INSTITUTE OF ACTUARIES OF AUSTRALIA

COURSE 1 INVESTMENTS **2005 EXAMINATIONS**

MARKING GUIDE

Level of Difficulty

Question	Syllabus Topic	Units	Knowledge & Understanding	Straight-forward Judgement	Complex Judgement	Total Marks
1a	1	1	2	0	0	2
1b	1	1	3	2	0	5
1c	1	1	0	3	0	3
2a	2	2	2	5	0	7
2b	2	7	0	0	2	2
2c	2	2	0	2	4	6
3a	2, 5	2, 3	2	2	0	4
3b	1, 2, 3	6	2	2	0	4
3c	2, 1	2, 3, 7	0	2	6	8
4a		4	0	2	2	4
4b		4, 5	0	1	2	3
4c		4, 5	0	2	3	5
4d		4, 5	0	1	4	5
4e		4, 5	0	0	5	5
5a	5	5	0	1	2	3
5b	5	5	0	1	2	3
5c	5	5	2	0	0	2
5d	5	5	1	2	0	3
5e	5	5	0	0	4	4
6a	1	8	3	0	0	3
6b	1	8	0	4	0	4
6c	1	8	2	4	0	6
6d	1	2, 8	1	3	2	6
6e	1	8	0	1	2	3
TOTAL			**20**	**40**	**40**	**100**

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QUESTION 1 **(10 Marks)**

As advisers working in an investment bank, you are discussing with a US colleague the major structural differences for investment in the USA compared with in Australia.

(a) **Explain the terms *market structure* and *institutional structure* in relation to a free-market economy such as Australia's.** **(2 marks)**

(b) **Describe the institutions which regulate, or significantly affect, the operation of the following markets in Australia, indicating at least one piece of supervisory legislation and its scope per market:** **(5 marks)**

- **Goods and services**
- **Capital**
- **Money**

(c) **Explain *briefly* the importance of the markets listed in (b) to the main forms of investment in Australia.** **(3 marks)**

QUESTION 1: SOLUTION **(10 Marks)**

(a)
The market structure of an economy refers to the organisation of markets, the rules that govern access and pricing, and their interaction. *1 mark.*

The institutional structure refers to the particular institutions that have been established to regulate, facilitate or motivate the operation of markets. The precise functions of institutions are specific to a particular country, but the markets are in concept the same. *1 mark.*

(b)
Goods and services: these comprise the bulk of those markets that people are most familiar with. In general they are regulated only to the extent necessary to ensure consumer protection and (domestic) fair competition. *1 mark.*

The main regulator is the ACCC, which enforces the *Trade Practices Act*. In addition there are many other regulators in specific markets (health, building construction, etc). *1 mark.*

Other possible regulator answers:

The entry of foreign goods is, however, regulated under the quotas and tariffs prescribed under the *Customs Act*. Economic activity can, in theory, also be regulated by varying the rate of GST under the *GST Act*.

Capital: The main providers of managed long term investment are insurance companies, superannuation funds, and unit trusts. *1 mark.*

The regulator for the operation of most of these entities is APRA, working under *SIS* or one of the insurance acts. *1 mark.*

Answers could also include:

Access to capital by public companies is regulated by the ASX. Consumer interests are protected by ASIC under the *Corporations Act* (which involves licensing of advisers) or the *Managed Investments Act*.

Money: These markets are the domain of banks, building societies and credit unions and other deposit-taking institutions, whose operations are regulated by APRA under the *Banking Act*. *1 mark.*

(c)
The production of goods and services affects investment markets in two ways. Firstly, increased production (over and above market expectations) generally leads to an improved operating performance for most companies. Secondly, the increased wealth from higher production often leads to greater demand for investments, particularly equity and property investments. *1 mark.*

To encourage investment, capital markets need to be liquid, reasonably efficient and effectively regulated. *1 mark.*

Money markets, as influenced by RBA operations, determine the level of short term interest rates which, together with interest rate and inflationary expectations, are the fundamental driver of debt and property performance. *1 mark.*

QUESTION 2 **(15 marks)**

You are the investment adviser to a large Japanese bank considering investing its reserves in Australian debt markets. As part of its standard submission on its management practices (the *IFSA questionnaire*), a major reputable debt manager has provided the following information on its approach:

