Institute of Actuaries of Australia

EMPIRICAL ANALYSIS OF INVESTOR UTILITIES IN INVESTMENT CHOICE

Prepared by John Livanas
C.E.O. AMIST Super

Presented to the Institute of Actuaries of Australia
Risk and Capital Management Research Conference 2008.
Sydney, Australia

The Institute of Actuaries of Australia
Level 7 Challis House 4 Martin Place
Sydney NSW Australia 2000
Telephone: +61 2 9233 3466 Facsimile: +61 2 9233 3446
Email: actuaries@actuaries.asn.au Website: www.actuaries.asn.au

INSTITUTE OF ACTUARIES RESEARCH CONFERENCE
22 SEPTEMBER
John Livanas

C.E.O. AMIST Super

EMPIRICAL ANALYSIS OF INVESTOR UTILITIES IN INVESTMENT CHOICE

This paper presents an empirical analysis of investor risk tolerances when choosing between balanced portfolios with differing risk, return and time horizon characteristics. The paper utilises techniques developed in consumer marketing, and employs commercial software to resolve partial utilities of investors to each of the three characteristics of the portfolio. This derives specific partial utility equations for risk, return and time horizon.

Further the paper then combines these partial utility equations to derive the aggregated risk – return characteristic of the investor group and comments on the stability of this characteristic versus varying time horizons. The paper reviews the implications of partial utilities for varying time horizon parameters within the context of current thinking of hyperbolic discounting and notes a similar outcome.

The empirical results for this paper are derived from an extensive telephonic survey of investors who actively had made (or hadn't made) an investment decision within the previous 12 months. A further study of a superset of members who made an investment selection over a period of 5 years provided the demographic and trend data.

INTRODUCTION

Classic MPT models of capital markets include two components at equilibrium: a constrained combination of risky assets along an efficient frontier; and an intersection of these with mean-variance maximizing investors each with specific expected utilities *E(U).* The analysis of the riskiness of various assets, and of their relative correlations has been subject to extensive focus; this has extended beyond equities to encompass credit products, infrastructure and other assets. The classic models (Markowitz 1952, Sharpe 1964, Fama 1971, Merton 1972), construct the efficient frontier and subsequently allow each investor to optimize their utility within the bounds of mean-variance efficient portfolios. Roll (1977) pointed out that the single testable hypothesis associated with the CAPM is that the market portfolio is mean-variance efficient. CAPM Sharpe 2007 (whose book postdated the research presented in this paper) utilizes principles of State/ Preference models to describe market equilibrium and to propose the Market Risk / Reward Theorem (MRRT): "Only market risk is rewarded with higher expected return". This State/Preference approach allows for quadratic utility functions, and discontinuous functions consistent with findings on utilities and choice by Kahneman-Tversky (1979) (2003)(KT).

More recently the rise of "behavioural finance" has sought to bring focus onto the construction of investor utilities. As is the case with new knowledge, much of the bedrock of behavioural finance has been developed through the aggregation of individual investor responses. Lo (2005) has made some important progress in unifying the various ideas under market equilibrium in his Adaptive Market Hypothesis (AMH). Summers (1986) comments that a "catholic approach should be taken to explaining the behavior of speculative prices…individual choice under uncertainty may provide guidance". However the responses of the sum-of-individual investors, each with their various biases, to the market constraints, is still a work in progress, and key amongst these questions remains the mechanisms for sudden shifts in aggregate expectations (to create bubbles and crashes, or changes in direction). Furthermore it is important to distinguish between changes in expectations through behavioural biases (which affect the expected utilities of individual investors), and new information (which is independent of investor biases, broadly based and consistent with EMH).

In order to present a model of investors that incorporates the principles of behavioural finance, it is important to construct a model that demonstrates how the aggregate preferences of investors creates equilibrium, and then test this equilibrium to determine whether the resulting choices are in aggregate mean-variant efficient.

This paper approaches this question using techniques developed for analysis of choices between consumer goods. In support of this, the general theories of utility (Fama, Miller 1971 Chapter 1 and others) presume that investment is a choice between consumption now and later (investment). Of course for any consumption "later" to be preferred where choice is freely possible, the utility for this "later" consumption must be superior to alternative consumptions "now". The paper focuses on alternatives of consumption "later" (investments) under uncertainty, and assesses whether in aggregate the preferences mirror mean-variance efficiency.

In testing equilibrium, the paper presumes unlimited 'supply' of alternative portfolios. This assumption is no different to that under general market conditions (all the way back to Marshall, Walrus), where a developed and demonstrated demand creates a supply response. Financial Markets are arguably becoming more 'demand responding'. One only has to track the relative changes in IPO's that seemingly arise more frequently when the demand (and risk appetite) exists for this type of investment.

An Outline of the Paper

A paper responds to a number of key questions:

- What is the form of the investor utility function? How do investor utilities combine to form an aggregate investor utility function, and does this create a mean-variance optimized universe?
- What are the factors that describe Investor utility? Are there differences according to personality or gender or education?
- How does investor utility change? Is there a way of describing the inertia of choice? What happens when an event triggers choice?

Furthermore, it is important to draw a distinction between utility at equilibrium – which is the point at which a decision to buy and sell is made; and the way that utility changes over time, allowing for disequilibrium.

This paper will propose a model based on empirical research at the point of equilibrium. A forthcoming paper will propose a model that defines movement of utility over time.

Section 1 proposed an experimental model that operates during instantaneous time and forced choice to estimate the *E(U)* for groups of investors.

Section 2 presents the empirical results of aggregate *E(U)* for the experiment of forced choice and makes a surprising discovery. Section 3 extends this analysis to test whether the information is correctly interpreted by the participants and whether *E(r)* is consistent, and whether we can identify homogenous sub-groups that can predict utility outcomes. Section 4 reviews the outcomes of a first-choice event, and Section 5 analyses data of choices actually made over a 6 year period.

Section 6 concludes.

SECTION 1: THE DERIVATION OF INVESTOR E(U) AND THEIR AGGREGATION.

UTILITY AND EQUILIBRIUM

The most common contemporary construction of expected utility *E(U)* is to define some form of mean-variance optimality in the form of $E(U) = f[E(r);\sigma]$. In MPT, the interaction of investor utility with the tangent to the efficient frontier defines the optimal portfolio for that investor.

The optimization of the aggregate of the individual portfolio choices occurs by setting a Lagrangian Function to maximize E(U) within a given an opportunity set in the form:

$$L = E(U_i)(q_1, q_2, \ldots q_n) - T(q_1, q_2, \ldots q_n)$$

Where q_i are factors pertaining to the satisfaction of consumption, T is the constraint set.

The optimizing requirements in MPT are between known utilities and constraints created by the efficient frontier.

THE EXPERIMENTAL MODEL: CHOICE BETWEEN TWO ALTERNATIVE PORTFOLIOS

The experimental set up removes the constraint of the efficient frontier and presumes that the market provides a combination of portfolio attributes to meet *E(U)*.

Furthermore we add time horizon as an optimizing factor for *E(U)*, requiring that the investor makes his or her decision based on their time horizon of investment. This is to match the available portfolio options in the Australian Financial Markets.

E(U) is required to be optimized as a function of Expected Return *E(r)*, Risk *f(σ)*, and Time Horizon *f(τ)* and can be presented in the form:

$$E(U) = f[E(r), f(\sigma), f(\tau)].$$

We construct an experiment that assumes a readily tradable financial market with bounded information and no implied transaction costs.

Alternative portfolios are created with specific limitations as to their attributes (q_i): specifically

$$q_i \in \{\text{Expected Return } E(r), \text{Risk } f(\sigma), \text{Time Horizon } f(\tau)\},$$

and limitations as to the values of these attributes:

$$E(r) \in \{x_1 \ldots x_n\},\ f(\sigma) \in \{y_1 \ldots y_n\}, \text{and } f(\tau) \in \{z_1 \ldots z_n\}.$$

A portfolio is created by a random draw from each of the three attribute sets.

$$P_x(q_1, q_{2,} q_3) :\ \{x_1 \ldots x_n\} \cap \{y_1 \ldots y_n\} \cap \{z_1 \ldots z_n\}. \qquad (1)$$

Where q_i is the attribute chose. The attributes in the experiment are chosen independently (orthogonal) to other attribute levels, so that each attribute's effect (utility) can be measured independently.

Pairs of portfolios are created by randomly drawing values from each set of attributes without replacement. A choice set is constructed as follows:

Choose between P_a and P_b $q_1 [P_a] \neq q_1 [P_b]$.

i.e. each attribute level is unique for each pair and all attributes are represented in the choice set.

Immediately the implication is that portfolios thus constructed are not necessarily mean-variance efficient. i.e. P_a may well have a set of attributes that are both higher risk and lower return, than for portfolio P_b. The portfolios that are presented to investors do not necessarily fall on the efficient frontier! (see Figure 1)

This is an important design implication because the first test will be to determine the relative importance of attributes and to test MRRT.

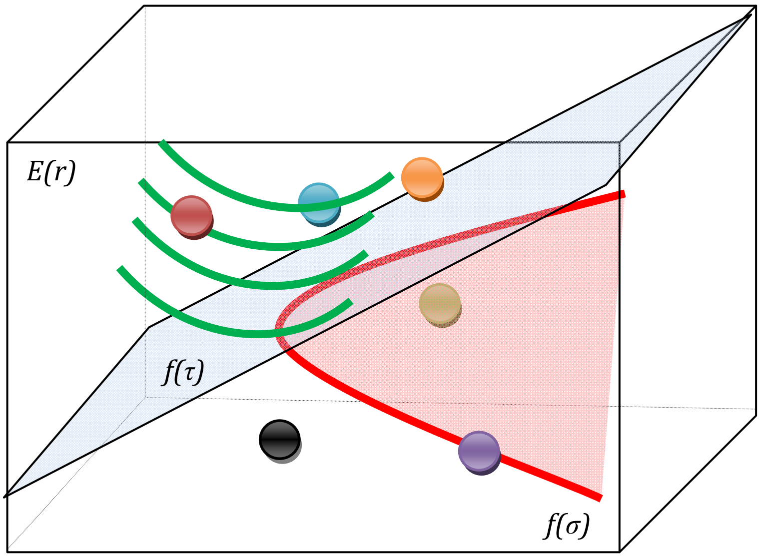

Figure 1 Stylised example of experiment

Each investor is required to transact in this market by making a choice between alternative portfolios, thereby stating his / her preference. There is no option to abstain from choice.

Choice-based conjoint analysis is used to estimate aggregate preferences for each of the attributes given the levels. The experiment also considers the estimated preferences segmented for a small number of sub-groups of investors.

Because the preferences are estimated at the aggregate level, the experiment presumes that a market is operating and that this market is in equilibrium at the point at which the aggregate of investor expected utility is optimised.

