THE INSTITUTE OF ACTUARIES OF AUSTRALIA

INVESTMENT MANAGEMENT **1998 EXAMINATIONS**

PAPER TWO **MODEL SOLUTIONS**

QUESTION 1 **(13 Marks)**

a)

- It is assumed that options can be replicated (without risk) by a portfolio of risk free assets (cash) and the physical (1 mark)
- The principal underpins both the risk-neutral valuation approach and binomial approach for valuing options (1 mark)
- It is assumed that there are no transaction costs, (½ mark)
  short-selling is allowed, (½ mark)
  and continuous trading is allowed (½ mark)
- The value of the portfolio required equates to the value of the option (1 mark)
- The delta of the option will be reflected in the weighting to the physical security in the portfolio (1 mark)
- Clearly explained example (1mark)

(To a maximum of 5 marks)

b)(i)

- As yield rises, the increased term causes an increase in duration that offsets the decrease in duration normally caused by higher yields (with the opposite happening if the yield falls) (2 marks)
- Convexity will be small or negative (2 marks)
  - if the student does not consider the possibility that convexity may still be positive, then (lose ½ mark)

b)(ii)

- As yields rise, the higher duration accelerates the impact of higher yields on reducing the asset price (1 mark)
- As yields fall, the lower duration diminishes the impact of lower yields on increasing the asset price (1 mark)
- In parallel yield movements, mortgage-backed securities will fall in value relative to fixed liabilities of the same modified duration (2 marks)
- Redington's theory requires the convexity of assets to be greater than liabilities – which may be more difficult to achieve if the asset portfolio contains assets with negative convexity. (1 mark)

(To a maximum of 4 marks)

QUESTION 2 **(14 Marks)**

a)

Full marks to any explanation that highlights the difference : outright measure vs relative measure.

- Standard deviation is an "outright" measure of risk, measuring the uncertainty of return (1½ marks)
- Beta is a relative measure of risk, measuring the exposure to market risk – the extent to which market fluctuations are likely to affect the return (1½ marks)
- The Beta of an asset (with respect to a portfolio) is a measure of the marginal change in portfolio variance ($sdev^2$) resulting from diversification to that asset. (1½ marks)
- Beta = [sdev (asset) / sdev (market)] x Correlation (½ mark)

(To a maximum of 3 marks)

b)

The circumstances are :

- Diversification to an asset with lower risk than the portfolio (1 mark)
- Diversification to an asset with less than perfect (or preferably negative) correlation with the portfolio (2 marks)
- Where necessary, it is possible to short sell (1 mark)
- In terms of standard deviation as a measure of risk, where short selling is not allowed, where [sdev (asset) / sdev (portfolio)] x Correlation (asset and portfolio) < 1 (i.e. $\beta < 1$) (1 mark)

(To a maximum of 4 marks)

c)(i)

$$\boxed{\beta_B(A) = C\frac{\sigma A}{\sigma_B} = 0.25\frac{0.20}{0.05} = 1}$$

- Correct formula (1½ marks)
- Using correct values (½ mark)

c)(ii)

Short Answer : (5 marks for this explanation)

- The beta of 1 implies there are no marginal benefits of diversifying from Asset B to Asset A.
- As Asset A has a higher standard deviation, the minimum standard deviation portfolio must be comprised totally of Asset B

Question 1 (continued)

Long Answer :

$$Min(stddev) \Rightarrow Min(Var)$$

$$Var = p^2\sigma_A{}^2 + (1-p)^2\sigma_B{}^2 + 2Cp(1-p)\sigma_A\sigma_B$$

$$= p^2\left(\sigma_A{}^2 + \sigma_B{}^2 - 2C\sigma_A\sigma_B\right) - 2p\left(\sigma_B{}^2 - C\sigma_A\sigma_B\right) + \sigma_B{}^2$$

$$\frac{\partial Var}{\partial p} = 2p\left(\sigma_A{}^2 + \sigma_B{}^2 - 2C\sigma_A\sigma_B\right) - 2\left(\sigma_B{}^2 - C\sigma_A\sigma_B\right)$$

$$\frac{\partial Var}{\partial p} = 0 \Rightarrow p = \frac{\sigma_B{}^2 - C\sigma_A\sigma_B}{\sigma_A{}^2 + \sigma_B{}^2 - 2C\sigma_A\sigma_B} = \frac{0.05^2 - 0.25\times 0.20\times 0.05}{0.20^2 + 0.05^2 - 2\times 0.25\times 0.20\times 0.05} = \frac{0}{0.0375} = 0$$

