Institute of Actuaries of Australia

# Australian Investment Performance 1960 to 2005 (and Investment Assumptions for Stochastic Models)

Prepared by Colin R. Grenfell FIA, FIAA, FASFA

Presented to the Institute of Actuaries of Australia
2005 Biennial Convention 8 May – 11 May 2005



The Institute of Actuaries of Australia
Level 7 Challis House 4 Martin Place
Sydney NSW Australia 2000
Telephone: +61 2 9233 3466 Facsimile: +61 2 9233 3446
Email: insact@actuaries.asn.au Website: www.actuaries.asn.au

# Table of Contents

## Index of Tables



## Index of Figures

# Australian Investment Performance 1960 to 2005 (and Investment Assumptions for Stochastic Models)

## Abstract

This paper analyses 44.75 years, or 179 quarters, of Australian investment performance for the period from 30 June 1960 (and earlier for some sectors) to 31 March 2005.

The analysis covers eleven investment classes plus four financial indicators:

| Growth Securities | Interest Income | Financial Indicators |
|---|---|---|
| Australian shares | Australian fixed interest | CPI |
| Int'l shares (unhedged) | Int'l fixed interest (hedged) | AWOTE |
| Property trusts | Government semis (0-3yrs) | 90 day bill rates |
| Direct property | Inflation linked bonds | 10 year bond rates |
| | Loans (floating rate) | |
| | Mortgage Trust | |
| | Cash | |

For each of these 15 "sectors" the annualised average results are tabulated and summarised for:

- risk margins (over 10 year bond rates),
- coefficients of variation,
- skewness,
- kurtosis,
- cross-correlations, and
- auto-correlations.

From these results, some assumptions are developed for the mean, standard deviation, skewness, kurtosis, cross-correlations and auto-correlations for each sector for use, during 2005 to say 2007, in stochastic investment models. The assumptions are intended for both medium-term (say 3 to 10 year) and long-term (say 10 to 40 year) modelling.

The assumptions, except those for correlations, are derived by examination of 38 year trends (and 23 year trends for risk margins and coefficients of variation), fitting a quadratic expression to the results and extrapolating forward the results by a couple of years. The assumptions for cross-correlations are derived using a similar approach. The assumptions for auto-correlations, in particular those for Australian Shares and 10 year bond rates, are derived using the same approach as for cross-correlations, but based on 40 year (rather than 38 year) periods.

## 1. Introduction

1.1 **Demand:** Most actuarial work requires assumptions about future investment returns. Often assumptions are also explicitly or implicitly made about future rates of inflation and/or salary increases.

1.2 **Supply:** However, often for commercial reasons, little information is publicly available about the data from which investment return assumptions can be derived.

1.3 This paper attempts to bridge, at least partly, the gap between demand and supply.

## 2. Methodology

2.1 **Firstly**, annual rates of return and forces of interest for the 15 sectors are sorted by years ending every quarter 31 March, 31 December, 30 September and 30 June.

2.2 **Secondly**, 38-year risk margins, coefficients of variation, skewness, kurtosis and 105 ranked and unranked cross-correlations are calculated and tabulated for each of the thirty-eight year running periods for each quarter ending in the seven years between 30/6/98 and 31/3/05. Some results are also calculated and tabulated for 23-year periods. The reasons for the use of 38 year and 23 year periods are explained in Section 5 (paragraphs 5.12 and 5.19).

2.3 **Thirdly**, the four quarter-ending results are averaged to give sets of 7 thirty-eight year running period results. That is:

**Table 2.1 Periods for 38 year calculations**

| Period | Average of four periods ending: |
|---|---|
| 6 | 30/6/98, 30/9/98, 31/12/98 and 31/3/99 |
| 5 | 30/6/99, 30/9/99, 31/12/99 and 31/3/00 |
| 4 | 30/6/00, 30/9/00, 31/12/00 and 31/3/01 |
| 3 | 30/6/01, 30/9/01, 31/12/01 and 31/3/02 |
| 2 | 30/6/02, 30/9/02, 31/12/02 and 31/3/03 |
| 1 | 30/6/03, 30/9/03, 31/12/03 and 31/3/04 |
| 0 | 30/6/04, 30/9/04, 31/12/04 and 31/3/05 |

Thus for example, as at 31 March 2005:

0 represents periods ending between 0 and 0.75 years ago, and
6 represents periods ending between 6 and 6.75 years ago.

Additional periods (7 to 18) were used for the 23 year calculations. Thus for these calculations the earliest period, period 18, relates to average results for the four 23 year periods ending 30/6/86, 30/9/86, 31/12/86 and 31/3/87.

2.4 **Fourthly**, a weighted quadratic EXCEL trend function is fitted to each of the 7 results and projected forward six years. For the cross and auto correlations this approach is slightly varied (see Sections 12 and 14). The number of fittings and projections is:

**Table 2.2 Number of fittings and projections**

| Projection | Fittings & projections | Number | Bases |
|---|---|---|---|
| Risk margins | Two per sector except bonds | 28 | 23 & 38 yrs |
| Coeff. of variation | Two per sector | 30 | 23 & 38 yrs |
| Skewness | One per sector | 15 | 38 years |
| Kurtosis | One per sector | 15 | 38 years |
| Cross correlations | One per combination | 105 | 38 years |
| Rank cross correln. | One per combination | 105 | 38 years |
| Auto correlations | 38 each for bonds and shares | 76 | 40 years |

2.5 **Fifthly**, recommendations of medium to long-term assumptions are made based primarily, but not totally, on the two-year projected results. The rationale for this is that these can be considered as a 'best estimate' of the 38 year (or 23 or 40 year) four-quarter averages expected in two years time. The two years might also be considered as the possible lifetime of these assumptions until they are subsequently updated. This weighted trend plus two year approach has been designed to give greatest weight to the latest period with recognition of past trends assumed to continue for at least the next two years. The projected results for year 2 are used as a guide (or "pointer") to set assumptions for the full term of the stochastic (or deterministic if applicable) modelling.

2.6 Pragmatically, the trend plus two year approach should also provide a reasonable response to the critic who argues that, because of such-and-such recent changes, use of (unadjusted) historical results is not appropriate for setting financial assumptions. The validity of this approach increases if there are no discernible significant data changes over the past 40 years that are "outside the norm".

2.7 The projected results for years 3 to 6 of the projection are used:

- to check that the results are not trending to ridiculous levels such as negative coefficients of variation or correlations greater than 0.99, and
- to get 'smooth' correlation assumptions at 2 years by averaging the results for years 0 to 4.

2.8 Consistent with accepted practice for input to stochastic projection models the following were calculated and tabulated:

(a) risk margins and means based on **arithmetic** averages of rates

(b) coefficients of variation and standard deviations based on **rates**

(c) skewness, kurtosis and correlations based on **forces**.

The possibility of inconsistencies arising from (a), (b) or (c) is discussed in Section 10.

2.9 Once assumptions relating to the results in 2.8 (a), (b) and (c) above have been set, it is of course possible to calculate equivalent compound average rates (and forces) for each sector.

2.10 A deliberate consequence of 2.3 above is that all individual results quoted in Sections 6 to 14 are the average of 4 results, **not** individual results for 38 or 23 years (or 40 years) ending on one date.

## 3. The 15 Sectors

3.1 The 15 "sectors" included in the analysis are:

**Table 3.1 Sector descriptions**

| Sector | Description |
|---|---|
| B | Bill rate (90 day bank) in middle of year |
| C | Cash and short term fixed interest sector |
| D | 10 year bonD rate in middle of year |
| F | Fixed interest sector |
| G | Government semis 0-3 years (SBC/UBS Warburg index SSG03) |
| I | International shares sector (MSCIAI prior 30/6/88) |
| J | International bonds sector |
| L | Loans sector (market valued, mainly floating rate) |
| M | Mortgage trust (valued on a hold to maturity basis) |
| N | Inflation linked bonds (all maturities) UBS index |
| P | Direct property (one third NM/AXA, two-thirds AMP) |
| Q | Property trust accumulation index (from 31/1/01 S&P/ASX 300, from 30/6/02 GICS) |
| S | Shares sector (All Ordinaries accumulation index prior 31/3/65) |
| W | AWOTE by quarter (= average 1.5 months lag), not seasonally adjusted. Full-time adults (post 9/81), Males original (pre 9/81), AWE Males (pre 1/75) |
| X | CPI indeX by quarter |

3.2 The codes B to X above are used throughout the paper to denote each of the sectors.

3.3 The 15 sectors comprise 4 growth (or "equity") sectors (I, P, Q and S), 7 interest income sectors (C, F, G, J, L, M and N) and 4 financial indicators (B, D, W and X). The main database contains **annual forces at quarterly intervals**.

3.4 The financial indicator data has been collected at the same intervals and for the same periods as the investment data because they are useful in both deterministic and stochastic models for valuing, projecting and/or illustrating liabilities.

3.5 Bills and bonds are taken at mid-year because this gives a far better correlation against annual cash, loan and mortgage rates. The lead between bond rates and annual investment returns for the F sector (Australian fixed interest) is considered in Section 6. The lag between bond rates and CPI and AWOTE is considered in Section 13.

## 4. "Backdating"

4.1 The available data for the various sectors commence from different dates. The following is a summary of the historical start dates for **quarterly** data (for bonds and CPI, yearly data are available from earlier dates, and for some other sectors, one or two month's data is available prior the date shown).

**Table 4.1 Historical start dates for data series**

| Series Start Date | Sector | Data series |
|---|---|---|
| 30/9/41 | W | AWE all males, total earnings |
| 30/9/48 | X | CPI (by quarter end) |
| 1950 | S | All ordinaries unweighted average dividend yield |
| 31/3/58 | D | Bond rate |
| 31/3/58 | S | All ordinaries price index |
| 31/12/59 | B | 13 week treasury notes (see section 4.6) |
| 31/3/65 | S | Australian equities (EFG system) |
| 31/3/65 | F | Australian government securities (EFG system "G" sector) |
| 30/9/69 | B | 90 day bank bills |
| 30/6/71 | P | AMP property sector |
| 31/12/71 | P | Property (EFG system) |
| 31/12/71 | S | Australian shares (EFG system) |
| 1974 | S | All ordinaries weighted average dividend yield |
| 31/12/74 | W | AWOTE males, ordinary time earnings |
| 31/3/77 | Q | Listed property trust accumulation index |
| 30/9/79 | C | Cash (EFG system) |
| 30/9/81 | W | AWOTE full-time adults, ordinary time, not seasonally adjusted |
| 30/9/82 | I | International shares (EFG system, see section 4.6) |
| 30/9/85 | F | Australian fixed interest (EFG system, previously "G" sector) |
| 30/6/86 | J | International bonds (ceased 30/6/87, recommenced 30/6/92) |
| 30/6/86 | L | Loans (market valued) |
| 30/6/88 | M | Mortgage trust (gross of MER's) |
| 31/12/89 | G | Semi-government bonds SBC index (0 to 3 years) |
| 31/3/91 | N | Inflation-linked bonds WDR index (all maturities) |

4.2 It is evident from the above summary that data definitions for six sectors, i.e. W, S, B, F, P and I have changed over time, though most of these changes relate to prior 1972.

4.3 It is also evident from the above summary that the National Mutual EFG investment unitisation system started on 31 March 1965. Sales brochures in the 1970's described it in these terms:

> *"Of major significance was the introduction in 1965 of a selective investment facility known as the EFG system. Evidence of the success and wide acceptance of this concept, which was pioneered by National Mutual in Australia, may now be seen in the fact that it has since been adopted by a number of other financial institutions as a medium for superannuation investment."*

The 40 year anniversary of the EFG system on 31 March 2005 was one incentive for the writing of this paper.

4.4 It is desirable with stochastic investment models to have consistency between assumptions both within sectors (eg risk margins and standard deviations) and across sectors (eg cross correlations). An efficient way to gain such consistency is to have a 'complete' database for all sectors back to the one date, hence the term "backdating".

4.5 The chosen common start date for the complete database was 30 June 1960. The backdating was achieved primarily but not solely by the method of least squares based on fitted parameters determined from data **after** the respective start dates and applying them to known data for other 'like' sectors **before** these start dates.

4.6 The formula used to backdate the annual forces for each sector, prior the start dates in Table 4.1 were:

Q = 52.06%F + 30.42%S + 18.59%M

P = 88.58%C + 50.02%X - 23.89%F

L = 89.90%C + 21.23%F + 1.50%P

M = 77.48%C + 34.49%L

G = 74.84%C + 37.27%F

J = 76.74%F + 19.25%C

C = 22.68%$B_{-2}$ + 27.44%$B_{-1}$ + 22.82%B + 25.76%$B_{+1}$
where $B_t$ = B in t quarter's time

N = 71.38%X + 62.99%F - 195.05%d
where d = delta D force

B = in nominal terms, 13 week Treasury Note rate + 1.37%
where 1.37% = median excess of 90 day bank bill rate over 13 week Treasury Note rate 30/9/69 to 30/9/79

F = prior 31/3/65,
87.09%D + 14.33%B - 673.02%d

S = approximate all ordinaries accumulation index prior 31/3/65. June values as published by the RBA. September, December and March values based on all ordinaries price indices plus unweighted average dividend yields less 1.75% per annum (being the average difference between unweighted and weighted dividend yields between 1974 and 1984).

I = MSCI accumulation index from 30/6/70 to 30/6/88. Prior to 30/6/70 the S&P500 series plus an assumed 3% per annum average dividend yield was used. All returns were then adjusted for \$AU/\$US exchange rate movements.

4.7 The process of creating historical data by regression analysis means that several series must be correlated, at least for the period over which the data is created. The correlation coefficients between the series for any 38-year period that extends back into the "backdated" period must therefore be illusionary in part. It is very difficult to unravel this effect. An examination was made to try and judge whether any significant bias was introduced. For the seven most effected sectors (i.e. G, J, L, M, N, P and Q) 16 cross correlations were calculated over the full period since 31/3/1965 to the calculation date and over the shorter periods from when each of these sectors commenced up to the calculation date. Each of these was compared with the relevant backdating parameter used to calculate the "backdated data". The analysis indicated that the parameters introduced very little, if any, bias overall.

