COURSE 1 INVESTMENTS OCTOBER 2012 EXAMINATIONS

Marking Guide

Level of difficulty

Question	Units	Learning Objectives	Knowledge and Understanding	Simple Judgement	Complex Judgement	Total Marks
Q1(a)	2	4.1-4.5	4	4		8
Q1(b)	2	4.4		1		1
Q1(c)	3	5.1-5.5			6	6
Q1(d)	3	5.1-5.5			5	5
Q2(a)	2	4.1,4.2		2		2
Q2(b)	2	4.1,4.2		3		3
Q2(c)	3	5.1-5.5			4	4
Q2(d)	3	5.1-5.5		4	4	8
Q2(e)	3	5.1-5.5		3		3
Q3(a)	1, 2	1.1, 4.1-4.7		4	4	8
Q3(b)	2	4.1-4.7		4	4	8
Q3(c)	2	4.1-4.7		2	2	4
Q4(a)	1	1.1,3.1	3			3
Q4(b)	1	1.1,3.1	4			4
Q4(c)	1	1.1,3.1	4	4		8
Q4(d)	1	1.1,3.1			5	5
Q5(a)	5	5.1-5.5	2			2
Q5(b)	5	5.1-5.5	2			2
Q5(c)	5	5.1-5.5	2			2
Q5(d)	5	5.1-5.5		4		4
Q5(e)	5	5.1-5.5			7	7
Q5(f)	1,5	2.1, 5.1-5.5		3		3
Total			**21**	**38**	**41**	**100**

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Answer all 5 questions.

QUESTION 1 **(20 Marks)**

You are an asset consultant advising a number of different investment funds, and you are planning to construct asset models to assist with asset allocation strategy for these funds. The asset classes you wish to include in the models are Australian equities, international equities (hedged and unhedged as distinct classes), Australian listed property, Australian fixed interest, international fixed interest (hedged) and Australian cash. International fixed interest may be subdivided into sovereign and non-sovereign (credit) debt.

Four possible types of asset return model are under consideration:

i) Multivariate normal (MVN) with iid returns (i.e. independent and identically distributed returns);

ii) MVN model with iid returns, with the returns for equities, listed property and credit modified by a mixture of normals to produce return distributions strongly skewed to the left, to allow for the large negative returns observed in major stock market downturns;

iii) Multivariate GARCH model;

iv) Vector autoregressive (VAR) model with iid multivariate normal innovation terms.

a) Compare the four classes of model in terms of their advantages and/or disadvantages in representing each of the following distributional properties of returns observed in practice:

- Form of marginal return distributions,
- Stability of the joint distribution over time, especially the stability of variances and covariances,
- Serial independence or dependence, and
- Comprehensive description of interaction between returns, past and present.

(8 Marks)

b) Why is the MVN still widely used in practice when it could be considered as a special case of each of the other three? **(1 Mark)**

Three investment funds for which you require asset return models are:

i) A closed pension fund where all members are pensioners aged 65 or older. The fund makes a fixed monthly payment to each pensioner.

ii) A defined contribution superannuation fund with active members only, most of whom are aged under 40.

iii) A savings fund for retail investors. The fund guarantees an annual return of 2% after tax. The trustees declare an annual crediting rate, which may be less than the actual earnings rate to pay for the guarantee, and an undistributed reserve fund may be maintained. Individual accounts are at call, although experience has indicated that very few investors redeem any of their account before age 65.

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c) For each of the three funds, outline the financial requirements for the fund and hence the characteristics of an asset return model to evaluate alternative asset allocations. Note that part (d) requires you to relate your answer to specific model types. (6 Marks)

d) For each of the three funds, which of the four model types discussed above would be an appropriate choice given the requirements outlined in part (c)? Explain briefly the reason for your choice in each case. You may nominate more than one model for any or all of your funds. (5 Marks)

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SOLUTIONS: QUESTION 1

a) Compare the four classes of model in terms of their advantages and/or disadvantages in representing each of the following distributional properties of returns observed in practice:

- **Form of marginal return distributions,**
- **Stability of the joint distribution over time, especially the stability of variances and covariances,**
- **Serial independence or dependence, and**
- **Comprehensive description of interaction between returns, past and present.**

(8 Marks)

MVN

Form All returns normal. In practice, many return distributions are long tailed or skewed to the left.

Stability Assumes distribution parameters are stable over time. This is not so: there are periods of high and low volatility, and correlations are unstable over time.

Serial dep Allows no serial dependence. For long term models, this is not serious because serial dependence tends to be short term. For a short term model, it may be important, especially for fixed interest and cash. It also fails to address volatility clustering in equity type investments.

Interaction Describes concurrent interaction, but may be inadequate for short term modelling if there is serial dependence.

Mixture of MVNs

Form Allows reasonable approximation to a range of marginal return distributions, including long tailed and skewed distributions.

Stability, serial dependence and interaction properties as for MVN.

GARCH

Form Returns assumed to be conditionally normal. This may be inadequate for long tailed or skewed distributions.

Stability Changing volatility and correlations are modelled explicitly. Unconditional means remain constant.

Serial dep Simple patterns of serial dependence are easily modelled with GARCH. This is probably adequate for most practical applications.

Interaction A certain amount of both concurrent and time series interaction is modelled. In principle, complex interaction patterns could be modelled, but the complexity of identifying the interaction forms and estimating the model usually dictates fairly simple relationships.

VAR

Form and stability as for MVN.

Serial dep VAR models can accommodate complex patterns of serial dependence.

Interaction Comprehensive interactions are managed routinely, both concurrent and time series.

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Marking Guide: (4KU, 4SJ)

- **2 marks per model with ½ mark per distributional property per model.**
- **Form and stability marks (4KU)**
- **Serial dependence and interaction marks (4 SJ).**

b) Why is the MVN still widely used in practice when it could be considered as a special case of each of the other three? (1 Mark)

The main reason is simplicity. The MVN model is easy to estimate and future returns can be simulated readily. The shortcomings of the MVN are not important for many long-term modelling applications.