The axioms as defined by Fama (1971) regarding comparability; transitivity; strong independence are presumed to be retained:

"Axiom 1: (Comparability). The individual can define a complete preference ordering over the set of prospects in S; that is for any two prospects x, y in S, he can say that x>y or y>x or x~y.

Axiom 2: (Transitivity). ..For example, x>y, and y>z imply that x>z....

Axiom 3: (Strong Independence). ..G(x,z:α) represents a gamble..in which the individual gets either x with probability α, or z, with probability 1-α....if x>y, then G(x,z:α)>G(y,z:α)..

Axiom 4: ...If x>y≥z or x≥y>z, there is a unique α such that y~G(x, z:α)...

Axiom 5: If x≥y≥z and x≥u≥z and y~G(x,z:α_1) and u~G(y,z:α_2), then $\alpha_1 > \alpha_2$ implies y>u...”

Axiom’s 1 and 2 allow us to presume that there is a consistent preference ordering; Axiom 3 allows us to assume that the expected utility of a choice is the sum of its constituent utilities. Axiom 4 allows the investor to tradeoff a gamble G(x,z:α) for a certain attribute, and Axiom 5 extends this to allow relative ordering to be preserved given uncertainty. The experiment relies on Axiom 3 to allow Multinomial Logit Analysis (MLA) and Axiom 5 for order preservation.

The market operates during time $t \rightarrow \delta t$ for which time period. This creates an instantaneous measure and specifically avoids Arrow-Debreu effects where the value of the traded item may vary according to the mode, time and place of delivery and allows preservation of Axiom 2 for observed attributes.

As proposed above, utilities are presumed in the form: $E(U) = f[E(r), f(\sigma), f(\tau)]$. The investor utility function under the constraint of the experiment, and written for a subgroup is as follows:

$$[E(U) \mid \Phi, \Omega] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)] \quad (2)$$

where Φ is information, such as the available choices and their attributes; Ω is a set of factors that are specific to groups of investors such as personality; age; education. For a single investor, Ω is not a condition.

pU denotes the partial utility relating to E(r), f(σ) and f(τ).a,b,c are weights.

The experiment proceeds with each participant requested to make a choice or tradeoff between two portfolios.

Section 2 utilises an experiment to estimate E(U). Section 3 tests evidence as to whether Φ is correctly interpreted.

Figure 1 and the experimental construction also infers that the *max E(U)* condition introduces a new orthogonal constraint ‘B’ to define time horizon.. In general terms this condition may relate to some temporal factor; in the case of the experiment proposed, this time factor may be the time horizon of the investor (for example – the need of the investment within a certain time period) and of the investment (such as a period during which the investment stays illiquid). However it may be extended readily to incorporate more general time referenced constraints and it will be shown in Section 4 that this constraint is necessary to respond to a specific event – such as the first availability of choice. In more general terms, this constraint may relate to tax years; week days; rebalancing requirements at the end of a quarter.

An important implication of introducing a temporal constraint is that the imputed tangent to the ‘efficient frontier’ must necessarily have differing risk free rates, dependant on the time horizon. This is clear in Figure 1 above with a trapezoidal ‘tangency condition’. Figure 1 Stylised example of experiment A 10 year bond rate might be the relevant risk free rate for a 10 year time horizon; T-Bills for immediate investment returns. Furthermore, this allows for the Debreu-Arrow conditions to be met – the value of a security may vary dependant on other factors – such as the specific day that its value is considered.

We define a Lagrange function as:

$$L = E(U_i)(q_1, q_2, \ldots q_n) - \lambda T(q_1, q_2, \ldots q_n) - \mu B(q_1, q_2, \ldots . q_n) \quad (3)$$

Maximising requires:

$\nabla E(U) = \lambda \nabla T(q) + \mu \nabla B(q)$

Choice-Based Conjoint Analysis

Conjoint analysis is ideally suited to quantify psychological attributes (see Luce 1964). (Binary) Choice-Based Conjoint (CBC) Analysis is used to analyse the outcomes from the experiment because the task of choosing a preferred portfolio is similar to what investors do in real life. However CBC does have limitations, principally that data from groups of homogenous respondents is required to be aggregated for analysis and that individual analysis is unreliable. In the experiment, this is assessed by reviewing the outcomes for sub groups within the experiment sample.

Assume each attribute value from (1) has a partial utility pU[.] and that the utility of the choice is the sum of the pU[.] as in (2).

In CBC the choice data is analysed by calculating the proportion of times each value is chosen versus the times it is presented.

Multinomial Logit Analysis (MLN) is then utilized to estimate parameters to develop "Utility values" for each partial Utility pU[.].

MLN assumes that data are case specific and that the dependent variable cannot be perfectly predicted from the independent variables. The variables are assumed to be uncorrelated (implying that mean-variance efficiency is not assumed), and this allows the assumption to be made of the independence of irrelevant alternatives. (This assumption states that choices made are not dependant on other alternatives that may be available. Of course in the real world this is less likely, as factors such as taxation, or time of year, or events may modify the way choices are made. This will be discussed separately in Section 3).

Formally: $P(x) = \frac{e_x^u}{e_x^u + e_y^u + e_z^u}$ and

(The specific software used was Sawtooth CBC v2.6. A description of the MLN used by this software is in Appendix 1)

The resulting partial utilities pU[.] are analysed individually and combined to determine joint effects.

SECTION 2: EMPIRICAL RESULTS FROM A CHOICE EXPERIMENT

QUESTIONNAIRE AND SAMPLE CONSTRUCTION

A questionnaire was delivered telephonically to 236 Superannuation Members, 186 of whom had recently made a choice. These members already held a portfolio within a Superannuation fund that corresponded to an 'Efficient Portfolio'. These portfolios and their characteristics are set out in Table 1 below:

Table 1 Portfolios already held by Superannuation Investors who participated in the Experiment

Portfolios	"High Growth"	"Diversified"	"Balanced"	"Capital Guarded"	"Cash"
Typical Assets held	*85-90% Equities, Property*	*65-70% Equities, Property*	*45-55% Equities, Property,*	*<25% Equities, Property*	*Largely Cash*
Time Horizon	5 - 10 yrs	5 yrs	3 yrs	3 yrs	1 yr
Return– Expected annual percentage return over the period of the portfolio	8.0% - 9.0% p.a.	7.2% – 8.1% p.a.	6.5% – 7.2% p.a.	6.0% – 6.3% p.a.	3.8% - 4.0% p.a.
Risk – The chance of a negative return for the year	1:3	1:4	1:5	1:7 to 1:8	Nil

A sample of investors (Table 2 below) was selected and asked to participate in the experiment and survey questions. Demographic data was also captured. The sample included members who had made an investment choice decision in the last 12 months prior to the questionnaire ('switchers') , and a smaller 'control' sample of members who had specifically not made a choice ('non-switchers'), who acted as a reference group. The telephonic interviews were conducted by Woolcott Research Pty Ltd.

Table 2: Investors taking part in experimental choice questionnaire

	Numbers
Investors who had recently made a change in investment portfolio (Switchers)	186
Investors who had not made a change in investment portfolio (Non-Switchers)	50
TOTAL	236

The investors were requested to make specific choices between standardized portfolios with differing risk, return and time horizon characteristics as follows:

$E(r) \in \{x_1 \dots x_n\}$, $f(\sigma) \in \{y_1 \dots y_n\}$, and $f(\tau) \in \{z_1 \dots z_n\}$.

The set of attributes *E(r), f(σ), f(t)* from which the values were drawn for the construction of the experimental portfolios are shown in Table 3 below and *are representative of the portfolios already held by investors.*

For the attribute relating to risk the values of this attribute were stated as the annualised chance of a negative return. For the attribute relating to the time horizon of investment, these time horizons were identified as being the minimum length of time that the investor had to lock away their money in that particular portfolio option.

Table 3 Values available to determine the attributes of the Experimental Portfolio

Return *E(r)*	Risk (Annualised Chance of a Negative Return) *f(σ)*	Time Horizon *f(τ)*
3.9%	no chance	1 year
6.0 - 6.3%	13% chance	3 year
6.5 - 7.2%	20% chance	5 year
7.2 - 8.1%	25% chance	10 year
8.0 - 9.0%	33% chance	

For the questionnaire, a set of two portfolios was presented at a time to investors and investors were asked to choose which of the two they preferred, as described in (1):

$$P_x(q_1, q_{2,} q_3) : \{x_1 \ldots x_n\} \cap \{y_1 \ldots y_n\} \cap \{z_1 \ldots z_n\}.$$

Each of the 236 investors was provided with a total of 16 paired choice options. This implied that sample was a partial task choice analysis. CBCv2.6 software was used and MLN derived the partial Utilities of each of the attributes as the basis of estimating (2):

$$[E(U) \mid \Phi, \Omega] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$$

The analysis also assessed the homogeneity of the group – an important consideration given the assumptions of CBC analysis. The respondents are grouped by age group (Ω = Age Group, shown in Table 4 below), by those who had recently switched versus those who had not (Ω= switching propensity) and by personality type (Ω = *f(MBTI test for personality)*) and these results are reviewed below.

PART UTILITIES BY ATTRIBUTE

The results of the MLN estimation of Partial Utilities are shown in Table 4 below:

Table 4 Results of partial Utilities *pU* by Attribute

Variables			Utilities by Respondent Segment			
Attribute	**Value**	**Description**	**All**	**18-34**	**35-54**	**55+**
E(r)						
1	1	3.9%	-1.352	-1.491	-1.522	-1.172
1	2	6.0 - 6.3%	0.121	0.415	-0.052	0.165
1	3	6.5 - 7.2%	0.083	-0.308	0.142	0.221
1	4	7.2 - 8.1%	0.375	0.579	0.417	0.242
1	5	8.0 - 9.0%	0.774	0.805	1.016	0.544
f(τ)						
2	1	1 year	-0.008	-0.411	0.194	-0.072
2	2	3 year	0.009	0.013	0.140	-0.118
2	3	5 year	0.177	0.327	0.083	0.235
2	4	10 year	-0.178	0.071	-0.417	-0.045
f(σ)						
3	1	no chance	1.425	1.074	1.403	1.681
3	2	13% chance	0.295	0.347	0.323	0.269
3	3	20% chance	-0.153	-0.060	-0.147	-0.223
3	4	25% chance	-0.479	-0.331	-0.460	-0.563
3	5	33% chance	-1.087	-1.031	-1.120	-1.164
4	1	Neither	-1.36218	-2.13867	-1.45955	-1.03366
Respondents			236	42	101	93

The partial Utilities for each value are plotted graphically in Figure 2, Figure 3, and Figure 4 below.

Partial Utility (pU)E(U) of Expected Returns

Figure 2 below shows the utility curve for investors' preferences plotted against expected annualised investment returns.