4.8 Any bias introduced will also 'wear off' over time due to the trend-fitting, weighting and extrapolation inherent in the methodology described in Section 2. Nevertheless, users of the *Austmod* investment simulation model (refer Section 20) and users of the cross-correlations assumptions tabulated in Sections 13 and 15, need to be aware that the "backdating" may have introduced some slight bias for those sectors which started at later dates (see Table 4.1).

## 5. Economic Cycles

5.1 This section considers economic cycles. Though closely related, matters relating to auto-correlations are considered later, in Section 14.

5.2 Dwonczyk (1993) identified 33 year cycles in Australian CPI inflation data. "A sine wave was fitted to the data which best estimated the actual [yearly rates] time series for the period 1956 to 1992" and actuals, seven year moving averages and time series estimates were graphed over the 91 year period from 1 July 1901 to 30 June 1992.

5.3 For comparison, using the data described in Section 3 above with annual forces at quarterly intervals, the following curve-fitting results were obtained. Each combined curve is **the sum of two sine waves**. The period of each sine wave is shown in Table 5.1 and only optimum results (with some minor rounding) are tabulated. The start points for Bonds (31/3/1958) and CPI (year ending September 1949) correspond with when quarterly data first became available.

5.4 To give greater significance to more recent data, the data behind Table 5.1 was discounted on a compound basis for each quarter prior 31 March 2005. Thus, for example, with a data discount of 0.5% per quarter, data before September 1959 was given a weight of less than 40% and data before December 1983 was given a weight of less than 65%. By changing the data discount rate it is possible to get an indication of whether or not results are period dependant.

**Table 5.1 Economic cycles**

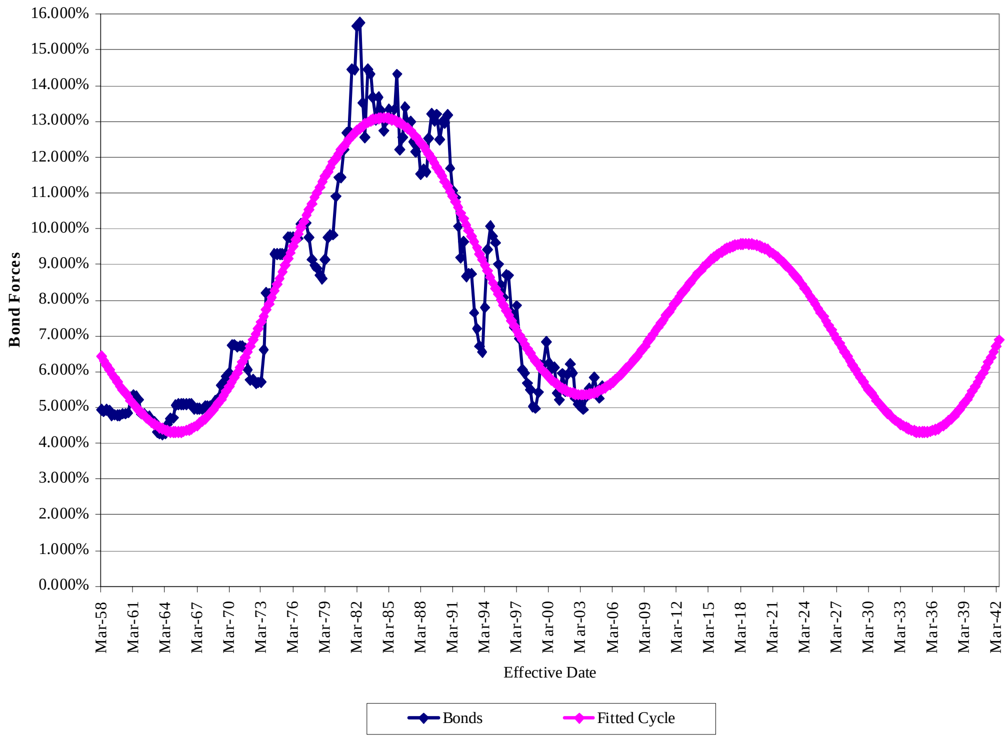
| **Sector** | **Data from** | **Period of Cycles (years)** | **Explained % of sum of squared residuals** — Data discount per quarter 0.5% | 0% | **Fitted curve min. date** | **Fitted curve next min.** | **Period between dates (years)** | **Refer para.** |
|---|---|---|---|---|---|---|---|---|
| Bonds | Mar 58 | 23 + 47 | 91.5 | 92.0 | (very flat at minimums) | | | 5.5 |
| | | 7 + 43 | 90.5 | 91.1 | Dec 64 | Dec 06 | 42.00 | 5.6 |
| | | 35 + 70 | 90.4 | 90.7 | Mar 65 | Jun 03 | 38.25 | 5.7 |
| CPI | Y/E Sep 49 | 6.25 + 34 | 63.2 | 59.0 | Dec 60 | Mar 98 | 37.25 | 5.8 |
| | | 34 + 136 | 59.2 | 55.8 | Dec 62 | Jun 97 | 34.50 | 5.9 |
| AWOTE | Y/E Sep 49 | 6.25 + 34.5 | 53.0 | 48.5 | Sep 60 | Dec 97 | 37.25 | 5.10 |
| | | 22 + 34 | 53.0 | 48.5 | Dec 61 | Mar 98 | 36.25 | 5.11 |

5.5 For bonds, the 23 plus 47 year curve explained a very high 91.5% of the sum of squared residuals from the mean. However the curve is a strange shape with very flat values at the minimums. For example, the curve has a low point with annual forces of between 5.1% and 4.7% for 10.5 years from June 2005 to December 2015. The curve then cycles to a maximum force of 13.4% in 2031. This is consistent with a **high** yield/ high inflationary future period very similar to the past (where the fitted curve also reaches a maximum force of 13.4% in December 1984).

5.6 For bonds, the 7 plus 43 year curve also explained a very high percentage of the sum of squared residuals. This curve is also consistent with a **high** yield/ high inflationary future period.

5.7 For bonds, the 35 plus 70 year curve explained about 90.5% of the sum of squared residuals for both data discount rates, indicating that it is less period dependant than the two previous curves. Unlike the two previous curves, it is smooth over time (and its first differences are also smooth) and it is consistent with a **low** yield / low inflationary future period. Figure 5.1 below graphs this curve and the actual data.

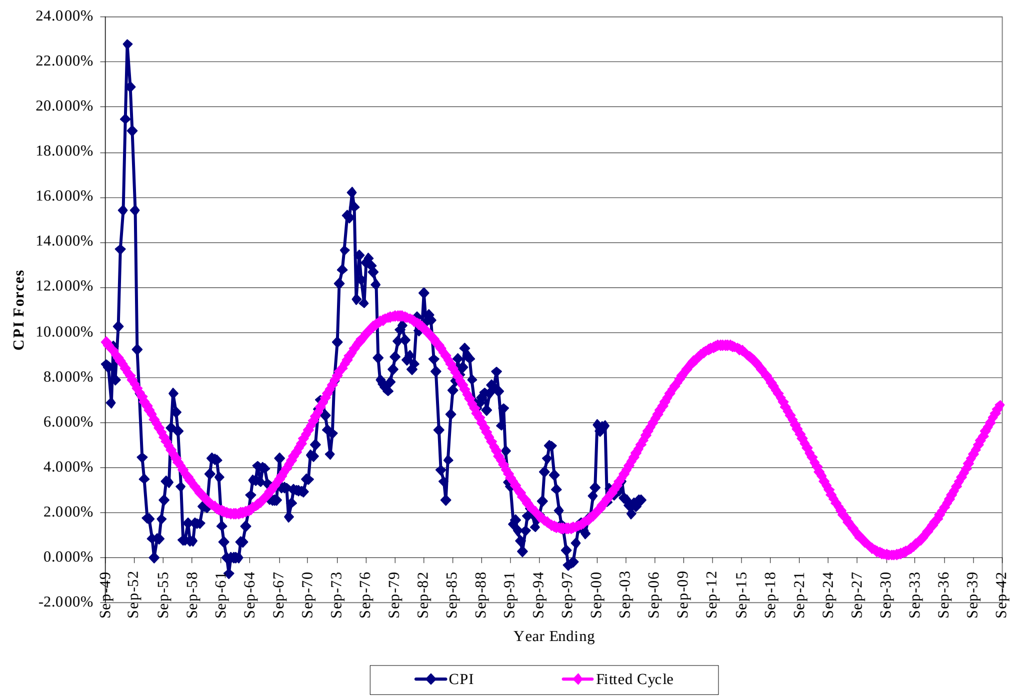
**Figure 5.1 Ten year bonds – economic cycle**




5.8 For CPI, the 6.25 plus 34 year curve explained 63.2% of the sum of squared residuals. This was the highest percentage explained of any one curve or two-curve combination; the CPI curve-fitting is significantly less close to the data than for bonds and is more period dependant. This curve is consistent with a **high** yield/ high inflationary future period very similar to the past (the fitted curve reaches a maximum force of about 11% in 2013 and in June 1982).

5.9 For CPI, the 34 plus 136 year curve explained 59.2% of the sum of squared residuals. This curve is smooth and is consistent with a **lower** yield/ lower inflationary future period (the fitted curve reaches a maximum force of about 9.5% in 2013 and a minimum force of 0.1% in 1931). Figure 5.2 below graphs this curve and the actual data.

**Figure 5.2 CPI – economic cycle**




5.10 For AWOTE, the 6.25 plus 34.5 year curve explained 53.0% of the sum of squared residuals. The AWOTE curve-fitting is also significantly less close to the data than for bonds and is more period dependent. This curve is consistent with a **high** yield/ high inflationary future period similar to the past (the fitted curve reaches a maximum force of about 13% in 2013 and in June 1976).

5.11 For AWOTE, the 22 plus 34 year curve also explained 53.0% of the sum of squared residuals. This curve is smooth and is consistent with a **high** yield/ high inflationary future period similar to the past but with a shorter wave length period (the fitted curve reaches a maximum force of about 13% in 2014 and a minimum force of 1.4% in 1929, just 15 years later).

5.12 Consideration of the second last column of Table 5.1 above shows that the period between **past** minimums has ranged from 35 to 42 years depending on the curve-fitting and whether one's focus is on bonds, CPI or AWOTE. Much of the work in the following sections is therefore based on 38 year past periods because:

- this is close to mid-way in the 35 to 42 years range, and
- **where-ever possible it is desirable for the setting of long-term assumptions to analyse results over a full economic cycle**, and
- the curve-fitting for bonds is clearly superior (in terms of the percentage of residuals explained and period dependency) to that for CPI and AWOTE, and
- the risk margins in Section 6 are defined relative to bonds, and
- for bonds, the 35 plus 70 year curve (which is consistent with a low inflationary/low yield future period) had a period between past minimums of 38.25 years.

5.13 However, for some sectors (eg, those denoted G, J, L, M and N) the 38 years chosen in Section 5.12 will often extend well back into the "back-dated data" periods described in Section 4. Thus it was considered desirable, for comparison, to also consider a shorter period, at least for means and coefficients of variation. A period of 23 years was chosen for this purpose because of the evidence of 22 and 23 cycle influences for bonds and AWOTE in the third column of Table 5.1 (and recognising that **past** cycle periods have generally been longer than the dominant cycle in the third column of that table).

## 6. Risk Margins

6.1 For the purposes of this paper, risk margins are calculated relative to the ten year bond rate. The risk margin is the excess of the sector annual investment return over the annualised effective point-in-time bond rate.

6.2 For sector D the bond rate is taken at mid-year, that is 6 months prior to the end of the period over which the annual investment returns are determined (but see paragraph 6.4 below).

6.3 Now consider the correlation between the F sector and D sector for various lags:

## Table 6.1 D and F sector correlations

**X yr Correlation of F to 31/3/05 against D lagged by Y years:**

| Y | 0 | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| X | | | | | | | | | |
| 40.00 | .435 | .513 | .587 | .600 | .594 | .592 | .572 | .552 | .535 |
| 39.75 | .435 | .512 | .588 | .600 | .595 | .593 | .573 | .553 | .535 |
| 39.50 | .434 | .512 | .588 | .600 | .595 | .593 | .573 | .553 | .535 |
| 39.25 | .433 | .512 | .588 | .600 | .595 | .594 | .573 | .554 | .536 |
| 39.00 | .433 | .512 | .588 | .601 | .596 | .594 | .574 | .554 | .536 |
| 38.75 | .432 | .511 | .588 | .601 | .596 | .594 | .574 | .554 | .536 |
| 38.50 | .431 | .511 | .588 | .601 | .596 | .594 | .574 | .554 | .536 |
| 38.25 | .430 | .511 | .588 | .601 | .596 | .594 | .574 | .554 | .536 |
| 38.00 | .429 | .510 | .588 | .601 | .596 | .594 | .574 | .554 | .535 |
| 37.75 | .429 | .510 | .589 | .602 | .596 | .595 | .574 | .554 | .535 |
| 37.50 | .428 | .510 | .589 | .602 | .597 | .595 | .574 | .554 | .535 |
| 37.25 | .427 | .510 | .590 | .603 | .597 | .595 | .574 | .554 | .535 |
| 37.00 | .428 | .511 | .592 | .605 | .599 | .598 | .576 | .555 | .537 |
| 36.75 | .429 | .513 | .593 | .607 | .601 | .599 | .578 | .557 | .538 |
| 36.50 | .424 | .508 | .589 | .603 | .597 | .595 | .574 | .553 | .534 |
| 36.25 | .417 | .503 | .585 | .599 | .593 | .591 | .569 | .548 | .528 |
| 36.00 | .411 | .496 | .579 | .593 | .588 | .586 | .563 | .542 | .522 |
| 35.75 | .402 | .489 | .573 | .586 | .580 | .578 | .556 | .533 | .513 |
| 35.50 | .395 | .482 | .567 | .580 | .574 | .572 | .549 | .526 | .506 |
| 35.25 | .388 | .476 | .562 | .575 | .568 | .566 | .542 | .519 | .498 |
| 35.00 | .383 | .470 | .556 | .570 | .562 | .559 | .535 | .512 | .491 |
| 34.75 | .379 | .467 | .553 | .566 | .559 | .556 | .531 | .508 | .486 |
| 34.50 | .378 | .466 | .553 | .567 | .559 | .556 | .532 | .508 | .487 |
| 34.25 | .378 | .466 | .553 | .567 | .560 | .557 | .532 | .509 | .488 |
| 34.00 | .382 | .470 | .557 | .571 | .565 | .564 | .539 | .516 | .495 |
| 33.75 | .394 | .483 | .571 | .585 | .578 | .577 | .557 | .534 | .513 |
| 33.50 | .409 | .496 | .585 | .599 | .592 | .591 | .571 | .551 | .531 |
| 33.25 | .425 | .512 | .599 | .613 | .607 | .606 | .585 | .565 | .549 |
| 33.00 | .433 | .521 | .608 | .620 | .613 | .613 | .592 | .572 | .555 |
| 32.75 | .433 | .521 | .608 | .621 | .613 | .613 | .591 | .572 | .555 |
| 32.50 | .423 | .513 | .602 | .614 | .608 | .609 | .587 | .567 | .550 |
| 32.25 | .410 | .506 | .600 | .614 | .607 | .610 | .592 | .571 | .553 |
| 32.00 | .405 | .498 | .599 | .614 | .607 | .610 | .594 | .577 | .557 |
| 31.75 | .417 | .500 | .600 | .615 | .608 | .612 | .594 | .578 | .563 |
| 31.50 | .429 | .518 | .608 | .616 | .607 | .612 | .593 | .576 | .563 |
| 31.25 | .428 | .517 | .609 | .611 | .600 | .605 | .585 | .568 | .554 |
| 31.00 | .431 | .517 | .609 | .612 | .595 | .597 | .577 | .560 | .546 |
| 30.75 | .435 | .521 | .610 | .612 | .595 | .592 | .569 | .551 | .537 |
| 30.50 | .434 | .521 | .610 | .612 | .595 | .591 | .570 | .553 | .539 |
| 30.25 | .435 | .522 | .611 | .613 | .593 | .590 | .568 | .549 | .534 |
| 30.00 | .435 | .522 | .611 | .614 | .594 | .588 | .567 | .548 | .531 |