Marking Guide: (1 SJ)

- **½ mark for simplicity**
- **½ mark for long-term (1 SJ)**

c) For each of the three funds, outline the financial requirements for the fund and hence the characteristics of an asset return model to evaluate alternative asset allocations. Note that part (d) requires you to relate your answer to specific model types. (6 Marks)

<u>Pension fund</u>
Financial requirements
- Adequate income to cover actuarial valuation rate of interest with a safety margin
- Sufficient stability of income to ensure ongoing solvency.

Model characteristics
- Accuracy of long term mean returns
- Description of effects of market downturns and increased volatility.

<u>DC superannuation fund</u>
Financial requirements
- Accurate description of long term contribution accumulations

Model characteristics
- Accuracy of long term mean returns and variances
- Although benefits are long term, members will be concerned with short-term progress. The model should therefore be capable of describing with reasonable accuracy the effects of short term variance changes and market downturns.

<u>Savings fund</u>
Financial requirements
- An average return sufficient to allow the crediting rate to be similar to other retail cash products, taking into account the cost of the guarantee.
- Short term solvency is important because investors' balances are at call. However, because calls are rare, this requirement is not over-riding.
- Because the guarantee is similar to a put option, its value (and cost) increases with volatility. The fund volatility must therefore be limited.

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Model characteristics

- Accuracy of short term mean returns and variances.
- Reliable description of the effects of volatility changes and market downturns.

Marking Guide: (6 CJ)

- **2 marks per fund split**
- **1 mark for financial requirement(s)**
- **1 mark for model characteristics.**
- **Award 1 mark each time if response covers at least one main issue correctly.**
- **Award ½ mark each time for a reasonable answer that is not eligible for 1 mark. (6 CJ)**

d) For each of the three funds, which of the four model types discussed above would be an appropriate choice given the requirements outlined in part (c)? Explain briefly the reason for your choice in each case. You may nominate more than one model for any or all of your funds. (5 Marks)

Pension fund

A MVN model will provide reasonable accuracy over the long term. However, short term changes in volatility and skewed or long-tailed distributions may affect solvency in the short term. Thus a mixture model would be preferable to assess the effect of non-normal distributions, and a GARCH model could be used to assess the effects of changing volatility. It may be possible to combine a mixture and GARCH model to get both characteristics.

DC superannuation fund

A MVN model would be adequate for assessing eventual retirement benefits, which are long term accumulations. However, to illustrate the short term effects of market anomalies on members' balances, a GARCH model and a mixture model would be required as above.

Savings fund

This fund is essentially short-term, and valuations should be frequent. At each valuation, the model must provide a forecast of mean and volatility over the next period, so that solvency and reserve requirements can be predicted with confidence. Because the benchmark is cash, equity and property investments are unlikely to be used, and market downturns will therefore not be so important. Credit may also be limited because of its susceptibility to downturns. A GARCH model would best fit these requirements.

Marking Guide: (5 CJ)

- **Pension fund (2 CJ)**
- **DC Super fund (1 CJ)**
- **Savings fund (2 CJ)**

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QUESTION 2 **(20 Marks)**

A new charitable trust in perpetuity has been created with the proceeds from the sale of a community asset. The purpose of the trust is to make grants on the basis of applications for financial assistance from community groups.

The amount realised from the sale was $300 million, compared with a book value of $250 million. The trustees view the book value of $250 million as the capital of the fund, and the remaining $50 million as retained earnings.

The trust deed requires the trustees to consider the competing claims of current and future generations in making grants from the trust fund. Trustees are appointed for a term of five years, and the current board has expressed a desire to pass on the fund to the next board in a similar financial position as it is now. Subject to these requirements, the trustees wish to maximise the total amount of grants provided by the fund.

The trustees have set out accordingly the following general objectives:

1. **Maintain the real value of the capital of the trust with regard to inflation.**
2. **Maintain equity between present and future generations in terms of the amounts available for distribution on an annual basis.**
3. **Maximise the total amount of income that can be provided by the investments of the trust over the long term subject to a prudent level of portfolio risk.**

a) **What limitations would you face in meeting the first objective (maintain the real value of the capital of the trust with regard to inflation) if the entire sale proceeds of $300 million were regarded as capital? (2 Marks)**

b) **At the initial meeting, one of the trustees has pointed out that the community is growing, and so maintaining the value of the fund in real terms will not adequately balance the claims of future against the current generation because the size of the fund per capita in real terms will diminish. How would you respond to this comment? (3 Marks)**

Two approaches to setting a funding strategy are to be examined. For either approach, the annual grant will be set at a fixed percentage (G%) of the capital, regardless of investment earnings and paid out of the notional capital fund.

***First approach*: The retained earnings (currently $50 million, or 20% of the capital) would be regarded as a reserve fund. Provided the reserve fund remains above 10% of capital, an amount would be transferred to capital each year to maintain its real value. If the reserve fund drops below 10%, the required transfer to capital would be deferred, and the annual grant reduced as necessary.**

***Second approach*: The capital would be invested using a conservative asset allocation to provide an income stream with low volatility. The reserve fund would be invested in a higher return-higher volatility portfolio. Provided the reserve fund is at least 10% of capital, transfers will be made to capital to maintain its real value. If the reserve fund drops below 10% of capital, no transfer will be made.**

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c) How should the fixed grant percentage G% be determined for each funding approach? (4 Marks)

d) Outline and compare the long-term dynamics you would expect for the two approaches. (8 Marks)

e) Briefly, what features should an asset return model have to compare the two funding approaches? (3 Marks)

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SOLUTIONS: QUESTION 2

a) What limitations would you face in meeting the first objective (maintain the real value of the capital of the trust with regard to inflation) if the entire sale proceeds of $300 million were regarded as capital? (2 Marks)

A perpetual fund must take a long term view. The only hope for maintaining the real value of the fund in the long term is to have considerable exposure to real assets such as shares and property. Treating $50m as retained earnings means, all else being equal, you can have higher volatility in the asset mix as we can use part or all of the retained earnings to shore up capital. The consequence of not treating $50m as retained earnings is the opposite, that is, we must have a lower overall volatility in our investments to meet the first objective.

