Institute
and Faculty
of Actuaries

Subject CM1
Actuarial Mathematics
Core Principles

Syllabus

for the 2019 exams

1 June 2018

CM1 – Actuarial Mathematics 1

Aim

The aim of the Actuarial Mathematics 1 subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Competences

On the successful completion of this subject, the candidate will be able to:

1 describe the basic principles of actuarial modelling.

2 describe, interpret and discuss the theories on interest rates.

3 describe, interpret and discuss mathematical techniques used to model and value cashflows which are contingent on mortality and morbidity risks.

Links to other subjects

Concepts are introduced in:

CS1 – Actuarial Statistics 1

Topics in this subject are further built upon in:

CM2 – Financial Engineering and Loss Reserving
CB1 – Business Finance
CP1 – Actuarial Practice
CP2 – Modelling Practice
SP1 – Health and Care Principles
SP2 – Life Insurance Principles
SP4 – Pensions and other Benefits Principles

Syllabus topics

1 Data and basics of modelling (10%)
2 Theory of interest rates (20%)
3 Equation of value and its applications (15%)
4 Single decrement models (10%)
5 Multiple decrement and multiple life models (10%)
6 Pricing and reserving (35%)

The weightings are indicative of the approximate balance of the assessment of this subject between the main syllabus topics, averaged over a number of examination sessions.

The weightings also have a correspondence with the amount of learning material underlying each syllabus topic. However, this will also reflect aspects such as:

- the relative complexity of each topic, and hence the amount of explanation and support required for it.
- the need to provide thorough foundation understanding on which to build the other objectives.
- the extent of prior knowledge which is expected.
- the degree to which each topic area is more knowledge or application based.

Skill levels

The use of a specific command verb within a syllabus objective does not indicate that this is the only form of question which can be asked on the topic covered by that objective. The Examiners may ask a question on any syllabus topic using any of the agreed command verbs, as are defined in the document “Command verbs used in the Associate and Fellowship written examinations”.

Questions may be set at any skill level: Knowledge (demonstration of a detailed knowledge and understanding of the topic), Application (demonstration of an ability to apply the principles underlying the topic within a given context) and Higher Order (demonstration of an ability to perform deeper analysis and assessment of situations, including forming judgements, taking into account different points of view, comparing and contrasting situations, suggesting possible solutions and actions, and making recommendations).

In the CM subjects, the approximate split of assessment across the three skill types is 20% Knowledge, 65% Application and 15% Higher Order skills.

Detailed syllabus objectives

1 Data and basics of modelling (10%)

1.1 Data analysis

1.1.1 Describe the possible aims of a data analysis (e.g. descriptive, inferential, and predictive).

1.1.2 Describe the stages of conducting a data analysis to solve real-world problems in a scientific manner and describe tools suitable for each stage.

1.1.3 Describe sources of data and explain the characteristics of different data sources, including extremely large data sets.

1.1.4 Explain the meaning and value of reproducible research and describe the elements required to ensure a data analysis is reproducible.

1.2 Describe the principles of actuarial modelling.

1.2.1 Describe why and how models are used including, in general terms, the use of models for pricing, reserving, and capital modelling.

1.2.2 Explain the benefits and limitations of modelling.

1.2.3 Explain the difference between a stochastic and a deterministic model, and identify the advantages/disadvantages of each.

1.2.4 Describe the characteristics of, and explain the use, of scenario-based and proxy models.

1.2.5 Describe, in general terms, how to decide whether a model is suitable for any particular application.

1.2.6 Explain the difference between the short-run and long-run properties of a model, and how this may be relevant in deciding whether a model is suitable for any particular application.

1.2.7 Describe, in general terms, how to analyse the potential output from a model, and explain why this is relevant to the choice of model.

1.2.8 Describe the process of sensitivity testing of assumptions and explain why this forms an important part of the modelling process.

1.2.9 Explain the factors that must be considered when communicating the results following the application of a model.

1.3 Describe how to use a generalised cashflow model to describe financial transactions.

1.3.1 State the inflows and outflows in each future time period and discuss whether the amount or the timing (or both) is fixed or uncertain for a given cashflow process.