6.4 Table 6.1 shows that when the D sector forces are lagged by a further 9 months (giving a total of 15 months) the correlation between D and F sector forces increases to a maximum. Further, this relationship appears to hold consistently over time. Therefore, to give greater stability to F sector risk margins, and to a lessor extent other sector risk margins, **all the following risk margins are defined as**:

(annual investment return) less (bond rate lagged 15 months)

## EXPLANATION

6.5 In the following figures and tables the meaning of "Period" is:

**Table 6.2 Statistics for each "period"**

| Period | Average statistic of four periods ending: |
|---|---|
| 6 | 30/6/98, 30/9/98, 31/12/98 and 31/3/99 |
| 5 | 30/6/99, 30/9/99, 31/12/99 and 31/3/00 |
| 4 | 30/6/00, 30/9/00, 31/12/00 and 31/3/01 |
| 3 | 30/6/01, 30/9/01, 31/12/01 and 31/3/02 |
| 2 | 30/6/02, 30/9/02, 31/12/02 and 31/3/03 |
| 1 | 30/6/03, 30/9/03, 31/12/03 and 31/3/04 |
| 0 | 30/6/04, 30/9/04, 31/12/04 and 31/3/05 |
| -1 | 30/6/05, 30/9/05, 31/12/05 and 31/3/06 (projections) |
| -2 | 30/6/06, 30/9/06, 31/12/06 and 31/3/07 (projections) |
| -3 | 30/6/07, 30/9/07, 31/12/07 and 31/3/08 (projections) |
| -4 | 30/6/08, 30/9/08, 31/12/08 and 31/3/09 (projections) |
| -5 & -6 | calculated as above, but not tabulated |

6.6 The Period is indicated on the x-axis of all figures.

6.7 The following codes are used:

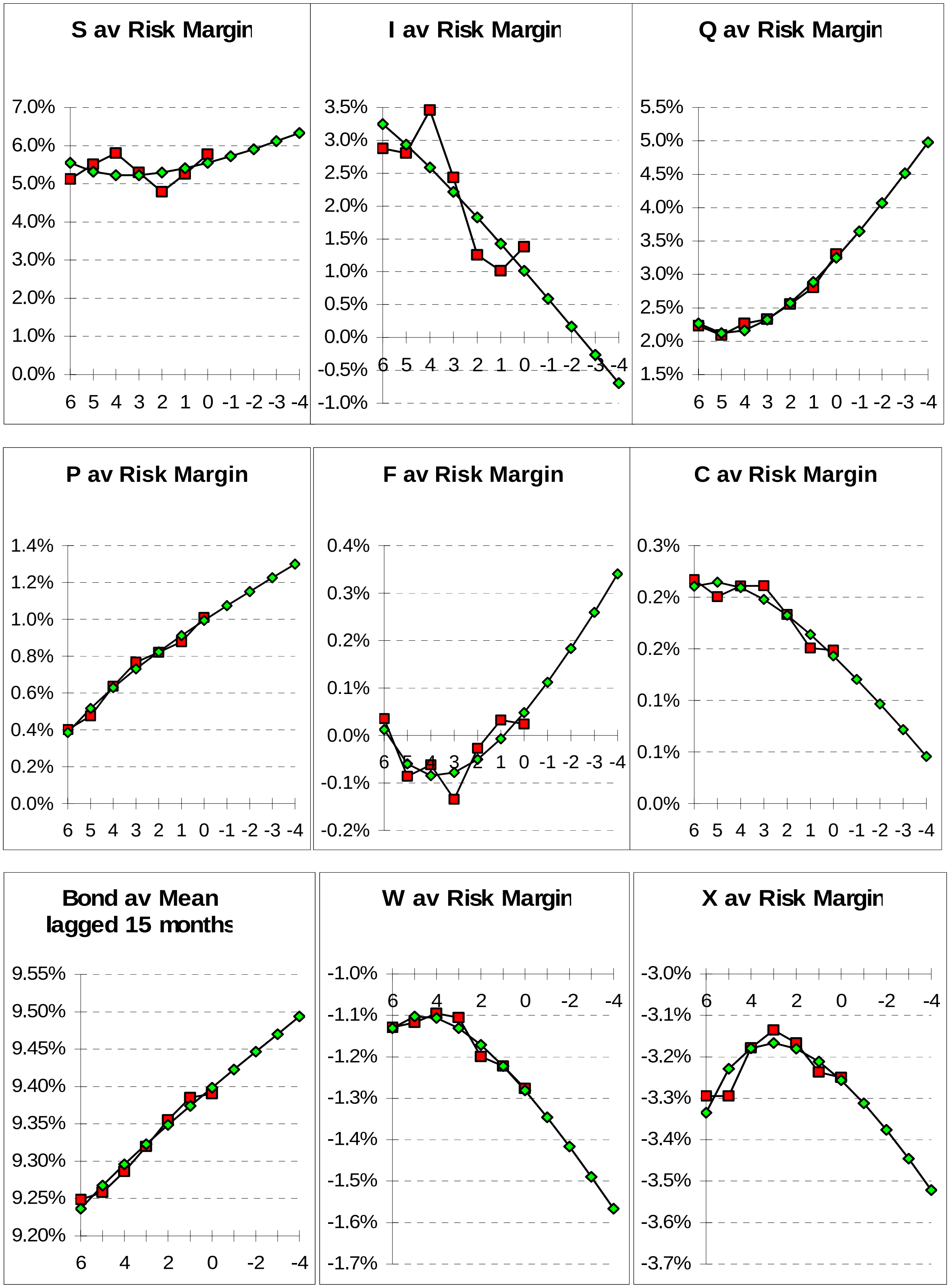
| **Sector** | **Relates to** |
|---|---|
| D | the ten year Bond Rate in the middle of the year (i.e. lagged by 6 months) |
| D-9 | the ten year Bond Rate lagged by 15 months (i.e. as for D but lagged by a further 9 months) |
| X | the annual increase in the Consumer Price Index |
| X-7 | the annual increase in the Consumer Price Index lagged by 7 years |
| W | the annual increase in AWOTE |
| W-7 | the annual increase in AWOTE lagged by 7 years |
| [square] | actual results |
| [diamond] | trend results |

6.8 The following "pointers" are referred to:

**Table 6.3 "Pointers"**

| Pointer | Refers to: |
|---|---|
| 38 – 2 | Average 38-year statistic for "Period -2" |
| 23 – 2 | Average 23-year statistic for "Period -2" |
| OLD | Previous assumptions based on data up to 31/12/2003 |
| NEW | Assumptions now proposed based on the preceding three "pointers". |

**Figure 6.1 Risk margins over 38 years**



## Table 6.4 Risk margins over 38 years

### Actual

| Period | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 6 | 5.1% | 2.9% | 2.2% | 0.4% | 1.4% | 1.9% | 0.0% | 1.2% |
| 5 | 5.5% | 2.8% | 2.1% | 0.5% | 1.4% | 1.9% | -0.1% | 1.1% |
| 4 | 5.8% | 3.5% | 2.3% | 0.6% | 1.4% | 1.9% | -0.1% | 1.1% |
| 3 | 5.3% | 2.4% | 2.3% | 0.8% | 1.4% | 1.9% | -0.1% | 1.1% |
| 2 | 4.8% | 1.3% | 2.6% | 0.8% | 1.5% | 1.9% | 0.0% | 1.1% |
| 1 | 5.3% | 1.0% | 2.8% | 0.9% | 1.4% | 1.8% | 0.0% | 1.0% |
| 0 | 5.8% | 1.4% | 3.3% | 1.0% | 1.4% | 1.8% | 0.0% | 1.0% |

| J | C | N | B | D-9 | W | X | Period |
|---|---|---|---|---|---|---|---|
| -0.4% | 0.2% | 1.0% | 0.3% | 9.25% | -1.1% | -3.3% | 6 |
| -0.5% | 0.2% | 0.9% | 0.3% | 9.26% | -1.1% | -3.3% | 5 |
| -0.5% | 0.2% | 1.0% | 0.3% | 9.29% | -1.1% | -3.2% | 4 |
| -0.5% | 0.2% | 1.0% | 0.3% | 9.32% | -1.1% | -3.1% | 3 |
| -0.3% | 0.2% | 1.1% | 0.3% | 9.36% | -1.2% | -3.2% | 2 |
| -0.2% | 0.2% | 1.2% | 0.2% | 9.39% | -1.2% | -3.2% | 1 |
| -0.2% | 0.1% | 1.3% | 0.2% | 9.39% | -1.3% | -3.2% | 0 |

### Quadratic Trend

| Period | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 6 | 5.5% | 3.2% | 2.3% | 0.4% | 1.4% | 1.9% | 0.0% | 1.2% |
| 5 | 5.3% | 2.9% | 2.1% | 0.5% | 1.4% | 1.9% | -0.1% | 1.1% |
| 4 | 5.2% | 2.6% | 2.2% | 0.6% | 1.4% | 1.9% | -0.1% | 1.1% |
| 3 | 5.2% | 2.2% | 2.3% | 0.7% | 1.4% | 1.9% | -0.1% | 1.1% |
| 2 | 5.3% | 1.8% | 2.6% | 0.8% | 1.4% | 1.9% | -0.1% | 1.1% |
| 1 | 5.4% | 1.4% | 2.9% | 0.9% | 1.4% | 1.8% | 0.0% | 1.0% |
| 0 | 5.5% | 1.0% | 3.2% | 1.0% | 1.4% | 1.8% | 0.0% | 1.0% |
| -1 | 5.7% | 0.6% | 3.6% | 1.1% | 1.4% | 1.8% | 0.1% | 1.0% |
| -2 | 5.9% | 0.2% | 4.1% | 1.2% | 1.4% | 1.8% | 0.2% | 1.0% |
| -3 | 6.1% | -0.3% | 4.5% | 1.2% | 1.4% | 1.8% | 0.3% | 0.9% |
| -4 | 6.3% | -0.7% | 5.0% | 1.3% | 1.4% | 1.7% | 0.3% | 0.9% |

| J | C | N | B | D-9 | W | X | Period |
|---|---|---|---|---|---|---|---|
| -0.5% | 0.2% | 0.9% | 0.3% | 9.24% | -1.1% | -3.3% | 6 |
| -0.5% | 0.2% | 0.9% | 0.3% | 9.27% | -1.1% | -3.2% | 5 |
| -0.5% | 0.2% | 1.0% | 0.3% | 9.30% | -1.1% | -3.2% | 4 |
| -0.4% | 0.2% | 1.0% | 0.3% | 9.32% | -1.1% | -3.2% | 3 |
| -0.4% | 0.2% | 1.1% | 0.3% | 9.35% | -1.2% | -3.2% | 2 |
| -0.3% | 0.2% | 1.2% | 0.2% | 9.37% | -1.2% | -3.2% | 1 |
| -0.2% | 0.1% | 1.3% | 0.2% | 9.40% | -1.3% | -3.3% | 0 |
| -0.1% | 0.1% | 1.3% | 0.1% | 9.42% | -1.3% | -3.3% | -1 |
| 0.0% | 0.1% | 1.4% | 0.1% | 9.45% | -1.4% | -3.4% | -2 |
| 0.1% | 0.1% | 1.5% | 0.1% | 9.47% | -1.5% | -3.4% | -3 |
| 0.3% | 0.0% | 1.6% | 0.0% | 9.49% | -1.6% | -3.5% | -4 |

### Pointers (risk margins)

| | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 38 -2 | 5.9% | 0.2% | 4.1% | 1.2% | 1.4% | 1.8% | 0.2% | 1.0% |
| 23 -2 | 4.2% | 5.6% | 4.8% | -0.6% | 1.5% | 1.4% | 2.7% | 1.3% |
| OLD | 4.0% | 4.0% | 3.0% | 1.2% | 1.0% | 1.0% | 1.0% | 0.5% |
| NEW | 4.0% | 4.0% | 3.5% | 1.2% | 1.0% | 1.0% | 1.0% | 0.5% |

| J | C | N | B | D-9 | W | X | |
|---|---|---|---|---|---|---|---|
| 0.0% | 0.1% | 1.4% | 0.1% | 9.4% | -1.42% | -3.38% | 38 -2 |
| 1.8% | -0.5% | 1.8% | -0.7% | 9.7% | -5.36% | -6.64% | 23 -2 |
| 0.8% | -0.5% | 1.0% | -0.5% | 6.0% | -2.25% | -3.50% | OLD |
| 0.8% | -0.5% | 1.0% | -0.5% | 6.0% | -2.25% | -3.50% | NEW |

6.9 In all cases the "NEW" risk margin assumptions are set in the range from:

a) the lowest of the first 3 pointers, to

b) the highest of the first 3 pointers.