Marking Guide: (2 SJ)

- **2 marks for point about long term view with suitable explanation (2 SJ)**

b) At the initial meeting, one of the trustees has pointed out that the community is growing, and so maintaining the value of the fund in real terms will not adequately balance the claims of future against the current generation because the size of the fund per capita in real terms will diminish. How would you respond to this comment? (3 Marks)

As with any business, growth (capital increase) must be financed through either retained earnings or new capital input.

Retained earnings should be able to finance inflationary increases in capital, at least in the long term, because expected returns have an expected inflation component.

The remaining income available for distribution as grants is the real investment income. Any further growth must be financed through a transfer from real income. If the expected real income was 4%, say, and the expected growth was 2%, then the trust could only expect to distribute 2% of capital each year. If the expected growth was 4%, nothing could be distributed!

Of course, if the investment strategy could deliver a higher expected real return, then some real growth could be financed while maintaining a reasonable net amount for distribution.

Put more simply, it is all about the objectives:

The fund stems from a community asset currently worth $300m. There is a maximum rate at which grants will be able to be made each year and still hope to maintain the real value of the fund (ie sticking to objectives). If this is sufficient to maintain per capita grants, great; if not too bad as it would be in conflict with objective 1.

Another argument is that there is a tie between the growing community and the inflation because some of the inflation is driven by demand expansion due to population. Therefore the objective aids help in term of maintain the real value of the distribution and the fund per capita.

Marking Guide: (3 SJ)

- **1 mark per point above to a maximum of 3 marks (3 SJ)**

c) How should the fixed grant percentage G% be determined for each funding approach? (4 Marks)

For the first approach, G% should be no more than the expected real return on the portfolio. It would be sensible to reduce G% somewhat from the expected real return to give a safety margin against both volatility and over estimation of the expected real return. The size of the safety margin can be evaluated using an asset-liability model.

For the second approach, G% should be no more than the expected real return on the portfolio *plus* an allowance for transfers from the reserve fund. Again, a safety margin is desirable. The size of the safety margin and the amount of the contribution from the reserve fund can be evaluated using an asset-liability model.

For the second approach, the G% should also account for the unexpected negative return due to aggressive investment strategy for the reserve fund which could result of no transfer for a few consecutive years.

Marking Guide: (4 CJ)

- **2 marks per approach (4 CJ)**

d) Outline and compare the long-term dynamics you would expect for the two approaches. (8 Marks)

The idea of using a reserve fund in either case is to allow the fund to maintain annual grants at an acceptable minimum level while investing in higher return-higher volatility securities to gain a higher expected real return, and thus make higher grants on average over the long term. The dynamics must be considered in this light.

<u>First approach</u>

Same asset allocation for capital and reserves.

- Higher risk allocation for the entire fund means higher average income over the long term.
- A 10% loss could easily be sustained with even a modest growth asset allocation following a market downturn over one or two years, eliminating the reserve fund. (In fact, this is inevitable.) Grants would have to be cut sharply in such circumstances.
- We would expect reserves to fall sharply or even disappear with market downturns from time to time, with the frequency depending on the risk level in the portfolio.
- In downturn grants would have to be cut sharply to maintain real capital and restore reserve levels.

<u>Second approach</u>

- Low risk asset allocation for capital, higher risk allocation for reserves.
- Because capital is much greater than reserves, the average portfolio risk is likely to be lower than under the first approach, and average grants over the long term would therefore be lower.
- In market downturns, the value of the reserve fund could fall sharply, so that transfers to capital to maintain its real value would have to be curtailed.

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- Low risk allocation for capital means that grants can be maintained at a constant nominal level even in quite severe downturns.
- The reserve fund may fall below 10% in downturns, but it would never disappear. The income for grants arising from the capital portfolio would be steady, usually rising with inflation, but probably having to be held at a constant nominal level in downturns.

The dynamics can be simulated with an asset-liability model, and the trustees can choose between the two approaches based on this demonstration.

Marking Guide: (4 SJ, 4 CJ)

- **Opening paragraph (1 SJ)**
- **First approach (3 SJ)**
- **Second approach (3 CJ)**
- **Final paragraph (1 CJ)**

e) Briefly, what features should an asset return model have to compare the two funding approaches? (3 Marks)

The fund will show problematic dynamics in downturns.

Therefore, the model must capture unusual market behaviour and the possibility of severe downturns.

This points strongly towards a GARCH model to capture changing volatility levels, and a marginal return distribution appropriately skewed to the left.

Marking Guide: (3 SJ)

- **1 mark per point above (3 SJ)**

QUESTION 3 **(20 Marks)**

You have been contracted by a financial planning firm to develop a series of model portfolios consisting of managed funds for use by their advisers.

A model portfolio sets out strategic asset allocations (SAAs) for a stated risk profile and the managed funds recommended for use within a sector specified for SAA.

In answering the questions below think not only of the technical investment issues you might encounter but also practical concerns, where appropriate, either the firm or its advisers might face in using the models you develop.

a) **Set out the information you would need for developing the SAAs both from the financial planning firm and sourced from your own resources. Why would you need this information? (8 Marks)**

b) **Describe the process you would undertake to determine the SAAs. (8 Marks)**

c) **What general principles should guide the choice of managed funds to populate the various sectors and/portfolios? (Do not repeat manager information you may have itemised under part (a) above and do not include a checklist of manager attributes or manager/fund specific requirements.) (4 Marks)**

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SOLUTIONS: QUESTION 3

a) Set out the information you would need for developing the SAAs both from the financial planning firm and sourced from your own resources. Why would you need this information? (8 Marks)

From the financial planning firm:

- Client segmentation data – Ages, proportions pre- and post-retirement, the distribution of investment amounts, cashflow certainty required (if possible) etc. These factors inform objective risk tolerance (that stemming from such matters as investment horizon, financial loss threshold etc as opposed to the attitudinal inputs that make up subjective risk tolerance). The firm may also need to decide, with your help, whether there should be a single or multiple series of portfolios, e.g. separate pre- and post-retirement series, separate superannuation/investment products. This is likely to be decided by a trade-off between the extent of genuine differences of need between the groups and administrative complexity.