Source of Value-added	Expected Contribution to Performance (%)
Asset Allocation – Bonds/Indexed Bonds	-
Duration Selection	60
Yield Curve Analysis – Maturity distribution based on current or forecast shape of yield curve	10
Sector Selection – Government bonds, semi-government bonds, corporate etc.	5
Issue Selection (Including Credit Analysis)	20
Arbitrage and spread anomaly using forwards, swaps etc.	5
Technical Analysis	-
TOTAL	100

(a) **List the drivers for each of the 7 sources of potential value-added in the table above, and the expertise you consider necessary to add value through each source. (7 marks)**

(b) **Using the table above, suggest any advantages that might result from the debt manager's position in the market. How would you go about assessing the extent of those advantages? (2 marks)**

(c) **You have been asked to design an investment mandate for the Japanese bank. List any special features that you would include in the mandate. How would you deal with each feature? (6 marks)**

QUESTION 2: SOLUTUION **(15 marks)**

(a)
Asset allocation: The main drivers in the bonds/indexed bonds decision are the view on interest rates and the relative liquidity of each market. The main expertise required is a familiarity with each of these markets. *1 mark.*

Duration selection: The main driver in the duration decision is the view on the change in the level of the yield curve. The expertise required is the ability to forecast the direction and magnitude of interest rate movements. An understanding of monetary policy and the formation of rate expectations is critical here. *1 mark.*

Yield curve analysis: The main driver in yield curve analysis is the *precise* nature of the expected yield curve changes (ie the expected change in the shape, not just the shift). The same skills as for duration are needed, only more finely tuned and more likely in a stochastic setting. In particular the factors affecting the shape of the yield curve under the various technical yield curve models must be understood. *1 mark.*

Sector selection: The main drivers of sector selection are credit risk and liquidity premiums. The required skills are credit assessment and liquidity assessment skills. *1 mark.*

Issue selection: The main drivers here are the special opportunities present on issue of a corporate debt, either in the form of underwriting or placement on special terms. In house credit assessment capabilities are vital, as well as market presence for participation.*1 mark.*

Arbitrage: The main driver here is the relationship between the traded price of an investment and its fair value. The aim is to exploit any differences in these values. The required skills are technical understanding of the various debt instruments and how to price them correctly. *1 mark.*

Technical analysis (AKA charting): The main driver here is the identification and exploitation of pricing trends, and requires tested theories of short term pricing behaviour. *1 mark.*

(b)
The debt manager claims that it can add a reasonable amount of value from issue selection. Such a source of value-added often comes from a manager's market position or size, if it can be used to affect prices of particular issues. *1 mark.*

This source of value-added could be confirmed by actual experience with new or past issues. *1 mark.*

(c)
A foreign bank is likely to be concerned with diversification of its risks across different economies and interest rate regimes. *1 mark.*

Given this, the Bank is less likely to be interested in management approaches that add value at the margin, such as arbitrage and sector selection in this case. Hence the investment mandate could state that the expected value from these sources of value-added should be zero (ie they are to be avoided). *1 mark.*

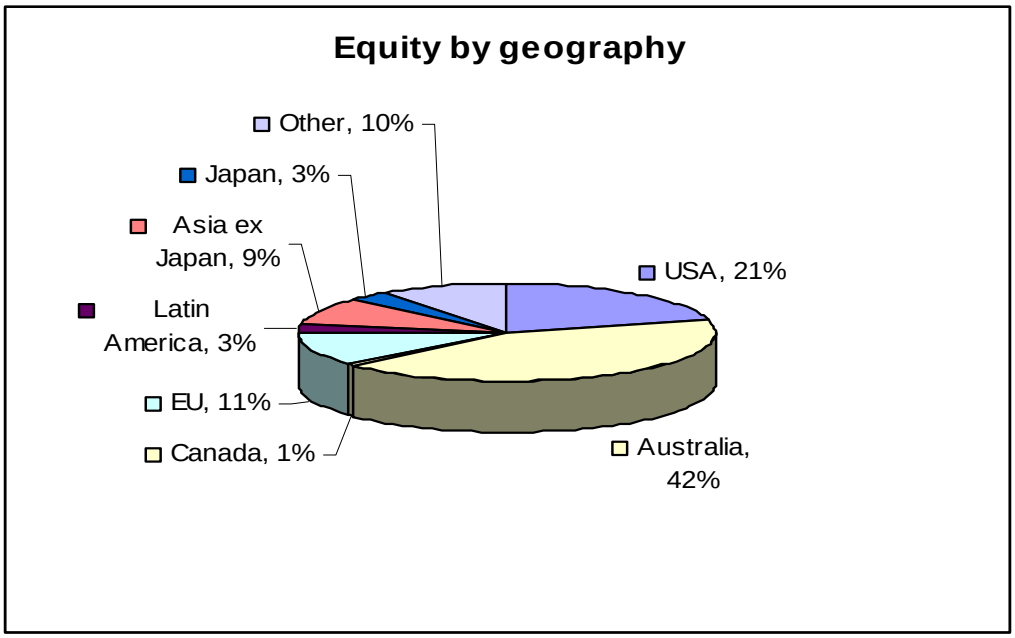
The creditworthiness and liquidity of Australian instruments will be of concern to a large Japanese bank. The mandate should therefore state the minium level of credit worthiness and liquidity for each instrument/sector. *2 marks.*