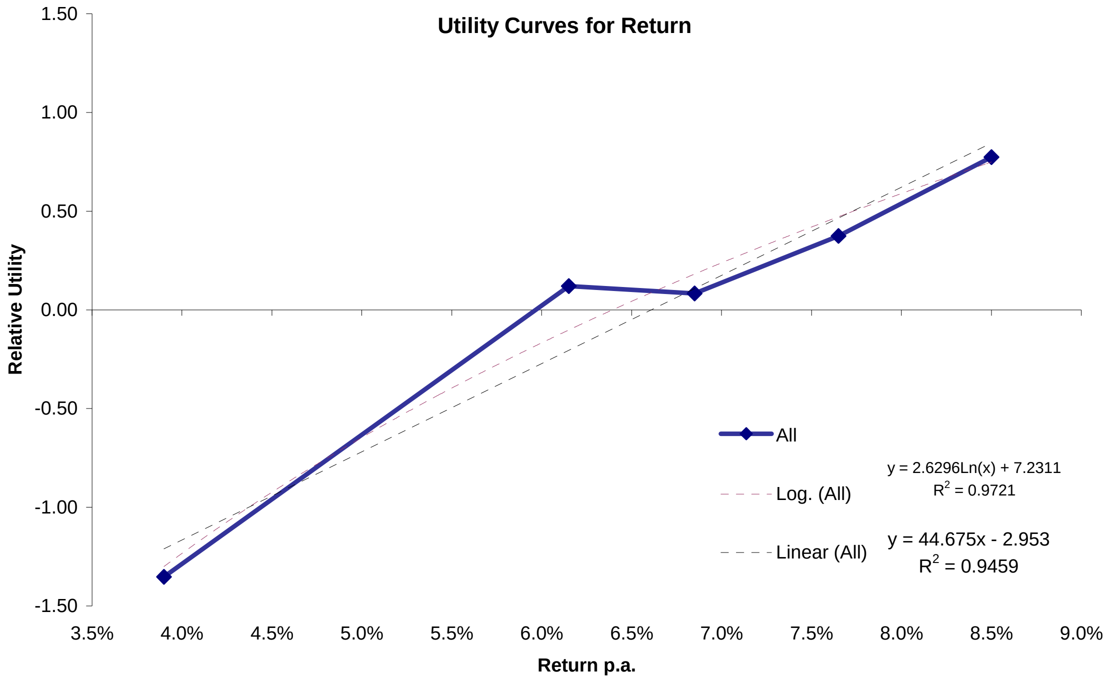

Figure 2 Utility curve for Return

The utility for return has a positively sloping utility function. A trend line fit using a log function yielded a slightly higher correlation that a linear function, but the difference in the correlations for the range of values studied do not allow sufficient confidence to draw any conclusions at this stage, other than slope is positively sloping.

The pattern of utilities did not vary significantly by age group.

Partial Utility (pU)(f(σ)) of Expected Risk

The second outcome of the choice modelling was in assessing the Partial Utility (pU) of expected risk to investors. Risk was identified to respondents as the chance of an average *annualised* negative return. Figure 3 below shows the Partial Utility (pU) curve for investor preferences given expectations of risk, expressed as a percentage probability.

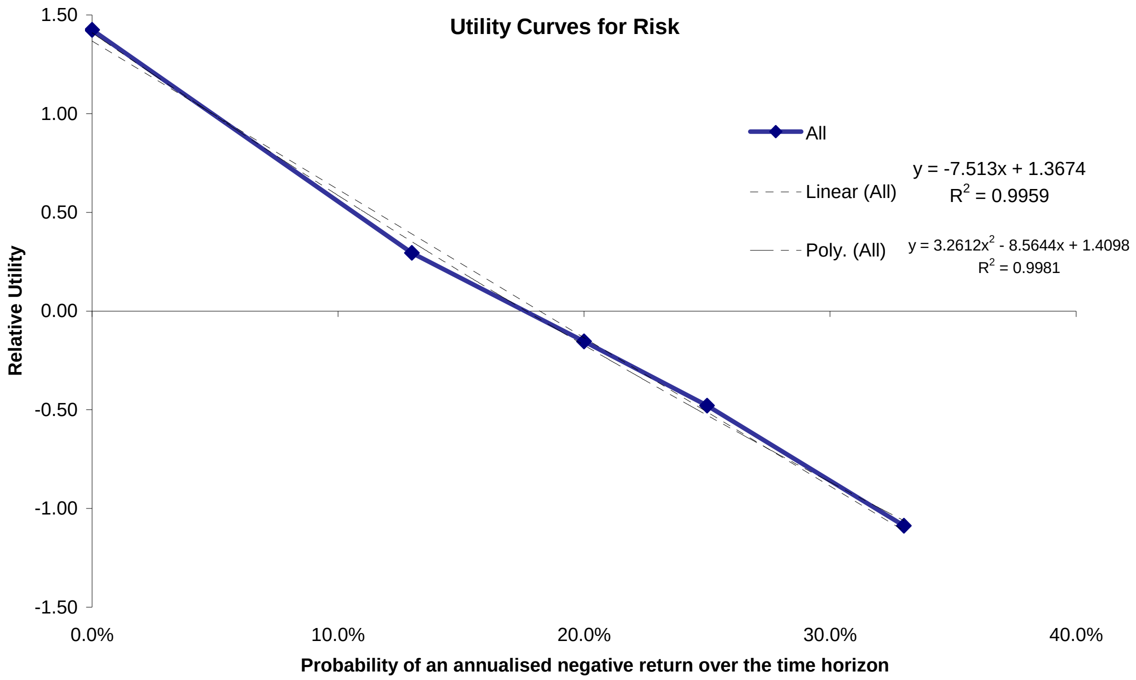

Figure 3 Utility curve for Risk

As expected, the slope of the Partial Utility (pU) function for risk is negative. Investors prefer less risk to more risk. The data is not sufficiently comprehensive to determine a best fit with a log trendline, although a quadratic function does provide a good correlation. The important finding is that there is a clearly negative slope as anticipated.

It must be noted again, that the measure of risk in this question is not standard deviation or variance. It would be highly unusual for investors to be able to accurately estimate expected standard deviation, or the Partial Utility (pU) derived from this. Rather, asking investors to consider, say a 1 in 3 chance of a negative return in any one year, provided a more consistent response.

Partial Utility (pU) of Expected Time Horizon

The third outcome of choice modelling was to assess Partial Utility (pU) of different time horizons. This is the expected time horizon that the investor would need to maintain his or her investment, in order to achieve an expected return, given an expected risk profile. Portfolios will generally have a time horizon over which the risk-return parameters are constructed. Holding onto investments for longer, should increase the likelihood of a particular outcome being achieved, given an expected variance of returns.

For time horizon, investors were asked to imagine that their money would be tied to a particular option. This was akin to a lock down period, with the investor encouraged to think of this commitment as a constraint on access to capital or choice.

Figure 4 below shows the utility curve for time horizon:

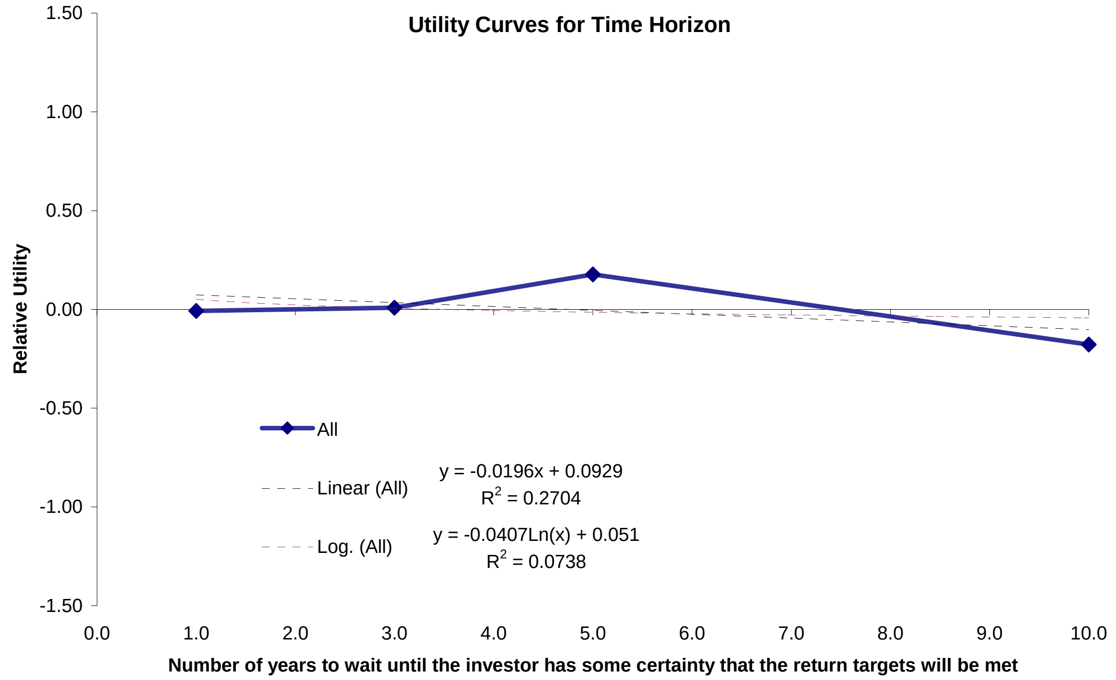

Figure 4 Utility curve for Time Horizon

There was no clear discernible relative utility for time horizon and neither a linear or logarithmic line provided any meaningful correlation with the trend. Investors were seemingly indifferent whether the time horizon constraint of the portfolio was 1 year, 5 years or even 10 years!

This ambiguity of investors in optimisation of *E(U)* given constraints was a significant surprise and immediately brought into doubt the temporal constraint imposed on the experiment.

However before removing this term it is instructive to review literature on hyperbolic discounting, and the shape of this function may be explained this way.

Error Terms

Table 5 below presents the error terms for the best fit curves that estimate the utility curves for *E(r)* and *f(σ).*

Table 5 Error Terms for pU of *E(r)* and *f(σ)*

partial Utility of E(r)			
y = 2.6296Ln(x) + 7.2311			
R^2 = 0.9721			
Expected Return	**Expected Value**	**Actual Value**	**Error**
3.9%	-1.300	-1.352	-0.052
6.0% - 6.3%	-0.102	0.121	0.223
6.5% - 7.2%	0.181	0.083	-0.098
7.3% - 8.0%	0.472	0.375	-0.097
8.0% - 9.0%	0.749	0.774	0.025
Mean			0.000
Sdev			0.1343

partial Utility of f(σ)			
y = 3.2612x2 - 8.5644x + 1.4098			
R^2 = 0.9981			
Expected Risk	**Expected Value**	**Actual Value**	**Error**
0.00%	1.410	1.425	0.015
13.00%	0.352	0.295	-0.057
20.00%	-0.173	-0.153	0.019
25.00%	-0.527	-0.479	0.049
33.00%	-1.061	-1.087	-0.026
Mean			0.000
Sdev			0.0414

PORTFOLIO INDIFFERENCE CURVES – ISOUTILITIES

Based on the utilities for each combination of portfolio, and rejecting time horizon as a constraint, requires that (2) be restated as follows:

$$[E(U)\,|\,\Phi,\Omega] = pU[aE(r)] + pU[bf(\sigma)] \qquad (2)$$

This is now in the form familiar to proponents of mean-variance optimised systems. There are two options to constructing the outcomes of 2: A simple sum of utility values is shown in Table 6 below.