- "Directional" information such as objectives (risk is dealt with below), preferred output/model structure, current model portfolios, if any (will also indicate extent of any transition required).

- Details of the risk profiles, the risk profiling process and whether the process is uniform across different client groups. If the process is biased to subjective factors, a process for harmonising them with objective factors may be required. The detail needs to be sufficient to allow you to use the profiles as inputs into the creation of differentiated portfolios either qualitatively or quantitatively or both. The profiles determine utility amongst other things. (See below for more detail on this.)

- Other risk data such as tolerance for illiquidity.

- Information on the relative sophistication of the planners and their clients. This may affect the type of managed fund you consider.

- Advice on the desire for dynamism in asset allocation (AA) processes - likely to be low other than for a lifestyle approach (dialling risk down as approach key dates such as retirement or lower FuM in advanced retirement) and the ability/preferred methods to deal with periodic rebalancing and significant environmental change. In general planners tend to deal with rebalancing etc at annual client reviews.

- Attitudes to fees, e.g. are they an issue in their own right or is it after-fee returns that are important?

- The extent to which tax should be built into the process – this may affect the pre vs post issue mentioned above or may be treated more generally in the sense of after fee and after tax returns.

- Any other regulatory or operational constraints that impact portfolio design.

- A risk metric which translates the key characteristics of investor's risk appetite, financial circumstance, investment experience etc into an quantifiable manner.

From your own or other resources:

For SAA modelling

- Historic return, risk and correlation data for relevant asset classes – Ideally risk data should cover volatility, downside volatility, negative return incidence, skewness and kurtosis. (Detail here will depend on the approach taken to modelling – see below.)
- Forecast return, risk and correlations for the chosen horizon/s.
- Key event and regime data either for modelling or portfolio resilience testing.
- Yield/return decompositions, especially if a post-retirement or cash generative series is required.
- Tax rates and related issues – Either as an ex ante input or as an ex post analysis.

For exposures within asset classes

- Manager research data for the relevant asset classes, both quantitative and qualitative. Qualitative research should include incorporate issues such as ownership, profitability, incentives, compliance and risk management practices, resourcing, client servicing capabilities and practices etc, investment style and processes (explanation, justification, process-based attribution, constraints) etc. Quantitative include returns, risk-reward measures, regime/period decomposition, holdings analysis, decision analysis etc.
- Product research data, including fees and general features such as unit pricing practices (e.g. forward vs historic pricing, buy-sell spreads, redemption requirements, especially if there is varying liquidity in the product or asset class etc).

(It is possible to think highly of an investment manager's investment skills but dislike product features, or regard a product as differentiated or innovative but to have reservations about the underlying manager.)

Marking Guide: (4SJ, 4 CJ)

- **Planner information – each significant point should have an attendant reason: client data & sophistication, risk profiles/profiling process, single vs multiple portfolios, tax & fees incorporation, rebalancing etc, iliquidity(4 SJ)**
- **Own/related resource information - each significant point should have an attendant reason : modelling data, manager/product data – 2 x 2 marks(4 CJ)**

b) Describe the process you would undertake to determine the SAAs. (8 Marks)

Overall

In general terms the portfolios that make up the SAAs should represent portfolios of equivalent risk-return efficiency where risk, however measured, is determined by the risk profile for the portfolio. Risk-return efficiency here is intended generally rather than necessarily indicating a mean-variance approach.

The portfolio design process will have both modelling and qualitative/acceptance aspects to it.

Objectives

While these are suggested as a planner firm input they need to be tested for:

- Attainability relative to resultant portfolios (another prompt for potential iterativeness is mentioned below)

- The extent they will produce differentiated portfolios, and
- How well they graduate the risk profile continuum.

This is likely to be part of the both the preparatory and results stages of the process.

Risk Measurement

The planning firm will have provided the risk profiles and the processes by which they were determined. Risk measures will need to be constructed for each profile to ensure that any modelling and choice of portfolio on risk criteria is appropriately referenced to the profiles. This may require some experimentation and testing on sample client data before the measure is selected.

As an example, planning firms often use a questionnaire approach, sometimes in conjunction with the objective risk tolerance data noted in part (a) above. These questionnaires often target loss and loss frequency tolerance as per behavioural finance findings. In such cases the probability of a negative return may an appropriate risk measure.

Modelling

A series of choices must be made before any modelling is undertaken:

- SAA structure – e.g. Sequential AA (find α within identified βs) vs α-β separation (no necessary relationship between invested as and βs), with the practical application at the planner-client level strongly influencing the choice. (It is possible to back-fit either approach with features of the other, making the starting point perhaps less critical. The greater point is to ensure the modelling accommodates the structure chosen.)
- Range of asset classes/opportunity sets to be included
- Modelling technique – e.g. Mean-variance, allowance for higher moments, deterministic/stochastic/simulations/distributional
- Modelling horizon/s
- Data – Historic vs forecast etc; how to apply same to return and risk measures
- Constraints to be applied to modelling to keep outcomes within acceptability, e.g. limiting illiquids or exotics to certain %s of portfolios. (Alternatively, these might be retro-fitted, e.g. models could be developed initially on an unconstrained basis and this might indicate where constraints may have to be imposed.)

Results

The process is likely to be iterative to some extent. The role of constraints in this is discussed above. In any event initial results will need to be discussed with the firm and this may throw up issues that need rethinking. Also the resilience/stress testing mentioned below may indicate the need for adjustments.