However in many cases this is a wide range. The following further brief explanations should give some indication of the setting process:

S Combined with I, the average of (5.9%, 0.2%, 4.2% and 5.6%) is 3.96%.

I Set equal to S because of the divergent results and trends.

Q The average difference between S and Q was (5.9% - 4.1% + 4.2% - 4.8%)/2 equals 0.61%. Hence, after rounding, the Q assumption was set .5% below S, that is 4% less .5% = 3.5%. Also for "38 – 2" the average of the S and P risk margins was (5.9% + 1.2%)/2 = 3.53%.

P Set equal to OLD and "38 – 2"

F The average of (0.2% and 2.7%) rounded down. Also, combined with J, approximately the average of (0.2%, 2.7%, 0.0% and 1.8%) = 1.18%

J Set equal to F less 0.2% to cover the cost of hedging (including the direct transaction costs and the indirect transaction costs arising from the re-investment of cash flows from the expiry of forward foreign exchange contracts).

N Set equal to F because of the short data term.

D The base was set equal to 6%

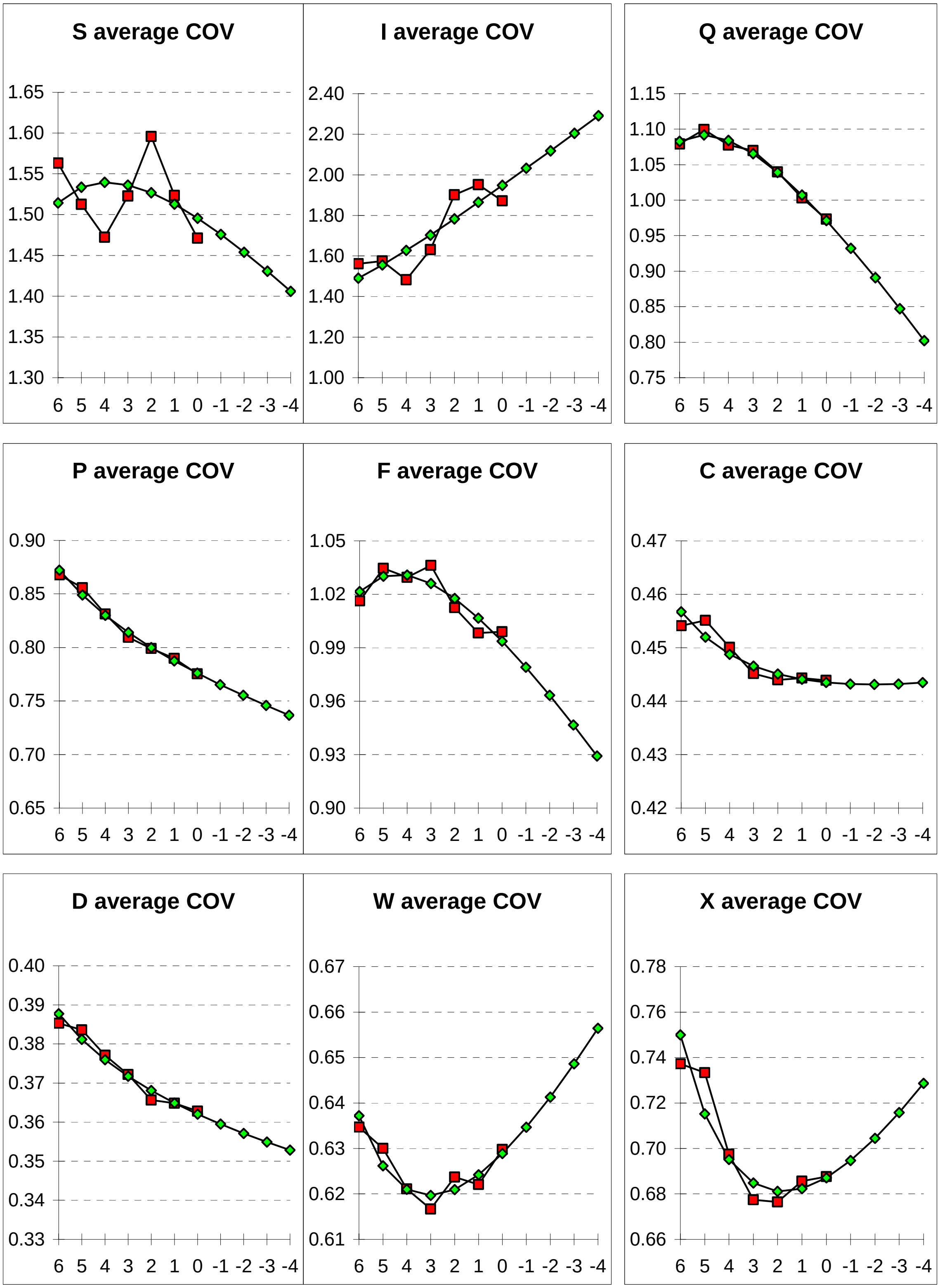
X Set equal to "38 – 2" rounded to -3.5%

W The "productivity" difference between W and X for "23 – 2" was (6.64% - 5.36%) equals 1.28%. Hence the W assumption was set 1.25% above X.

## 7. Coefficients of Variation

7.1 The coefficient of variation is equal to the mean divided by the standard deviation. Figure 7.1 and Table 7.1 show the results.

**Figure 7.1 Coefficients of variation over 38 years**



## Table 7.1 Coefficients of variation over 38 years

### Actual

| Period | S | I | Q | P | L | M | F | G | J | C | N | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 1.56 | 1.56 | 1.08 | 0.87 | 0.48 | 0.44 | 1.02 | 0.56 | 0.87 | 0.45 | 0.83 | 0.47 | 0.39 | 0.63 | 0.74 |
| 5 | 1.51 | 1.58 | 1.10 | 0.86 | 0.48 | 0.44 | 1.03 | 0.57 | 0.89 | 0.46 | 0.83 | 0.48 | 0.38 | 0.63 | 0.73 |
| 4 | 1.47 | 1.48 | 1.08 | 0.83 | 0.48 | 0.44 | 1.03 | 0.56 | 0.88 | 0.45 | 0.82 | 0.47 | 0.38 | 0.62 | 0.70 |
| 3 | 1.52 | 1.63 | 1.07 | 0.81 | 0.47 | 0.44 | 1.04 | 0.57 | 0.88 | 0.45 | 0.82 | 0.47 | 0.37 | 0.62 | 0.68 |
| 2 | 1.60 | 1.90 | 1.04 | 0.80 | 0.47 | 0.43 | 1.01 | 0.56 | 0.85 | 0.44 | 0.79 | 0.47 | 0.37 | 0.62 | 0.68 |
| 1 | 1.52 | 1.95 | 1.00 | 0.79 | 0.47 | 0.43 | 1.00 | 0.56 | 0.83 | 0.44 | 0.78 | 0.47 | 0.36 | 0.62 | 0.69 |
| 0 | 1.47 | 1.87 | 0.97 | 0.78 | 0.47 | 0.43 | 1.00 | 0.57 | 0.83 | 0.44 | 0.78 | 0.48 | 0.36 | 0.63 | 0.69 |

### Quadratic Trend

| Period | S | I | Q | P | L | M | F | G | J | C | N | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 1.51 | 1.49 | 1.08 | 0.87 | 0.48 | 0.44 | 1.02 | 0.56 | 0.88 | 0.46 | 0.83 | 0.48 | 0.39 | 0.64 | 0.75 |
| 5 | 1.53 | 1.56 | 1.09 | 0.85 | 0.48 | 0.44 | 1.03 | 0.56 | 0.88 | 0.45 | 0.83 | 0.47 | 0.38 | 0.63 | 0.72 |
| 4 | 1.54 | 1.63 | 1.08 | 0.83 | 0.48 | 0.44 | 1.03 | 0.56 | 0.88 | 0.45 | 0.82 | 0.47 | 0.38 | 0.62 | 0.70 |
| 3 | 1.54 | 1.70 | 1.07 | 0.81 | 0.47 | 0.44 | 1.03 | 0.56 | 0.87 | 0.45 | 0.81 | 0.47 | 0.37 | 0.62 | 0.68 |
| 2 | 1.53 | 1.78 | 1.04 | 0.80 | 0.47 | 0.44 | 1.02 | 0.56 | 0.86 | 0.45 | 0.80 | 0.47 | 0.37 | 0.62 | 0.68 |
| 1 | 1.51 | 1.86 | 1.01 | 0.79 | 0.47 | 0.43 | 1.01 | 0.56 | 0.84 | 0.44 | 0.79 | 0.47 | 0.36 | 0.62 | 0.68 |
| 0 | 1.50 | 1.95 | 0.97 | 0.78 | 0.46 | 0.43 | 0.99 | 0.56 | 0.82 | 0.44 | 0.77 | 0.48 | 0.36 | 0.63 | 0.69 |
| -1 | 1.48 | 2.03 | 0.93 | 0.77 | 0.46 | 0.43 | 0.98 | 0.56 | 0.80 | 0.44 | 0.76 | 0.48 | 0.36 | 0.63 | 0.69 |
| -2 | 1.45 | 2.12 | 0.89 | 0.76 | 0.46 | 0.43 | 0.96 | 0.56 | 0.79 | 0.44 | 0.75 | 0.48 | 0.36 | 0.64 | 0.70 |
| -3 | 1.43 | 2.20 | 0.85 | 0.75 | 0.45 | 0.43 | 0.95 | 0.56 | 0.77 | 0.44 | 0.73 | 0.48 | 0.35 | 0.65 | 0.72 |
| -4 | 1.41 | 2.29 | 0.80 | 0.74 | 0.45 | 0.43 | 0.93 | 0.56 | 0.74 | 0.44 | 0.72 | 0.49 | 0.35 | 0.66 | 0.73 |

### Pointers (COV)

| | S | I | Q | P | L | M | F | G | J | C | N | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 38 -2 | 1.45 | 2.12 | 0.89 | 0.76 | 0.46 | 0.43 | 0.96 | 0.56 | 0.79 | 0.44 | 0.75 | 0.48 | 0.36 | 0.64 | 0.70 |
| 23 -2 | 1.38 | 1.45 | 0.75 | 0.98 | 0.50 | 0.52 | 0.54 | 0.55 | 0.47 | 0.56 | 0.65 | 0.59 | 0.44 | 0.53 | 0.81 |
| OLD | 1.630 | 1.630 | 1.500 | 1.361 | 0.457 | 0.457 | 0.714 | 0.631 | 0.691 | 0.490 | 0.800 | 0.527 | 0.366 | 0.640 | 0.660 |
| NEW | 1.600 | 1.600 | 1.221 | 1.000 | 0.472 | 0.472 | 0.700 | 0.600 | 0.691 | 0.490 | 0.800 | 0.527 | 0.384 | 0.600 | 0.700 |

7.2 In all cases the "NEW" coefficient of variation assumptions are set in the range from:

a) the lowest of the first 3 pointers, to
b) the highest of the first 3 pointers.

The following brief explanations should give some indication of the setting process:

S Combined with I, the average of (1.45, 1.38, 2.12 and 1.45) is 1.602

I Set equal to S (as for risk margins).

Q Set equal to the average of the S and P highest COV's, (1.45 + 0.98)/2 = 1.22

P Set approximately equal to "23 – 2"

F Approximately the average of (0.96, 0.54, 0.79 and 0.47) = 0.69, rounded

J Set equal to "OLD" (which is between "23 – 2" and "38 – 2")

N Set equal to F plus 0.100 (being approximately N-F for "23 – 2")

D Approximately the average of (0.36 and 0.44) =.398, rounded down

W Approximately the average of (0.64 and 0.53) =.584, rounded

X Set equal to "23 – 2".

## 8. Means and Standard Deviations

8.1 When formulating assumptions for stochastic models, the risk margins and coefficients of variation from the previous two sections are used to determine corresponding means and standard deviations. The resulting assumptions are summarised in Section 15.

8.2 In this section the 38 year means and 38 year standard deviations are tabulated, purely for historical interest. It should be noted that all these results, and all results up to and including Section 15, are **gross of tax and gross of fees**.

8.3 In the following three tabulations the sectors denoted G, J, L, M and N have been omitted because they started after 1980. The 38 year statistics for these sectors are heavily dependant on the backdating described in Section 4 and might be misleading.