Table 6 E(U) as the arithmetic sum of pU

	pU of *f(σ)*	1.425	0.295	-0.153	-0.479	-1.087
pU of *E(r)*		0	13%	20%	25%	33%
-1.352	3.9%	0.07	-1.06	-1.51	-1.83	-2.44
0.121	6.0% - 6.3%	1.55	0.42	-0.03	-0.36	-0.97
0.083	6.5% - 7.2%	1.51	0.38	-0.07	-0.40	-1.00
0.375	7.3% - 8.0%	1.80	0.67	0.22	-0.10	-0.71
0.774	8.0% - 9.0%	2.20	1.07	0.62	0.29	-0.31

Correspondingly, constructing E(U) as a combination of the estimated utility functions of E(r) and *f(σ)* results in (3), and is shown in :

$$E(U) = 2.6296\ln(E(r)) + 3.2612 f(\sigma)^2 - 8.5644 f(\sigma) + 8.6409 \qquad (3)$$

Table 7 E(U) determined utilizing the best fit equations for *E(r)* and *f(σ)*

	pU of *f(σ)*	1.425	0.295	-0.153	-0.479	-1.087
pU of *E(r)*		0	13%	20%	25%	33%
-1.352	3.9%	0.11	-0.95	-1.47	-1.83	-2.36
0.121	6.0% - 6.3%	1.31	0.25	-0.27	-0.63	-1.16
0.083	6.5% - 7.2%	1.59	0.53	0.01	-0.35	-0.88
0.375	7.3% - 8.0%	1.88	0.82	0.30	-0.06	-0.59
0.774	8.0% - 9.0%	2.16	1.10	0.58	0.22	-0.31

The differences are minor, and for simplicity Table 6 is used to determine the conjoint utilities. Figure 5 and Figure 6 show a plot of the partial utilities as isoutilities against traditional risk and return planes. As highlighted above, the effects of time horizon are explicitly excluded, and this can be seen as a two dimensional representation of Figure 1above. Any point on an isoutility will correspond to the equivalent investor utility for those combinations of attributes of a portfolio that appear on the isoutility.

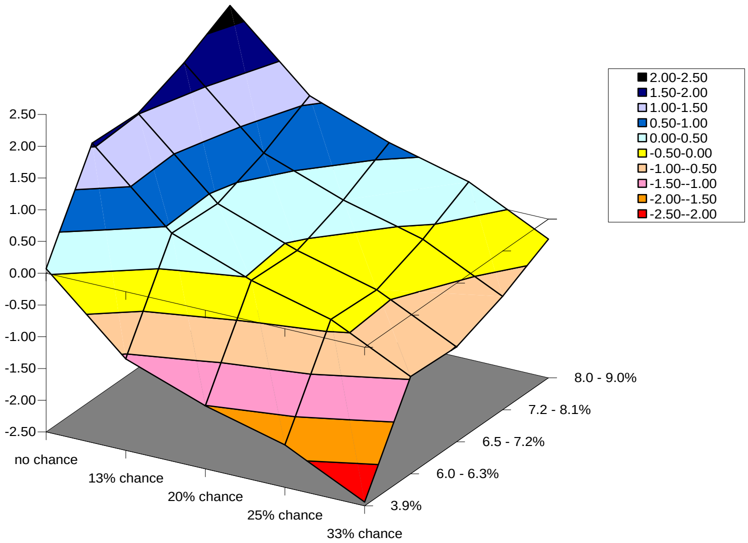

Figure 5 Investor E(U) generated from Partial Utilities

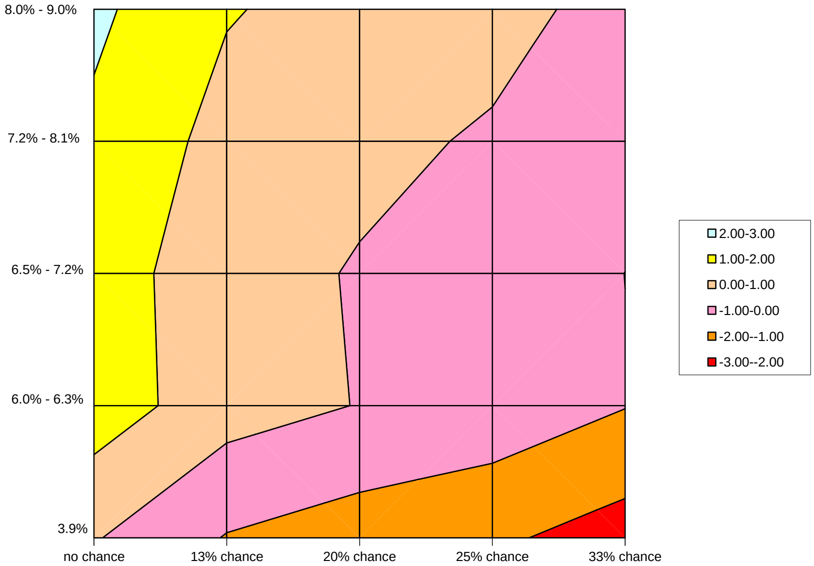

Figure 6 Investor E(U) as a Surface

Implications from the Findings of the Study: MRRT Holds

The experimental outcome provided an estimate of E(U) for a group of investors. The portfolio choices were not mean-variance optimized, nor did they offer Market Risk / Reward tradeoffs. The portfolios were randomized draws from the set of attributes and therefore it would have been common for a portfolio set to include portfolios that were offering inconsistent tradeoffs.

The result in aggregate however, shows monotonic pU's for *E(r)* and *f(σ)* that hold for MRRT, providing very significant implications for the construction of the market and for its efficiency.

The implications are that the model for market can be derived from Investor preferences without necessitating the assumption that the portfolios offered in the first place, are necessarily on the efficient frontier.

SECTION 3: TESTS OF EFFICIENT INTERPRETATION OF INFORMATION AND TESTS OF THE INFLUENCE OF Ω = DEMOGRAPHIC GROUPS OF INVESTOR

Equation (2) requires an estimation by investors of Risk, Return and a given time horizon. The analysis presented in Section 2 indirectly evaluates the groups' optimisation of the state preference task. The outcome was to provide proof that, even without the establishment of an efficient frontier, MRRT holds. Investors in aggregate will make choices that will force MRRT.

However this did not provide proof of that optimisation only occurs on the Efficient Frontier.

The test as to whether the chosen portfolios exist only on the Efficient Frontier requires that the following conditions to be met:

> *Investors, in making state preference choices, interpret the information of E(r); f(σ) and f(τ) correctly; or*
>
> *Only portfolios that exist on the Efficient Frontier are available in the market.*

The second condition is possible, and can be argued on the basis that only portfolios that exist on the Efficient Frontier survive. However even then, the process of failure may take a considerable period.

The alternative condition is therefore a test of Φ, in equation 2, which restated here:

$$[E(U) \mid \Phi, \Omega] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$$

Ω is maintained as a choice frame .

A survey of investors was undertaken to relate their expectations (and their dispersion) of the various attributes of alternative portfolios. The sample used is the same as for Section 2 as presented in Table 2.

In the survey, the common terms used in Superannuation were tested. i.e. the terms for High Growth versus Medium Growth versus Low Growth were stated and investors were asked to propose their ex ante expectations of the value of the attributes *E(r); f(σ) and f(τ)* of each of these theoretical portfolios. This was compared to the attribute values of portfolios that could (and were) be held by these investors in their Superannuation Fund. (The investors in the survey were all superannuation members holding one of the portfolios shown in Table 1.

The test is as follows:

> *Assume a Kahneman-Tversky type of transform that modifies information in accordance with behavioural biases of each investor. Assume Φ as the unfiltered and non-transformed information, we define Φ* as the probability density function of a group of investors such that:*

$$\Phi_i^* = \Pr[KT_i . \Phi \mid \Omega] \qquad (4)$$

(2) then becomes:

$$[E(U) \mid \Phi^*, \Omega] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$$

Because the survey tests expectations, the survey compares the expectations of the value of the attributes *E(r), f(σ) and f(τ)* of the investors in the sample, to expectations of professionals who constructed the portfolios. Assuming that professionals optimised the construction of the portfolios on some efficient frontier, the test of investor expectations, is also a test of whether these investors are able to interpret correctly the characteristics of portfolios on the efficient frontier.

INVESTOR EXPECTATIONS OF RETURNS FOR DIFFERENT PORTFOLIO TYPES

Figure 7 **Error! No bookmark name given.**below shows the range of respondent responses to their interpretation of E(r) given the portfolio descriptions. The shaded portion represents the actual range of investment returns for each corresponding type of 'real' portfolios.

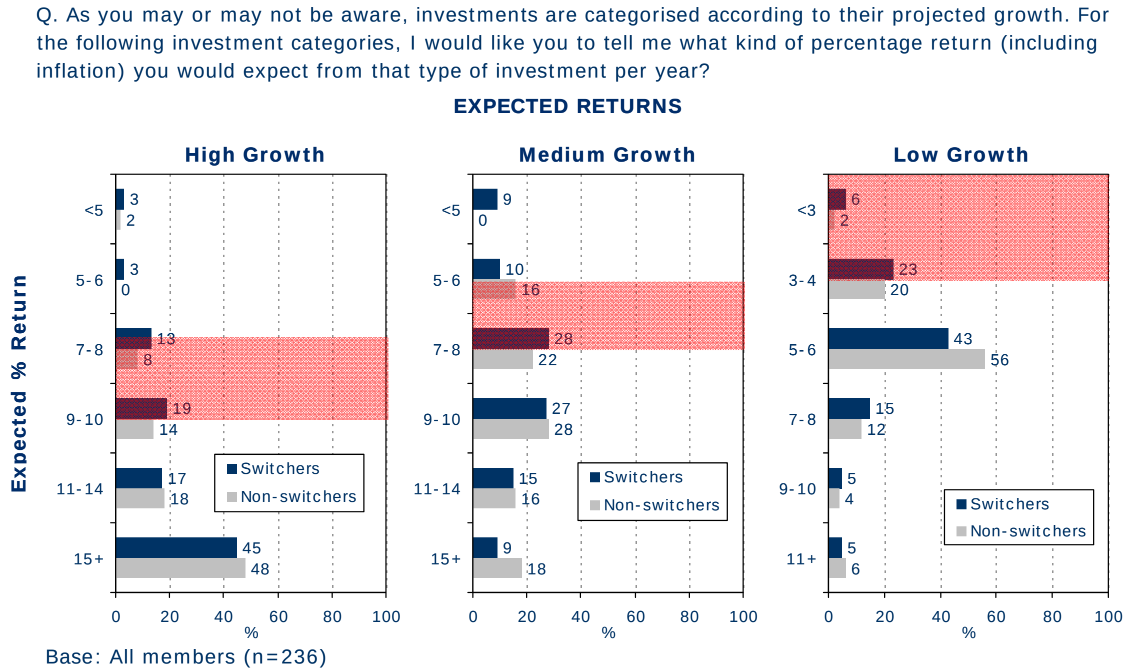

Figure 7 Investor perceptions of return characteristics

Setting Ω conditional on people switching, there seemed to be no real difference between investors that elected to shift their risk by making an investment change ('switchers') and those that didn't.