1.3.2 Describe in the form of a cashflow model the operation of financial instruments like a zero coupon bond, a fixed interest security, an index-linked security, cash on deposit, an equity, an interest only loan, a repayment loan, and an annuity certain; and an insurance contract like endowment, term assurance, contingent annuity, car insurance and health cash plans.

2 Theory of interest rates (20%)

2.1 Show how interest rates may be expressed in different time periods.

2.1.1 Describe the relationship between the rates of interest and discount over one effective period arithmetically and by general reasoning.

2.1.2 Derive the relationships between the rate of interest payable once per measurement period (effective rate of interest) and the rate of interest payable p (> 1) times per measurement period (nominal rate of interest) and the force of interest.

2.1.3 Calculate the equivalent annual rate of interest implied by the accumulation of a sum of money over a specified period where the force of interest is a function of time.

2.2 Demonstrate a knowledge and understanding of real and money interest rates.

2.3 Describe how to take into account time value of money using the concepts of compound interest and discounting.

2.3.1 Accumulate a single investment at a constant rate of interest under the operation of simple and compound interest.

2.3.2 Define the present value of a future payment.

2.3.3 Discount a single investment under the operation of a simple (commercial) discount at a constant rate of discount.

2.4 Calculate present value and accumulated value for a given stream of cashflows under the following individual or combination of scenarios:

2.4.1 Cashflows are equal at each time period.

2.4.2 Cashflows vary with time which may or may not be a continuous function of time.

2.4.3 Some of the cashflows are deferred for a period of time.

2.4.4 Rate of interest or discount is constant.

2.4.5 Rate of interest or discount varies with time which may or may not be a continuous function of time.

2.5 Define and derive the following compound interest functions (where payments can be in advance or in arrears) in terms i, v, n, d, δ, $i(p)$ and $d(p)$:

2.5.1 $a_{\overline{n}|}$, $s_{\overline{n}|}$, $a^{(p)}_{\overline{n}|}$, $s^{(p)}_{\overline{n}|}$, $\ddot{a}_{\overline{n}|}$, $\ddot{s}_{\overline{n}|}$, $\ddot{a}^{(p)}_{\overline{n}|}$, $\ddot{s}^{(p)}_{\overline{n}|}$, $\bar{a}_{\overline{n}|}$ and $\bar{s}_{\overline{n}|}$.

2.5.2 ${}_{m|}a_{\overline{n}|}$, ${}_{m|}a^{(p)}_{\overline{n}|}$, ${}_{m|}\ddot{a}_{\overline{n}|}$, ${}_{m|}\ddot{a}^{(p)}_{\overline{n}|}$ and ${}_{m|}\bar{a}_{\overline{n}|}$.

2.5.3 $(Ia)_{\overline{n}|}$, $(I\ddot{a})_{\overline{n}|}$, $(\bar{I}a)_{\overline{n}|}$ and $(\bar{I}\bar{a})_{\overline{n}|}$ and the respective deferred annuities.

2.6 Show an understanding of the term structure of interest rates.

2.6.1 Describe the main factors influencing the term structure of interest rates.

2.6.2 Explain what is meant by, derive the relationships between and evaluate:

- discrete spot rates and forward rates.
- continuous spot rates and forward rates.

2.6.3 Explain what is meant by the par yield and yield to maturity.

2.7 Understanding duration, convexity and immunisation of cashflows

2.7.1 Define the duration and convexity of a cashflow sequence, and illustrate how these may be used to estimate the sensitivity of the value of the cashflow sequence to a shift in interest rates.

2.7.2 Evaluate the duration and convexity of a cashflow sequence.

2.7.3 Explain how duration and convexity are used in the (Redington) immunisation of a portfolio of liabilities.

3 Equation of value and its applications (15%)

3.1 Define an equation of value.

3.1.1 Define an equation of value, where payment or receipt is certain.

3.1.2 Describe how an equation of value can be adjusted to allow for uncertain receipts or payments.

3.1.3 Understand the two conditions required for there to be an exact solution to an equation of value.

3.2 Use the concept of equation of value to solve various practical problems.

3.2.1 Apply the equation of value to loans repaid by regular instalments of interest and capital. Obtain repayments, interest and capital components, the effective interest rate (APR) and construct a schedule of repayments.