8.4 For sectors which started before 1980 the 38 year average arithmetic means have been:

**Table 8.1 Arithmetic means over 38 years**

| Period | S | I | Q | P | F | C | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 14.4% | 12.1% | 11.5% | 9.7% | 9.3% | 9.5% | 9.6% | 9.3% | 8.1% | 6.0% |
| 5 | 14.8% | 12.1% | 11.3% | 9.7% | 9.2% | 9.5% | 9.6% | 9.3% | 8.1% | 6.0% |
| 4 | 15.1% | 12.7% | 11.6% | 9.9% | 9.2% | 9.5% | 9.6% | 9.3% | 8.2% | 6.1% |
| 3 | 14.6% | 11.8% | 11.7% | 10.1% | 9.2% | 9.5% | 9.6% | 9.3% | 8.2% | 6.2% |
| 2 | 14.1% | 10.6% | 11.9% | 10.2% | 9.3% | 9.5% | 9.6% | 9.4% | 8.2% | 6.2% |
| 1 | 14.6% | 10.4% | 12.2% | 10.3% | 9.4% | 9.5% | 9.6% | 9.4% | 8.2% | 6.1% |
| 0 | 15.2% | 10.8% | 12.7% | 10.4% | 9.4% | 9.5% | 9.6% | 9.4% | 8.1% | 6.1% |

8.5 The 38 year average compound means have been:

**Table 8.2 Compound means over 38 years**

| Period | S | I | Q | P | F | C | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 12.3% | 10.6% | 10.8% | 9.3% | 8.9% | 9.4% | 9.5% | 9.2% | 8.0% | 5.9% |
| 5 | 12.7% | 10.5% | 10.7% | 9.4% | 8.8% | 9.4% | 9.5% | 9.2% | 8.0% | 5.9% |
| 4 | 13.0% | 11.2% | 10.9% | 9.6% | 8.8% | 9.4% | 9.5% | 9.3% | 8.1% | 6.0% |
| 3 | 12.5% | 10.2% | 11.0% | 9.8% | 8.8% | 9.5% | 9.5% | 9.3% | 8.1% | 6.1% |
| 2 | 12.0% | 8.8% | 11.2% | 9.9% | 8.9% | 9.5% | 9.5% | 9.3% | 8.0% | 6.1% |
| 1 | 12.5% | 8.6% | 11.5% | 10.0% | 9.0% | 9.5% | 9.5% | 9.3% | 8.1% | 6.1% |
| 0 | 13.1% | 9.0% | 12.0% | 10.1% | 9.0% | 9.5% | 9.5% | 9.3% | 8.0% | 6.1% |

8.6 The 38 year average standard deviations have been:

**Table 8.3 Standard deviations over 38 years**

| Period | S | I | Q | P | F | C | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 22.5% | 18.9% | 12.4% | 8.4% | 9.4% | 4.3% | 4.6% | 3.6% | 5.2% | 4.4% |
| 5 | 22.4% | 19.0% | 12.5% | 8.3% | 9.5% | 4.3% | 4.6% | 3.6% | 5.1% | 4.4% |
| 4 | 22.3% | 18.9% | 12.4% | 8.2% | 9.5% | 4.3% | 4.5% | 3.5% | 5.1% | 4.3% |
| 3 | 22.3% | 19.2% | 12.5% | 8.2% | 9.5% | 4.2% | 4.5% | 3.5% | 5.1% | 4.2% |
| 2 | 22.6% | 20.2% | 12.4% | 8.1% | 9.4% | 4.2% | 4.5% | 3.4% | 5.1% | 4.2% |
| 1 | 22.3% | 20.3% | 12.2% | 8.1% | 9.4% | 4.2% | 4.6% | 3.4% | 5.1% | 4.2% |
| 0 | 22.3% | 20.2% | 12.4% | 8.1% | 9.4% | 4.2% | 4.6% | 3.4% | 5.1% | 4.2% |

8.7 For comparison with Table 8.2, the 23 year average compound means **for all sectors** have been:

**Table 8.4 Compound means over 23 years**

| Period | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 6 | 16.5% | 14.8% | 15.0% | 11.0% | 13.3% | 13.6% | 12.9% | 13.2% |
| 5 | 16.7% | 14.9% | 14.5% | 10.6% | 13.1% | 13.4% | 12.6% | 12.9% |
| 4 | 17.0% | 15.4% | 14.4% | 10.4% | 12.8% | 13.1% | 12.4% | 12.7% |
| 3 | 16.0% | 14.3% | 13.8% | 10.2% | 12.6% | 12.9% | 11.9% | 12.3% |
| 2 | 13.8% | 12.3% | 13.8% | 10.0% | 12.5% | 12.6% | 12.2% | 12.2% |
| 1 | 11.8% | 11.7% | 13.7% | 9.7% | 12.2% | 12.3% | 12.4% | 12.0% |
| 0 | 13.1% | 12.4% | 14.0% | 9.4% | 11.9% | 11.9% | 12.4% | 11.6% |

| J | C | N | B | D | W | X | Period |
|---|---|---|---|---|---|---|---|
| 11.9% | 11.3% | 13.2% | 11.4% | 11.3% | 7.4% | 6.3% | 6 |
| 11.7% | 11.2% | 12.7% | 11.3% | 11.1% | 6.9% | 5.8% | 5 |
| 11.5% | 11.0% | 12.4% | 11.1% | 10.9% | 6.7% | 5.6% | 4 |
| 11.2% | 10.8% | 11.9% | 10.8% | 10.8% | 6.6% | 5.4% | 3 |
| 11.6% | 10.6% | 12.1% | 10.6% | 10.6% | 6.4% | 5.1% | 2 |
| 11.7% | 10.3% | 12.2% | 10.2% | 10.3% | 6.1% | 4.8% | 1 |
| 11.7% | 9.9% | 12.3% | 9.8% | 10.0% | 5.7% | 4.5% | 0 |

8.8 For each of the 10 sectors in Table 8.2, the average 38 year returns are more stable than the average 23 year returns in Table 8.4. This is partly due to the longer averaging periods underlying Table 8.2 but, importantly, it is also due to Table 8.4 comprising primarily the down-slope of the economic cycles identified in Section 5.

8.9 For the reasons explained in the previous paragraph, the 38 year statistics form better indicators of long-term returns. Where-ever possible it is desirable for the setting of long-term assumptions to analyse results over a full economic cycle

8.10 It should be noted that all individual results quoted in Sections 6 to 14, including those in this section, are the average of 4 annual results at quarterly intervals, **not** individual results for 38 or 23 years ending on one date.

## 9. Outliers

9.1 Outliers can introduce bias into the sample means and standard deviations and in particular can have a very significant affect on skewness and kurtosis. To illustrate the sensitivity of results to one extreme outlier, consider the following 38-year annual forces for years ending 30 June, 30 September, 31 December and 31 March for the F sector (Australian fixed interest securities). The period chosen corresponds to "Period 0" in section 2.3.

**Table 9.1 F sector annual forces**

| Year Ending | 30-Jun | 30-Sep | 31-Dec | 31-Mar | Year Ending | 30-Jun | 30-Sep | 31-Dec | 31-Mar |
|---|---|---|---|---|---|---|---|---|---|
| 1967/68 | 5.1% | 5.1% | 5.0% | 5.4% | 1986/87 | 17.5% | 14.3% | 22.2% | 14.0% |
| 1968/69 | 5.4% | 5.3% | 7.1% | 6.8% | 1987/88 | 15.3% | 22.8% | 18.0% | 19.0% |
| 1969/70 | -0.8% | -1.3% | -3.2% | -7.1% | 1988/89 | 17.4% | 12.7% | 10.5% | 5.8% |
| 1970/71 | -4.3% | -3.5% | -3.0% | 1.5% | 1989/90 | 5.5% | 6.9% | 13.0% | 15.8% |
| 1971/72 | 6.7% | 7.0% | 12.7% | 20.4% | 1990/91 | 16.1% | 15.0% | 18.0% | 22.1% |
| 1972/73 | 20.2% | 19.9% | 14.0% | 6.1% | 1991/92 | 22.5% | 27.2% | 26.2% | 18.6% |
| 1973/74 | -6.2% | -22.5% | -21.9% | -21.4% | 1992/93 | 21.6% | 15.9% | 9.5% | 16.7% |
| 1974/75 | -18.7% | -1.3% | -1.3% | -0.7% | 1993/94 | 13.4% | 15.5% | 15.3% | 6.1% |
| 1975/76 | 9.0% | 4.7% | 5.7% | 4.8% | 1994/95 | -1.7% | -5.3% | -5.3% | 2.0% |
| 1976/77 | 5.7% | 10.0% | 7.6% | 7.3% | 1995/96 | 11.2% | 15.4% | 17.7% | 12.9% |
| 1977/78 | 7.3% | 7.4% | 16.8% | 20.2% | 1996/97 | 9.9% | 11.9% | 12.0% | 12.3% |
| 1978/79 | 20.6% | 21.0% | 13.5% | 6.6% | 1997/98 | 16.2% | 14.7% | 11.5% | 14.8% |
| 1979/80 | 1.0% | 1.2% | -0.4% | -1.2% | 1998/99 | 10.6% | 8.9% | 9.5% | 6.6% |
| 1980/81 | 3.2% | 1.1% | 0.4% | 5.9% | 1999/00 | 3.5% | 1.6% | -0.8% | 1.9% |
| 1981/82 | 5.5% | 3.2% | 7.5% | 4.5% | 2000/01 | 6.4% | 6.3% | 11.5% | 11.9% |
| 1982/83 | 7.9% | 20.0% | 21.2% | 18.8% | 2001/02 | 7.5% | 9.7% | 4.8% | 1.8% |
| 1983/84 | 18.6% | 15.0% | 15.1% | 18.7% | 2002/03 | 4.7% | 4.8% | 7.9% | 8.9% |
| 1984/85 | 18.3% | 15.2% | 11.4% | 11.9% | 2003/04 | 9.5% | 6.2% | 3.9% | 4.9% |
| 1985/86 | 12.0% | 10.6% | 7.9% | 17.6% | 2004/05 | 3.2% | 5.3% | 6.9% | 5.0% |

| Statistics for all 38 years: | | | | | Average across |
|---|---|---|---|---|---|
| 8.60% | 8.63% | 8.64% | 8.62% | mu | 8.62% |
| 8.61% | 9.17% | 9.00% | 8.72% | sigma | 8.87% |
| -72% | -81% | -94% | -95% | skewness | -85% |
| 138% | 240% | 239% | 238% | kurtosis | 214% |

| Modified statistics for all 38 years: | | | | | Average across |
|---|---|---|---|---|---|
| 8.60% | 8.63% | 8.64% | 8.62% | mu | 8.62% |
| 8.61% | 8.70% | 9.00% | 8.72% | sigma | 8.76% |
| -72% | -55% | -94% | -95% | skewness | -79% |
| 138% | 144% | 239% | 238% | kurtosis | 190% |

9.2 The annual force for the year ending 30 September 1973 was –22.5% (boxed). This represents 3.395 standard deviations below the mean. If the F sector had a normal distribution this value has about 1 chance in 3000 of occurring. It compares with the most extreme outliers for Australian Shares, in September 1974, of 2.697 standard deviations below the mean and, in September 1987, of 2.546 standard deviations above the mean. The F sector annual forces for the years ending 31 December 1973, 31 March 1974 and 30 June 1974 all represents more than 3 standard deviations below the mean.

9.3 The F sector skewness and kurtosis for the 38 years ending 2004/2005 averaged **-85% and 214%** respectively. However the outlier of -22.5% (for the year ending 30 September 1973) significantly affects these results.

9.4 To indicate the sensitivity of the skewness and kurtosis to the 30 September 1973 outlier, if this force had been 4% greater (i.e. –18.5%) and the force for the previous year had been 4% less (i.e. 15.9%), then the modified average skewness and kurtosis for the 38 years ending 2004/2005 would have been **-79% and 190%** respectively.

9.5 In the previous paragraph, the F sector average skewness for "Period 0" reduced from –85% to -79% (and the average kurtosis reduced from 214% to 190%) with only a 4% change in one outlier. These reductions have **not** been reflected in the following two sections, or in subsequent sections, because the unmodified results do not seem to be abnormal. For example, the unmodified skewness for the 38 years ending 30 September 2004 is –81% which is within the range of the –72% to –95% results for other 38-year periods ending 2004/2005. Even the unmodified kurtosis for the 38 years ending 30 September 2004 is only marginally outside (by 1%) the range of results for other 38-year periods ending 2004/2005.

9.6 It can also be noted from Table 9.1 that the average sigma reduced from 8.87% to 8.76%) with only a 4% change in the one outlier. This change (of about 0.1%) was not considered significant since in equates to the usual round-off for quoted annual standard deviations.

9.7 The conclusion drawn from paragraphs 9.1 to 9.6 above is that the analysis has sufficient data and the methodology is sufficiently robust, to not be overly concerned about outliers.

## 10. Skewness

10.1 As explained in Section 2.8, the skewness results shown in this section, and the kurtosis, cross-correlation and auto-correlation results shown in Sections 11 to 14 are based on annual forces.

10.2 When using stochastic models it is often desired to convert assumptions and/or model results from rates to forces and vice versa and to combine the results from a number of sectors to form or approximate results for a portfolio or composite sector. Such changes are often based on the assumption, or an implied assumption, of a normal distribution for the forces and a lognormal distribution for the rates. However if non-zero values for skewness and kurtosis are used as input to a stochastic model, then formulae (based on a normal distribution for the forces and a lognormal distribution for the rates) will not hold.

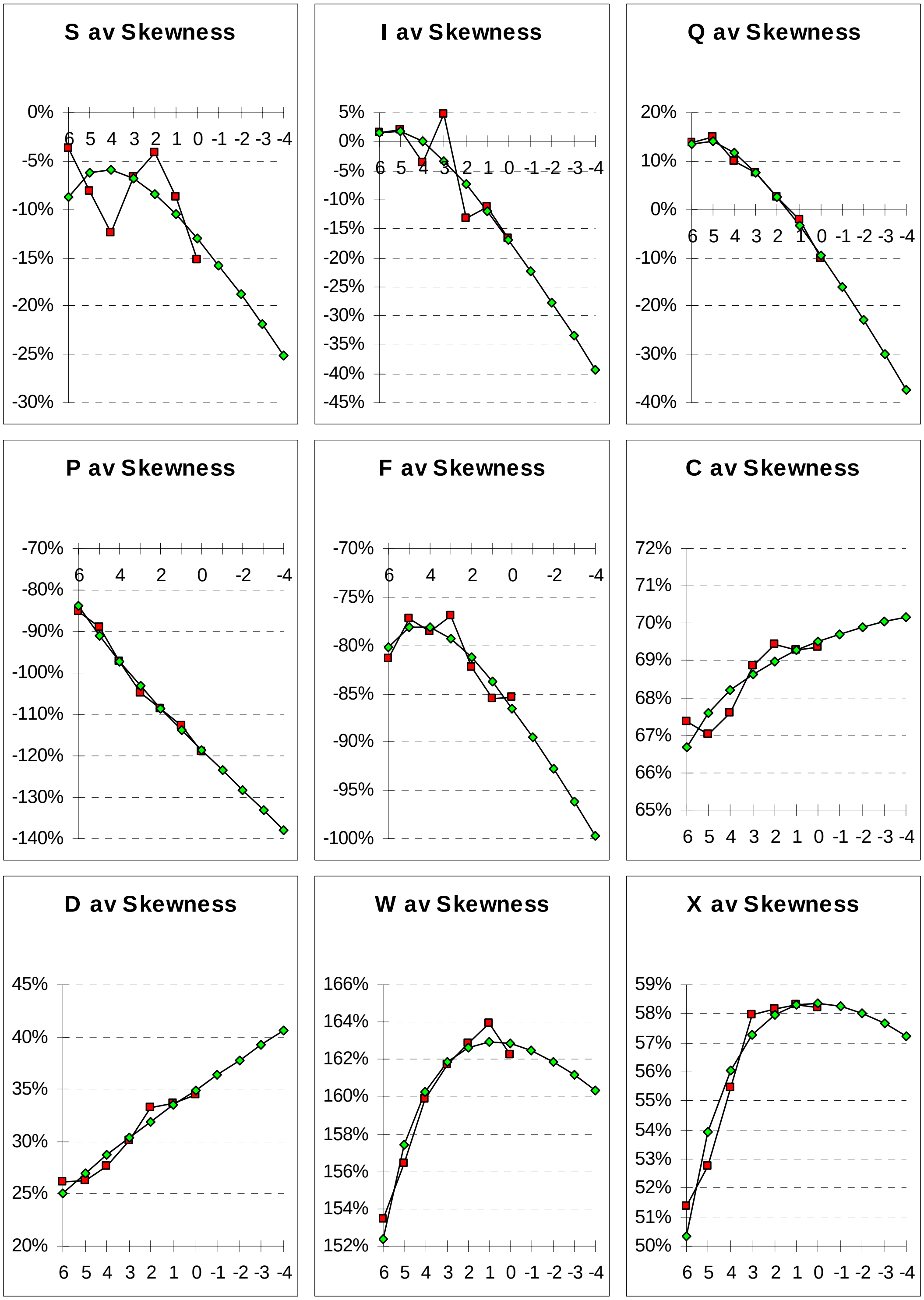
10.3 For example, when using dependant variables and non-zero values for skewness and kurtosis, it is not easy to determine:

- Long-term estimates of means and/or standard deviations of portfolio or composite sector **rates or forces**, from long-term estimates of means and/or standard deviations of **rates or forces** for each individual sector, or
- Long-term estimates of means and/or standard deviations of **forces** for each individual sector from long-term estimates of means and/or standard deviations of **rates** for each individual sector.

