CAPITAL RESERVING FOR CREDIT RISK

FOR

INSURERS (LIFE & GI) AND OTHER INSTITUTIONS

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OVERVIEW

Credit risk is a large and multifaceted subject that is impacting increasingly on the core work of many insurance actuaries, and often arises in many newer fields of work in which actuaries are becoming increasingly involved, such as funds management, structured finance and the management of various “bank” products. It also has the potential to become an issue that will increasingly affect the work of actuaries in other areas, such as superannuation and health insurance.

This paper focuses on the subject of capital reserving for credit risk. The intention of the paper is to provide an introduction to the subject, to help facilitate discussion amongst Australian actuaries on the issues involved, and to outline some actuarial credit risk reserving models that other actuaries may find of value.

The discussion, techniques and examples in the paper mostly focus on Australian life insurance. Nonetheless, the issues, approaches and conclusions of the paper are, in the authors’ view, equally applicable to the capital reserving and allocation assessment needs of many other financial structures and institutions, and in jurisdictions inside and outside Australia.

AUTHORS

Greg Martin and Martin Paino

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1. INTRODUCTION

1.1 Background to Paper

1.1.1 Introduction

Credit risk is a large and multifaceted subject that:

- Is impacting increasingly on the core work of many insurance actuaries (both life or general) through a combination of changes to investment approaches, product structures, general business and regulator awareness of the issue, and the expanding role of the actuary in the general insurance industry.
- Often arises in many newer fields of work in which actuaries are becoming increasingly involved, such as funds management, structured finance and the management of various “bank” products (e.g. the management of derivative products).
- Has the potential to become an issue that will increasingly affect the work of actuaries in other areas, both traditional and new, such as superannuation and health insurance.

The issues involved potentially range from those concerned with "credit rating" assessment, credit risk pricing, identifying and managing credit risk (both in general and for specific exposures), through to risk capital assessment, measurement and allocation.

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The discussion, techniques and examples in the paper mostly focus on Australian life insurance. Nonetheless, the issues, approaches and conclusions of the paper are, in the authors’ view, equally applicable to the capital reserving and allocation assessment needs of many other financial structures and institutions, and in jurisdictions inside and outside Australia.

1.1.2 Life Insurance

Australian life insurers have been subject to reasonably comprehensive regulatory risk based capital reserving requirements since late 1996, specifically the Solvency and Capital Adequacy Standards of the Life Insurance Actuarial Standards Board (the “LIASB”). These standards have been updated and added to since 1996, including the recent introduction of the new Management Capital Standard from July 2002.

The focus of these standards has historically been on those risks perceived to be the key potential risks facing the life insurance industry in Australia. On this basis the standards primarily have addressed issues such as:

- The major areas of liability experience risk (e.g. adverse claims, policy options and guarantees, lapse/surrender rates and expense levels);
- Excessive asset concentration (poor counterparty diversification) and related party risks; and
- Asset/liability mismatch risks.

Under the third element above, the dominant driver of asset / liability mismatch capital requirements under the standards (the "resilience reserve") historically has been, what may be called, "crude mismatch risks". For example, mismatch risks arising from backing guaranteed, dollar liabilities with equity assets or duration mismatched fixed interest assets. However, other “secondary mismatch risks”, such as convexity mismatch and credit risk mismatch, have not been addressed substantially in the standards.

In recent years life insurers have, in general, been reducing their exposure to crude mismatch risk, and consequently reducing their statutory capital requirements. However, in the search for ways to maintain their overall investment returns, this has often been accompanied by a move away from the use of sovereign and very high grade ("A" rated) fixed interest securities to higher yielding lower grade securities, and so called "high yield" (below investment graded) securities.

Concurrent with this, there has been a general:

- Relative reduction in liabilities related to products such as discretionary credit (or asset return linked) investment account business; and an
- Increase in competitively priced fixed rate, fixed term annuities, with often modest profit margins, and/or increasing exposure to insurance IBNR and outstanding claims reserves, which may have little built-in margins.

The result is that for many current portfolios, the current resilience reserve requirements and blunt inadmissible asset reserve requirements (focusing only on large, single counterparty exposures) may not generate sufficient capital reserves relative to the asset risks involved.

Reflecting this trend the most recent Solvency and Capital Adequacy Standards (AS2.03 10.4.2 and AS3.03 10.3.2) direct actuaries "… where ...the overall portfolio of assets … has too little diversification, is too illiquid or ***has too great an exposure to obligators of low credit standing*** ..." to hold appropriate additional capital reserves to "… protect the interests of the policy owners". In this context the Standards note that "the impact of credit risk may not be adequately provided for through the prescribed asset concentration limits [nor specified resilience reserve requirements], and proper consideration should be given to adopting lower limits or ***employing other recognised credit reserving bases***". {***emphasis*** and [qualifier] added}.

Following on from this, one of the principal aims of this paper is to facilitate discussion within the actuarial profession on this requirement and assist with the development of a "recognised credit reserving basis".

1.1.3 General Insurance

In the case of a general insurance business, it is generally the case that the liability risks will be the principal and major driver of the overall capital requirements of the entity. It is presumed by the authors that it is on this basis that asset/liability mismatch risks are only superficially and, in the authors view, defectively, addressed in the current APRA capital requirement standard “GPS 110”.

Nonetheless, while accepting the dominance of the liability risks, asset/liability mismatch risks can be just as important, in absolute terms, for a general insurer as for a life insurer and should be taken into consideration by an actuary involved in the overall risk capital management of a general insurer (irrespective of the current regulatory requirements).

The focus and examples in this paper would seem to the authors to have equal application to general insurance.

1.1.4 Superannuation & Health Insurance

As with general insurance, for a health insurer the accrued liability risks and the contribution renewal risks of the business will generally comprise the dominant capital reserving risks to be considered. Nonetheless, as the role of actuaries increases in the health insurance industry, and in particular if the appointed actuary role materialises with capital reserving sign-off responsibilities, then credit risk reserving will become an issue to be considered by actuaries advising in that industry.

In superannuation, asset/liability mismatch is an issue that actuaries have taken into account in a general manner and typically:

- In the context of defined benefit funds with respect to longer term contribution rate and funding level impacts; and
- In the context of defined contribution funds with respect to member account crediting rate strategies and formulae.

However, the role of the actuary in superannuation has been changing in recent years with obligations involving the provision of solvency certificates, advice on "unsatisfactory financial position" and the like. In the current corporate governance, compliance and regulatory environment, combined with recent economic experience (poor/negative equity returns) and coming issues such as the reporting of superannuation funds' net asset position on employer balance sheets (under IAS19), the focus on asset/liability mismatch is inevitably going to become heightened and more direct.

In this case, "resilience reserve" measures and management, perhaps similar in structure (if not level) to the LIASB standards, may well become part of the future actuarial advice and certification regime. Furthermore, if part of the response to the future environment by trustees (and employers) is to reduce the existing level of crude mismatch risk in their superannuation funds, following a similar pattern in life insurance as discussed above, it is quite likely that credit risk reserving measurement and management could also become an increasing issue for many superannuation actuaries in future.

1.1.5 Other Financial Institutions

For actuaries involved in other institutional areas, such as banking, structured finance and funds management, credit risk and capital management of credit risk reserving are issues that arise frequently. However, in the authors' experience, the existing credit risk exposure and reserving models used by these institutions are often framed in the context of simple bank style products and/or reflect the existing regulatory capital regimes and requirements imposed on them. As a consequence, the models and techniques are often insufficient for the new and/or more complex products or situations upon which actuaries are advising.

The discussions and conclusions of this paper may well be of interest and use to actuaries operating in these fields.

1.2 Objective & Scope of Paper

As noted above, this paper focuses on the subject of capital reserving for credit risk, with the intention of providing an introduction to the subject, to help facilitate discussion amongst Australian actuaries on the issues involved, and to outline some actuarial credit risk reserving models that other actuaries may find of value.

As such, the discussion in the paper aims to provide a "brief wander through the credit reserving countryside" and provide many readers with an initial overview of the topic, but not an exhaustive review of the subject.

The discussion, techniques and examples in the paper mostly focus on Australian life insurance and the central asset/liability situation considered is a portfolio of fixed interest assets held to fund a portfolio of non-asset-linked liabilities. Nonetheless, the issues, approaches and conclusions should be equally applicable to many other financial structures and institutions.