Resilience/Stress Testing

In addition to the modelling, which will be assumption driven to some extent even when stochastic/simulation/other distributional approaches are taken, it will generally be useful to test the performance of portfolios under a range of different scenarios and to test whether they generate amongst other things the cashflow required by the risk profiles. This

information could be useful in communicating with the planning firm to ensure they fully appreciate the potential behaviour of the portfolios, and may assist planners when communicating with their clients.

Report

A report should be provided that describes the:

- Nature of the brief
- Define the investment objectives and key constraints
- Process followed, including the data and assumptions used
- Results and comments (see section on Objectives above), and
- Any limitations and advice on the use of the results.

In addition any relevant professional standard should be observed and it may be useful to comment on any differences that have emerged relative to previous model portfolios, where relevant.

Marking Guide: (4 SJ, 4 CJ)

- **The essential elements of the answer are: getting the objectives right, setting out the modelling process, testing the results, communication with the client, reporting & commenting on how the results should be used. Up to (4 SJ)**
- **Differentiating elements include: converting the risk profile information into usable risk measures, detail on the modelling choices, ensuring the portfolios graduate by risk profile, overall quality. Up to (4 CJ)**

c) What general principles should guide the choice of managed funds to populate the various sectors and/portfolios? (Do not repeat manager information you may have itemised under part (a) above and do not include a checklist of manager attributes or manager/fund specific requirements.) (4 Marks)

- The SAA structure noted above may have an influence, e.g.an α-β separation approach will influence whether α β (market/factor) or α or both are being targeted in the fund/s in question; an asset sector, then exposure within sector approach may engender a more open consideration of the type/s of fund/s.
- While the individual funds themselves must be selected carefully, it is the portfolio of funds that is more important - noting that where α β is targeted, a single fund may suffice. The portfolio should capture the objectives and beliefs set for the sector or factor in question, e.g. a value approach may be preferred for equities for capital preservation reasons; a sector neutral combination might be preferred to ensure that at least one fund is firing at all stages of the cycle. Issues such as inter-fund correlations, up/down market performance per fund etc, either overall or at the α level, as appropriate, may play a role. As with the asset allocations, period or regime specific analysis may assist.
- Value-for-money (lowest cost for highest expected return).

Marking Guide: (2 SJ, 2 CJ)

- **The key issue is that question calls for "general" principles. (1 SJ)**
- **Possible principles include choices must be consistent with strategy(1SJ), a portfolio emphasis (1 CJ), value-for-money. (1 CJ)**

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QUESTION 4 **(20 Marks)**

a) Explain any relationship between the price volatility and duration of a bond. If you believe there is no relationship, provide your reason/s. (3 Marks)

b) Are there any other bond attributes that might be useful in managing bond portfolios in volatile or changing markets? Give reasoning for your answer. (4 Marks)

c) A colleague has informed you that he pursues a trading strategy for his bond portfolio that adjusts portfolio duration by small, frequent, selective amounts after movements in yields. On what would such a strategy be premised? Describe three ways in which you could implement this strategy. (Note: you are not required to indicate shortcomings on the approach relative to attributes you might have mentioned in part (b) above.) (8 Marks)

d) Another colleague tells you she is developing a strategy that will benefit or protect a fund from volatility that uses options. What feature/s of options, if any, make/s them amenable to such a strategy? How might that/those features be built into such a strategy? If you believe there are no strategies, provide your reason/s. (5 Marks)

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SOLUTIONS: QUESTION 4

a) Explain any relationship between the price volatility and duration of a bond. If you believe there is no relationship, provide your reason/s. (3 Marks)

Mathematical Explanation

Duration (D) is the weighted average timing of bond cashflows (coupon and principal, both represented by CF below) with the weights being the present value of those cashflows. It can be thought of as the discounted mean term of the bond cashflows and is a useful measure of a bond's sensitivity to changes in interest rates (see below). In formula terms: D = Σ t.CFt.vt/ Σ CFt.vt

D is sometimes known as Macauley's duration. The other form of duration, modified duration (D*), indicates the change in the bond price for a given change in yield to maturity. In formula terms: D* = D/(1+i), where i is yield to maturity.

Price volatility is defined as -(1/P).(dP/di), where P is price and i is interest rate.

Using the cashflow terminology above:

P = Σ CFtvt , dP/di = -(1+i)-1 Σ tCFt (1+i)-t = -v Σ tCFt vt

Therefore -(1/P).(dP/di) = v Σ tCFt vt / Σ CFtvt = vD = D*

General Reasoning Explanation

Price is a discounted cashflow calculation. It depends on the amount and timing of the cashflows and the discount rate. The discount factor for each cashflow is 1/(1 + discount rate)^cashflow date (where ^ is the Excel exponentiation operator).

In the fixed interest context replace "discount rate" with interest rate and replace "cashflow date" with n. The further out the cashflow date, the larger n is.

Price volatility is the responsiveness of price to changes in the interest rate while holding the other elements constant.

If the interest rate goes up, (1 + interest rate) increases. The larger n is, the larger the increase in (1 + interest rate)^n. Therefore, the larger n is, the larger the decrease in 1/(1 + interest rate)^n.

If the interest rate goes down, then (1+ interest rate) decreases. The larger n is, the larger the decrease in (1 + interest rate)^n. Therefore, the larger n is, the larger the increase in 1/(1 + interest rate)^n.

Since duration is a measure of the size of n, it provides a measure of sensitivity to interest rate change. When interest rates increase, you would prefer to be holding short duration securities; when interest rates go down, you would prefer to be holding long duration securities.

Marking Guide: (3 KU)

- **Statement of duration as a measure of price volatility (1 KU)**
- **Reasoning -either a mathematical or general reasoning approach is acceptable. (2 KU)**
- **Note that more detail is given in the solution than is required for the marks above.**

b) Are there any other bond attributes that might be useful in managing bond portfolios in volatile or changing markets? Give reasoning for your answer. (4 Marks)

Duration deals with parallel shifts in the yield curve but takes no account of changes in the shape of the yield curve. The sensitivity to same is known as convexity. Convexity is effectively derived as a measure of changing duration and may be regarded as more dynamic in orientation to this extent. "Volatile or changing markets" may well affect convexity. For example, whether a change in the short end of the curve will induce a proportionate change at the long end will, in part, depend on whether further changes are expected.