3.2.2 Calculate the price of, or yield (nominal or real allowing for inflation) from, a bond (fixed-interest or index-linked) where the investor is subject to deduction of income tax on coupon payments and redemption payments are subject to deduction of capital gains tax.

3.2.3 Calculate the running yield and the redemption yield for the financial instrument as described in 3.2.2.

3.2.4 Calculate the upper and lower bounds for the present value of the financial instrument as described in 3.2.2 when the redemption date can be a single date within a given range at the option of the borrower.

3.2.5 Calculate the present value or yield (nominal or real allowing for inflation) from an ordinary share or property, given constant or variable rate of growth of dividends or rents.

3.3 Show how discounted cashflow and equation of value techniques can be used in project appraisals.

3.3.1 Calculate the net present value and accumulated profit of the receipts and payments from an investment project at given rates of interest.

3.3.2 Calculate the internal rate of return, payback period and discounted payback period and discuss their suitability for assessing the suitability of an investment project.

4 Single decrement models (10%)

4.1 Define various assurance and annuity contracts.

4.1.1 Define the following terms:

- whole life assurance
- term assurance
- pure endowment
- endowment assurance
- whole life level annuity
- temporary level annuity
- guaranteed level annuity

- premium
- benefit

including assurance and annuity contracts where the benefits are deferred.

4.1.2 Describe the operation of conventional with-profits contracts, in which profits are distributed by the use of regular reversionary bonuses, and by terminal bonuses. Describe the benefits payable under the above assurance-type contracts.

4.1.3 Describe the operation of conventional unit-linked contracts, in which death benefits are expressed as combination of absolute amount and relative to a unit fund.

4.1.4 Describe the operation of accumulating with-profits contracts, in which benefits take the form of an accumulating fund of premiums, where either:

- the fund is defined in monetary terms, has no explicit charges, and is increased by the addition of regular guaranteed and bonus interest payments plus a terminal bonus; or

- the fund is defined in terms of the value of a unit fund, is subject to explicit charges, and is increased by regular bonus additions plus a terminal bonus (Unitised with-profits).

In the case of unitised with-profits, the regular additions can take the form of (a) unit price increases (guaranteed and/or discretionary), or (b) allocations of additional units.

In either case a guaranteed minimum monetary death benefit may be applied.

4.2 Develop formulae for the means and variances of the payments under various assurance and annuity contracts, assuming constant deterministic interest rate.

4.2.1 Describe the life table functions l_x and d_x and their select equivalents $l_{[x]+r}$ and $d_{[x]+r}$.

4.2.2 Define the following probabilities: ${}_n p_x$, ${}_n q_x$, ${}_{n|m} q_x$, ${}_{n|} q_x$ and their select equivalents ${}_n p_{[x]+r}$, ${}_n q_{[x]+r}$ ${}_{n|m} q_{[x]+r}$, ${}_{n|} q_{[x]+r}$.

4.2.3 Express the probabilities defined in 4.2.2 in terms of life table functions defined in 4.2.1.

4.2.4 Define the assurance and annuity factors and their select and continuous equivalents. Extend the annuity factors to allow for the possibility that payments are more frequent than annual but less frequent than continuous.

4.2.5 Understand and use the relations between annuities payable in advance and in arrear, and between temporary, deferred and whole life annuities.

4.2.6 Understand and use the relations between assurance and annuity factors using equation of value, and their select and continuous equivalents.

4.2.7 Obtain expressions in the form of sums/integrals for the mean and variance of the present value of benefit payments under each contract defined in 4.1.1, in terms of the (curtate) random future lifetime, assuming:

- contingent benefits (constant, increasing or decreasing) are payable at the middle or end of the year of contingent event or continuously.
- annuities are paid in advance, in arrear or continuously, and the amount is constant, increases or decreases by a constant monetary amount or by a fixed or time-dependent variable rate.
- premiums are payable in advance, in arrear or continuously; and for the full policy term or for limited period.