In this context, it is assumed throughout this paper that the underlying asset portfolio under discussion comprises a diversified portfolio of investments, and does not involve any material single, large counterparty exposures. Attempting to determine or hold statistical based reserves for such individual credit risk exposures is, to say the least, problematic, if not meaningless.

We have used S&P rating scales and categories throughout this paper. This is not to reflect any lesser regard for other rating agencies' scales and categories, but merely to simplify the content of the paper.

2. CREDIT RISK ELEMENTS

2.1 Overview of Credit Risk Relevance

2.1.1 Identifying Credit Risk

As noted in Section 1, Credit risk is a subset of the broader subject of asset/liability mismatch risk.

Asset/liability mismatch risks arise when a given liability (or series of liabilities) is funded by a given portfolio of assets, but where either:

- The realisable quantity, or “value”, of the cash-inflows receivable from the asset portfolio will not match the quantity, or “value”, of the liability cash-outflow with certainty; or
- The quantity, or “value”, of the liability is not precisely linked to the realised cash-flows or “value” of the funding assets.

In the simple case of a fixed (known) set of dollar liability obligations, one may categorise the potential asset/liability mismatch risks as comprising four aspects:

- The risk that fixed asset cash-flows (e.g. fixed interest coupons and maturity values) are not received as expected as a result of defaults occurring at a different (higher) rate than originally budgeted.
- The risk that variable (uncertain) asset cash-flows (e.g. equity dividends or property rents) are received at a level different (below) that originally budgeted.
- The risk that assets which need to be sold into the marketplace pre-maturity to fund the liability cash-flows are sold at an actual price different (below) that originally budgeted.
- The risk that asset cash-flows (e.g. interest, dividends, maturities) received ahead of the liability cash-flows can only be reinvested at a rate different (below) that originally budgeted in funding the liability.

The specific reserving formula set out in the existing LIASB solvency and capital adequacy “resilience reserve” calculations largely address the crude elements of the last three aspects above. That is, as mentioned in Section 1, for example:

- Backing dollar based liabilities with equity or property based assets; and/or
- Fundamental duration mismatch between the assets and liabilities.

Other asset/liability profile mismatch risks, such as convexity, are also contained within the last three aspects above. While these are not specifically addressed in the LIASB resilience reserve formula, they are addressed under the requirements of AS2.03 and AS3.03. Reserving for these risks can generally be readily established via scenario testing of the impact on the relative value of the portfolio's assets and liabilities of factors such as yield curve "twists" and market volatility (valuation) parameter variations to an extent consistent with the interest rate and other prescribed yield variations set out in the resilience tests.

On this basis, one may therefore conclude that it is only the first asset/liability mismatch risk aspect above that relates specifically to credit risk and need be considered.

2.1.2 Regulatory Reality

On this basis, it may be initially thought that for a nominally matched asset/liability portfolio (i.e. liability cash-flows plus asset/liability spread margin = asset cash-flows), the key credit risk reserving issue is reserving for the risk of actual, realised adverse (worse than expected) default experience.

This may be true if the liabilities (plus spread margin) can be, initially and thereafter, valued at a discount rate equal to the "expected (net of default) fund earning rate" on the assets held.

However, it is typical practice that for technical/regulatory solvency, the liabilities are not discounted on this basis:

- Under LIASB solvency and APRA GPS110, the liabilities are required to be discounted and valued at sovereign debt rates.
- While the LIASB capital adequacy standard requires liabilities to be discounted at a rate related to the anticipated fund earning rate, the increasing margins required to be taken from these rates as the reliability of the expected returns reduces, means that in practice the actual nominal expected earning rate on the assets and that used for the liability valuation are heavily disconnected.
- Furthermore, if evolving liability fair value doctrine continues appropriately to move away from using fund earning rate based discount rates towards "risk free" or "replicating portfolio" discount rates, a trend which is also reflected in the principles of emerging international accounting standards, then even the financial reporting basis of the liabilities will not react to changes to the nominal net yield on the backing assets held.

In this case, credit risk reserving relates not only to potential actual (realised) default experience, but also to any asset/liability valuation impacts linked to credit risk issues, for example changes in market credit spreads impacting the asset portfolio.

Indeed, this reality is also reflected in the requirements of AS2.02 11.5 and AS3.03 11.6 that require the actuary to consider and allow for not only changes in the shape of the yield curve, but also changes in market credit and liquidity margins and changes in market spreads.

2.2 Technical Credit Risk Elements

On the basis of the above, the elements that would seem to need to be included in a credit risk reserving model include:

- The impact of potential actual defaults.
- The impact of potential transition of assets held from one credit rating category to another (lower) credit rating category with a different (higher) market credit spread applying.
- The impact of potential adverse variation in overall market credit spread levels (relative to the liability discount rate basis).

Nonetheless, to the extent the discount rate adopted for the liabilities is less than that expected to be earned on the assets over time, then the reserves determined allowing for all three of the above variation effects, would seem to be able to be reduced by the ex-ante net out-performance expectation of the asset returns over the defined liability discount rate.

3. BANKING INDUSTRY APPROACHES

3.1 Overview

In this section of the paper we aim to briefly review the credit risk reserving approaches adopted under the banking industry Basel accord and some other common approaches adopted within the banking industry. This includes a comparison of these methods with the credit reserving elements discussed in Section 2.

3.2 1988 Basel

3.2.1 Summary of Credit Risk Reserving Basis

The 1988 Capital Accord introduced by the Basel Committee on Bank Supervision (the “BCBS”) requires banks to hold credit risk capital of at least 8% of risk-weighted assets.

The BCBS 1988 paper requires that a weighting be applied to all the assets, and off-balance sheet exposures of a bank. The weighting applied to each asset depends on its counterparty category, however only 5 weights are used, 0%, 10%, 20%, 50% and 100%.

Low credit risk assets such as OECD Government Bonds have a risk weighting of zero, whereas corporate debt has a 100% weighting. Appendix A contains more details on the risk weights for balance sheet assets.

3.2.2 Comment & Observations

The practical effect of the 1988 Basel accord credit risk reserving basis is that all corporate exposures have a capital charge of 8% of exposure value, irrespective of the credit rating of the individual counterparties, or the overall credit rating of the bank’s total corporate book.

This is a very blunt reserving basis.

- It would seem to grossly over reserve for a diversified portfolio of highly rated corporate debt, yet likely materially under reserve for a portfolio of well below “investment grade” (below S&P BBB) debt.
- In addition, it is not clear how or to what extent the overall 8% reserve addresses the underlying reserving elements theoretically relevant as set out in Section 2.2 above.

3.3 Basel II

3.3.1 Summary of New Basis

A New Basel Accord is currently being developed in response to demands for more flexibility and risk sensitivity in the reserving basis.

Under the new accord banks will have the choice of three formulaic approaches for assessing credit risk capital reserving needs:

- The "Standardised Approach";
- The "Foundation Internal Rating Based Approach"; or
- The "Advanced Internal ratings Based Approach".

These are briefly discussed in turn below.

3.3.2 The Standardised Approach.

This approach is very similar to the 1988 Accord, except that the risk weightings applied to each counterparty exposure are based on credit ratings from an external credit assessment institution. Under the previous approach the weighting was based on the type of counterparty (e.g. government, bank or corporate). Under the new approach corporate exposures will have following risk weightings:

Credit Assessment (S&P Scale)	AAA to AA-	A+ to A-	BBB+ to BB-	Below BB-	Unrated
Risk Weighting	20%	50%	100%	150%	100%

Appendix A contains more details on the risk weights for on-balance sheet assets.

3.3.3 The Foundation Internal Rating Based Approach.

Under this approach the Bank estimates its own risk weight for each counterparty exposure (*RWc*). This is determined via a specified formula which takes into account:

- The probability of default (*PD*) on the asset; and
- In combination with the:
  - Exposure at default (*EAD*);
  - Loss Given Default (*LGD*) parameter supplied by an external party; and
  - The time to maturity (*M*);

- A risk weighting (RWc) is determined.

The advantage of this approach is that greater granularity of the risk weights is possible.

Under the Foundation approach *M* is specified to be taken as 3 years (and in practice *M* does not actually enter the final formula defined below).