10.4 However formulae can be determined to resolve these issues. For example, Appendix A contains formulae for statistical conversion between rates and forces of return for use when skewness and/or kurtosis are not zero.

10.5 The following Figure 10.1 and Table 10.1 illustrate the skewness results for each sector.

**Figure 10.1 Skewness over 38 years**



## Table 10.1 Skewness over 38 years

### Actual

| Period | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 6 | -4% | 2% | 14% | -85% | 53% | 65% | -81% | 9% |
| 5 | -8% | 2% | 15% | -89% | 53% | 65% | -77% | 10% |
| 4 | -12% | -4% | 10% | -97% | 53% | 66% | -79% | 9% |
| 3 | -7% | 5% | 8% | -105% | 53% | 67% | -77% | 9% |
| 2 | -4% | -13% | 3% | -109% | 55% | 68% | -82% | 9% |
| 1 | -9% | -11% | -2% | -113% | 55% | 68% | -86% | 9% |
| 0 | -15% | -17% | -10% | -119% | 55% | 68% | -85% | 9% |

| J | C | N | B | D | W | X | Period |
|---|---|---|---|---|---|---|---|
| -68% | 67% | -33% | 83% | 26% | 153% | 51% | 6 |
| -65% | 67% | -32% | 82% | 26% | 156% | 53% | 5 |
| -67% | 68% | -37% | 83% | 28% | 160% | 55% | 4 |
| -68% | 69% | -38% | 83% | 30% | 162% | 58% | 3 |
| -76% | 69% | -43% | 83% | 33% | 163% | 58% | 2 |
| -81% | 69% | -46% | 82% | 34% | 164% | 58% | 1 |
| -83% | 69% | -49% | 82% | 35% | 162% | 58% | 0 |

### Trend

| Period | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 6 | -9% | 2% | 14% | -84% | 53% | 64% | -80% | 10% |
| 5 | -6% | 2% | 14% | -91% | 53% | 65% | -78% | 9% |
| 4 | -6% | 0% | 12% | -97% | 53% | 66% | -78% | 9% |
| 3 | -7% | -3% | 8% | -103% | 54% | 67% | -79% | 9% |
| 2 | -8% | -7% | 3% | -109% | 54% | 67% | -81% | 9% |
| 1 | -11% | -12% | -3% | -114% | 55% | 68% | -84% | 9% |
| 0 | -13% | -17% | -9% | -119% | 55% | 68% | -87% | 9% |
| -1 | -16% | -22% | -16% | -124% | 56% | 68% | -90% | 9% |
| -2 | -19% | -28% | -23% | -128% | 56% | 69% | -93% | 9% |
| -3 | -22% | -33% | -30% | -133% | 57% | 69% | -96% | 9% |
| -4 | -25% | -39% | -37% | -138% | 58% | 69% | -100% | 9% |

| J | C | N | B | D | W | X | Period |
|---|---|---|---|---|---|---|---|
| -66% | 67% | -32% | 82% | 25% | 152% | 50% | 6 |
| -66% | 68% | -34% | 83% | 27% | 157% | 54% | 5 |
| -67% | 68% | -36% | 83% | 29% | 160% | 56% | 4 |
| -70% | 69% | -39% | 83% | 30% | 162% | 57% | 3 |
| -74% | 69% | -42% | 83% | 32% | 163% | 58% | 2 |
| -79% | 69% | -46% | 82% | 33% | 163% | 58% | 1 |
| -84% | 70% | -49% | 82% | 35% | 163% | 58% | 0 |
| -89% | 70% | -53% | 81% | 36% | 162% | 58% | -1 |
| -95% | 70% | -57% | 81% | 38% | 162% | 58% | -2 |
| -100% | 70% | -60% | 80% | 39% | 161% | 58% | -3 |
| -106% | 70% | -64% | 79% | 41% | 160% | 57% | -4 |

10.6 The quadratic trend Period –2 results are the skewness assumptions that are tabulated in Section 15.

## 11. Kurtosis

11.1 The Institute's Guidance Note 552, "Economic Valuations" (July 2004), states at paragraph 5.3:

> "Stochastic valuation techniques
>
> In applying stochastic valuation techniques, the Member should be satisfied that the economic and probability models are reasonable and make appropriate allowance for the impact of correlations and that the tails of distributions are suitably modelled."

11.2 The tails of a distribution are significantly influenced by the kurtosis, and to a lessor extent the skewness.

11.3 The following Figure 11.1 and Table 11.1 illustrate the kurtosis results for each sector.

**Figure 11.1 Kurtosis over 38 years**

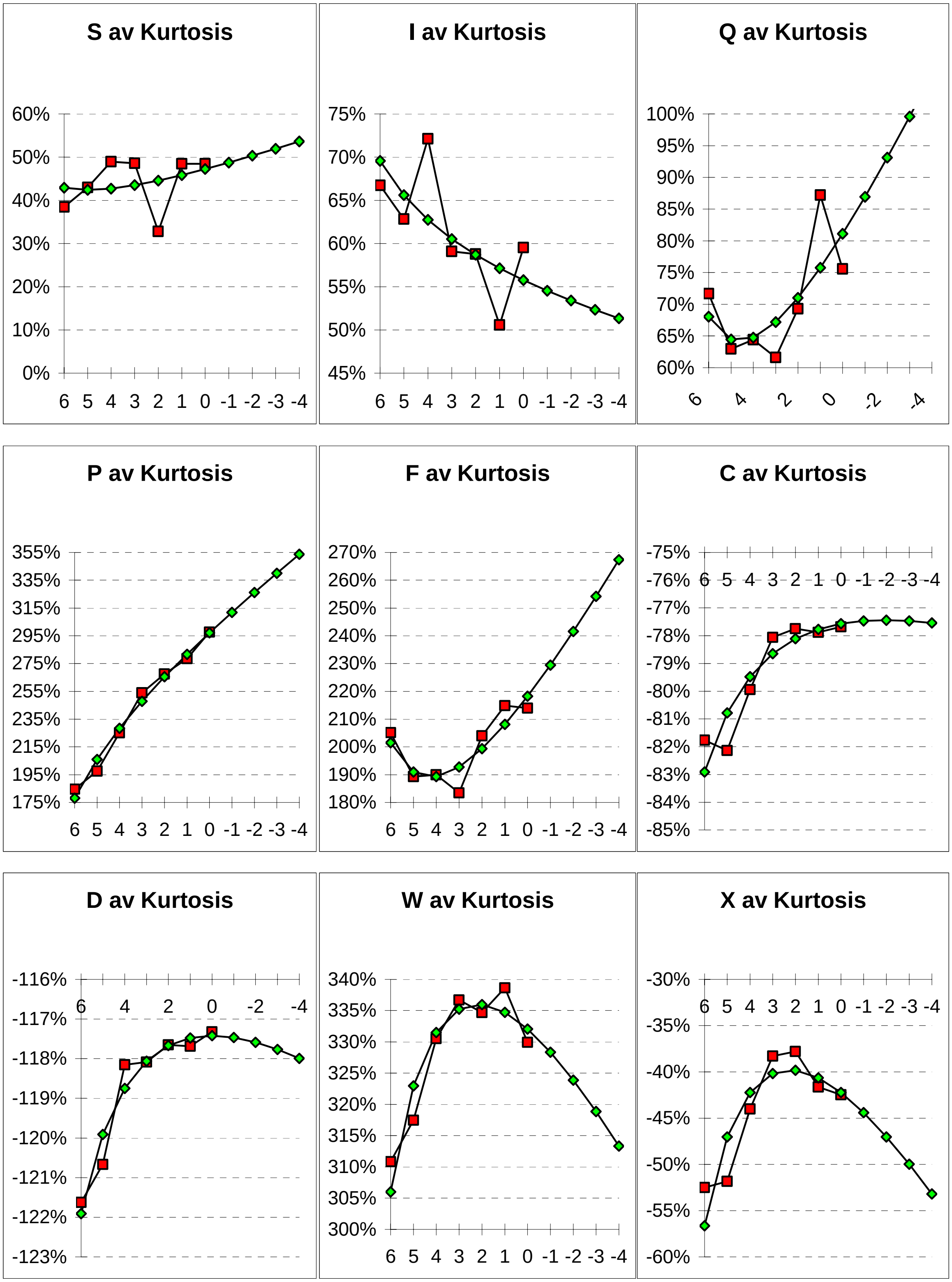


**Table 11.1 Kurtosis over 38 years**

**Actual**

| Period | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 6 | 38% | 67% | 72% | 185% | -44% | -76% | 205% | -25% |
| 5 | 43% | 63% | 63% | 198% | -44% | -76% | 189% | -33% |
| 4 | 49% | 72% | 64% | 225% | -43% | -75% | 190% | -33% |
| 3 | 49% | 59% | 62% | 254% | -39% | -73% | 183% | -33% |
| 2 | 33% | 59% | 69% | 267% | -34% | -73% | 204% | -29% |
| 1 | 48% | 51% | 87% | 279% | -33% | -73% | 215% | -30% |
| 0 | 48% | 60% | 76% | 298% | -33% | -73% | 214% | -35% |

| J | C | N | B | D | W | X | Period |
|---|---|---|---|---|---|---|---|
| 163% | -82% | 16% | -35% | -122% | 311% | -52% | 6 |
| 149% | -82% | 11% | -36% | -121% | 317% | -52% | 5 |
| 152% | -80% | 18% | -34% | -118% | 331% | -44% | 4 |
| 152% | -78% | 19% | -33% | -118% | 337% | -38% | 3 |
| 178% | -78% | 37% | -34% | -118% | 335% | -38% | 2 |
| 194% | -78% | 44% | -35% | -118% | 339% | -42% | 1 |
| 198% | -78% | 49% | -36% | -117% | 330% | -42% | 0 |

**Trend**

| Period | S | I | Q | P | L | M | F | G |
|---|---|---|---|---|---|---|---|---|
| 6 | 43% | 70% | 68% | 178% | -46% | -77% | 201% | -29% |
| 5 | 42% | 66% | 64% | 206% | -43% | -75% | 191% | -30% |
| 4 | 43% | 63% | 65% | 228% | -41% | -74% | 189% | -31% |
| 3 | 43% | 61% | 67% | 248% | -38% | -74% | 193% | -31% |
| 2 | 45% | 59% | 71% | 265% | -36% | -73% | 199% | -32% |
| 1 | 46% | 57% | 76% | 282% | -34% | -73% | 208% | -32% |
| 0 | 47% | 56% | 81% | 297% | -32% | -73% | 218% | -33% |
| -1 | 49% | 55% | 87% | 312% | -30% | -73% | 229% | -33% |
| -2 | 50% | 53% | 93% | 326% | -28% | -73% | 242% | -33% |
| -3 | 52% | 52% | 100% | 340% | -26% | -72% | 254% | -34% |
| -4 | 54% | 51% | 106% | 354% | -24% | -72% | 267% | -34% |

| J | C | N | B | D | W | X | Period |
|---|---|---|---|---|---|---|---|
| 158% | -83% | 13% | -36% | -122% | 306% | -57% | 6 |
| 151% | -81% | 14% | -34% | -120% | 323% | -47% | 5 |
| 153% | -79% | 18% | -34% | -119% | 332% | -42% | 4 |
| 161% | -79% | 25% | -34% | -118% | 335% | -40% | 3 |
| 173% | -78% | 33% | -34% | -118% | 336% | -40% | 2 |
| 187% | -78% | 42% | -35% | -117% | 335% | -41% | 1 |
| 203% | -78% | 51% | -35% | -117% | 332% | -42% | 0 |
| 220% | -77% | 61% | -36% | -117% | 328% | -44% | -1 |
| 237% | -77% | 71% | -38% | -118% | 324% | -47% | -2 |
| 256% | -77% | 82% | -39% | -118% | 319% | -50% | -3 |
| 275% | -78% | 93% | -40% | -118% | 313% | -53% | -4 |

11.4 The quadratic trend Period –2 results are the kurtosis assumptions that are tabulated in Section 15.

## 12. Cross-correlations

12.1 The methodology used for risk margins, COV's, skewness and kurtosis is detailed in Section 2. However for correlations it is essential that projected statistics are entirely consistent with each other over sectors (for cross-correlations) or over time (for auto-correlations). To obtain this greater consistency, the methodology was therefore slightly varied for correlations.

12.2 After step 1 in section 2.1 the database was extended by adding on 6 years of "projected" annual forces for each quarter date. For this purpose the "projected" values were obtained from the projected trend risk margins (for example see Table 6.4) by adding back the fixed Bond base value (6% per annum) and converting to forces.

12.3 After the adjustments in 12.2 above, the extended database of each sector covered 50 years for each of the four quarter end dates. Steps 2 and 3 in Section 2 were then applied and step 4 was replaced by central 5 year running averages.