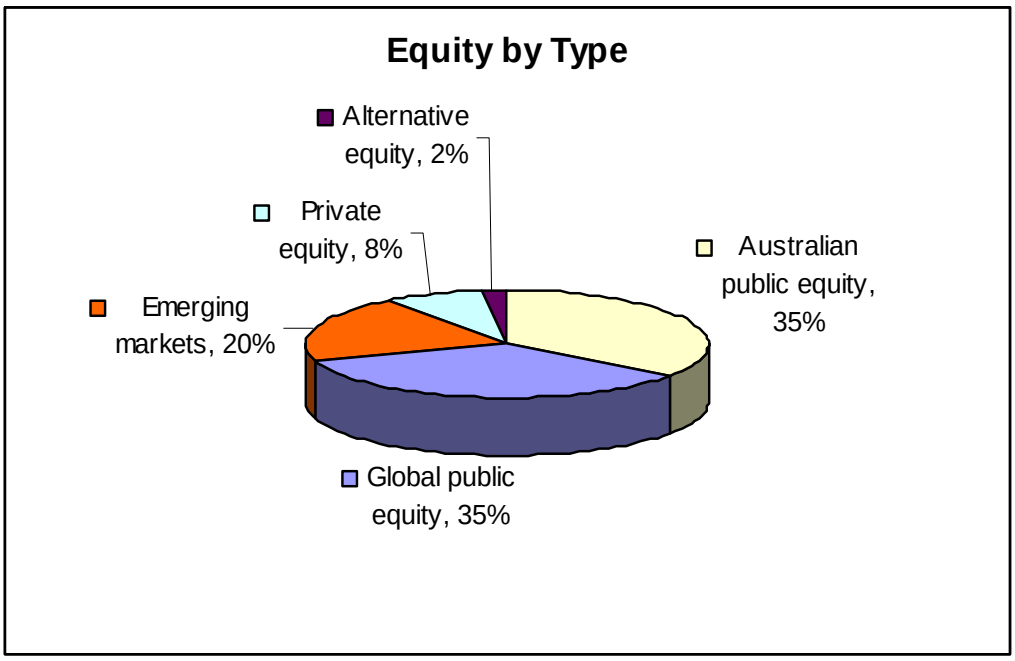
The Bank is likely to have a view about currency risk, especially if the investment (or disinvestment) can itself move currency markets. Therefore the mandate should also discuss the currency hedging policy to be adopted for the portfolio. *1 mark.*

Finally the Bank may have a view as to the duration required for the portfolio. Hence the mandate should also discuss the allowable range for the portfolio duration. *1 mark.*

QUESTION 3 **(16 marks)**

You are the Chief Investment Officer (CIO) of a large Australian corporation, whose superannuation plan has equity assets diversified in terms of both type and geography as follows:





The equity assets cover public equity, private equity and emerging markets and also include a limited exposure to infrastructure and hedge funds. The exposure to private equity is through limited partnerships. The management of each equity type is contracted out to a number of investment managers that each adopt a different investment approach.

You are concerned, not with the merits of the various equity classes/types, but with *the management time and effort required in monitoring the many investment approaches adopted.* You consider it worthwhile to rationalise the approaches taken in the light of these concerns.