- Thus the portfolio which minimises standard deviation is comprised totally of Asset B
- Using correct formula for variance (2½ marks)
- Doing mathematics correctly (differentiating to find minimum) (1½ marks)
- Getting right answer (1 mark)
- **Bonus** mark if student uses long approach, but then demonstrated an understanding of the result (i.e. the short approach) (1 mark)

QUESTION 3 **(16 Marks)**

RISK :
- Credit/default risk associated with non-payment or deferral of interest and/or principal payments (2 marks)

MANAGING RISK
- Yield deduction in the valuation of liabilities and pricing (equal to the expected default cost) (1½ marks)
- Enter into a credit swap (1½ marks)

RISK
- Floating rate asset vs fixed rate liability (2 marks)

MANAGING RISK
- Use interest rate swap to swap floating rate asset to fixed rate (2 marks)

RISK
- Term risk associated with fixed liability payment at 2 years vs value of 15 year term asset at year 2 (1½ marks)

MANAGING RISK
- Swap back to floating beyond 2 years since the value of floating rate debt is largely immune to changes in nominal interest rates (1½ marks)

Question 3 (continued)

RISK

- Concentration risk associated with default of a large single credit exposure (1½ marks)

MANAGING RISK

- Restrict exposure to asset in relation to portfolio size (1½ marks)

RISK

- Liquidity risk associated with the need to sell the asset at year 2 if insufficient new annuity business written to enable the asset to be rolled-back to back new annuity business. (1½ marks)

MANAGING RISK

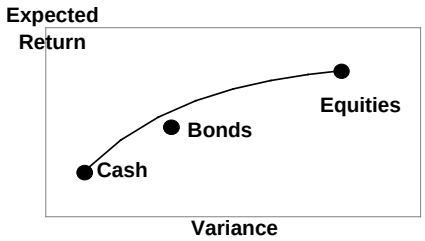
- Enter into an agreement that enables you to sell the asset to another party at your option in 2 years time, or ensure that you can swap the asset to support other liabilities in exchange for cash. 1½ marks)

(To a maximum of 16 marks)

QUESTION 4 **(22 Marks)**

a)

- Label axes : Expected Return / Standard Deviation or Variance of return (1 mark)
- Cash as minimum variance / Equities as maximum variance (1 mark)
- Bonds within (not on) efficient frontier, with variance between cash and equities (1 mark)



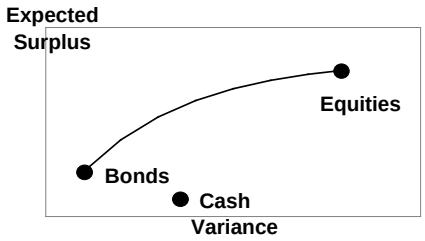
- There should be a 1-to-1 mapping between Variance and expected return - if the efficient frontier "curls around" then (lose 1 mark)
- Students may suggest that equities do not lie on the efficient frontier, if they argue that a subset of the equities market may be more efficient.

b)

- Label axes : Expect Return or Surplus / Standard Deviation or Variance of return or surplus (1 mark)
- Bonds as minimum variance / shares as maximum variance (2 marks)
- Cash within (not on) efficient frontier, with variance between bonds and equities (1 mark)

Question 4 (continued)

- Cash with lower expected surplus than bonds (1 mark)



- There should be a 1-to-1 mapping between Variance and expected return - if the efficient frontier "curls around" then (lose 1 mark)

c)(i)

- The emphasis here is on demonstrating a clear understanding of how ALM is actually used. The following is an example. Marks setting out as a letter will be awarded in part (ii).
- Clarity of explanation - use your judgement (0, ½ or 1 mark)
- Select a wide range of "candidate" portfolios (1 mark)
- Perform simulations of future asset & liabilities - both values and cashflows (1 mark)
- Determine an appropriate quantifiable risk measure, such as, say "probability of breaching the capital requirement within 10 years" (1 mark)
- Determine an appropriate maximum level of risk - say probability less than 5% - this may be determined or reviewed after an examination of the simulation results (1 mark)
- Choose the portfolio(s) which maximise expected shareholder value subject to meeting the risk requirement - alternatively one may draw an "efficient frontier" (1 mark)
- Use the best portfolios to fine-tune the results (performing new simulations) (½ mark)
- The results will be presented as a best portfolio, and a selected number of good portfolios, identifying their expected return to shareholders and risk (½ mark)

(To a maximum of 6 marks)

Question 4 (continued)

c)(ii)