No mention is made of credit in the question. Even in a sovereign only fund the last few years have made clear that credit rating cannot be taken for granted. Changes in credit rating will affect the market credit spread of a security and overall yield. Note also that credit duration may not be the same as the interest rate duration of the portfolio. Credit duration refers to the sensitivity of the price to changes in its credit rating. A portfolio view aggregates security views.

(Other factors might include changes in demand or supply, the influence of or arbitrage opportunities through derivatives or the use of currency as a bond proxy. The first would presumably be taken up in price and duration positioning. Credit should be allowed for the latter two depending on the credibility of the case made.)

Marking Guide: (4 KU)

- **Convexity, credit, currency, derivatives (1 mark for convexity and ½ mark for each of the others to a maximum of 2 (KU))**
- **Reasoning (1 mark per explanation given) (2 KU)**

c) A colleague has informed you that he pursues a trading strategy for his bond portfolio that adjusts portfolio duration by small, frequent, selective amounts after movements in yields. On what would such a strategy be premised? Describe three ways in which you could implement this strategy. (Note: you are not required to indicate shortcomings on the approach relative to attributes you might have mentioned in part (b) above.) (8 Marks)

There are essentially four elements to the strategy set out in the question – changing duration in response to changes in yields and changes that are small, frequent and selective.

Why Change Duration?

Generally, for unchanged convexity, a shorter duration portfolio will outperform a longer duration portfolio when yields increase, and a longer duration portfolio will outperform a shorter duration when yields decrease.

To measure the strategy's effectiveness on this score would require a reference duration of some kind. For a relative investment approach the relevant index would provide such a duration. Portfolio duration would be made shorter or longer than index duration in the circumstances mentioned above. Otherwise the pre-adjustment duration could provide the reference duration.

The proviso "for unchanged convexity" requires some attention. For the strategy to be effective, a duration-focussed approach requires changes in convexity to have either a neutral effect on returns available from solely adjusting duration or for the changes in duration to somehow incorporate convexity. Where a relative approach is taken changes in both portfolio and index convexity must be considered.

Small, Frequent, Selective Changes

Small changes suggest a low relative risk approach, e.g. a quarter or half-year adjustment in duration.

If the colleague believes he has the skills to profit by such an approach, then it makes sense he uses those skills as opportunities present themselves. The fundamental law of active management suggests an investors provides themselves with sufficient breadth (no. of times they exercise their skill) to exploit information opportunities. It is in this sense that "frequent" could/should be interpreted.

The fact that the changes will be selective means that the colleague will respond differentially to market changes or may not respond at all. This will allow the colleague to assess whether there is an information/return opportunity available in the market change. For example, a rise in yields may not automatically induce an equal and opposite reduction as the next movement in yields.

This should also help in the management of transaction costs. In particular any fixed costs will have greater proportionate effect on small portfolio movements. Presumably, though, return after costs will be one of the "selective" criteria.

Implementation

Possible methods include:

- Transactions in physical bonds – buying cash, shorter- and longer dated-bonds as required. Adding/reducing cash will shorten/lengthen portfolio duration. Its effectiveness will depend on the extent of cash the portfolio is allowed to hold.

- Differentially directing incoming cashflow to shorter or longer bonds or selling shorter or longer bonds to fund redemptions. This requires a portfolio open to new subscriptions and redemptions and its effectiveness will depend on the extent and direction of net cashflow. No information is provided within the question to allow comment on whether this might apply.

- Buying/selling bill, 3 year and 10 year futures as required. Bill and 3 year futures will generally shorten duration; 10 year futures will generally lengthen duration

- Another possibility is using interest rate derivative such as interest rate swap to reposition the cash flow, hence the duration.

Marking Guide: (4 KU, 4 SJ)

- **Premise – why changing duration might work, reasoning for small/frequent selective changes (4 KU)**
- **Implementation – physical, cashflow crossing, derivatives, cashflow or derivatives as cheapest (1 mark per approach given) (4 SJ)**

d) Another colleague tells you she is developing a strategy that will benefit or protect a fund from volatility that uses options. What feature/s of options, if any, make/s them amenable to such a strategy? How might that/those features be built into such a strategy? If you believe there are no strategies, provide your reason/s. (5 Marks)

According to the Black-Scholes formula option prices depend on:

- The strike price
- The current price of the associated physical (the difference between the two being the intrinsic value)
- Time to expiry of the option (Indicating the time value)
- Risk free rate, and
- Expected price volatility.

(Expected cash payments can also be built into the formula for dividend and income paying assets.)

Options can thus be used to estimate and price volatility. Vega ($\delta V/ \delta\sigma$) is the change in option value relative to changes in volatility and, therefore, can be used to test changes in option price relative to expected changes to volatility and as the basis for trading strategies.

Such strategies may involve the buying and selling of put and call options, depending on directional views of volatility and relative pricing, e.g. when put-call parity does not seem to apply for constant volatility.

Or they may involve the associated physical as well bearing in mind that volatility derived from option prices (known as implied volatility) may exhibit basis variation from that experienced in the physical markets.

Marking Guide: (5 CJ)

- **Role of volatility in option pricing. (1 CJ)**
- **Strategies – vega, relative pricing, directional views, options/physical arbitrage. (4 CJ)**

QUESTION 5 **(20 Marks)**

a) **You wish to analyse the performance of a managed equity portfolio. The fund has 2 active management styles. Individual Stock Selection where stocks are actively weighted in the portfolio and Sector Selection where industry sectors are actively weighted in the portfolio. The equity fund has 3 industry sectors and 20 stocks. The monthly returns for the portfolio and benchmark of the 3 sectors are tabulated below in Table 1. Perform a Brinson arithmetic attribution to calculate the Sector Selection active return and the Stock Selection active return. (2 Marks)**

Table 1.