(a) **Outline the main alternative management approaches for each of the (four) equity classes listed above (ie Alternative equity, Private equity, Emerging markets and Public equity). Your answer should cover: (4 marks)**

- **The issues associated with active management.**
- **The alternative styles for stock selection.**

(b) **List the advantages and disadvantages of investing in: (4 marks)**

- **a fund-of-funds, and**
- **a limited partnership.**

(c) **For each of the (four) equity classes listed above (ie Alternative equity, Private equity, Emerging markets and Public equity) discuss: (8 marks)**

- **the benefits and limitations to the superannuation plan, and**
- ***your* plan *for applying* a uniform approach**

in terms of (i) the reliance to be placed on active management and (ii) the styles to be adopted for stock selection.

QUESTION 3: SOLUTION **(16 marks)**

(a)
There are essentially four equity classes that need to be differentiated: public market equity (in developed countries); emerging market equity; private equity; and alternative equity (including hedge funds and infrastructure).

Public equity

Since investible benchmarks/indices exist, it is possible for public equity to be passively managed. Hence both active and passive management is possible. *½ mark.*

A variety of active stock selection styles exist that can be selected from. Such styes include value, growth, thematic, small/large capitalisation, quantitative and fund-of-funds. *½ mark.*

Emerging market equity

Passive management is possible if an appropriate benchmark/index can be agreed upon. Since this is likely, both active and passive approaches are possible. *½ mark.*

If active management is chosen, stock selection decisions/styles are likely to be narrowly focused on country or region. The investible universe is too large and uneven to be researched closely by a single manager. Local laws and customs would also be problematic. *½ mark.*

Private equity

A suitable benchmark/index is unlikely to exist. Further, the investible universe is likely to be very large and diverse. Hence passive management is not an option, leaving only active management. *½ mark.*

Stock selection alternatives are limited due to availability of data eg the quantitative approach is unlikely to be adopted. Value, growth, large/small capitalisation approaches are all possible however some flexibility/compromise may have to be allowed for. *½ mark.*

Alternative equity

Passive management is not possible given the lack of a suitable benchmark/index. Hence active management is the only option. *½ mark.*

There are many stock selection styles, including long-short, thematic, arbitrage, fund-of-funds etc. Investments can be focussed on hedge funds, infrastructure or other types of investments. *½ mark.*

(b)

Fund of Funds

Advantages:

- Diversification across styles, regions
- Pooling of funds can result in lower management fees

Disadvantages

- Relies on expertise of fund-of-fund manager, not just expertise of underlying managers
- Another layer of management fees can result in higher fees

½ mark for each point. Maximum of 2 marks.

Limited partnerships

Advantages

- Gives you access to investments that are otherwise inaccessible

Disadvantages

- Commitment period is generally quite long
- Difficult to benchmark performance
- Low liquidity
- High management fees, including performance incentives

½ mark per point. Maximum of 2 marks.

(c)

Public Equity

It is in developed markets, especially the USA, where market efficiency is most likely to hold. Hence the benefits of active management may not justify the fees payable nor the monitoring costs. A uniform, passive management approach should be adopted in this case. *1 mark*

Some exceptions might be made for smaller markets such as Australia, where local expertise resides and the Plan's investment is significant. The styles adopted should be chosen such that the overall portfolio is diversified and that monitoring costs are not too significant. *1 mark.*

Emerging Markets Equity

If a suitable index/benchmark does not exist then active management is the only alternative. However, even if a suitable benchmark/index does exist, active management may be chosen as emerging markets are likely to be less efficient than developed markets. *1 mark.*

Since a uniform investment approach may introduce a style bias into the portfolio, a number of different investment approaches should be selected. Monitoring costs and fees can be kept under control by investing in a fund-of-funds instead of directly with fund managers. This has the added benefit to the Plan of achieving greater diversification with less funds invested in the equity class. *1 mark.*

Private Equity

The lack of a suitable benchmark/index rules out passive management of this equity class, so the reliance will be entirely on active management. This equity class is likely to be less efficient than public equity given the limited amount of publicly available information. This additional risk should result in a higher expected return than public equity. *1 mark.*

In respect of uniformity, each private equity manager is likely to adopt a different investment approach. Monitoring costs can be kept to a minium by selecting a minimal number of managers or investing in a fund-of-funds. This shouldn't create much of a style bias since only 8% of the portfolio is allocated to this equity class. *1 mark.*

Alternative Equity

The lack of a suitable benchmark/index rules out passive management of this equity class, so the reliance will be entirely on active management. *1 mark.*

In respect of uniformity, each alternative equity manager is likely to adopt a different investment approach. However, since only 2% of the portfolio is allocated to this equity class, monitoring costs can be kept to a minium by selecting just the one manager or investing in a fund-of-funds. *1 mark.*

QUESTION 4 **(22 marks)**

As actuary to the employer and trustee of a large defined benefit superannuation fund in Australia, you have been consulted on the introduction of a new accounting standard (*AASB 119*) for employer superannuation costs.

