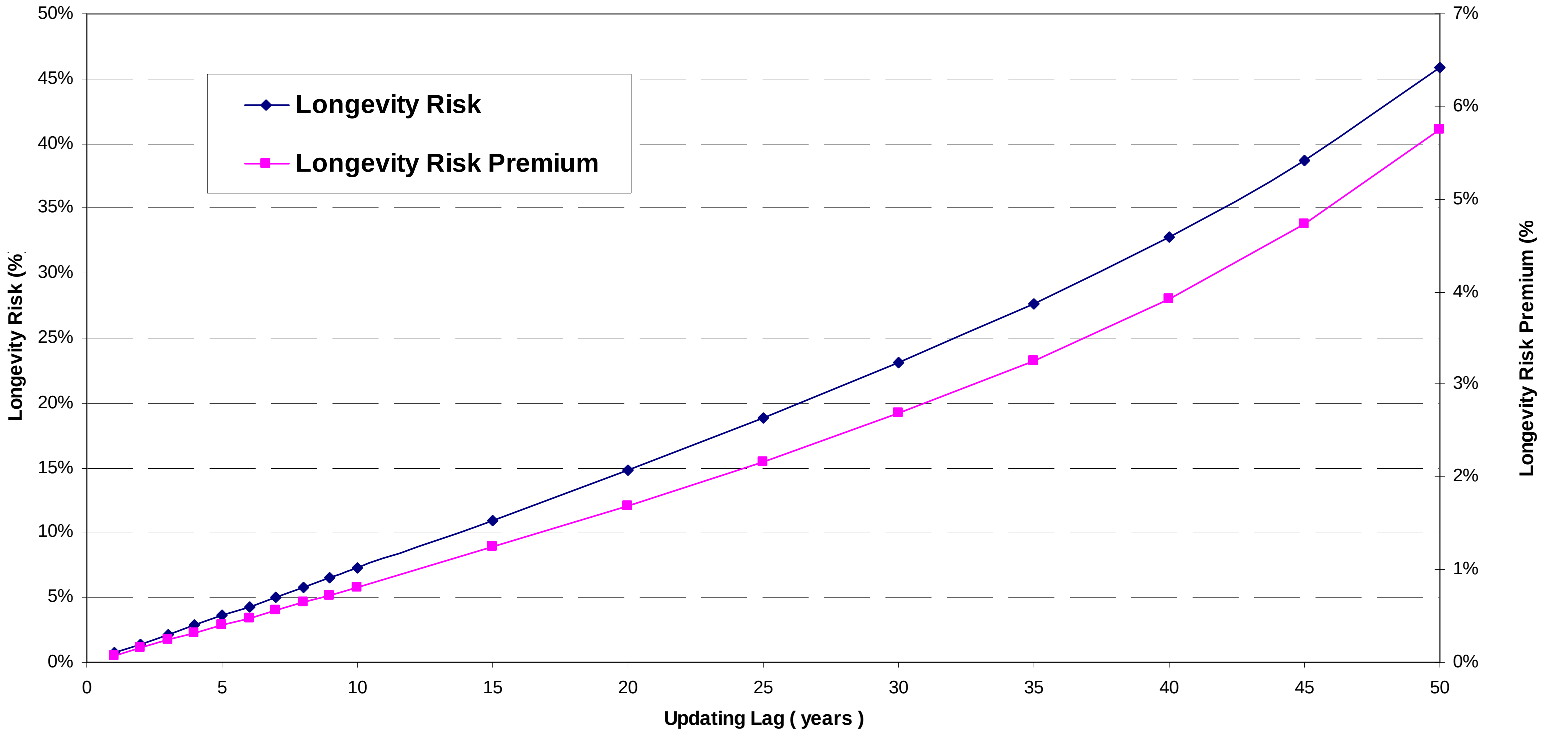
Source: Author’s Calculation. Australian Life Table 1995-97

Equivalent Longevity Term Structure (UK and AUS)



Source: author's calculation based on ALT1995-97 and UK Interim Life Table

Accumulative Effect of Longevity Risk



Source: Author's calculation. The risk free term structure as of 31 May 2004 is used. The mortality benchmark is the cohortised life table for Australian male aged 60 from ALT1995-97.

Conclusions

- Force of Longevity is the key concept to bring longevity and investment together, which can be easily offset against term structure in life annuity pricing.
- Longevity experiences in UK and AUS over the last two decades coincide closely.
  - Forces of longevity over retirement ages are quite “consistent” over time, while the bigger contributions of longevity observed over the older ages.
  - The weighted average annual force of longevity would be in the order of 0.1% for male cohort aged 60 in 2004, and around half for female.
  - Longevity risks accumulate proportionately over time.
- Most of the exposure to longevity shortfall can be reduced by updating the cohortised life tables on a regular basis.

Further Discussions

- Diversification is possible if we can understand the correlation pattern between these risks.
- The diverging longevity trends in China and Russia over last decade might be a good example of the negative correlation between investment risk and longevity risk.
- An empirical study of the correlation between longevity and investment in different stages of business cycle might be an interesting topic.
- But it is beyond the scope of this paper.

Any Question?