Sector	Benchmark		Portfolio	
	Weight	Return	Weight	Return
1	20%	2%	10%	2%
2	30%	3%	30%	4%
3	50%	4%	60%	9%
Total	100%	3.3%	100%	6.8%

b) **Assume that both portfolio managers have equal investment skill (information coefficients) and update their portfolios once a month. What is the expected *ratio* of the information ratios of Stock Selection over the Sector Selection active investment style? (2 Marks)**

c) **The Unit Prices of the managed equity portfolio are priced daily and investors can buy and sell investment units at the daily unit price. Portfolio returns are calculated using the Internal Rate of Return (IRR) with daily cash flows. Give one advantage and one disadvantage of this monthly portfolio performance calculation. (2 Marks)**

d) **We record 5 years of monthly returns from the Stock Selection and Sector Selection management styles. The standard deviation of Stock Selection returns is 3% annualised and the standard deviation Sector Selection returns is 5% annualised. Both active management styles have an average annual return of 1.5% over the 5 year period. Can we be 95% confident that the active managers achieved these returns through investment skill rather than luck? Show all formulae and working.**

[Hint: Use the Normal distribution approximation to the t distribution. Normal cdf values are given below in Table 2.] (4 Marks)

Table 2.

x	Pr(X<x) where X~N(0,1)
0	0.5000
0.5	0.6915
1	0.8413
1.5	0.9332
2	0.9772

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e) Assume the active returns of the Stock Selection and Sector Selection management styles are uncorrelated and that the ex-ante (forward) risk and return of the active styles will be the same as the previous 5 years risk and return (part d). You wish to maximise the information ratio of combined active styles by assigning a proportion $0 \leq \delta \leq 1$ of the value of the underlying equity fund to Sector Selection and a proportion $1 - \delta$ of the value of the underlying equity fund to Stock Selection. Find the value of the optimal proportion $0 \leq \delta \leq 1$.

[Hint: Minimise the variance of the combined active returns.] (7 Marks)

f) Eugene Fama and Kenneth French (1992) found that over the long term (1926-1992) a well-diversified equity portfolio of stocks with a current high price to book ratio (the ratio of the market price of equity to the book/accounting value of equity) has significantly lower returns than a well-diversified portfolio of stocks with a current low price to book ratio. What specific implications does this have for each of the categories of market efficiency in the Efficient Markets Hypothesis? (3 Marks)

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SOLUTIONS: QUESTION 5

a) You wish to analyse the performance of a managed equity portfolio. The fund has 2 active management styles. Individual Stock Selection where stocks are actively weighted in the portfolio and Sector Selection where industry sectors are actively weighted in the portfolio. The equity fund has 3 industry sectors and 20 stocks. The monthly returns for the portfolio and benchmark of the 3 sectors are tabulated below in Table 1. Perform a Brinson arithmetic attribution to calculate the Sector Selection active return and the Stock Selection active return. (2 Marks)

The Brinson attribution formula is written as follows (p – portfolio, b – benchmark):

$$Q1 = \sum w_i^b \times r_i^b = 20\% \times 2\% + 30\% \times 3\% + 50\% \times 4\% = 3.3\%$$

$$Q2 = \sum w_i^p \times r_i^b = 10\% \times 2\% + 30\% \times 3\% + 60\% \times 4\% = 3.5\%$$

$$Q3 = \sum w_i^b \times r_i^p = 20\% \times 2\% + 30\% \times 4\% + 50\% \times 9\% = 6.1\%$$

$$Q4 = \sum w_i^p \times r_i^p = 10\% \times 2\% + 30\% \times 4\% + 60\% \times 9\% = 6.8\%$$

$$\text{Active Stock Selection} = Q3 - Q1 = 2.8\%$$

$$\text{Active Sector Selection} = Q2 - Q1 = 0.2\%$$

Marking Guide: (2 KU)

- **The question tests student knowledge of portfolio performance attribution formula. The formula and correct answer are sufficient for the marks (Q4 is not necessary for the answer). (2 KU)**

b) Assume that both portfolio managers have equal investment skill (information coefficients) and update their portfolios once a month. What is the expected ratio of the information ratios of Stock Selection over the Sector Selection active investment style? (2 Marks)

Grinold and Kahn propose the following conceptual model of the information ratio of a managed investment portfolio, $IR = IC\sqrt{Br}$, where IR is the Information Ratio, IC is the Information Coefficient and is a proxy for manager skill and Br is the Breadth or number of independent investment decisions in the portfolio. Therefore since the equity portfolio contains 20 stocks and there are 3 portfolio sectors the ratio is:

$$\frac{IC\sqrt{20}}{IC\sqrt{3}} \approx 2.58$$

Marking Guide: (2 KU)

- **The question tests student knowledge of the conceptual model of the information ratio. The correct ratio is sufficient for the marks. (2 KU)**

c) The Unit Prices of the managed equity portfolio are priced daily and investors can buy and sell investment units at the daily unit price. Portfolio returns are calculated using the Internal Rate of Return (IRR) with daily cash flows. Give one advantage and one disadvantage of this monthly portfolio performance calculation. (2 Marks)

The advantage of the Internal Rate of Return (Money Weighted Rate of Return) is that it calculates the actual investment returns of the portfolio.

The disadvantage of the Internal Rate of Return (Money Weighted Rate of Return) is that it does not remove cash flow effects from the portfolio returns. If investors in the equity portfolio can buy or redeem equity investment units daily, the investment manager does not control cash flows in and out of the equity portfolio and Time Weighted Rates of Return (TWRR) should be used to calculate investment portfolio returns to remove cash flow effects from the portfolio returns.