The new standard will have the effect of including a significant element of the fund's asset return in any year as part of the employer's revenue for that year. At the same time the standard will bring to account the change in the discounted value of accrued benefit liabilities during the year, with discounting at a rate related to current bond yields.

You have been asked to review the fund's investment strategy in the light of your employer's concerns about the standard's effect on both short and long term revenue. A stochastic asset/liability model is to be applied. As a first step, you are required to produce an outline of your approach as follows:

Model Specifications

(a) **Discuss how the asset model will be used as well as the key economic variables that are to be modelled. (4 marks)**

(b) **Describe the main features of the liability model that you would give particular attention to. You are not required to give details of benefit projections. (3 marks)**

Inputs:

(c) **In calibrating the asset model, list the current economic/financial parameters you see as critical and discuss how you would source those parameters. (5 marks)**

Outputs:

(d) **List the outputs of your asset and liability model. How would you use the outputs in formulating an investment strategy? (5 marks)**

(e) **One of the company directors, who is also a trustee director of the fund, expresses concern that the fund's primary objectives are based on solvency and may differ from those of the employer. Explain how your asset liability modelling will change if you were to pursue both sets of objectives in formulating an investment strategy, ie ensuring fund solvency and employer cost minimisation. (5 marks)**

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QUESTION 4: SOLUTION **(22 marks)**

(a)
The asset model will be used to simulate asset returns (1/2 *mark)* and to model salary inflation and bond yields for the projection and valuation of benefit liabilities.
1/2 mark. (maximum of 1 mark)

Debt as an asset class will be a natural hedge for the liabilities to the extent that they behave like a bond. Thus the key economic variables to be modelled are:

Yield curve: this should be modelled consistently with inflation, debt returns and equity returns. (*1 mark*) Mean reversion is also necessary in yields to give sensible results. The presence of serial correlation in yields is also important for modelling the volatility of liabilities. *1 mark. (maximum of 2 marks.)*

Inflation: price and wage inflation should be modelled consistently, the difference between them representing productivity growth in wages. This should be consistent with industry expectations. *1 mark.*

(b)
The main features of the liability model that I would give particular attention to are as follows:

Fund membership. The overall membership should be projected in a way that is consistent with the employer's expectations. This involves of course the level of new employees in the long term. *1 mark.*

Consistency with the asset model. For the liability model, it is critical that benefits can be projected consistently with the elements of the asset model. Benefits related to salary can be projected using simulated wage inflation. Benefits related to asset returns (accumulation elements) need to be disentangled. *1 mark.*

Funding strategy. Though short-term superannuation cost is not sensitive to funding strategy, there is long-term dependence, as funding strategy dictates the level of assets invested. Therefore a realistic projection of long-term funding strategy is needed. *1 mark.*

(c)
The current economic/financial parameters I see as critical are current inflation, interest rate and yield levels. *1 mark.*

Current data could be sourced from any data provider such as Bloomberg or Reuters.
1 mark

Another significant assumption is the equity risk premium. *1 mark.*
A justifiable approach would be to adopt a long-term historical level, adjusted for current equity pricing levels. This might involve the assumption of mean reversion in PE ratios.
1 mark.

Property assumptions should be consistent with the ERP and the debt assumptions.

1 mark.

Maximum of 5 marks.

(d)

Given a particular asset allocation, the outputs of the asset/liability model would be simulated streams of variables of the following type:

- Aggregate salaries
- Benefit payments
- Contributions
- Asset values
- Discounted benefit liabilities
- Current service cost

½ mark per point. Maximum of 3 marks.

In order to formulate an investment strategy, the outputs should be used to project the employer's cost of superannuation in accordance with the rules of the accounting standard. Through simulation, an appropriate investment strategy can be determined such that the employer's expected future superannuation costs are minimised. *1 mark.*

The simulations should measure the expected value and standard deviation of the employer's expected superannuation costs so that the employer not only knows the firm's expected future superannuation costs, but the expected variability of these costs also.