The specified formula for a corporate exposure is:

$$RW_C = \min(LGD/50 * BRWc(PD), 12.5 * LGD)$$

$$BRWc(PD) = 976.5N(1.118 \times G(PD) + 1.288) \times (1 + 0.047 \times (1 - PD)/PD^{0.44})$$

Where:

- *PD* is entered as a decimal (1% = 0.01).
- *N* is cumulative standard normal distribution.
- *G* is inverse cumulative standard normal distribution.

It should be noted that the above equations:

- Are structured to generate a capital requirement sufficient to cover a loss at a 99.5% confidence level (i.e. a 0.5% probability of ruin); and
- They are based on an underlying assumption that individual counterparties' supporting asset values have an average 20% return correlation, irrespective of the credit rating of the counterparty (the meaning of this is discussed further in Section 4).

3.3.4 Advanced Internal Ratings Based Approach.

The Advanced approach is similar to the Foundation approach, except that the bank also provides the *LGD* parameter, and the maturity *M* parameter adopted is based on the actual asset exposure being risk weighted.

The specified formula for a corporate exposure is:

$$RW_C = \min(LGD/50 * BRWc(PD) \times (1 + b(PD) \times (M - 3)), 12.5 * LGD)$$

$$b(PD) = \frac{0.0235 \times (1 - PD)}{PD^{0.44} + 0.0470 * (1 - PD)}$$

These formulae are based on the same probability of ruin and underlying asset values return correlations as the Foundation method.

3.3.5 Comments & Observation

The new Basel accord approaches represent a significant improvement on the 1998 Basel accord basis. Nonetheless:

- None of the three alternative approaches set out above take into account the actual dimensions and level of diversification of the portfolio being considered. The bases presuppose a very large and very well diversified portfolio is applicable in each case.
- In addition, it is not clear how or to what extent these approaches address all of the underlying reserving elements theoretically relevant as set out in section 2.2 above.

3.4 Other Approaches Used by Banks

In contrast to the Basel approach of applying a risk weighting to exposures to reserve for credit risk, a number of banks around the word have developed alternate credit reserving models.

The Basel Committee on Banking Supervision (1999) paper provides a review of practises in credit risk modelling that had been developed by banks at that time. The BCBS 1999 paper suggests that banks generally employ one of two paradigms to define credit loss:

- The default mode (DM) paradigm. Under the DM paradigm, a credit loss only occurs when a borrower defaults.
- The mark-to-market (MTM) paradigm. Under the MTM paradigm, a credit loss can also arise from a reduction in market value associated with a credit rating downgrade.

3.4.1 The Default Mode (DM) Paradigm

Under the default mode Paradigm a loss only occurs when a borrower defaults. The DM paradigm can be thought of as a two state model. Either the bond defaults, with the loss equal to the banks exposure minus the present value of future recoveries, or no loss occurs.

The "Default Risk Reserving Component" of the "Adjusted Default Based Model" discussed in section 4 is an example of a DM model.

3.4.2 The Mark-to-Market (MTM) Paradigm.

In addition to losses due to default, the MTM paradigm also allows for value loss due to deterioration in the creditworthiness of the borrower. The probability of credit rating migrations, in addition the probability of default, is taken into account.

The loss due to migration is equal to the value of the bond if there had been no migration minus the value of the bond at the new credit rating. Under this approach, default is one of the potential credit migration outcomes.

The value of the bond is at each credit rating is determined by calculating the present value of future cashflows. Two theories have been developed to determine the value of the bond. The Discounted contractual cash flow approach discounts contractual payments, while the Risk-neutral valuation approach discounts contingent payments.

As the name suggests, the "Adjusted MTM Transition Model" discussed in Section 4 is an example of a MTM model and uses the Discounted contractual cash flow approach.

3.4.3 Comments & Observation

JP Morgan, Credit Suisse Financial Products, KMV and McKinsey have all developed Credit Risk Reserving methodologies. Each method is based on a different approach. Crouhy, Galai and Mark (2000) explain that:

- The JP Morgan CreditMetrics approach is based on a credit migration approach;
- KMV uses a methodology based on Merton's (1974) asset value model;
- CreditRisk+, developed by Credit Suisse Financial Products (CSFP), uses a so called "actuarial" approach; and
- McKinsey uses an approach based on modelling default rates using macroeconomic variables.

The CSPF CreditRisk+ model and the McKinsey CreditPortfolioView are DM models, while the J.P. Morgan CreditMetric model and KMV's model are examples of MTM models.

Benchmark testing by the International Institute of Finance and the International Swap Dealers Association against various portfolios indicates that they are all reasonable internal models for determining credit risk reserves for straight bonds and loans without option features, but are inappropriate for swaps and other derivative products. It is argued in Crouhy (2000) that this is because all the above models assume deterministic interest rates and exposures.

That is, these models are largely "book value balance sheet" base models that do not directly deal with actual mark-to-market effects as reflected in the third credit risk reserving element set out in paragraph 2.2 (i.e. stochastic movements in overall market credit spreads).

The models developed in the following section 4 attempt to improve on these models in this area by allowing for stochastic credit spreads, as well as allowing for technical present value effects such as the expected net return out-performance of the asset portfolio over the liability discount rate basis used.

3.5 Other Issues To Be Considered

Any credit reserving model must consider the following issues.

3.5.1 What Time Horizon?

Of the banks surveyed by the Basel Committee, most employ a one-year time horizon for measuring credit risk. The reasons given for using this period reflected the belief that it was a typical period over which:

- New capital could be raised.
- Loss mitigating action could be taken to eliminate future risk.
- New information about the counter party would surface.
- Default rate data is published.
- Capital planning decisions are made.
- Exposures to counter parties are reviewed.

Some banks however, use a 5-year horizon or the time to maturity. Basel II reserves implicitly cover a one-year horizon as the *PD* estimates relate to a one year horizon.

In developing the models set out in section 4, the authors have adopted a one year horizon as a reasonable time frame for the average Australian life insurer to identify and implement appropriate action with respect to adverse credit risk outcomes.

3.5.2 What Probability of Ruin?

The confidence level can be selected to be consistent with the financial institutions/life insurance companies desired credit rating (BCBS 1999), while the Basel risk weights are based on a 99.5% confidence rate.

In section 4 we provide indicative results on a range of confidence levels (probabilities of ruin). Nonetheless, the authors would suggest that a reversing level based on a 12 month time horizon with a confidence level less than 99% (probability of ruin greater than 1%) would not be appropriate for capital adequacy reserving.

4. TWO "ACTUARIAL" MODELS

4.1 Introduction

As set out in Section 2, credit risk reserves can be considered as comprising four components or elements:

- The impact of potential adverse default experience.
- The impact of potential transition from one credit rating category to another.
- The impact of potential adverse variation in overall market credit spreads.
- Less the expected net out-performance of the asset portfolio return over the liability discount rate basis used.

However, as noted in Section 3, the Basel reserving bases do not address all these points well (neither all nor individually). While some of the other banking industry models address some of the individual elements reasonably, the published models do not address all the components.

We have therefore built on some of the models briefly reviewed in Section 3, and have developed two general models for consideration:

- An "Adjusted Default Based" Model (the ADB model); and
- An "Adjusted MTM Transition" Model (the AMTMT model).

These two models are discussed in turn below (with the first being the simpler of the two). Subsequently, the results of applying each to the example asset/liability portfolio set out in Appendix F are indicated, as well as a comparison with the results that would emerge under the Basel I and II rules.

4.2 Adjusted Default Based Model

The Adjusted Default Based model is a "deterministic" model that is based on the determination of four reserve components:

- A default risk reserving model (that deals with the risk of actual default experience);
- An approximate migration reserving model;
- An approximate credit spread reserving model; and
- An out-performance reserving reduction estimate.

These components are discussed in turn below.

4.2.1 Default Risk Reserving Component

The reserve for default risk is calculated using the mean/standard deviation approach as discussed in Basel 1999. This approach is an example of the default mode paradigm, and is similar to calculating the value at risk of an equity portfolio.

When calculating the value at risk of an investment portfolio the mean and standard deviation of the return on the portfolio is determined by aggregating the mean and standard deviation of the individual stocks of the portfolio (and allowing for correlation). Based on the assumption that investment returns on the portfolio are normally distributed, and using the portfolio mean and standard deviation the range in portfolio values can be determined for a certain confidence interval.