- Set out as a letter (1 mark)
- Clarity of explanation - use your judgement (0, ½ or 1 mark)
- Variance or standard deviation is not a relevant risk measure to management, since it doesn't adequately capture the risk of breaching the regulatory capital requirement in the medium term (1½ marks)
- Asset return or return in excess of expected liability payments is not a relevant measure of value to management, since it doesn't adequately capture future profit margins, or the cost of capital (1½ marks)
- Part (a) ignores liabilities and asset-liability interactions entirely (1 mark)
- Part (b) treats liability payments as fixed, which is inappropriate for participating life insurance business where bonus/crediting rates are managed over time, and fails to capture asset-liability interactions (1½ marks)
- The shareholders value approach captures future profit margins, cost of capital requirements and the asset-liability interactions that are relevant to management (1 mark)

(To a maximum of 8 marks)

QUESTION 5 **(35 Marks)**

The first 4 parts of this question require students to have a practical understanding of :

- delta hedging
- the fact that options have both a cash and an equity exposure
- the main factors affecting option prices – without the need to understand Black-Scholes, binomials, martingales or other higher mathematics.

It is possible that, due to a combination of the length of the paper and the length of the last question, the majority of students did not have enough time to give the last part of this question the time it deserved. To assist in the final marking process, it would be of assistance if you could give each student an indicative grade based on parts (a) to (d), in addition to an overall grade. It will not be necessary to agree with your co-marker on this grade.

To the extent that the later parts of the question rely on the answers to earlier parts, students should be awarded marks for the correct approach, and not be penalised for assuming incorrect calculations done earlier.

a)

This question requires a practical understanding of delta hedging. To get full marks, students should be aware that the SFE contract for $25 per point (this is in the course notes).

- Trade involves selling call options (1 mark)

Question 5 (continued)

- Reduce exposure by $50 mil (1 mark)
- Number of contracts = Exposure / Delta / Spot / $25 = $50mil/0.316/2000/$25 = 3165 (mark)
- Option premium received = 3165 x 56.7 x $25 = $4.49 mil (1 mark for correct approach)
- If the only error is that the option premium is out by a factor of 10 or 100, then only lose (½ mark)
- If the only error is the number of contracts of 2877 (dividing by 2200 instead of 2000), then only lose (½ mark)
- If the only error is that the number of contracts is calculated at $1 per point (to give 78,114 contracts) but the answer is otherwise correct, then only lose (½ mark)

Some students chose to either :

- buy a put option (a valid approach in real life, but against the instructions of the question, and for which there was no details) or
- buy a call option and sell all the physical securities.

The second approach is against the instructions of the question, but it is reasonable to award marks as appropriate. In that case, the trade is to buy 3165 call options. Part (c) is simplified in that the gamma then works in the portfolio's favour, and part (d) is also simplified – adjust marks as appropriate.

b)

Students should be able to demonstrate an understanding that the option position has both an equity and a cash exposure. Part of the purpose of this question is to help students set out the information from part (a) in such a way as to make it easier for them to answer parts (d) and (e).

- Recognition that option includes both cash and equities exposure (1 mark)
- Calculation of physical cash exposure (1 mark)
- Calculation of the option equity exposure (1 mark)
- Calculation of the cash exposure of the option as the equity exposure of the call less the premium, as per their results from part (a) (1 mark)
- Effective equity exposure for option position : -0.316 x 3165 x $25 x 2000 = $50.007 mil
- Effective cash exposure for option = Equity exposure - premium = $50.007 mil -$4.486 mil = $45.521

Asset	$mil
Physical equities	+100.000
Cash	+4.486
Options	-4.486
Actual	+100.00

Question 5 (continued)

	Equity Exposure	Cash Exposure	Total Exposure
	$mil	$mil	$ mil
Physical equities	+100.000	0.000	+100.000
Cash	+0.00	+4.486	+4.486
Options	-50.007	+45.521	-4.486
Actual	+40.993	+50.007	+100.000

c)

Gamma risk : (maximum 2½ marks)

- Delta rises as All Ords rises - leading to loss, and delta falls as All Ords falls, also leading to loss (2½ marks)
- Loss at expiry if the price falls (1 mark)
- Loss at expiry if the price rises through 2200 (1 mark)
- Loss due to changes in delta (without specifying how this might happen) (½ mark)

Vega risk : (maximum 2 marks)

- Risk of loss if volatility rises (2 marks)
- Risk due to changes in volatility (without specifying direction) (1 mark)

Hedging risks : (maximum 2 marks)

- Stock selection : Equity values do move in line with All Ords (1 mark)
- Basis risks : Futures out of line (or move out of line) with All Ords (1 mark)
- Equity values move out of line with futures (1 mark only)

Other points : (maximum 1½ marks)

- Need to rebalance (risk of gapping) (1 mark)
- Liquidity risk due to margin calls (½ mark)