1 mark.

(e)

If solvency is a concern as well as the cost to the employer, then the asset model should be concerned not only with the relationship between asset returns, inflation and bond yields, but also with the incidence of asset returns. *1 mark.*

This means that the year-by-year structure of returns is critical, and simple mean variance asset models are unlikely to be adequate. Greater focus should thus be placed on:

- The return distribution, particularly for equity returns
- Serial correlation between different years' returns

1 mark per point, max of 2 marks.

This would considerably increase the number of parameters required for modelling. Most of these parameters need to be determined historically. *1 mark*

If dynamic volatility is part of the model, then current levels of volatility would need to be determined. These might be obtained from fitting the model to past data, or could be inferred from current option pricing. *1 mark.*

QUESTION 5 **(15 marks)**

As an adviser within the hedging section of a trading bank, you are assisting the risk manager of a major manufacturing company producing hardware supplies. The risk manager's usual responsibilities relate to the cash and currency exposures of the company, but in recent times his responsibilities have broadened to cover other risk factors, as well as the use of derivative instruments.

The company is vertically diversified and wholly owns several subsidiaries in various industries that it is now looking to divest.

(a) **Suggest other risks that the company may face.** **(3 marks)**

(b) **The manager understands that (economic) exposures relating to futures and options on an underlying physical asset S are given as follows:** **(3 marks)**

- **For a bought futures position on asset S, exposure = S**
- **For a call option on asset S, exposure = Δ.S where Δ is the current delta of the option.**

Set out the general definition (formula) of economic exposure for any derivative instrument based on the asset S associated with a given single risk factor. Explain (using formulas) how this definition results in the exposures for futures and option positions as given by the manager.

(c) **Explain the change in exposure arising from entering a swap between fixed and floating interest rates.** **(2 marks)**

(d) **The company is looking to sell in 12 months time a small subsidiary operating in the materials industry, currently valued at $8.2 million. It wishes to hedge the price to be achieved on this divestiture in terms of its current valuation. The manager estimates that the performance of the materials industry follows the following historical relationship:** **(3 marks)**

$$R = 0.2R_f + 0.7R_m$$

where R, R_f, R_m **are returns on the industry index, the return on cash, and the return on the S&P/ASX 200 index, respectively.**

Explain how you would hedge the sale price by means of the SPI 200 futures contract, given the following information:

- **The SPI contract is based on an exposure of $25 for every point in the S&P/ASX 200 index;**
- **The current level of the S&P/ASX 200 index is 4,100 points.**

(e) **Explain the term *basis risk* in relation to the hedge in (d) and how you would estimate it. Suggest other ways of hedging that might reduce the basis risk.** **(4 marks)**

QUESTION 5: SOLUTION (15 marks)

(a)
Other exposures are to the market risk inherent in its subsidiary investments. *1mark.*

In addition, the company will be exposed to risks on the commodities used as inputs to manufacturing, eg. oil, steel, gas. *1 mark.*

The company will also be exposed to factors that affect demand for its products, eg. building construction. *1 mark.*

(b)
The exposure of a portfolio of assets (or a business) with value V to the risk is given by $\partial V / \partial S$, measured in units of S. That is, it is the number of units of S which have the same response as the portfolio to small changes in the price of the risky asset.
1 mark.

A long futures contract on asset S at a price F has the value:

$$V = S - F/(1+r)$$

where r is a discount factor to the contract date. [This is because the combination of the contract, together with a cash amount of $F/(1+r)$, invested to pay the futures price on the contract date, is identical to holding the risky asset.] Thus the exposure of the contact is $\partial V / \partial S = 1$ unit of S. *1 mark.*

A call option on S, of value C, may be combined with a short position of δ units of S to give an instantaneously riskless portfolio (the Black Scholes arbitrage argument). This occurs when:

$$\frac{\partial}{\partial S}[C - \delta S] = 0,$$

i.e. when $\partial C / \partial S = \delta$, giving by definition the delta of the option. However this is also the exposure of the option according to the definition above, and thus the exposure of a call option is δ units of S. *1 mark.*

Markers should use discretion in allocating marks, subject to a maximum of 3 marks.