Where appropriate, simplify the above expressions into a form suitable for evaluation by table look-up or other means.

4.2.8 Define and evaluate the expected accumulations in terms of expected values for the contracts described in 4.1.1 and contract structures described in 4.2.7.

5 Multiple decrement and multiple life models (10%)

5.1 Define and use assurance and annuity functions involving two lives.

5.1.1 Extend the techniques of objectives 4.2 to deal with cashflows dependent upon the death or survival of either or both of two lives.

5.1.2 Extend the technique of 5.1.1 to deal with functions dependent upon a fixed term as well as age.

5.2 Describe and illustrate methods of valuing cashflows that are contingent upon multiple transition events.

5.2.1 Define health insurance, and describe simple health insurance premium and benefit structures.

5.2.2 Explain how a cashflow, contingent upon multiple transition events, may be valued using a multiple-state Markov Model, in terms of the forces and probabilities of transition.

5.2.3 Construct formulae for the expected present values of cashflows that are contingent upon multiple transition events, including simple health

insurance premiums and benefits, and calculate these in simple cases. Regular premiums and sickness benefits are payable continuously and assurance benefits are payable immediately on transition.

5.3 Describe and use methods of projecting and valuing expected cashflows that are contingent upon multiple decrement events.

5.3.1 Describe the construction and use of multiple decrement tables.

5.3.2 Define a multiple decrement model as a special case of multiple-state Markov model.

5.3.3 Derive dependent probabilities for a multiple decrement model in terms of given forces of transition, assuming forces of transition are constant over single years of age.

5.3.4 Derive forces of transition from given dependent probabilities, assuming forces of transition are constant over single years of age.

6 Pricing and reserving (35%)

6.1 Define the gross random future loss under an insurance contract, and state the principle of equivalence.

6.2 Describe and calculate gross premiums and reserves of assurance and annuity contracts.

6.2.1 Define and calculate gross premiums for the insurance contract benefits as defined in objective 4.1 under various scenarios using the equivalence principle or otherwise:

- contracts may accept only single premium.
- regular premiums and annuity benefits may be payable annually, more frequently than annually, or continuously.
- death benefits (which increase or decrease by a constant compound rate or by a constant monetary amount) may be payable at the end of the year of death, or immediately on death.
- survival benefits (other than annuities) may be payable at defined intervals other than at maturity.

6.2.2 State why an insurance company will set up reserves.

6.2.3 Define and calculate gross prospective and retrospective reserves.

6.2.4 State the conditions under which, in general, the prospective reserve is equal to the retrospective reserve allowing for expenses.

6.2.5 Prove that, under the appropriate conditions, the prospective reserve is equal to the retrospective reserve, with or without allowance for expenses, for all fixed benefit and increasing / decreasing benefit contracts.

6.2.6 Obtain recursive relationships between successive periodic gross premium reserves, and use this relationship to calculate the profit earned from a contract during the period.

6.2.7 Outline the concepts of net premiums and net premium valuation and how they relate to gross premiums and gross premium valuation respectively.

6.3 Define and calculate, for a single policy or a portfolio of policies (as appropriate):

- death strain at risk
- expected death strain
- actual death strain
- mortality profit

for policies with death benefits payable immediately on death or at the end of the year of death; for policies paying annuity benefits at the start of the year or on survival to the end of the year; and for policies where single or non-single premiums are payable.

6.4 Project expected future cashflows for whole life, endowment and term assurances, annuities, unit-linked contracts, and conventional/unitised with-profits contracts, incorporating multiple decrement models as appropriate.

6.4.1 Profit test life insurance contracts of the types listed above and determine the profit vector, the profit signature, the net present value, and the profit margin.

6.4.2 Show how a profit test may be used to price a product, and use a profit test to calculate a premium for life insurance contracts of the types listed above.

6.4.3 Show how gross premium reserves can be computed using above the cashflow projection model and included as part of profit testing.

6.5 Show how, for unit-linked contracts, non-unit reserves can be established to eliminate ("zeroise") future negative cashflows, using a profit test model.

Assessment

Combination of a computer based modelling assignment and a three hour written examination.

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