The default risk mean/standard deviation approach is very similar to this. A portfolio's expected loss from default (μ) is calculated as the sum of the expected losses on individual credit facilities (*ELi*) (BCBS 1999):

$$\mu = \sum_{i=1}^{N} EL_i \qquad (1)$$

The standard deviation of the default loss incurred by the portfolio ($\sigma_{Portfolio}$), is based on the individual credit standard deviations (σ_i) and covariances between individual credits ($\rho_{i\,j}$), and is given by:

$$\sigma_{Portfolio} = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} \sigma_i \sigma_j} \qquad (2)$$

However, the above needs to allow for the distribution of default losses. An individual security's impact on the portfolio is a function of the institution's exposure to it (*EXP*), the probability of default (*EDF*) and in the event of default, the loss given default (*LGD*). The expect loss on security i (*ELi*) is therefore given by:

$$EL_i = EDF_i \times EXP_i \times \overline{LGD}_i \qquad (3)$$

As noted in BCBS (1999), if:

- The exposure to each security is known;
- Defaults are independent of the *LGD*; and
- *LGD*'s are independent across securities;

Then the standard deviation of credit losses for the i[th] security can be expressed as:

$$\sigma_i = EXP_i \sqrt{EDF_i(1 - EDF_i)\overline{LGD_i}^2 + EDF_i \sigma_{LGD,i}^2} \qquad (4)$$

Where $\sigma_{LGD,i}$ is the volatility (standard deviation) of *LGD*.

BCBS 1999 suggests that the probability density function of credit losses is typically assumed to be approximated by a beta distribution that is parameterised by the mean and variance of the default losses.

Given the simulation results of the AMTMT model discussed below, we approximated this by using the normal distribution, but multiply the z factors by 4.5/2.57=1.75 to allow for skewness and kurtosis (the 4.5/2.57 being the ratio of our simulation result 99.5% confidence interval to a standard z factor 99.5% confidence interval).

Based on the above model and default assumptions (as indicated in Appendices B to D), a credit risk default reserve at the desired probability of confidence (ruin) can be determined.

4.2.2 Credit Migration Reserving Component

This reserve has been calculated in a very approximate way under this model. Using the migration matrix in Appendix B, an approximation for the standard deviation of interest rate change due to credit migration, for each credit rating, was calculated using the equation below:

$$CMSD_i = \sqrt{\sum_{j=1}^{j=N} p_{i,j}\left(cs_i - cs_j\right)^2}$$

Where:

$CMSD_i$ = Credit migration standard deviation for rating i.

$p_{i,j}$ = migration probability from rating i to rating j (excluding default).

cs_i = credit spread for rating i.

Based on the migration matrix and credit spreads in the appendices, the following migration standard deviations were determined:

Approximate Credit Migration Standard Deviations Rating	
AAA	0.1%
AA	0.3%
A+	0.4%
A	0.5%
A-	0.6%
BBB	1.4%
BB	1.6%
B	1.2%
CCC	2.0%

Credit spread migrations were then assumed to be independent and normally distributed, for simplicity, when determining the portfolio credit migration standard deviation.

The credit spread migration reserve is then equal to the modified duration of the portfolio at the end of the year multiplied by the required z factor from the standard normal distribution and the portfolio credit spread standard deviation. The equation being:

$$CMR = CMSDP \times Z \times Dur$$

Where:

CMR = Credit migration reserve.

CMSDP = Credit migration standard deviation for the portfolio.

Z = Z factor from standard normal distribution for required confidence level (ruin probability).

Dur = Modified duration of the portfolio.

In practice migration is subject to correlation effects. However:

- For a typical portfolio of diversified credit ratings these are not large; and
- The above model, in assuming migration is a continuous process (rather than discrete), tends to produce an offsetting overstatement effect.

Nonetheless, for a portfolio concentrated at the low credit rating end, some allowance for the underlying actual correlation effects may be appropriate.

4.2.3 Credit Spread Risk Reserving Component

If it is assumed that overall market credit spread movements are perfectly correlated (i.e. all category market credit spreads move up and down consistently), then the standard deviation of credit spread for the portfolio equals the sum of the individual stock standard deviations. If it is further assumed that market credit spread variations are normally distributed, then the credit spread reserve for the required confidence interval can be calculated as:

$$CSR = CSSDP \times Z \times Dur$$

Where:

CSR = Credit spread volatility reserve.

CSSDP = Credit spread standard deviation for the portfolio.

Z = Z factor from standard normal distribution for required confidence level (ruin probability).

Dur = Modified duration of the portfolio.

4.2.4 Out-Performance Reserve Offset

Subtracted from the sum of the above three reserve components is the expected default adjusted yield to be earned on the portfolio of assets less the discount rate adopted for the liability discount rate (i.e. the expected asset return out-performance over the liabilities return required).

If the liabilities are valued at sovereign discount rates, then the reserve offset is equal to the value of the portfolio at the beginning of the period multiplied the average net of expected default credit spread margin (i.e. credit spread on the portfolio minus the expected default cost).

Appendix E contains the net credit spreads we have used for each credit rating in the indicative results in Section 4.4.

4.3 Adjusted MTM Transition Model

The Adjusted MTM Transition Model is a stochastic simulation based model that has two components:

- A credit risk model, that reserves for default, migration, and credit spread risk.
- An out-performance reserving reduction estimate.

These components are discussed in turn below.

4.3.1 The Credit Risk Model

The credit risk model is based on JP Morgan's CreditMetrics model, and is a stochastic (simulation) MTM model. The CreditMetric's model as discussed by Crouhy (2000) is based on "estimation of the forward distribution of the changes in value of a portfolio of loan and bond type products at a given time horizon, usually one year. The changes in value are related to the eventual migration in credit quality of the obligator, both up and down, as well as default."

Unlike the CreditMetrics model, the model described in this paper also allows for the effect of overall market credit spread movements.

4.3.2 Valuing a Single Bond

To help explain this model, the value distribution of a single bond is first discussed.

Consider a BBB rated bond. At the end of the year, using the transition matrix in Appendix B, the probability of the bond having a certain credit rating is:

Probability of BBB Bond having the following ratings	
Rating Year End	%
AAA	0.04
AA	0.25
A+	0.37
A	0.98
A-	3.17
BBB	89.12
BB	4.70
B	0.81
CCC	0.27
Default	0.30
Total	**100.00**

If the bond at the beginning of the period has 5 years to maturity and the coupon rate is 6.1%, the value of the bond can be determined at the end of the year for each potential credit rating. Assuming the yield curve is flat at 5% and the credit spreads in Appendix E apply, the value of the bond at year end could be:

Value of BBB Bond with a Market Value of 100	
Rating Year End	$
AAA	102.67
AA	102.06
A+	101.41
A	101.34
A-	101.15
BBB	100.14
BB	84.48
B	78.99
CCC	69.27
Default	50.00

Note that the value of the bond remaining in the BBB category increases over the year reflecting the narrowing of market spreads as duration reduces.

The default value is equal to the face value multiplied by the default bond price in Appendix C. Combining the probability and value information, the mean expected default loss is $0.15 and the expected migration loss is $0.94:

Distribution of Bond Value at year end		
Value at beginning of year		100.00
Nominal value at end of year		100.14
Rating Year End	Probability (%)	Value ($)
AAA	0.04	102.67
AA	0.25	102.06
A+	0.37	101.41
A	0.98	101.34
A-	3.17	101.15
BBB	89.12	100.14
BB	4.70	84.48
B	0.81	78.99
CCC	0.27	69.27
Default	0.30	50.00
Expected Value at Year End		99.05
Nominal Spread Margin		1.10
Spread Narrowing Gain		0.14
Expect Default Loss		-0.15
Expect Migration Loss		-0.94
Expect Profit		**0.15**

To estimate the distribution of potential bond prices, including default loss variation and market credit spread volatilities, a stochastic simulation is run. The distribution of default bond prices is assumed to follow a beta distribution as discussed in Appendix C, while credit spreads are assumed to be normally distributed, and correlated based on the matrix in Appendix E

4.3.3 Valuing a Bond Portfolio

The value of a bond portfolio is not calculated simply by adding the individual bond results, as bond migrations are known to be correlated. As discussed in Appendix D, JP Morgan use an Asset Value Model to allow for correlation. Using this approach, and a choleski decomposition matrix, correlated random migrations are simulated by calculating correlated underlying counterparty asset returns and converting these to correlated migrations.