(c)
A swap is the payment of one type of security in return for another type. Thus the exposure to one type of security is reduced and to the other type increased. *1 mark*

For a fixed/floating rate swap, it is the payment of a fixed rate bond in return for a floating rate bond of the same maturity (and redemption value). Thus interest rate exposure is reduced and cash exposure (to floating rates) increased. *1 mark*

(d)
The performance relationship says that each $100 in value of the subsidiary behaves (in expectation) like a portfolio of $20 in cash and $70 in the market index. *1 mark.*

Thus the subsidiary may be hedged by selling futures contracts on the SPI 200 contract in 12 months time. *1 mark*

Since the exposure on a SPI contract is based on $25 for each current index point, the required number of contracts is 0.7 x 8.2 m / (25 x 4100) = 56 contracts. *1 mark*

(e)
In general, basis risk refers to the divergence between the asset to be hedged and the hedging instrument.

In the present case, such risk arises from two possible sources:

(i) The divergence between the subsidiary's performance and that of the materials sector, of which it is part; *1 mark*

(ii) The divergence between the materials sector and the whole market (i.e. the ASX/S&P 200) *1 mark*

The basis risk from (ii) can be quantified as the standard deviation of the error in the historical performance relationship. This risk might be reduced by introducing other hedgeable factors that affect the performance of the materials industry, i.e.

$$R = \alpha R_f + \beta_m R_m + \beta_n R_n + \ldots$$

where R_n might be the price movement on oil, energy, etc. *1 mark*

The basis risk from (i) is much harder to measure or control, as presumably there are no easy means of valuing the subsidiary with any regularity. As in (ii), proxy factors particular to the subsidiary might be useful. *1 mark*

QUESTION 6 **(22 marks)**

As the asset consultant to a large industry superannuation fund, you have just presented to the Board of Trustees the performance results for the fourth quarter in relation to each of the fund's investment choices. The choices relate to units in:

- **Australian equity;**
- **Australian debt;**
- **Australian direct property;**
- **Alternative investments;**
- **Cash.**

These choices are managed by specialist managers. Managers have limited discretion to vary from their sectoral benchmarks in terms of industries or other sub-sectoral groupings.

The default choice is a balanced investment unit, invested across the asset classes listed above and managed by the superannuation fund's own staff. Derivatives are used to help adjust the exposures of the balanced unit to the asset classes. You assist from time to time in this respect.

After your presentation, you are approached by a new trustee director who is not familiar with the benchmarking and performance measures you have presented.

(a) **Explain the purpose of performance benchmarks in relation to the investment choices.** **(3 marks)**

(b) **Set out the criteria you would use for choosing an appropriate benchmark and discuss how you would determine the benchmark for the default choice.** **(4 marks)**

(c) **For each investment choice and each period, you have decomposed the manager's actual return into the weighted sum of sub-sectoral returns** $r^A = \sum_i w_i^A r_i^A$ **and similarly for the benchmark return** $r^B = \sum_i w_i^B r_i^B$**.**

In attributing performance between sub-sectoral allocation and stock selection, explain how you would deal with the compounding effects that arise in attribution over many periods. **(6 marks)**

(d) **At the start of a particular year, the balanced investment unit has exposure to three other units, representing equity, debt and property in equal proportions. The expected returns and covariance matrix for these asset classes are as follows:**

$$E(\mathbf{r}) = [9,5,7]^T$$

$$\text{cov}(\mathbf{r}) = \begin{bmatrix} 400 & 20 & 24 \\ 20 & 25 & 0 \\ 24 & 0 & 64 \end{bmatrix}$$

where $\mathbf{r}$ is the vector of equity, debt and property returns in that order, expressed as a percentage per annum. In addition, the current cash rate is 4.5% pa.

Show how the expected return and volatility for the balanced unit would vary if the exposure to equity were doubled by means of a futures contract. You may ignore dividend effects and assume that performance on the futures contract matches exactly that of the equity unit. (6 marks)

(e) **The benchmark for the default choice is a static weighted sum of the benchmarks for the other specific choices. Discuss how you would assess the performance of the fund's management of this choice. (3 marks)**

QUESTION 6: SOLUTION **(22 marks)**

(a)
The purposes of performance benchmarks are to:

- Express the trustee's risk/return objectives for each choice in a concrete way
- Provide a framework for manager objectives and performance assessment
- Provide a framework for investment controls.

1 mark per point, maximum of 3 marks.