There is however a marked overestimation as to the return expectations of portfolios for all risk characteristics. Investors consistently overestimated the returns targeted and available from investment portfolios. (As an ex post assessment, the performance of the portfolios for the following year were in fact closer in estimate to those of the surveyed investors than to the professionals constructing the portfolios. An interpretation of this may be that professionals will construct the estimates based on longer term outlooks).

INVESTOR EXPECTATIONS OF RISKS FOR DIFFERENT PORTFOLIO TYPES

The following question was to assess investor understanding of risk characteristics for each of these portfolios. Risk was characterized as the probability of a year of negative returns. In the survey the question was asked to assess the number of years out of 10 during which investors would experience a negative return. Figure 8 below shows the respondent perceptions while the shaded portion represents the actual risk of investments for each corresponding type of 'real' portfolios.

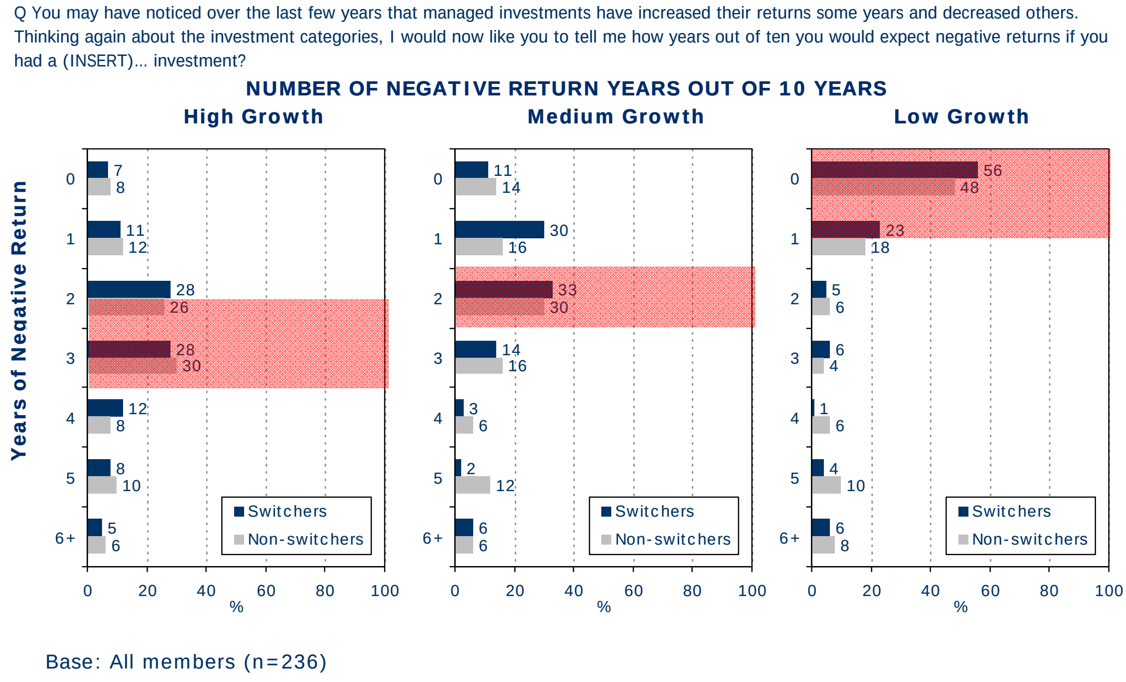

Figure 8 Investor perceptions of risk characteristics

Surprisingly, investors were fairly accurate in their assessment of the likelihood of investment portfolios returning a year with negative investment returns, across all three investment portfolio types.

Investor Expectations of Time Horizon for different portfolio types

The final question was to assess investor understanding of the time horizon for which investment objectives were constructed, for each of these portfolios. Time horizon was characterized as the likely number of years investors would have to wait before being fairly sure that they would experience the return characteristics of the particular portfolio.

Figure 9 below shows the respondent perceptions while the shaded portion represents the time horizon objective for each corresponding type of 'real' portfolio.

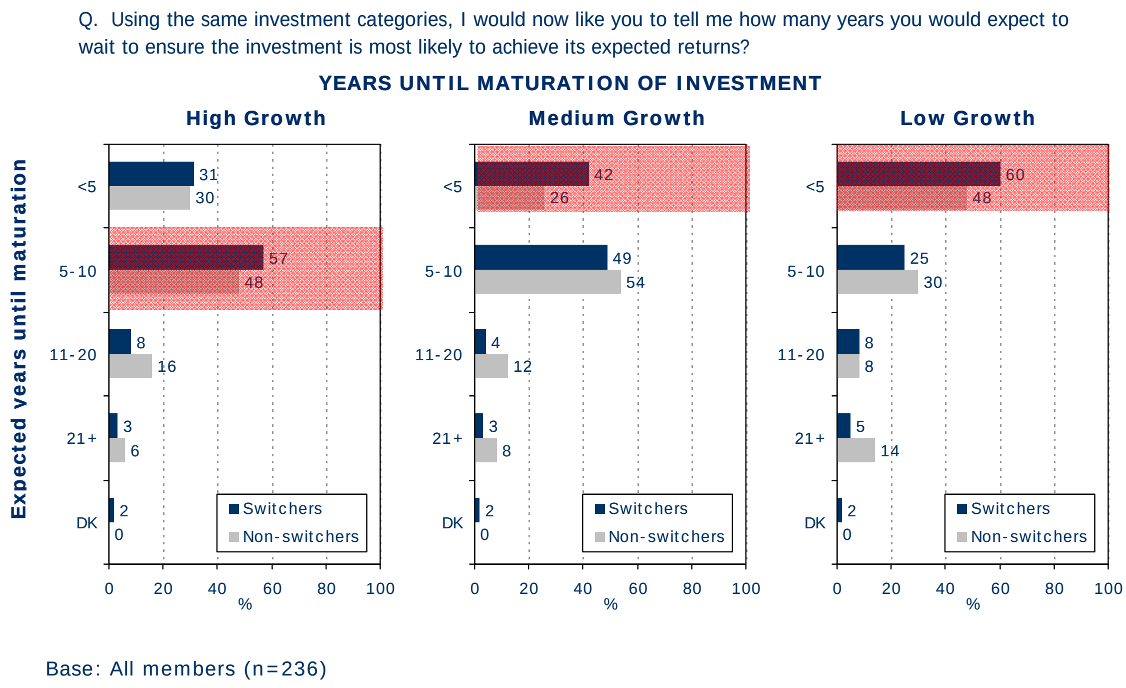

Figure 9 Investor perceptions of time horizon characteristics

Interestingly, investors again were able to reasonable reflect the time horizon of the portfolios discussed. However, the data is insufficiently robust to draw any specific conclusions.

Ω = f (PERSONALITY): CORRELATION OF DECISION MAKING WITH INVESTOR PERSONALITY

Investor personality is rarely reviewed in aggregate. The survey attempted to assess whether investor personalities provided some relationship to investor behaviour. In this test the condition *Ω = f (Personality Type)*

A comprehensive personality test for each of the respondents was not possible. Specifically, the Meyers-Briggs Personality Type Instrument (AMBIT) was reviewed as to whether it could be used. In the survey questionnaire, an abbreviated set of questions were asked of survey respondents to assess personality type. These questions were loosely based on the AMBIT and on the Jung 16 personality types. However, because these questions were necessarily abbreviated relative to a full scale AMBIT, no conclusions can be drawn as to individual personality types. However in aggregate, the anticipated MBTI standard response pattern was observed in the spread of the responses, and as such, in aggregate some conclusions can be drawn as to personality types and investment behaviour.

Specifically, to limit overstretching the capability of the survey to draw conclusions, the only dimension assessed was whether investors who actively engaged in investment switching had any differing personality characteristics. A demographic overlay relating specifically to education level provided some further clues. Figure 10 below highlights the results of the survey in relation to investor personality profiles. The conventional MBTI investor characteristics are used as short hand reference points, but it must be noted that the MBTI instrument was not used:

SWITCHERS

	S	S	N	N	
I	13%	3%	3%	6%	J
I	7%	3%	2%	2%	P
E	11%	3%	1%	3%	P
E	19%	5%	4%	16%	J
	T	F	F	T	

NON-SWITCHERS

	S	S	N	N	
I	18%	4%	12%	4%	J
I	8%	4%	2%	2%	P
E	10%	2%	2%	4%	P
E	16%	2%	4%	6%	J
	T	F	F	T	

Base: All respondents (n=236)

Figure 10 Personality profiles of Switchers and Non-switchers

The figure above highlights three areas of possible personality difference between investors who implemented investment choice and those who didn't.

Ω = f(Education): Correlation of Decision making with investor Education

To complement personality typing, demographic questions were correlated against propensity to action investment decisions.Table 8 below highlights demographic information relating to investors' education levels, and switching propensity. This highlights that 'switchers' may be more likely to possess a higher level of education.

Table 8 Education Levels

Education Levels	**Switchers %**	**Non-Switchers %**
Some secondary school	4	8
Intermediate/School Certificate	11	16
Leaving Certificate/HSC	14	18
Trade qualification/Diploma	35	40
University Undergraduate Degree	19	12

Who and what provided investors with the greatest influence?

In assessing the transform of Φ, it is useful to consider the influence of external factors that shape the specific transform as a result of investor behavioural biases 'KT'

The final series of questions in the survey attempted to assess which events, or who, influenced investors in their investment decision. This completes the theme of assessing the underlying factors in investor decision making.