4.3.4 Allowing for Credit Spread Correlation with Defaults

As noted above, in addition to the effects of credit rating migration, the impact of variable market credit spreads is also taken into account. In including this factor, we would expect credit spreads to widen (shorten) when the overall level of migrations is relatively high (low). Consequently, we have assumed that there is correlation between market credit spread movements and the overall level of migrations. However, there appears to be little market data and/or published analysis relating to such correlation. On this basis we have adopted a 50% correlation as a not unreasonable allowance for the indicative numerical analysis below. This is implemented in the model as follows:

- The correlated stochastic asset returns underlying the migration model are added together and divided by the number of stocks to determine a portfolio average "migration" (z) factor.
- This stochastic result is then correlated with random variables for credit spread movements (for each credit rating category), using a choleski decomposition matrix, to determine correlated market credit spread random variables. It is assumed that credit spreads are normally distributed (and correlated as per the assumption in Appendix E).

These random variables are used to impact the market credit spreads at the end of the period for the particular simulation iteration.

4.3.5 Creating a Distribution of Bond Portfolio Values

The simulation is then run multiple times (in the usual fashion) to determine a distribution of MTM results, with the reserve taken at the required probability level.

For readers who would like more detail, JP Morgan has released a technical document explaining the mechanics of CreditMetrics in considerable detail, while Crouthy (2000) provides a more condensed summary of the methodology.

4.3.6 Out-Performance Reserve Offset

Subtracted from the reserve calculated above is the net credit spread margin expect to be earned over the year. This is determined in the same way as for the Adjusted Default Based Model above.

4.4 Some Indicative Results

4.4.1 Applying the Models to an Example Portfolio

The results of applying the ADB and AMTMT models to the example asset/liability portfolio set out in Appendix F are outlined below:

Calculated Reserves for example portfolio	95% CI		99% CI		99.5% CI		99.9% CI	
	ADB	AMTMT	ADB	AMTMT	ADB	AMTMT	ADB	AMTMT
Default Risk	2.4%	2.3%	3.4%	3.9%	3.7%	4.5%	4.5%	5.6%
Migration	1.5%	0.6%	2.1%	1.3%	2.4%	1.4%	2.9%	1.9%
Spread	1.4%	1.6%	2.0%	2.1%	2.2%	2.3%	2.7%	2.7%
Credit Risk Reserve	**5.3%**	**4.6%**	**7.5%**	**7.2%**	**8.4%**	**8.2%**	**10.0%**	**10.1%**
Outperformance	-1.6%	-1.6%	-1.6%	-1.6%	-1.6%	-1.6%	-1.6%	-1.6%
Total Reserve	**3.8%**	**3.0%**	**6.0%**	**5.7%**	**6.8%**	**6.6%**	**8.5%**	**8.5%**

The results are based on the assumptions outlined in Appendix A to D, and 7,000 simulations.

4.4.2 Basel Results

The table below compares the reserve required under Basel I and II (for the Standardised Approach) with the reserves calculated by the two models as above:

Calculated Reserves for example portfolio	**Credit Risk Reserve**
BASEL I	8.0%
BASEL II (Standardised Approach)	3.8%
ADBM*	6.8%
AMTMT*	6.6%
*99.5% confidence interval	

4.5 Brief Comments

We would make the following comments and observations on the above example portfolio results:

- The example asset portfolio considered has a nominal average S&P credit rating of about "A". Out of a total of 70 bonds held, only 21 (30%) are below an "A-" rating and 14 (20%) below BBB. Nonetheless, the reserving models considered above have generated some significant credit reserving requirements (possibly 7% to 8% for capital adequacy reserving for a life insurer).
- This partly reflects the fact that while the nominal average credit rating may appear to be about "A", the default rate weighted average rating is closer to "BB". It does not require a large exposure to below BBB rated bonds to significantly impact the average rating of a total portfolio. However, even without exposure to low credits, reserving requirements can still be significant if fully assessed.
- Interestingly, even for an asset portfolio of duration 5 years where the impact of credit spread variation (through either overall market spread movement or rating migration effects) would be expected to be a major reserve driver (as it is), the reserve for outright default remains the largest individual reserve driver. Nonetheless, as the impact of credit spread variation relates to the duration of the assets (and liabilities), its significance would increase for a long duration portfolio.
- For a relatively straightforward asset/liability structure such as the example, a simple "deterministic" credit reserving model similar to the Adjusted Default Based Model can produce satisfactory results relative to a more sophisticated stochastic based model. However, some of the simplifying and "offsetting" assumptions made for the simpler model need to be borne in mind, especially when interpreting the result emerging. Also, the ability of a simple model to deal with any optionality characteristics of the asset/liability structure need to be considered.
- While the Basel I reserving basis produces a result similar to the two models for the example portfolio, this is substantially a co-incidence. The Basel I reserve would not change with the underlying specific credit quality of the overall portfolio, nor any change in the duration of the portfolio.
- The Basel II Standardised Approach reserving basis assumes a fixed 3 year bond maturity profile and does not allow for the market value effect of market spread volatilities. Other Basel II approaches may address the first point, but not the second. Nonetheless, comparing the Basel II result to the 99.5% CI result with the Default and Migration elements above suggests the assumptions set out in the Appendices may be a little conservative overall.

5. CLOSING REMARKS & CONCLUSIONS

5.1 Closing Remarks & Observations

In developing a statistical based capital reserving model, allowance for the following risk elements needs to be made:

- The impact of potential adverse default experience.
- The impact of potential transition from one credit rating category to another.
- The impact of potential adverse variation in overall market credit spreads.
- Less the expected net out-performance of the asset portfolio over the liability discount rate basis used.

Section 4 outlined two general models that address these elements and some indicative assumptions that could be used to drive them are summarised in the Appendices. The indicative results of applying the models to a simple asset and liability portfolio were also provided.

Nonetheless, there are a number of important issues to consider in both the application of these, or similar, models, and the general interpretation of the results. Some of these issues are briefly noted below.

5.1.1 Individual Credit Exposures

In the introduction it was noted that reserving for single large credit exposures is problematic and outside the scope of this paper. Nonetheless, while an asset portfolio may not have any large single credit exposures, it is not uncommon for it to comprise a limited number of holdings (less than say 100).

In such circumstances it is possible for a statistical based reserve to be smaller than some of the single counterparty exposures. A practical rule used by one of the authors is to hold the greater of the statistical reserve and the sum of the two biggest single counterparty exposures with a rating lower than the rating of the institution involved (e.g. if the insurer has a A+ rating, hold a reserve no less than the two largest exposures rated A or lower).

5.1.2 Junk Bonds As Quasi Equity

In many respects, low rated bonds ("high yield" and "junk") have risk and market value characteristics not dissimilar to equity. The credit risk reserves determined for them should therefore approach similar dimensions to equity. Any credit reserving model not producing such results should be carefully considered.

5.1.3 Parameter Variability

Many, if not all, of the model parameters discussed in this paper (default rates, migration rates, correlations, severities etc) are not fixed, but vary over time and are likely to move together – particularly all adversely in times of economic down turn. While holding reserves at a confidence level greater than 99% (ruin below 1%) will help cover some of the modelling issues this gives rise to, it is important to consider if a reasonably foreseeable event could lead to a worse outcome than the amount of reserves determined by a model.

5.1.4 Parameter "Accuracy"

As noted in the comments at the end of Section 4, the assumptions set out in the Appendices may well, in total, be a little conservative. This can be seen in areas such as the small implied profit margin for the BBB bond considered in section 4.3.2, as well as the results of the comparison against Basel II. This highlights the need to check such implied results from credit reserving models for reasonableness so as to not inappropriately over or under reserve for credit risk.

5.1.5 Time Horizons & Ruin Probabilities

As discussed, the model results set out in Section 4 are based on a 12 month time horizon and a range of ruin probabilities are considered. These are important parameters that can have a significant impact on the total reserve determined. These need to be carefully considered and the ramifications of the basis selected needs to be appreciated by the users of the reserving results determined (e.g. the Board of the company). This may include consideration of the compliance, monitoring and automatic action trigger points regime that needs to be in place to support the reserving basis adopted.

5.2 Acknowledgements

In preparing this paper the authors enjoyed considerable support from their employer and from actuarial colleagues who reviewed this paper pre-publication and provided valuable feedback, comments and observations that have been taken into account in the paper.