(b)
The criteria for choosing an appropriate benchmark are that the benchmark should be:

- Achievable
- Objective
- Easily ascertained
- Consistent with the relevant investment objectives
- Representative of the investible assets

½ mark per point, max. of 2.5 marks.

The benchmark for the default choice could be a weighted sum either of other benchmarks or of other performances. *½ mark.*

In either case the weights should:

- reflect the trustee's objectives in balanced management *½ mark*
- be consistent with the management controls, asset allocation ranges and rebalancing rules *½ mark*

Maximum of 4 marks

(c)
Let $\Delta w_i = w_i^A - w_i^B$ and $\Delta r_i = r_i^A - r_i^B$ be the variation of actual weights and returns from benchmark and for simplicity, drop the superscript on benchmark variables.
1 mark.

Then the usual one period attribution is to decompose as follows:

$$\sum_i \left[(w_i + \Delta w_i)(r_i + \Delta r_i) - w_i r_i\right] = \underbrace{\sum_i w_i \Delta r_i}_{\text{stock selection}} + \underbrace{\sum_i r_i \Delta w_i}_{\text{allocation}} + \underbrace{\sum_i \Delta w_i \Delta r_i}_{\text{interaction}}$$

1 mark.

The interaction term can be added to either the allocation or stock selection effect, but may tend to disappear if attribution is done frequently (see below). *1 mark.*

For compounding over multiple periods, a variety of methods can be used. The dollar attribution weights the attribution for the current period by the actual return factor $\Pi\left(1+\sum_i w_i^A r_i^A\right)$ for other periods. *1 mark.*

The simplest method is to interpret the returns as instantaneous log-returns over short periods. Thus the portfolio value increase factor, relative to that of the benchmark, is:

$$e^{\sum_i [(w_i+\Delta w_i)(r_i+\Delta r_i)-w_i r_i]} = \underbrace{e^{\sum_i w_i \Delta r_i}}_{\text{stock selection}} \cdot \underbrace{e^{\sum_i r_i \Delta w_i}}_{\text{allocation}} \cdot \underbrace{e^{\sum_i \Delta w_i \Delta r_i}}_{\text{interaction}}$$

which lends itself to geometric compounding. *1 mark.*

When summed over many periods, the lack of correlation between Δw_i and Δr_i may over time tend to reduce the significance of the interaction term. *1 mark.*

Markers should use discretion, subject to a maximum of 6 marks.

(d)
Before doubling the equity exposure, the expected return and volatility are given by:

$$\mathbf{w}^T E(\mathbf{r}) = 7\%$$
$$\mathbf{w}^T \operatorname{cov}(\mathbf{r})\mathbf{w} = 8.0\%$$

where $\mathbf{w} = [1/3, 1/3, 1/3]^T$. *1 mark*

After doubling, it is necessary to account for the performance of the futures contract. Let $V = S - F/(1+i)^t$ denote the value of the contract at any time, where S is the exposure under the contract, F is the (fixed) contract price, and $(1+i)^t$ is a discount for the term to the contract date using the risk-free rate. *1 mark*

At outset, $S = S_0 = 1/3$, and since the initial value must be zero, $F = (1+r)S_0$. *1 mark*

On the contract date, the profit/loss on the contract is $S - F = (S - S_0) - rS_0$, which is simply the expected return (in dollars) on the equity exposure, less a return at the cash rate. *1 mark*

Thus the effect of the futures is to lengthen equity, with portfolio weights changing to $\mathbf{w} = [2/3, 1/3, 1/3]^T$, but to shorten cash exposure. *1 mark*

The expected return and volatility on this basis are:

$$\mathbf{w}^T E(\mathbf{r}) - i/3 = 8.5\%$$
$$\mathbf{w}^T \operatorname{cov}(\mathbf{r})\mathbf{w} = 14.4\%$$

1 mark

Markers should use discretion, subject to a maximum of 6 marks.

(e)
The fund's investment of the balanced unit is through asset allocation across the other units. Thus the allocation effect in an attribution of overall fund performance against benchmark could be adopted, in a similar way to that described for (c). *1 mark*

However two special features should be noted:

- Rebalancing across units cannot be continuous in practice, and this should be allowed for in the benchmark (on the grounds of achievability) as well as in actual performance. *1 mark*

- The profit or loss on derivatives should be entirely attributed to asset allocation, as their function is to manage exposures. This means that they should be disregarded in the attribution process but their effect on overall return included with the final asset allocation effect. *1 mark*