Marking Guide: (2 KU)

- **The question tests student knowledge of portfolio performance calculation and the difference between the calculation Money Weighted Rate of Return (IRR) and Time Weighted Rate of Return.**
- **One mark for correctly identifying the advantage and one mark for correctly identifying disadvantage of using IRR performance calculations. (2 KU)**

d) We record 5 years of monthly returns from the Stock Selection and Sector Selection management styles. The standard deviation of Stock Selection returns is 3% annualised and the standard deviation of Sector Selection returns is 5% annualised. Both active management styles have an average annual return of 1.5% over the 5 year period. Can we be 95% confident that the active managers achieved these returns through investment skill rather than luck? Show all formulae and working.

[Hint: Use the Normal distribution approximation to the t distribution. Normal cdf values are given below in Table 2.] (4 Marks)

Since we are choosing between skill and luck, this is equivalent to a one-tail t-test where the Null hypothesis is a mean (return) of zero: $H_0: \mu = 0$. Rejecting the Null hypothesis implies the returns are due to skill. The t-statistic for this test is:

$$t = \frac{\bar{\mu}}{\bar{\sigma}/\sqrt{n}}$$

Where:

$\bar{\mu}$ is the sample mean (average return monthly).

$\bar{\sigma}$ is the sample standard deviation (monthly).

n is the number of samples (60 months, 5 years).

The twist in the question is that these are given as annual values, converting to monthly risk and return numbers gives the following t-statistic for Sector Selection:

$$t = \frac{0.13\%}{1.44\%/\sqrt{60}} = 0.67$$

For the Stock Selection:

$$t = \frac{0.13\%}{0.87\%/\sqrt{60}} = 1.12$$

The 95% confidence level is a 1-tail probability of the t-distribution with degrees of freedom 59 and it is reasonable to approximate this using the Normal distribution. From Table 2 we see that the t-statistic should be above 1.5 for 95% confidence that the Null hypothesis is rejected. Therefore we cannot reject the Null hypothesis at the 95% confidence level and thus cannot be 95% confident that the Stock Selection and Sector Selection returns were generated by skill rather than luck.

Marking Guide: (4 SJ)

- **The question tests student understanding of the statistical significance of reported active portfolio performance, in particular the requirement for long periods of reported outperformance before this can be accepted as evidence of manger investment skill rather than a random outcome.**
- **Two marks for correctly identifying the problem as a one-tail t-test where rejecting the Null hypothesis implies the returns are due to skill rather than luck, and an additional 2 marks for correctly calculating the resultant t-statistics and reasoning that we cannot reject the Null hypothesis. (4 SJ)**

e) Assume the active returns of the Stock Selection and Sector Selection management styles are uncorrelated and that the ex-ante (forward) risk and return of the active styles will be the same as the previous 5 years risk and return (part d). You wish to maximise the information ratio of combined active styles by assigning a proportion $0 \leq \delta \leq 1$ of the value of the underlying equity fund to Sector Selection and a proportion $1 - \delta$ of the value of the underlying equity fund to Stock Selection. Find the value of the optimal proportion $0 \leq \delta \leq 1$.

[Hint: Minimise the variance of the combined active returns.] (7 Marks)

We can formulate the combined information ratio using δ :

$$\text{IR} = \frac{(\delta + (1-\delta)) \times 1.5\%}{\sqrt{(\delta \times 5\%)^2 + ((1-\delta) \times 3\%)^2}} = \frac{1.5\%}{\sqrt{(\delta \times 5\%)^2 + ((1-\delta) \times 3\%)^2}}$$

It is clear from the formulation above that maximizing the IR is equivalent to minimizing the combined variance: $(\delta \times 5\%)^2 + ((1-\delta) \times 3\%)^2$.

For notational convenience; $m = (5\%)^2$ and $n = (3\%)^2$, substituting and expanding gives:

$$\delta^2 \text{m} + (1-\delta)^2 n = n - 2n\,\delta + (m+n)\delta^2 = f(\delta)$$

Taking the first order optimality condition on the resultant quadratic:

$$\frac{d\,f(\delta)}{d\,\delta} = -2n + 2(m+n)\delta = 0$$

$$\delta = \frac{n}{m+n} = \frac{(3\%)^2}{(5\%)^2 + (3\%)^2} \approx 0.26$$

Thus optimal proportion of Sector Selection should be 26% and Stock Selection should be 74%.

Marking Guide: (7 CJ)

- **The question tests the concepts of risk budgeting and the optimal composition of active management styles in an actively managed portfolio.**
- **Three marks for correctly identifying that maximising the IR is equivalent to minimizing the combined variance.**
- **Three marks for correctly formulating the first order optimality condition and an additional mark for the correct answer. (7 CJ)**

f) Eugene Fama and Kenneth French (1992) found that over the long term (1926-1992) a well-diversified equity portfolio of stocks with a current high price to book ratio (the ratio of the market price of equity to the book/accounting value of equity) has significantly lower returns than a well-diversified portfolio of stocks with a current low price to book ratio. What specific implications does this have for each of the categories of market efficiency in the Efficient Markets Hypothesis? (3 Marks)

Semi-strong market efficiency states that current stock prices reflect all public information about stocks. Since the book value of shares can readily be calculated from company financial statements, this is public information. Fama and French show that well-diversified portfolios of low price to book ratio stocks significantly outperform well-diversified portfolios of high price to book ratio stocks. This outperformance is evidence against Semi-Strong market efficiency since stock prices do not reflect the future outperformance of low price to book ratio stocks. Since the 3 levels of market efficiency are hierarchical, evidence against Semi-strong market efficiency is also evidence against Strongly Efficient Markets. Conversely, Fama and French did not design the well diversified portfolios based on historical price to book ratios and this is not evidence against Weakly Efficient Markets.

Marking Guide: (3 SJ)

- **The question tests student knowledge of the hierarchy of the 3 levels of the Efficient Market Hypothesis.**
- **One mark for each correct implication about weakly efficient, semi-strongly efficient and strongly efficient markets.(3 SJ)**

END OF PAPER

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