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Nonetheless, the views expressed in this paper remain those of the authors and should not in anyway be taken to represent the views of their employer or those of any of the individual reviewers.

6. BIBLIOGRAPHY

Basel Committee on Banking Supervision. "International Convergence of Capital Measurement and Capital Standards", Basel (1988).

Basel Committee on Banking Supervision. "Credit Risk Modelling: Current Practices and Applications", Basel (1999).

Basel Committee on Banking Supervision. "Consultative Document: The New Basel Capital Accord", Basel (2001).

Basel Committee on Banking Supervision. "Consultative Document: The Internal Ratings-Based Approach", Basel (2001).

Basel Committee on Banking Supervision. "Consultative Document: The Standardised Approach to Credit Risk", Basel (2001).

Brady, B., and Bos, R.J. "Record Defaults in 2001 the Result of Poor Credit Quality And a Weak Economy," Standard & Poor's Special Report, (2002).

Crouchy, M., Galai, D., and Mark, R. "A comparative analysis of current credit risk models," Journal of Banking and Finance, (2000).

Credit Suisse First Boston. "CreditRisk+ A Credit Risk Management Framework", Credit Suisse First Boston International, (1997).

Griep, C.M. "Higher Ratings Linked to Stronger Recoveries," Standard & Poor's Ratings Direct (2002).

Gupton, G.M., Finger, C.C., and Bhatia, M.. "CreditMetrics – Technical Document," J.P. Morgan & Co. Incorporated, (1997).

Hamiliton, D.T., Gupton, G., and Berhault, A. "Default and Recovery Rates of Corporate Bond Issuers: 2000," Moody's Investor Service, Global Credit Research, (2001).

Kiesel, R., Perraudin, W., and Taylor, A. "The structure of credit risk: spread volatility and ratings transitions," Bank of England (2001).

Lucas, D.J. "Default correlation and credit analysis," The Journal of Fixed Income, (1995).

Merton, R.C. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," Journal of Finance, (1974).

Reserve Bank of Australia. “Bulletin February 2003,” RBA (2003).

Rice, J.A. “Mathematical Statistics and Data Analysis, Second Edition”, Wadsworth, Inc., (1995).

Zhou, C. “Default Correlation: An Analytical Result,” Federal Reserve Board, Finance and Economics Discussion Series, (1997).

A. BASEL (I & II) RISK WEIGHTS

A.1 Basel I

The following table summarises the 1998 Basel specified risk weights:

	Risk Weights by Category of On-Balance Sheet Assets
0%	➢ Cash. ➢ Claims on central government governments and central banks denominated in national currency and funded in that currency. ➢ Other Claims on OECD central governments and central banks. ➢ Claims collateralised by cash of OECD central-government securities or guaranteed by OECD central governments.
0%, 10%, 20%, or 50% (at national discretion)	➢ Claims on domestic public-sector entities, excluding central government, and loans guaranteed by such entities.
20%	➢ Claims on multilateral development banks and claims guaranteed by, or collateralised by securities issued by such banks. ➢ Claims on banks incorporated in countries of the OECD with a residual maturity of up to one year and loans with a residual maturity of up to one year guaranteed by banks incorporated in countries outside the OECD. ➢ Claims on non-domestic OECD public-sector entities, excluding central government, and loans guaranteed by such entities. ➢ Cash items in the process of collection.
50%	➢ Loans fully secured by mortgage on residential property that is or will be occupied by the borrower or that is rented.

	Risk Weights by Category of On-Balance Sheet Assets
100%	➢ Claims on the private sector. ➢ Claims on banks incorporated outside the OECD with a residual maturity over one year. ➢ Claims on central governments outside the OECD (unless denominated in national currency – and funded in that currency – see above). ➢ Claims on commercial companies owned by the public sector. ➢ Premises, plant and equipment and other fixed assets. ➢ Real estate and other investments (including non-consolidated investment participations in other companies). ➢ Capital instruments issued by other banks (unless deducted from capital). ➢ All other assets.

A.2 Basel II – Standardised Approach

The following table summarises the Basel II specified risk weights that apply under the "Standardised" approach:

	Risk Weights by Category of Exposure						
Sovereign (Central Government) Exposures	Credit Assessment (S&P Scale)	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to BB-	Below B-	Un-Rated
	Risk Weighting	0%	20%	50%	100%	150%	100%
Non-Central Government Public Sector Entities (PSEs)	Same as Banks in that country, but they may be treated as sovereigns, or corporate, at the direction of the national regulator,						

<table>
<tr><td></td><td>Risk Weights by Category of Exposure</td></tr>
<tr><td>Multilateral Development Banks (MDB)</td><td>Same as Option 2 for Banks, but a risk weighting of zero will apply to highly rated MDM’s that meet certain criteria</td></tr>
<tr><td>Banks</td><td>Two options are available:
➢ Option 1: The risk weighting is based on the sovereign rating of the country the bank is incorporated in:
<table>
<tr><td>Credit Assessment of Sovereign</td><td>AAA to AA-</td><td>A+ to A-</td><td>BBB+ to BBB-</td><td>BB+ to BB-</td><td>Below B-</td><td>Un-Rated</td></tr>
<tr><td>Bank Risk Weighting</td><td>20%</td><td>50%</td><td>100%</td><td>100%</td><td>150%</td><td>100%</td></tr>
</table>
➢ Option 2: The risk weighting is based on the rating of the bank, with a lower weighting available to claims with an original maturity of three months or less that are not expected to be rolled over:
<table>
<tr><td>Credit Assessment of Bank</td><td>AAA to AA-</td><td>A+ to A-</td><td>BBB+ to BBB-</td><td>BB+ to BB-</td><td>Below B-</td><td>Un-Rated</td></tr>
<tr><td>Risk Weighting</td><td>20%</td><td>50%</td><td>50%</td><td>100%</td><td>150%</td><td>100%</td></tr>
<tr><td>Short Term Claims</td><td>20%</td><td>20%</td><td>20%</td><td>50%</td><td>150%</td><td>20%</td></tr>
</table></td></tr>
<tr><td>Securities Firms</td><td>Same as Banks</td></tr>
<tr><td>Corporates
(includes insurance companies)</td><td>
<table>
<tr><td>Credit Assessment</td><td>AAA to AA-</td><td>A+ to A-</td><td>BBB+ to BB-</td><td>Below BB-</td><td>Un-Rated</td></tr>
<tr><td>Risk Weighting</td><td>20%</td><td>50%</td><td>100%</td><td>150%</td><td>100%</td></tr>
</table></td></tr>
<tr><td>Retail Assets</td><td>TBA</td></tr>
</table>

	Risk Weights by Category of Exposure
Claims secured by Residential Property	50% - Same as Basel I
Claims secured by Commercial real estate	100%
Higher Risk Weights	150% - In addition assets above receiving a weighting of 150%, securitisation tranches that are rated between BB+ and BB- as set out in the Basel document "Asset Securitisation" and the unsecured portion of past due assets net of specific provisioning
Other Assets	100% - Same as Basel I

B. RATINGS TRANSITION & DEFAULTS

B.1 Historic Data & Published Research

Both Standard and Poor's and Moodys regularly publish historical credit rating migration rates by rating. Standard and Poor's (Brady, and Bos, 2002) set out the following table of average annual migration rates for the period 1/1/1981 to 31/12/2001:

Average One-Year Transition Rates by Rating

	Rating Year End																		
Rating Start	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D	N.R.
AAA	89.62	3.24	2.28	0.40	0.14	0.17	0.11	0.06	0.03	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	3.93
AA+	1.99	80.85	9.32	3.13	0.28	0.85	0.07	0.00	0.21	0.07	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00	3.13
AA	0.62	1.21	83.01	6.96	2.34	1.42	0.26	0.45	0.21	0.09	0.05	0.02	0.02	0.02	0.00	0.02	0.05	0.00	3.24
AA-	0.05	0.29	3.07	79.83	8.21	3.26	0.48	0.19	0.11	0.03	0.03	0.00	0.00	0.00	0.13	0.00	0.00	0.03	4.31
A+	0.00	0.06	0.60	3.97	80.09	7.42	2.49	0.58	0.35	0.12	0.04	0.10	0.02	0.10	0.06	0.00	0.02	0.06	3.93
A	0.06	0.09	0.47	0.72	4.57	79.05	5.35	3.04	1.11	0.31	0.17	0.17	0.13	0.14	0.01	0.00	0.03	0.04	4.55
A-	0.12	0.05	0.14	0.42	1.00	6.76	75.62	7.13	2.64	0.74	0.21	0.32	0.12	0.16	0.02	0.02	0.07	0.05	4.42
BBB+	0.03	0.03	0.05	0.15	0.46	1.63	6.82	73.89	7.90	2.92	0.49	0.36	0.15	0.23	0.15	0.00	0.13	0.18	4.42
BBB	0.02	0.02	0.09	0.09	0.40	0.73	1.75	6.45	74.68	5.40	1.86	1.00	0.40	0.31	0.22	0.00	0.11	0.29	6.18
BBB-	0.06	0.00	0.09	0.18	0.18	0.42	0.48	1.98	7.14	72.14	5.13	2.82	1.05	0.66	0.36	0.36	0.51	0.33	6.12
BB+	0.10	0.00	0.00	0.05	0.15	0.29	0.29	0.73	3.29	10.35	64.59	5.13	3.43	1.45	1.06	0.19	0.97	0.48	7.45
BB	0.00	0.00	0.08	0.04	0.00	0.23	0.11	0.15	1.15	3.53	6.02	66.77	6.67	2.64	1.15	0.69	1.03	1.07	8.66
BB-	0.00	0.00	0.00	0.03	0.06	0.03	0.17	0.23	0.32	0.75	2.68	7.12	65.25	7.67	2.68	1.21	1.21	1.76	8.82
B+	0.00	0.02	0.00	0.08	0.00	0.06	0.15	0.08	0.12	0.19	0.41	1.49	4.57	69.35	5.42	2.31	2.34	3.24	10.17
B	0.00	0.00	0.09	0.00	0.00	0.21	0.21	0.00	0.17	0.09	0.47	0.60	1.70	6.65	60.76	4.35	4.77	9.29	10.65
B-	0.00	0.00	0.00	0.00	0.10	0.00	0.10	0.19	0.10	0.10	0.19	0.29	0.38	3.33	5.99	55.76	9.61	11.89	11.99
CCC	0.11	0.00	0.00	0.00	0.11	0.00	0.11	0.45	0.22	0.00	0.22	0.34	0.89	1.90	3.02	4.03	51.34	24.72	12.53

N.R.—Rating withdrawn.

There are however, a number of other published studies available, each showing results with some variation from the above.

B.2 Assumptions Adopted for this Paper

For the purposes of the example modelling set out in this paper we have adopted the following simplified migration matrix and default rates:

Transition Matrix Adopted

	Rating Year End									
Rating Start	AAA	AA	A+	A	A-	BBB	BB	B	CCC	D
AAA	93.27	6.16	0.15	0.18	0.11	0.09	0.03	0.00	0.00	0.01
AA	0.92	92.48	3.76	1.92	0.28	0.47	0.04	0.08	0.02	0.03
A+	0.00	4.82	83.38	7.72	2.59	1.09	0.17	0.17	0.02	0.04
A	0.06	1.34	4.79	82.79	5.60	4.67	0.49	0.16	0.03	0.06
A-	0.13	0.64	1.05	7.07	79.07	10.99	0.68	0.21	0.07	0.10
BBB	0.04	0.25	0.37	0.98	3.17	89.12	4.70	0.81	0.27	0.30
BB	0.04	0.07	0.08	0.20	0.21	7.39	82.54	6.81	1.16	1.50
B	0.00	0.07	0.04	0.10	0.17	0.39	3.77	80.16	6.30	9.00
CCC	0.13	0.00	0.13	0.00	0.13	0.80	1.73	10.70	61.37	25.00

C. DEFAULT SEVERITIES (LOSSES)

C.1 Historic Data & Published Research

There is a large body of research showing a correlation between recovery rates and the level of seniority of the debt involved. Hamiliton, Gupton, and Berhault (2001) reviewed historical default bond prices for the period 1970 to 2000. The table below summarises their findings.

Default Bond Prices, 1970 – 2000

	Median	Average	St Dev
Senior/ Secured Bonds	53.8	52.6	24.6
Senior/ Unsecured Bonds	44.0	46.9	28.0
Senior/ Subordinated Bonds	29.0	34.7	24.6
Subordinated Bonds	28.5	31.6	21.2
Junior/ Subordinated Bonds	15.1	22.5	18.7

Standard & Poors (Griep 2002) investigated the relationship between rating and recoveries on defaulted securities. Standard and Poor's concluded that debt with higher ratings and higher seniority usually have higher recoveries, but the effect of seniority is stronger than rating.

C.2 Assumptions Adopted for this Paper

The purposes of the example modelling set out in this paper we have adopted the following simplified matrix of bond default prices (or realisation values):

Default Bond Price Assumption

	Mean	St Dev
Senior Debt	50%	25%
Other	30%	20%

Gupton, Finger, and Bhatia (1997) suggested that default bond prices may be reasonably models via the beta distribution. We have adopted this distribution assumption for this paper

D. CREDIT RATING AND DEFAULT CORRELATIONS

D.1 Historic Data & Published Research

Researchers in the past have attempted to estimate credit rating category transition, and default, correlations by a number of different approaches, including:

- Observing historical credit ratings and bond spreads movement data.
- Using Monto Carlo simulations of various models of entity based default risk.
- Assuming bond issuer's net asset values drive credit rating changes, and therefore model credit rate changes via net asset value based models.

The first approach is generally considered unsuitable for estimating credit rating and default correlations principally owing to a lack of appropriate data.

Chunsheng Zhou (1997) argues that the third approach is has limited appeal given its time consumption and that the results are often difficult to interpret.

Gupton, Finger, and Bhatia (1997) and Chunsheng Zhou (1997) have developed models based on the bond issuer's net asset value driving credit ratings and therefore credit rating movement correlations. Both approaches imply that the correlation between asset values drives credit correlations.

D.1.1 Default Correlation Results

Chunsheng Zhou (1997) calculates implied default correlations over 1, 2, 3, 5, and 10 periods. Zhou's results for a one-year time horizon is replicated below, and are based on an asset correlation factor of 0.4.

One Year Default Correlations (%)

	AA	A	BBB	BB	B
AA	0.00				
A	0.00	0.00			
BBB	0.00	0.00	0.00		
BB	0.00	0.00	0.01	1.32	
B	0.00	0.00	0.00	2.47	12.46

Zhou (1997) finds that default correlations for highly rated bonds are virtually zero at the short to middle investment horizons, but default correlations for lowly rated bonds are high for even short investment horizons.

Zhou (1997) findings are similar to Lucas (1995) over short to medium investment horizons.

D.1.2 Credit Rating Category Transition Correlations

While the Chunsheng Zhou (1997) model only looks at default correlations, the Gupton, Finger, and Bhatia (1997) model can be used to calculate both credit rating transition and credit default correlations. The Gupton, Finger, and Bhatia (1997) model has been used in this paper owing to this advantage.

Gupton, Finger, and Bhatia (1997) explain that their model is based on Merton (1974). They argue that credit rating migration is driven by the net asset value of the bond issuer, and therefore movements in asset values will describe changes in credit rating.

If the return on the bond issuer's net assets breaks certain thresholds of return, the credit rating will change accordingly.

For a given net asset return model, the various thresholds can be estimated (back fitted) using a migration matrix (as per Appendix B). The credit rating transition and default rate correlations are then derived as a function of the bond issuers' net asset return correlations.

Based on their analysis, Gupton, Finger, and Bhatia (1997) estimate that across a portfolio bonds the average asset correlation is likely to be between 20% and 35%.

Gupton, Finger, and Bhatia (1997) also explain that a correlation matrix can be calculated using the following equation:

$$ {}_{ij}\rho_D = \frac{p_{ij} - p_i p_j}{\sqrt{p_i(1-p_i)p_j(1-p_j)}} $$

Where:

${}_{ij}\rho_D$ = the default correlation between rating i and rating j.

p_{ij} = the joint probability of two bonds rated i and j both defaulting.

p_j = the probability of bonds i defaulting.

Gupton, Finger, and Bhatia (1997) calculate the joint probability assuming asset returns on security i and j and normally distributed and correlated.