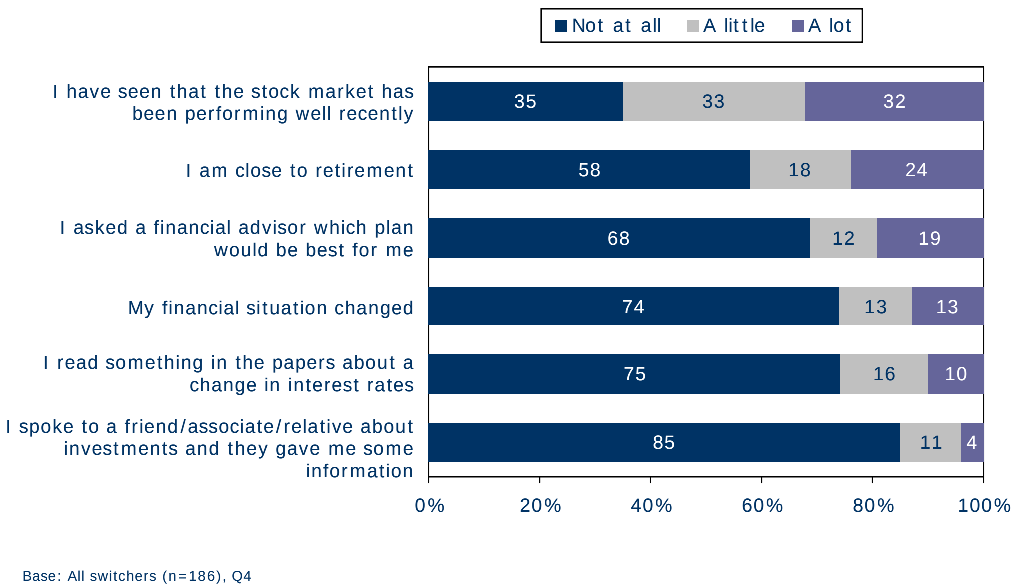

Figure 11 Factors influencing Investor Behaviour

The survey asked investors to rate a variety of factors in their decision making process. These factors included events, as well as influences of family and friends.

The main, but not majority factors, seemed to be relating to those few for whom retirement was approaching (and where consequently older) and for those who had seen a financial adviser. However, the largest single factor seemed to be that the direction of the stock market provided information to investors, which they used to effect a shift in their portfolio risk.

This then seems to be the first clear evidence in this research, where the quantitative analysis of aggregate data showing investor behaviour once a trend in the market was confirmed, corresponding to quantitative survey data of investor behaviour. However with just a 1/3 of respondents confirming that this was a major factor, it would be interesting to conduct this research amongst much larger sample sizes.

Implications of Tests of Efficient Interpretation of Information and Influence

The survey presented findings that indicate that investor expectations differed to a greater or lesser extent from those of 'Professional' investors who created the portfolios. This is not in itself a surprise. Importantly however the expectations showed a dispersion which may be important in understanding the equilibrium process, and specifically price discovery. It is this dispersion that may result in successive thresholds of 'action' being reached at different times, and provides an insight into the construction of the investor utility function.

Ω, the conditioning of E(U) based on demographics or other investor characteristics, is interesting in understanding causal factors. However it is debatable whether in aggregate, this characterizes the effects of the transform of Φ any differently to that which a probability density function would.

For example, setting Ω = education, may provide some clues as to the relative evaluation of information, or to the characteristic of E(U) for a given set of attributes. However, a probability density function transform of Φ would incorporate those factors within the characteristic of the probability density function.

Given that we are concerned with aggregate effects and with market equilibrium, we can ignore Ω for a large enough group.

The second implication of the tests here, is that the transform of Φ can be characterized by a probability density function (pdf) with given characteristics for each of the attributes. The specific pdf has not been attempted, but is readily quantifiable. This is a very important outcome, because the kurtosis and the skewness of the transform will change the manner of trade, characterizing the speed at which thresholds are reached. Enticingly later work seems to show that as kurtosis increases (i.e. there is less agreement as to specific values of attributes), so volatility increases and rapid jumps in prices occur. This will be explored in a forthcoming paper.

For now (2) can be written as:

$$[E(U) \mid \Pr[\Phi^*]] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)] \quad (5)$$

SECTION 4: EVENT ANALYSIS

This section reviews the outcomes from analysis of choices made by investors when they are allowed to make these choices for the first time. It can be imputed that the choices made when the constraint is removed, can provide us with information as to the manner in which E(U) is optimized and equilibrium is achieved.

In section 1, we proposed that the optimization process of E(U) given constraints be expanded to include an additional constraining variable B. This section expands on this:

Equations 2 (5) is:

$$[E(U) \mid \Pr[\Phi^*]] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)] \qquad (5)$$

It has been noted that the *pU[cf(τ)]*is likely to approach zero, but it has been retained for completeness.

From section 3, E(U) for a group of investors follows a probability distribution determined by investor interpretation of Φ.

Let $E(U_i^*)$ be the expected utility function of investor *i* at the moment of trade, given information Φ. For investor to commence and conclude trade $E(U_i^*) > E(U_i)$. i.e. the *E(U)* of the investor must increase in order for that investor to initiate trade. Without an increase in utility, there is no impulsion on the investor to engage in any trade and the market equilibrium is preserved without price discovery. Define $E(U_i^*)$ as:

$$E(U_i^*) > E(U_i)$$

At the limit, with no changes to $E(r), \sigma, \tau, \Omega$ this resolves as follows:

$$E(U_1) - E(U_i^*) = \frac{\partial E(U_1)}{\partial \Phi} = \frac{\partial E(U_1)}{\partial (E(r), \sigma, \tau)} \cdot \frac{\partial (E(r), \sigma, \tau)}{\partial \Phi} \qquad (6)$$

Assuming that no new information enters the market, and that the market is efficient, this does not provide sufficient conditions for trade. Trade will not proceed once equilibrium has been achieved.

However this does not provide any information about optimizing E(U), subject to constraint, and E(U) may exist less than E(U*) where trade is not possible or allowed.

As noted in section 1, we introduce a new orthogonal constraint B (such as constraint arising at the end of a tax year; rebalancing requirements; cashflow requirements; constraint on choice).

Trade then can occur for two reasons:

1. A change in constraint B can create a new equilibrium, where the prior system was in equilibrium
2. A change in constraint B can allow an equilibrium to be achieved where previously the investor cold not optimise E(U). horizon.

The set of constraints defined by B may interact with individual investor utilities in different ways to risk, return and time horizon.

In the case of the event study in this section, B simply allows or constrains trade. We write (3):

$$L = E(U_i)(q_1, q_2, \ldots q_n) - \lambda T(q_1, q_2, \ldots q_n) - \mu B(q_1, q_2, \ldots q_n)$$

Maximising requires:

$$\nabla E(U) = \lambda \nabla T(q) + \mu \nabla B(q) \qquad (7)$$

Assuming a constraint (such as inability to trade) is removed, then the investors who have not optimised their utility may chose to trade.

Importantly however most investors do not actually engage in a trade. There are two probably reasons:

1. Inertia or friction to trade
2. Those investors are in equilibrium already

We define a Bernoulli variable *X*, where

$$E(X) = 1_{[E(U^*)-E(U)]>\theta} \text{ for trade to occur:} \qquad (8)$$

We define some threshold θ that is set endogenously by each investor. The test becomes whether a change in information or in a constraint is sufficient to overcome this threshold value.

Where E(U) and E(U*) for the group can be represented by pdf's, then dependant on the hurdle set by each (which could also be a pdf), the numbers of choices made will vary from 0% to 100%.

Investment Decisions of Investors having a choice for the first time

A subset of investors, who were given Investment Choice from the 1st October 2005 for the first time, was analysed to assess whether this 'event' created a specific bias in investor behaviour.

Investor had not been able to exercise investment choice prior to 1 October 2005, and had been invested according to 'Trustee Selection', a portfolio with a risk profile between High Growth and Diversified. From 1 October 2005, these investors were offered the opportunity to change their investment options as shown in Table 9 below. For the purposes of analysis, these options were assigned 'Risk Values', with a choice that led to an increase in risk, resulting in a positive number.

Table 9 Investment Choices Available and Relative Risk Values

Pre 1 October 2005	Post 1 October 2005	'Risk Values'
	High Growth	+1
Trustee Selection	Trustee Selection	0
	Diversified	-1
	Balanced	-2
	Cap Guarded	-3
	Cash	-4

As of May 2006, a number of members had elected to switch out of their default option (i.e. Trustee Selection) into one of these investment options (i.e. shift their investment

These risk shifts were analysed in aggregate to establish whether there was any correlation between switch decision and age or gender. Because of the very small sample size, the sample was stratified by age, with each age cohort assigned an aggregate outcome based on a simple count of individual investor risk shifts. This allowed a better comparison between say 58 year olds, and 48 year olds, where the number of investors aged 58 and making an investment choice differed in number, when compared to those aged 48[i]. This approach provided a surprisingly strong correlation between age and investment decision. In fact, the closer the member to retirement, the more conservative the investment option members switched into. This correlation was sustained for both male and female members.

Figure 12 below shows a scatter diagram relating age to risk bias that closely mirrored conventional wisdom that presumes investors to reduce risk as they move colder to retirement. A linear trendline provides a correlation coefficient R^2 of 0.716 .

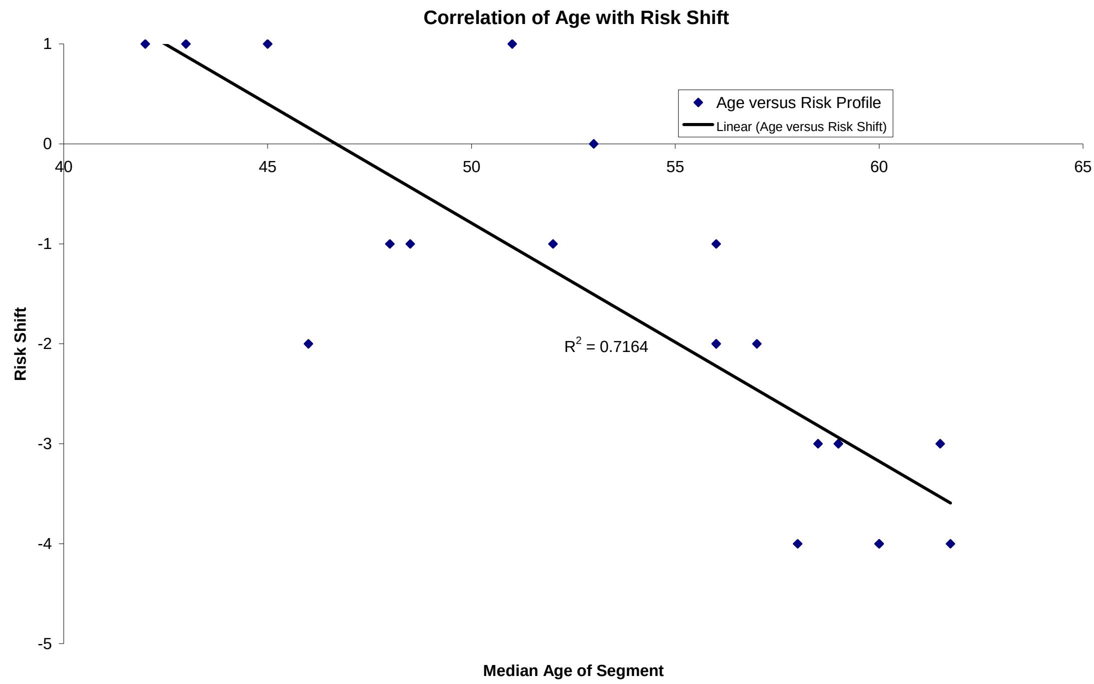

Figure 12 Median age of member segments versus risk

The implications of this event study, is that the information received by the investors was consistently interpreted and that the choices made were entirely reliant on the removal of a constraint (B). Furthermore, an aggregate study of the numbers of investors that actually made a choice was small, implying that either they were not in disequilibrium, or that θ provided a substantial hurdle. Given that the investors who actually made a choice, made a consistent choice to reduce risk with age, one must presume that the investors who did not chose, could not also all be in equilibrium (i.e. have optimised E(U)). Furthermore, given the consistent interpretation of the information, it must also be presumed that Φ is correctly interpreted, but that θ must be quite large.