12.4 It is well documented that sample correlations can be distorted by outliers and non-zero skewness and kurtosis. To overcome this problem, for cross-correlations, 50 year ranks of the extended database forces were used as well as the actual forces. Thus two sets of cross-correlations were obtained, one based on ranks and one based on actual (i.e. actual plus backdated plus projected) forces. In the *Austmod* investment simulation model, the rank cross-correlations are used because they are not distorted by outliers and the impacts of skewness and kurtosis are separately allowed for in the model.

12.5 Figure 12.1 and Table 12.1 below illustrate the above process and show the results obtained for the S (Shares) sector rank cross-correlations.

**Figure 12.1 Cross-correlations over 38 years**

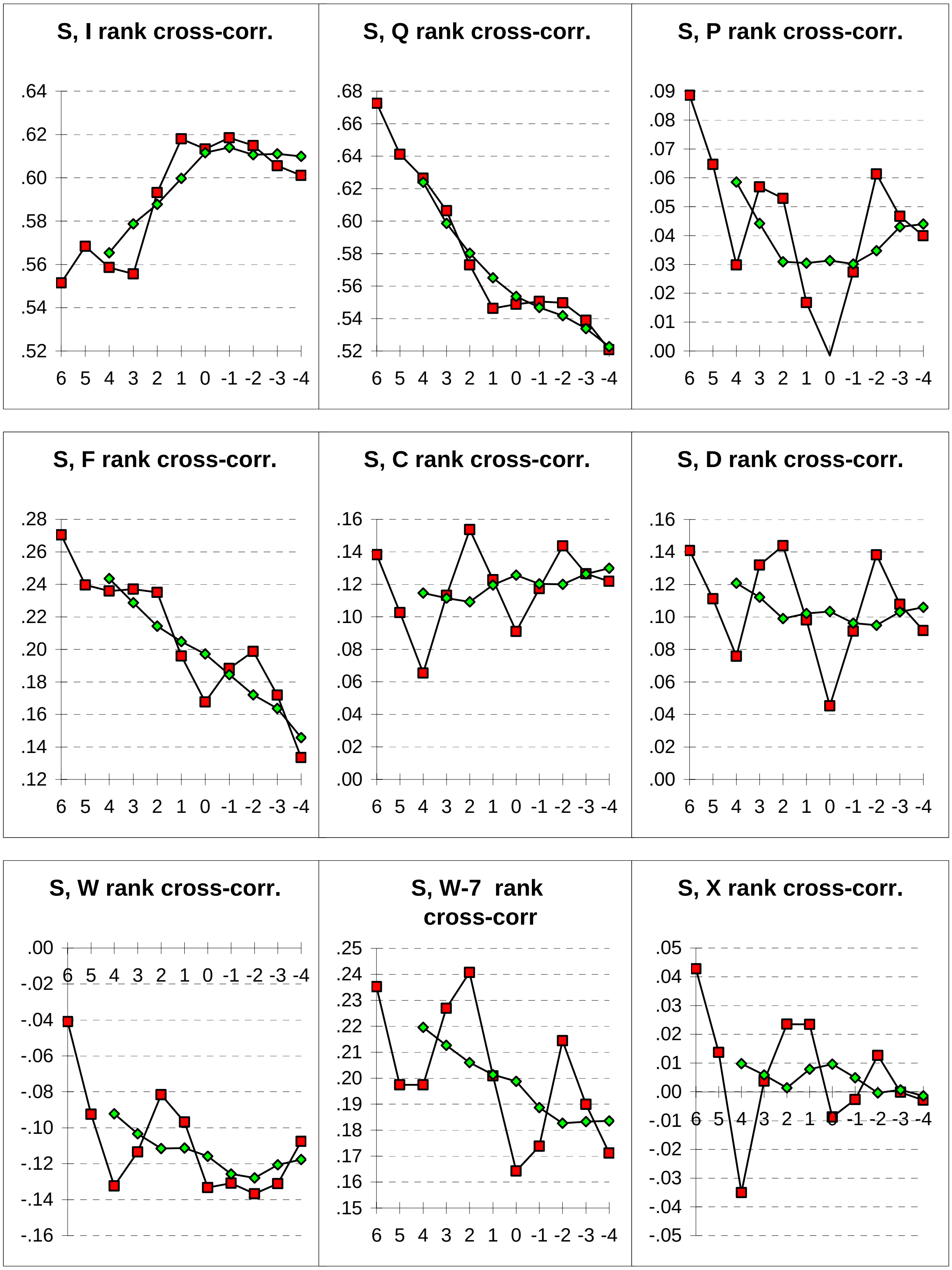


**Table 12.1 Rank cross-correlations over 38 years**

**Actual**

| Period | S I | S Q | S P | S L | S M | S F | S G | S J |
|---|---|---|---|---|---|---|---|---|
| 6 | .55 | .67 | .09 | .28 | .24 | .27 | .32 | .27 |
| 5 | .57 | .64 | .06 | .24 | .20 | .24 | .28 | .24 |
| 4 | .56 | .63 | .03 | .22 | .17 | .24 | .26 | .23 |
| 3 | .56 | .61 | .06 | .25 | .23 | .24 | .28 | .24 |
| 2 | .59 | .57 | .05 | .27 | .27 | .24 | .30 | .21 |
| 1 | .62 | .55 | .02 | .23 | .23 | .20 | .27 | .17 |
| 0 | .61 | .55 | .00 | .19 | .19 | .17 | .22 | .15 |

| S C | S N | S B | S D | S W | S X | S W-7 | S X-7 | Period |
|---|---|---|---|---|---|---|---|---|
| .14 | .27 | .12 | .14 | -.04 | .04 | .24 | .31 | 6 |
| .10 | .23 | .09 | .11 | -.09 | .01 | .20 | .26 | 5 |
| .07 | .21 | .05 | .08 | -.13 | -.04 | .20 | .25 | 4 |
| .11 | .23 | .11 | .13 | -.11 | .00 | .23 | .25 | 3 |
| .15 | .22 | .16 | .14 | -.08 | .02 | .24 | .24 | 2 |
| .12 | .18 | .13 | .10 | -.10 | .02 | .20 | .19 | 1 |
| .09 | .16 | .10 | .05 | -.13 | -.01 | .16 | .13 | 0 |

**Five-year central averages**

| Period | S I | S Q | S P | S L | S M | S F | S G | S J |
|---|---|---|---|---|---|---|---|---|
| 4 | .57 | .62 | .06 | .25 | .22 | .24 | .29 | .24 |
| 3 | .58 | .60 | .04 | .24 | .22 | .23 | .28 | .22 |
| 2 | .59 | .58 | .03 | .23 | .22 | .21 | .27 | .20 |
| 1 | .60 | .57 | .03 | .23 | .23 | .20 | .26 | .19 |
| 0 | .61 | .55 | .03 | .23 | .23 | .20 | .26 | .18 |
| -1 | .61 | .55 | .03 | .22 | .22 | .18 | .25 | .17 |
| -2 | .61 | .54 | .03 | .21 | .22 | .17 | .23 | .16 |
| -3 | .61 | .53 | .04 | .21 | .22 | .16 | .23 | .15 |
| -4 | .61 | .52 | .04 | .20 | .22 | .15 | .22 | .13 |

| S C | S N | S B | S D | S W | S X | S W-7 | S X-7 | Period |
|---|---|---|---|---|---|---|---|---|
| .11 | .23 | .10 | .12 | -.09 | .01 | .22 | .26 | 4 |
| .11 | .22 | .11 | .11 | -.10 | .01 | .21 | .24 | 3 |
| .11 | .20 | .11 | .10 | -.11 | .00 | .21 | .21 | 2 |
| .12 | .20 | .12 | .10 | -.11 | .01 | .20 | .19 | 1 |
| .13 | .19 | .12 | .10 | -.12 | .01 | .20 | .17 | 0 |
| .12 | .18 | .12 | .10 | -.13 | .00 | .19 | .15 | -1 |
| .12 | .18 | .11 | .09 | -.13 | .00 | .18 | .12 | -2 |
| .13 | .17 | .12 | .10 | -.12 | .00 | .18 | .11 | -3 |
| .13 | .16 | .12 | .11 | -.12 | .00 | .18 | .10 | -4 |

12.6 The five year central average Period –2 results are the S sector cross-correlation assumptions that are tabulated in the first of the two half-page matrices in Table 15.3.

## 13. Leads and Lags

13.1 Consider the correlation between the D sector and CPI for various lags:

**Table 13.1 D sector and CPI correlation**

X yr Correlation of D to 31/3/05 against CPI lagged by Y years:

| X \ Y | 0 | 2 | 4 | 6 | 7 | 7.25 | 7,5 | 7.75 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 40.00 | .499 | .605 | .672 | .772 | .803 | .806 | .807 | .803 | .793 |
| 39.75 | .498 | .601 | .669 | .771 | .802 | .805 | .806 | .802 | .791 |
| 39.50 | .496 | .598 | .665 | .771 | .800 | .803 | .804 | .800 | .789 |
| 39.25 | .493 | .596 | .661 | .773 | .799 | .802 | .802 | .798 | .787 |
| 39.00 | .489 | .594 | .657 | .774 | .798 | .800 | .801 | .796 | .785 |
| 38.75 | .486 | .593 | .653 | .776 | .797 | .799 | .800 | .794 | .783 |
| 38.50 | .483 | .593 | .648 | .776 | .798 | .798 | .799 | .793 | .781 |
| 38.25 | .482 | .591 | .644 | .774 | .800 | .799 | .797 | .792 | .780 |
| 38.00 | .480 | .590 | .640 | .772 | .802 | .801 | .798 | .791 | .779 |
| 37.75 | .477 | .590 | .636 | .769 | .804 | .803 | .800 | .791 | .777 |
| 37.50 | .474 | .588 | .633 | .766 | .804 | .805 | .802 | .793 | .778 |
| 37.25 | .469 | .585 | .631 | .762 | .802 | .805 | .803 | .795 | .779 |
| 37.00 | .465 | .582 | .630 | .759 | .800 | .803 | .804 | .796 | .781 |
| 36.75 | .461 | .579 | .628 | .756 | .797 | .801 | .802 | .797 | .782 |
| 36.50 | .458 | .576 | .628 | .752 | .795 | .798 | .800 | .795 | .782 |
| 36.25 | .454 | .576 | .626 | .749 | .792 | .795 | .797 | .792 | .780 |
| 36.00 | .451 | .573 | .626 | .746 | .789 | .793 | .795 | .790 | .778 |
| 35.75 | .447 | .571 | .625 | .744 | .787 | .791 | .792 | .788 | .775 |
| 35.50 | .445 | .569 | .623 | .742 | .785 | .788 | .790 | .785 | .773 |
| 35.25 | .442 | .565 | .620 | .740 | .782 | .786 | .788 | .783 | .771 |
| 35.00 | .441 | .562 | .618 | .739 | .781 | .785 | .788 | .783 | .770 |
| 34.75 | .440 | .560 | .616 | .738 | .780 | .784 | .786 | .782 | .769 |
| 34.50 | .439 | .557 | .613 | .737 | .778 | .783 | .785 | .781 | .768 |
| 34.25 | .441 | .555 | .613 | .736 | .777 | .781 | .784 | .780 | .767 |
| 34.00 | .444 | .552 | .611 | .735 | .776 | .780 | .782 | .778 | .766 |
| 33.75 | .447 | .550 | .608 | .735 | .775 | .779 | .781 | .777 | .764 |
| 33.50 | .449 | .547 | .606 | .733 | .775 | .778 | .780 | .775 | .761 |
| 33.25 | .451 | .544 | .602 | .731 | .774 | .778 | .779 | .774 | .759 |
| 33.00 | .450 | .544 | .598 | .729 | .774 | .777 | .779 | .772 | .758 |
| 32.75 | .451 | .543 | .595 | .726 | .773 | .777 | .778 | .772 | .757 |
| 32.50 | .459 | .543 | .592 | .724 | .772 | .776 | .777 | .771 | .756 |
| 32.25 | .472 | .547 | .588 | .724 | .770 | .775 | .777 | .771 | .755 |
| 32.00 | .490 | .551 | .585 | .723 | .768 | .773 | .776 | .770 | .754 |
| 31.75 | .502 | .552 | .584 | .722 | .768 | .773 | .776 | .769 | .753 |
| 31.50 | .517 | .552 | .583 | .722 | .768 | .773 | .776 | .769 | .753 |
| 31.25 | .539 | .552 | .582 | .722 | .768 | .773 | .776 | .769 | .753 |
| 31.00 | .553 | .552 | .582 | .724 | .769 | .774 | .778 | .772 | .755 |
| 30.75 | .573 | .552 | .582 | .725 | .771 | .776 | .779 | .775 | .758 |
| 30.50 | .593 | .553 | .582 | .727 | .773 | .777 | .781 | .775 | .761 |
| 30.25 | .601 | .555 | .582 | .729 | .777 | .779 | .783 | .778 | .762 |
| 30.00 | .616 | .561 | .583 | .730 | .779 | .783 | .785 | .780 | .764 |

13.2 Also consider the correlation between the D sector and AWOTE for various lags:

**Table 13.2 D sector and AWOTE correlation**

X yr Correlation of D to 31/3/05 against AWOTE lagged by Y years:

| X \ Y | 0 | 2 | 4 | 6 | 7 | 7.25 | 7,5 | 7.75 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 40 | .269 | .381 | .458 | .644 | .736 | .754 | .768 | .766 | .764 |
| 39 | .256 | .378 | .442 | .662 | .731 | .748 | .761 | .758 | .756 |
| 38 | .252 | .375 | .428 | .648 | .752 | .768 | .779 | .764 | .752 |
| 37 | .245 | .362 | .425 | .636 | .741 | .763 | .780 | .778 | .775 |
| 36 | .256 | .360 | .423 | .626 | .733 | .755 | .772 | .768 | .765 |
| 35 | .262 | .354 | .412 | .625 | .726 | .746 | .763 | .761 | .758 |
| 34 | .290 | .361 | .408 | .624 | .725 | .747 | .760 | .757 | .754 |
| 33 | .298 | .370 | .403 | .617 | .726 | .752 | .769 | .764 | .756 |
| 32 | .342 | .415 | .415 | .618 | .721 | .745 | .764 | .763 | .761 |
| 31 | .429 | .416 | .416 | .615 | .721 | .745 | .763 | .762 | .761 |
| 30 | .541 | .426 | .423 | .616 | .723 | .748 | .765 | .764 | .763 |

13.3 Table 13.1 and Table 13.2 show that when CPI and AWOTE forces are lagged by about 7 to 7.5 years their correlation with D forces increases to a maximum. Further, these relationships appear to hold reasonably consistently over time. A visual comparison of Figure 5.1 and Figure 5.2 also gives some support to this view.