Do not award marks for these points :

- Counterparty risk (0 marks - SFE risk is negligible)
- Time decay (theta risk) (0 marks - time works in favour of OTM call writer)

(To a maximum of 6 marks)

Question 5 (continued)

d)

- Physical value $100 x 1.055 = $105.50 mil (1½ marks)
- Cash value unchanged at $4.49 mil (1 mark)
- Option value is -106.8/56.7 x (-premium in part (a)) = 106.8/56.7*-$4.49m=$8.45m
  (1 marks for correct approach)
  plus (1 mark for using correct premium at end : 106.8)
- Total = Physical + Cash + Options = $105.5m + $4.49m - $8.45m = $101.54m (½ mark for correct formula)
- A maximum of **(3½ marks)** for the above if the student estimated the option value at the end using the delta and change in All Ords : $4.49m + (2100-2000) x 3165 x 25 = $6.99m

Asset	$mil
Physical equities	+105.50
Cash	+4.49
Options	-8.45
Actual	+101.54

- **Bonus** mark if the student identifies the effective equity exposure at the end = $105.5m-3165 x 0.450 x 25 x 2100 = $30.73m (1 mark)

e)

This is a difficult question that requires students to have a clear understanding of the impact of factors affecting the portfolio, in the context of performance measurement. Given both the difficulty and size of this question, an allowance will be made for students who fail this part but do well on the rest of the paper.

There are really two parts to this question :

- identification of sources of under/over- performance relative to benchmark, and
- quantification of the impact

Students will get substantial marks for simply identifying the sources of under/over-performance.

General points (maximum 3 ½ marks)

- Recognition that an attribution analysis is required (1½ marks)
- Calculation of benchmark result (1 mark if correct approach used)
- Calculation of underperformance relative to the benchmark
  (1 mark if correct approach used)
- Benchmark at end : $50 + $50 x 2100/2000= $102.50

Question 5 (continued)

- Under-performance : Portfolio at end, from part (d) - benchmark at end = $101.54 - $102.50 = -$0.96

Physical
Stock selection (maximum 4 marks)
- Recognition that the performance of physicals was a source of outperformance (2 marks)
- Calculation of benchmark physical performance (1 mark)

Options
Effect of increased volatility (Vega) (maximum 4½ marks)
- Recognition that an increase in volatility was a source of underperformance (2 marks)
- Calculation of change due solely to change in volatility (2½ marks for correct approach)
  - if include the interactive effect then a maximum of (2 marks)
- Option value if volatility and price unchanged : -67.5/56.7 x $4.49 = $5.34
- Loss due to higher volatility = Option value if only volatility changed - starting option value = $5.34 - $4.49 = $0.85
- Including interactive effect : (106-8-94.4)x3165x25 = $0.98

Effect of changing delta as the All Ords increased (Gamma) (maximum 5 marks)
- Recognition that an increase in the All Ords was a source of underperformance (2 marks)
- Calculation of expected change in option price if volatility unchanged, based on delta (1½ marks if correct approach used)
- Calculation of change in option price due to change in All Ords (1½ marks if correct approach used)
- Calculation of effect of change in delta (1 mark if correct approach used)
- Expected change in option price if volatility unchanged (based on delta) = 0.316 x (2200-2000)/56.7 x $4.49 (=50/2000 x $50) = $2.50
- Change in option price due to change in All Ords (without considering change in volatility) = (94.4/56.7-1) x $4.49 = $2.98
- Effect of change in delta = $2.98 - $2.50 = $0.48

Interactive effect : (higher price and higher volatility) (maximum 3 marks)
- Recognition that the interaction of an increase in price combined with higher volatility caused underperformance (1½ marks)
- Calculation of interactive effect (1½ marks for correct approach)
- Actual change in option price : $8.45 - $4.49 = $3.96

Question 5 (continued)

- Interactive effect : Actual change in option price less change due solely to the change in volatility, less the change due to the change in delta = $3.96 - $0.85 - $2.98 = $0.13

Item	$mil
Benchmark	+102.50
+ Stock selection (physicals vs All Ords)	+0.50
+ Delta increase due to price increase (Gamma)	-0.48
+ Option price increase due to volatility increase (Vega)	-0.85
+ Interactive effect : higher price and higher volatility	-0.13
= Actual	101.54

(To a maximum of 16 marks)

General comments

The exam consisted of 2x2 hour papers, each of which was for a maximum of 100 marks. The shorter duration means that the points/mark ratio is higher than for other IAA exams. The exam was sat under open book conditions – hence the lack of bookwork questions.

END SOLUTIONS PAPER TWO