SECTION 5: CONTINUOUS TIME ANALYSIS: A REVIEW OF RISK SHIFTS AS A RESULT OF INVESTMENT DECISIONS MADE BETWEEN 1 JULY 2002 AND 20 APRIL 2006

A final review of investor behaviour over time was attempted by reviewing the choices investors made over a period of time, and attempting to assess whether there was correlation with any factor.

As for Section 4, risk changes were normalised by assigning monotonically increasing values to greater 'risks' (as per Table 9) and counting the numbers of investors who made a change.

WAS THERE ANY CORRELATION BETWEEN RISK SHIFTS AND THE DIRECTION OF THE MARKET?

The first analysis attempted to assess whether, for investors who had exercised choice, there was any correlation between the resulting shifts in risks and the direction of the market. The risk shifts, calculated for each investor in the manner described above, were aggregated for all investors for each day. This provided a composite view of the aggregate shift in investor attitudes on a daily basis.

To understand whether the absolute value of investment had any impact on decision making, the analysis was also modified to account for money flows, providing a money weighted basis of assessing risk shifts. Money weighting also removed the potential bias of small switches in the aggregate direction of the risk shifts.

The resulting unweighted and money weighted risk shifts were compared to the performance of the Australian Equities Market. For the purposes of comparison, the unit price series of the FuturePlus Super Australian Equities portfolio managed by FuturePlus Financial Services.

A 30 day trend line was calculated for both analyses, to assess pattern of switches more accurately[ii]. Furthermore a linear best fit line was attempted, to assess whether there was any bias in the direction of risk shifts over the period analysed.

The resulting plot of money weighted switches against the Australian Equities Unit Price Series is shown in Figure 13 and Figure 14 below:

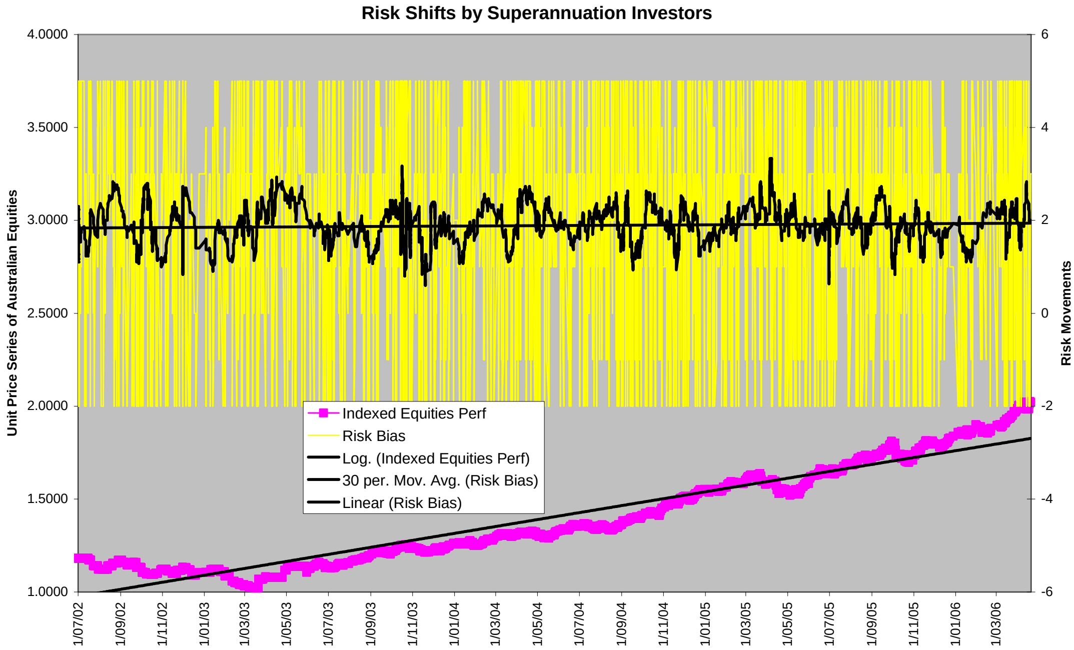

Figure 13 Unweighted Risk Shifts by Superannuation Investors

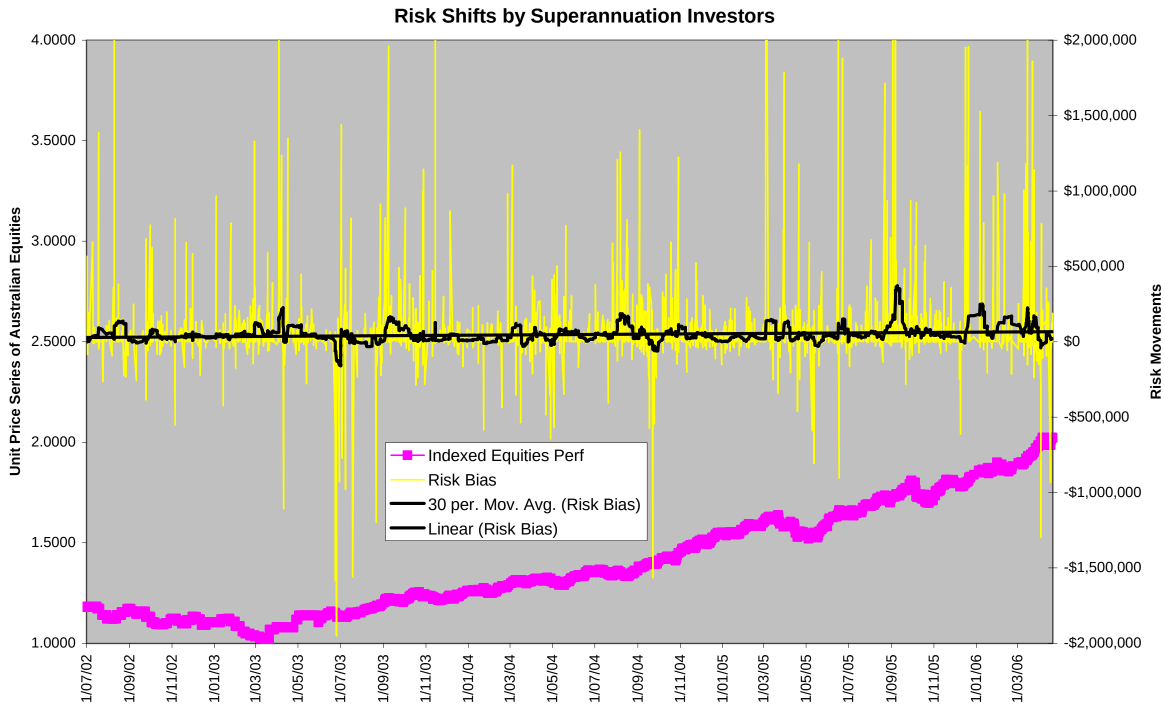

Figure 14 Money Weighted Risk Shifts by Superannuation Investors

MONEY WEIGHTING AS A PROXY FOR ABSOLUTE WEALTH

The money weighted data series provides an important analytical tool. The data set used was of investors switching their entire investment between portfolios. The data set specifically excludes a small subset of investor where investments were split between numbers of target portfolios.

The implication of this important observation is that the absolute value of investments can be deemed to provide clues as to absolute wealth. i.e. the higher the investment value switched, the more likely the investor had a higher level of wealth. While this assumption is not true in all cases, on aggregate, the more investors had in Superannuation, and were then able to switch, the more likely they had accumulated greater wealth through higher income or longer duration of savings.

The data shows a number of outcomes:

Firstly there is no discernible correlation between the aggregate direction of the risk shifts and the direction of the market. The data cannot demonstrate that investors were making investment choices that bore any relationship to the direction of the market during the period of study, for either unweighted or weighted risk shifts. Correlation between the risk shifts and the unit price series reflecting the market, are shown inTable 10 as follows:

Table 10 Correlation of Risk Shifts with the Australian Equities Market

Correlation between Aggregate Risk Shift Series and Australian Equities Unit Price Series	0.00898
Correlation between Aggregate Weighted Risk Shift Series and Australian Equities Unit	0.05558

Secondly, there was no correlation between the Aggregate Weighted Risk Shifts and Australian Equities Unit Price Series. There is a higher correlation however between the Unweighted Risk Shift Series and the Weighted Risk Shift Series, with a correlation of 0.24. However this is to be expected as the former is a major factor of the latter. Nevertheless it is surprising that the correlation is so low and additional analysis may be merited.

The importance of the money weighted risk shift series having an equally low correlation to the unweighted risk shift series may support Simon (1955) and Kahneman (1979) that investors are more likely to act on changes in value of investments rather than absolute wealth.

Figure 15 below shows comparison of age versus switching patterns on an unweighted basis (to avoid any bias from age based increases in wealth), while Figure 16 shows correlation with gender.