D.2 Assumptions Adopted for this Paper

D.2.1 Default Based Model

For the purposes of the simplified default based model illustrated in this paper, we have used the following indicative default rate correlation matrix:

Default rate correlation matrix

	AAA	AA	A+	A	A-	BBB	BB	B	CCC
AAA	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AA		0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
A+			0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
A				0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
A-					0.0%	0.0%	0.0%	0.0%	0.0%
BBB						0.8%	0.3%	0.0%	0.3%
BB							3.0%	2.0%	1.0%
B								7.4%	5.0%
CCC									11.0%

This matrix is based on an asset correlation assumption of 20% and the equation above. This asset correlation figure is consistent with the Basel Consultative document on the internal ratings-based approach which notes that 0.2 "is broadly consistent with industry practise and research carried out by the committee ".

It should be noted that the bivariate normal distribution does not have an analytical solution, and the above matrix was calculated using a simulation approach. As the simulation approach does not provide accurate results when the probabilities are low, some adjustments were made to bring it into line with the shape of the Chunsheng Zhou (1997) matrix.

D.2.2 MTM Transition Based Model

For the more complicated MTM Transition based model illustrated in this paper, we have assumed a net asset model based on a return correlation level of 20% for the same reason outlined above.

E. CREDIT SPREADS & SPREAD VOLATILITY

E.1 Historic Data & Published Research

As most credit risk reserving models do not take into account the impact of changes in market credit spreads, not a great amount of research has been published on this subject and relevant historic credit spread data is difficult to obtain.

Nonetheless, the RBA bulletin publishes historical credit spreads for corporate bonds with 2 to 4 years to maturity, although this information is only available for highly rated bonds. The following table summarises some of the RBA published series:

Credit Spreads (Bps) - RBA Bulletin

	AAA	AA	A
Jun-99	32	56	78
Jun-00	53	86	86
Jun-01	37	58	80
Jun-02	33	54	68
Jan-03	35	61	85

Furthermore, Kiesel et al (2001) calculated annual standard deviations in credit spreads, and the correlation between credit spreads:

Historical Credit Spread Standard Deviations and Correlations

	AAA	AA	A	BBB	BB	B
Std Dev(pa)	0.08%	0.09%	0.19%	0.25%	0.48%	0.95%
	AAA	AA	A	BBB	BB	B
AAA	1.00	0.75	0.63	0.67	0.51	0.53
AA		1.00	0.76	0.73	0.49	0.57
A			1.00	0.91	0.62	0.59
BBB				1.00	0.71	0.57
BB					1.00	0.62
B						1.00

In addition, Reuters has published details on credit spreads which provide some indication about how credit spreads vary with duration to maturity and credit rating:

Credit Spreads - Reuters 22/4/2003							
Rating	**1 yr**	**2 yr**	**3 yr**	**5 yr**	**7 yr**	**10 yr**	**30 yr**
Aaa/AAA	18	23	34	39	57	79	89
Aa1/AA+	27	39	43	48	67	80	100
Aa2/AA	29	44	46	52	70	82	103
Aa3/AA-	31	47	48	57	74	86	112
A1/A+	55	63	67	72	90	104	127
A2/A	58	66	69	74	91	106	131
A3/A-	62	69	72	78	95	109	132
Baa1/BBB+	75	88	94	103	138	160	186
Baa2/BBB	78	96	102	110	143	166	191
Baa3/BBB-	85	101	107	114	148	173	196
Ba1/BB+	585	595	605	615	635	655	675
Ba2/BB	595	605	610	625	645	665	685
Ba3/BB-	605	615	625	635	655	675	695
B1/B+	755	765	775	805	845	885	935
B2/B	765	775	785	815	855	895	945
B3/B-	775	785	795	825	865	905	955
Caa/CCC	1175	1185	1195	1220	1250	1310	1260

E.2 Assumptions Adopted for this Paper

The credit-spread assumptions adopted below can only be regarded as broadly indicative due to the lack of available data. In particular, determining an appropriate estimate for the annual standard deviation for credit spreads based on a monthly standard deviation estimate is difficult as assuming the time series of credit spreads is independent and random may not hold.

The credit-spread assumptions adopted for this paper are as follows:

Credit Spreads (Bps) Assumptions					
Rating	**5Yr Spread**	**4Yr Spread**	**5Yr/ 4Yr**	**SD p.a.**	**Spread 4Yr/ SD**
AAA	37.5	35.0	1.07	8.2	4.3
AA	55.0	52.0	1.06	9.0	5.8
A+	72.5	70.0	1.04	15.2	4.6
A	75.0	72.0	1.04	19.0	3.8
A-	80.0	77.5	1.03	22.8	3.4
BBB	110.0	106.0	1.04	25.0	4.2
BB	610.0	600.0	1.02	48.0	12.5
B	810.0	800.0	1.01	95.0	8.4
CCC	1210.0	1200.0	1.01	150.0	8.0

Credit Spread Correlation Matrix									
	AAA	AA	A+	A	A-	BBB	BB	B	CCC
AAA	1.00	0.75	0.63	0.63	0.63	0.67	0.51	0.53	0.50
AA	0.75	1.00	0.76	0.76	0.76	0.73	0.49	0.57	0.53
A+	0.63	0.76	1.00	0.95	0.95	0.91	0.62	0.59	0.59
A	0.63	0.76	0.95	1.00	0.95	0.91	0.62	0.59	0.59
A-	0.63	0.76	0.95	0.95	1.00	0.91	0.62	0.59	0.59
BBB	0.67	0.73	0.91	0.91	0.91	1.00	0.71	0.57	0.59
BB	0.51	0.49	0.62	0.62	0.62	0.71	1.00	0.62	0.57
B	0.53	0.57	0.59	0.59	0.59	0.57	0.62	1.00	0.62
CCC	0.50	0.53	0.59	0.59	0.59	0.59	0.57	0.62	1.00

Based on the credit spreads above, the default rates adopted in appendix B, the recovery rates in appendix D, the following out performance rates were adopted for senior debt.

Expected Outperformance Table (Bps)				
	Spread	Expected Default Rate	Loss Given Default	Expected Outperform
AAA	37.5	1	50%	37
AA	55	3	50%	54
A+	72.5	4	50%	71
A	75	6	50%	72
A-	80	10	50%	75
BBB	110	30	50%	95
BB	610	150	50%	535
B	810	900	50%	360
CCC	1210	2500	50%	-40

The above results for "B" and "CCC" are somewhat curious and may suggest, inter alia, that the market credit spreads applicable in early 2003 summarised above may be below longer term averages relative to the default rates set out in Appendix B. This highlights the importance of the comments made in sections 4.5 and 5.1.4.

F. ILLUSTRATIVE ASSET / LIABILITY PORTFOLIO

F.1 Liability Structure

The liability portfolio considered is a simple fixed rate, fixed term annuity portfolio, described as follows:

- Term to maturity of annuities = 5 years.
- All annuities are all "100% return of capital" type. That is, they pay half yearly interest, however the initial capital sum is not paid until the end of the annuity term (i.e. in 5 years).
- The present value of the future annuity obligations, associated administration expenses and final maturity has been calculated on the Australian Commonwealth Treasury Bond curve as $1.4 billion.
- For the purposes on this example the Treasury Bond yield curve is specified as a flat 5% p.a. effective.

F.2 Asset Portfolio

The table below describes the corporate bond portfolio that has been established to fund the above liability obligation. All are assumed to be 5 year bonds, paying interest half-yearly, that precisely match the timing of the annuity portfolio obligations. The current market yield for each bond equals the coupon on the bond. All bonds are assumed to be Senior/Secured debt.

Bond Portfolio

Rating	No. of Holding	Face Value of Holding	Coupon Rate	Term to Maturity
AAA	7	20,000,000	5.4%	5 yrs
AA	21	20,000,000	5.6%	5 yrs
A+	7	20,000,000	5.7%	5 yrs
A	7	20,000,000	5.8%	5 yrs
A-	7	20,000,000	5.8%	5 yrs
BBB	7	20,000,000	6.1%	5 yrs
BB	7	20,000,000	11.1%	5 yrs
B	7	20,000,000	13.1%	5 yrs
CCC	0	20,000,000	17.1%	5 yrs
Total	70	1,400,000,000		

Rating of Portfolio (based on nominal credit rating)	A
Rating of Portfolio (based on weighted average default rate)	BB