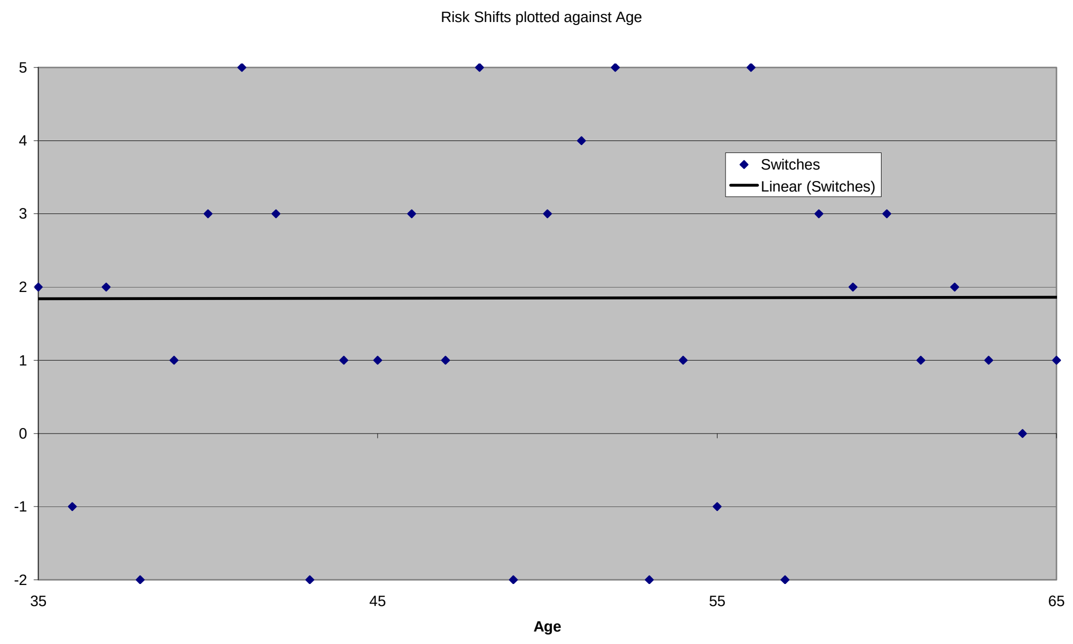

Figure 15 Risk Shifts plotted against age

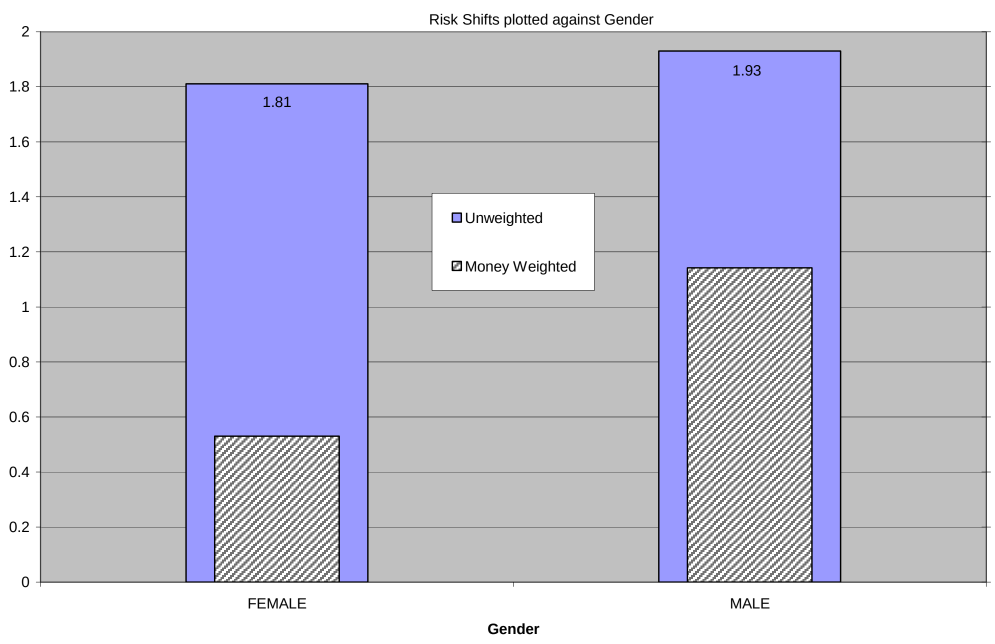

Figure 16 Risk Shifts plotted against gender

In both cases there is no discernible change. As a result, we must remove Ω from the aggregate market utility as it does not provide any information relating to the market equilibrium. However, Section 4 continues to be instructive as to first time shifts under change in constraint, and as discussed in Section 4 Ω does not need to be conditional.

SECTION 6: CONCLUSION

The paper presented an analysis of Investor Utility using state preference or choice-based conjoint analysis under several conditions.

It proposed a number of observations:

Firstly, the utility function of the aggregation of investors can be written in the form:

$$[E(U) \mid \Pr[\Phi^*]] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$$

Where: $\Phi_i^* = \Pr[KT_i . \Phi \mid \Omega]$

This proposes that the aggregate of E(U) forms a pdf according to the combined probability of interpretation of information. Furthermore it proposes a three dimensional optimization space, and comments on the relevance of the risk free rate.

The utility functions underlying the partial utilities for risk and return may be linear over the experiment. No clear estimation was provided of the partial utility for time horizon.

Importantly , investors who chose to optimise their utility follow the MRRT (Market Risk / Reward Theorem), but do not necessarily choose portfolios on the Efficient Frontier!

Secondly, there is no evidence that demographic factors are conditions on aggregate utility, even though they may be instructive on smaller groups.

Thirdly, event studies show that trade occurs for reasons other than changes in information, or changes in utility for the attributes noted. This can occur by introducing a further constraint B such that optimization involves estimating the tangents as follows:

$$L = E(U_i)(q_1, q_2, \ldots q_n) - \lambda T(q_1, q_2, \ldots q_n) - \mu B(q_1, q_2, \ldots . q_n)$$

Under even studies a Bernoulli variable can act to create a hurdle θ that creates friction or inertia in choice as follows:

$E(X) = 1_{[E(U^*)-E(U)]>\theta}$ for trade to occur:

Finally, on aggregate, analysis of data for switches over time confirmed few investors participated in choice, and that demographic factors did not play a role.

APPENDIX 1: DATA FROM CHOICE-BASED CONJOINT ANALYSIS OF STATE PREFERENCE TASKS

Main Effects
Tasks Included: All Random

n= 236

	Effect	Std Err	Lower Bound	Upper Bound	t Ratio	Attribute	Level	
1	-1.35204	0.13597	-1.619916	-1.084164	-9.94365	1	1	3.90%
2	0.12071	0.10705	-0.0901904	0.331610387	1.12769	1	2	6 to 6.3%
3	0.08303	0.10856	-0.1308453	0.296905255	0.76482	1	3	6.5 to 7.2%
4	0.37466	0.10256	0.17260541	0.576714589	3.65294	1	4	7.2 to 8.1%
5	0.77364	0.10343	0.56987141	0.977408585	7.47999	1	5	8 to 9%
6	-0.00811	0.09069	-0.1867794	0.170559371	-0.08942	2	1	1 year
7	0.00876	0.0922	-0.1728842	0.190404238	0.09506	2	2	3 year
8	0.17736	0.09348	-0.006806	0.36152598	1.89728	2	3	5 year
9	-0.17801	0.0899	-0.355123	-0.00089702	-1.98014	2	4	10 year
10	1.42478	0.12252	1.18340199	1.666158005	11.62907	3	1	no chance
11	0.2949	0.09942	0.09903156	0.49076844	2.96602	3	2	13% chance
12	-0.15328	0.10716	-0.3643971	0.057837099	-1.43042	3	3	20% chance
13	-0.47894	0.11069	-0.6970116	-0.26086841	-4.32699	3	4	25% chance
14	-1.08745	0.11619	-1.3163572	-0.8585428	-9.35924	3	5	33% chance
15	-1.36218	0.11314	-1.5850784	-1.13928164	-12.03924			None

References

Cochrane, J., 1997, "Where is the market going? Uncertain facts and novel theories." NBER, Working Paper 6207.

Fama, E. 1970, "Efficient Capital Markets; A Review of Theory and Empirical Work", Journal of Finance

Fama, E., 1997, 'Market Efficiency, Long-Term Returns and Behavioural Finance', Journal of Financial Economics, 49, pp. 283-306.

Fama,E. And M. Miller, 1971, "Theory of Finance", Chapter 1, pp11-39, Drysden Press, Hinsdale, Illinois

Kritzman, M.P., 2003, 'The portable financial analyst: what practitioners need to know', John Wiley & Sons, Inc., 2nd ed.

Kahneman, D., 2003, 'Maps of Bounded Rationality: A Perspective on Intuitive Judgement and Choice', American Economic Review, pp. 1449-1475.

Kahneman, D., Tversky, A., 1979, 'Prospect Theory: An Analysis of Decision under Risk', Econometrica, Volume 47, Issue 2, pp.263-292

Livanas, J., 2006A, 'How can the market be efficient if investors are not rational? ', JASSA (Journal of the Securities Institute of Australia), No. 2, Winter, 2006, pp. 20-24.

Livanas, J., 2006B, 'Are investors rational and does it matter? ', Paper presented to the 14th Colloquium of Superannuation Researchers "Choice in Retirement Funding", 20–21st July 2006

Lo, A. 2005, "Reconciling Efficient Markets with Behavioral Finance: The Adaptive Market Hypothesis", The Journal of Investment Consulting, Vol 7, no 2 pp21-29

Louviere, J. J., and G. G. Woodworth, 1983, "Design and Analysis of Simulated Consumer Choice or Allocation Experiments: An Approach Based on Aggregate Data," *Journal of Marketing Research* 20, 350-367.

Luce, R.D. and Tukey, J.W. 1964, Simultaneous conjoint measurement: a new scale type of fundamental measurement. Journal of Mathematical Psychology, 1, 1–27.

Merton, Robert, 1972, "An Analytic Derivation of the Efficient Portfolio Frontier", The Journal of Financial and Quantitative Analysis, Vol 7, No.4 (Sep., 1972), pp1851-1872.

Markowitz, H. 1952, "Portfolio Selection", The Journal of Finance, Vol. VII, No.1, March 1952

McCulloch, B. and D. Leonova, (May 2005) "The Market Equity Risk Premium', New Zealand Treasury

Russell, B., 1991, 'History of Western Philosophy', George Allen & Unwin Ltd

Roll, R., 1977, "A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory", Journal of Financial Economics **4**(2): 129–176,

Simon, H.A. 1955, 'A Behavioral Model of Rational Choice', Cowles Foundation Paper 98, The Quarterly Journal of Economics, Vol LXIX, pp. 99-118.

Sharpe, W.F, 1964, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", Journal of Finance 19: pp 425-442

Sharpe, W. F., 2006A, 'Investors and Markets: Portfolio Choices, Asset Prices and Investment Advice ', Princeton: Princeton University Press

Sharpe, W. F., 2006B, "Expected Utility Asset Allocation", unpublished paper www.wsharpe.com

Sharpe, W.F., 2007, "Investors and Markets: Portfolio Choices, Asset Prices and Investment Advice", Princeton University Press

Summers, L. 1986, "Does the Stock market Rationally Reflect Fundamental Values?", The Journal of Finance, vol XLI, No 3, pp. 591-601, July 1986

Thompson, J.A.K, 1976, 'Aristotle: Ethics', Penguin, Revised Edition 34, pp. 65

---

[i] The sample in Section 4 was deemed sufficiently large to avoid swamping by different sized age cohorts, although it may be useful to assess whether even in this sample, stratification may change the conclusions.

[ii] Care should be taken in assigning relevance to the actual day recorded as the day of the switch. The time to administratively implement the decision may be up to 1 week, with perhaps a delay of a further week to effect the investment changes and to finalise reporting. When also considering the time delay between the actual investment decision taken by the investor, and the actual direction provided by the investor, the granularity of the data is unlikely to be able to support analysis at periods shorter than 4 weeks to 30 days. Consequently a 30 day trend line probably provides the shortest period on analysis possible.