13.4 The increase in correlation with the D sector as the lag changes from zero to say 7 years is significant for both CPI and AWOTE. The W-7 and X-7 columns of Table 13.3 show that a similar significant increase occurs when lagged CPI (sector X) and lagged AWOTE (sector W) are correlated against the forces of most other sectors.

13.5 The following table compares a wider range of correlations both with and without the 7 year lag.

**Table 13.3 Alternative cross-correlation assumptions**

**RANK** CROSS-CORRELATIONS
@ 2 YEARS (5-point average, rounded)

| | W | X | W-7 | X-7 |
|---|---|---|---|---|
| S | -.13 | .00 | .18 | .12 |
| I | -.10 | .01 | .22 | .17 |
| Q | -.12 | -.10 | .17 | .24 |
| P | .49 | .59 | .48 | .34 |
| L | .28 | .42 | .77 | .76 |
| M | .36 | .51 | .81 | .73 |
| F | -.13 | -.05 | .38 | .44 |
| G | .28 | .39 | .72 | .68 |
| J | -.07 | .02 | .42 | .48 |
| C | .53 | .66 | .74 | .58 |
| N | .02 | .14 | .38 | .36 |
| B | .53 | .66 | .74 | .57 |
| D | .37 | .54 | .82 | .77 |
| W | 1 | .85 | .41 | .15 |
| X | .85 | 1 | .52 | .26 |
| W-7 | .41 | .52 | 1 | .79 |
| X-7 | .15 | .26 | .79 | 1 |

**STANDARD** CROSS-CORRELATIONS
@ 2 YEARS (5-point average, rounded)

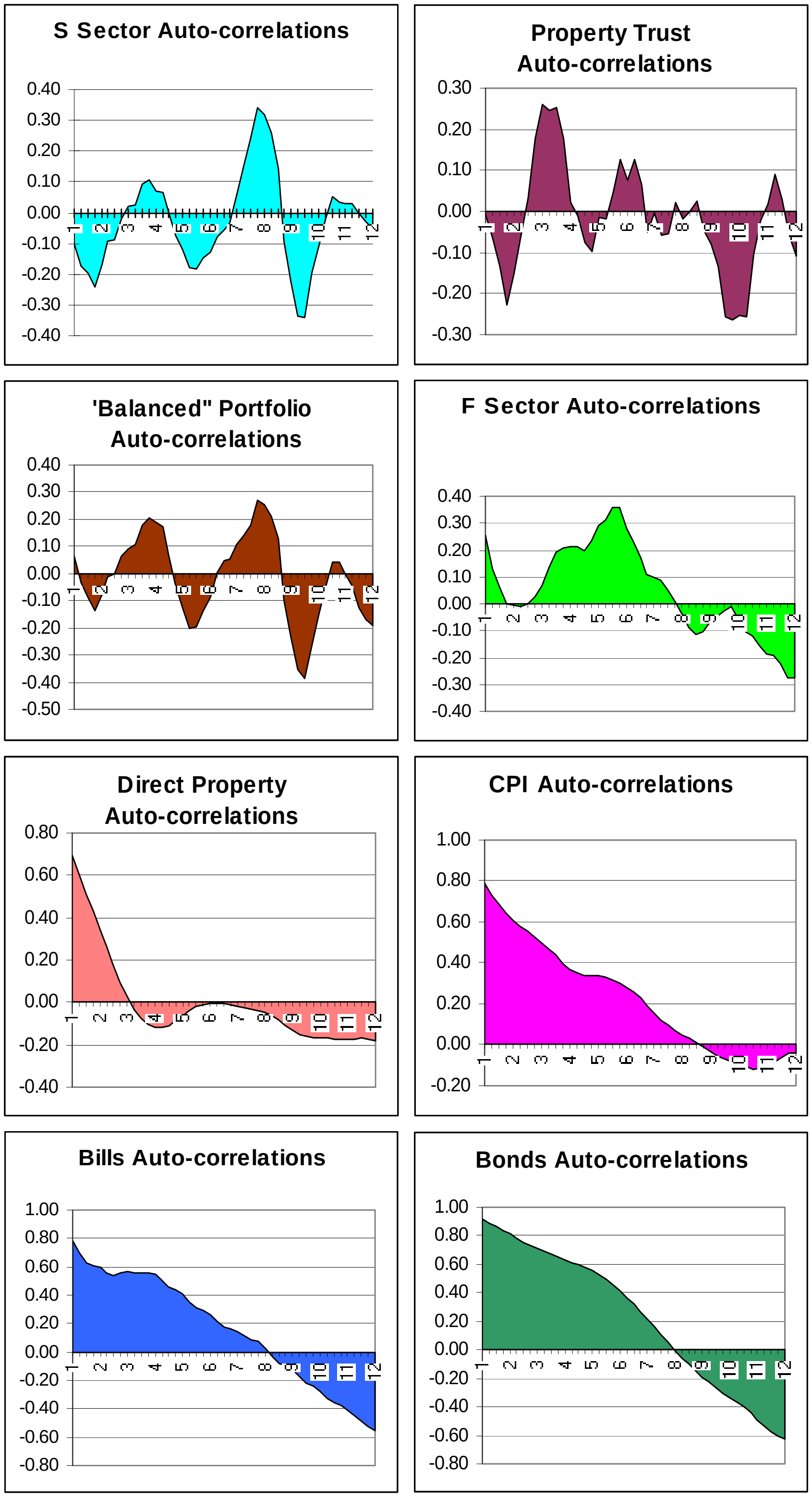
| | W | X | W-7 | X-7 |
|---|---|---|---|---|
| S | -.29 | -.05 | .02 | .10 |
| I | -.18 | .00 | .12 | .21 |
| Q | -.28 | -.14 | .16 | .30 |
| P | .30 | .48 | .45 | .32 |
| L | .16 | .36 | .66 | .70 |
| M | .31 | .51 | .74 | .75 |
| F | -.29 | -.16 | .25 | .36 |
| G | .06 | .26 | .59 | .64 |
| J | -.24 | -.10 | .29 | .38 |
| C | .39 | .59 | .74 | .71 |
| N | -.07 | .09 | .25 | .29 |
| B | .39 | .58 | .71 | .67 |
| D | .28 | .49 | .75 | .80 |
| W | 1 | .82 | .33 | .10 |
| X | .82 | 1 | .45 | .23 |
| W-7 | .33 | .45 | 1 | .80 |
| X-7 | .10 | .23 | .80 | 1 |

13.6 The results in this section suggest that stochastic investment models might be more realistic if allowance for a seven year lag is built in for CPI and AWOTE. Some further comments on this are included in Appendix B.

## 14. Auto-correlations

14.1 Auto-correlations vary significantly between the 15 sectors examined in this paper. To produce the following auto-correlation charts the annual forces for each sector were tabulated at quarterly intervals for the forty year period from 31/3/1965 to 31/3/2005. From this, forty year or 160 quarter database, auto-correlations were calculated for lags of 4 to 48 quarters for seven of the sectors plus a "balanced portfolio" composite sector. Figure 14.1 shows the results. The x-axis of the charts is the lag in years.

**Figure 14.1 Auto-correlations over 40 years**



14.2 Auto-correlations are also dependent on the period over which they are measured. For some distributions this applies even if the underlying distribution has been consistent over time. For example, if annual forces follow say an exact 35 year sine-wave over an infinite number of years, then auto-correlations measured over any say 30, 35 or 40 year periods will produce different auto-correlations for any given lag.

14.3 When the *Austmod* investment simulation model was first developed in the early 1990's, it was based on 32 year projections partly because at that time less than 30 years' investment performance data was available for the main sectors. With the longer data period now available it is desired to produce at least 40 year projections.

14.4 For these reasons it was decided to measure auto-correlations over the following 40 year periods:

**Table 14.1 Periods for auto-correlations**

| Period | Average of four periods ending: |
|---|---|
| 4 | 30/6/00, 30/9/00, 31/12/00 and 31/3/01 |
| 3 | 30/6/01, 30/9/01, 31/12/01 and 31/3/02 |
| 2 | 30/6/02, 30/9/02, 31/12/02 and 31/3/03 |
| 1 | 30/6/03, 30/9/03, 31/12/03 and 31/3/04 |
| 0 | 30/6/04, 30/9/04, 31/12/04 and 31/3/05 |

14.5 The results were then projected forward using the same methodology (but with two less periods) to that used for cross-correlations in Section 12.

14.6 Appendix B explains that the auto-correlations for Australian Shares and 10 Year Bond Rates "form two extremes". The following Figures 14.2 and 14.3 and Tables 14.2 and 14.3 therefore illustrate the auto-correlations results for only the S and D sectors.

**Figure 14.2 S sector auto-correlations over 40 years**

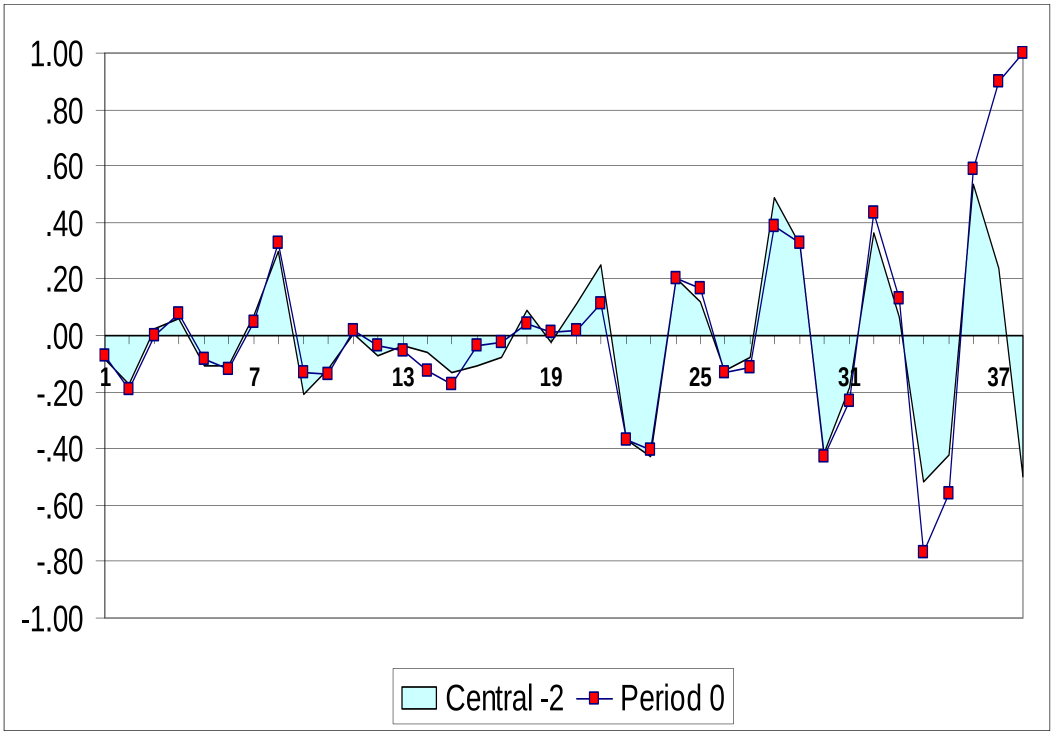



**Table 14.2 S sector auto-correlations over 40 years**

| Period: | 4 | 3 | 2 | 1 | 0 | -1 | -2 | -3 | -4 | -2 |
|---|---|---|---|---|---|---|---|---|---|---|
| Lag | | | | | | | | | | central |
| 1 | -.09 | -.09 | -.08 | -.06 | -.07 | -.09 | -.08 | -.09 | -.08 | -.08 |
| 2 | -.18 | -.18 | -.18 | -.17 | -.19 | -.18 | -.17 | -.17 | -.16 | -.17 |
| 3 | .01 | .02 | -.01 | -.01 | .00 | .02 | .02 | .03 | .04 | .02 |
| 4 | .10 | .08 | .09 | .07 | .08 | .06 | .05 | .05 | .04 | .06 |
| 5 | -.08 | -.08 | -.08 | -.08 | -.09 | -.11 | -.12 | -.12 | -.10 | -.11 |
| 6 | -.12 | -.12 | -.11 | -.10 | -.12 | -.13 | -.12 | -.11 | -.06 | -.11 |
| 7 | .02 | .04 | .03 | .04 | .05 | .07 | .05 | .10 | .10 | .07 |
| 8 | .33 | .32 | .32 | .32 | .33 | .30 | .30 | .29 | .29 | .30 |
| 9 | -.07 | -.08 | -.10 | -.11 | -.13 | -.24 | -.23 | -.23 | -.22 | -.21 |
| 10 | -.12 | -.14 | -.11 | -.09 | -.14 | -.12 | -.12 | -.11 | -.12 | -.12 |
| 11 | -.03 | .00 | -.04 | -.01 | .02 | .02 | .01 | .00 | -.02 | .01 |
| 12 | .02 | .01 | .00 | -.02 | -.04 | -.06 | -.05 | -.07 | -.13 | -.07 |
| 13 | -.04 | -.07 | -.06 | -.05 | -.05 | -.03 | -.02 | -.05 | -.04 | -.04 |
| 14 | -.14 | -.16 | -.13 | -.12 | -.12 | -.09 | -.06 | -.03 | .02 | -.06 |
| 15 | -.15 | -.16 | -.19 | -.19 | -.17 | -.10 | -.11 | -.11 | -.15 | -.13 |
| 16 | .09 | .05 | -.01 | -.05 | -.04 | -.08 | -.12 | -.16 | -.15 | -.11 |
| 17 | .09 | .11 | .05 | .02 | -.02 | -.10 | -.08 | -.07 | -.11 | -.08 |
| 18 | -.04 | -.07 | -.01 | .02 | .04 | .12 | .12 | .10 | .06 | .09 |
| 19 | -.02 | .04 | -.08 | -.04 | .01 | -.02 | .00 | -.04 | -.08 | -.02 |
| 20 | .02 | .04 | .08 | .03 | .02 | .09 | .13 | .16 | .17 | .11 |

**Figure 14.3 Bond auto-correlations over 40 years**

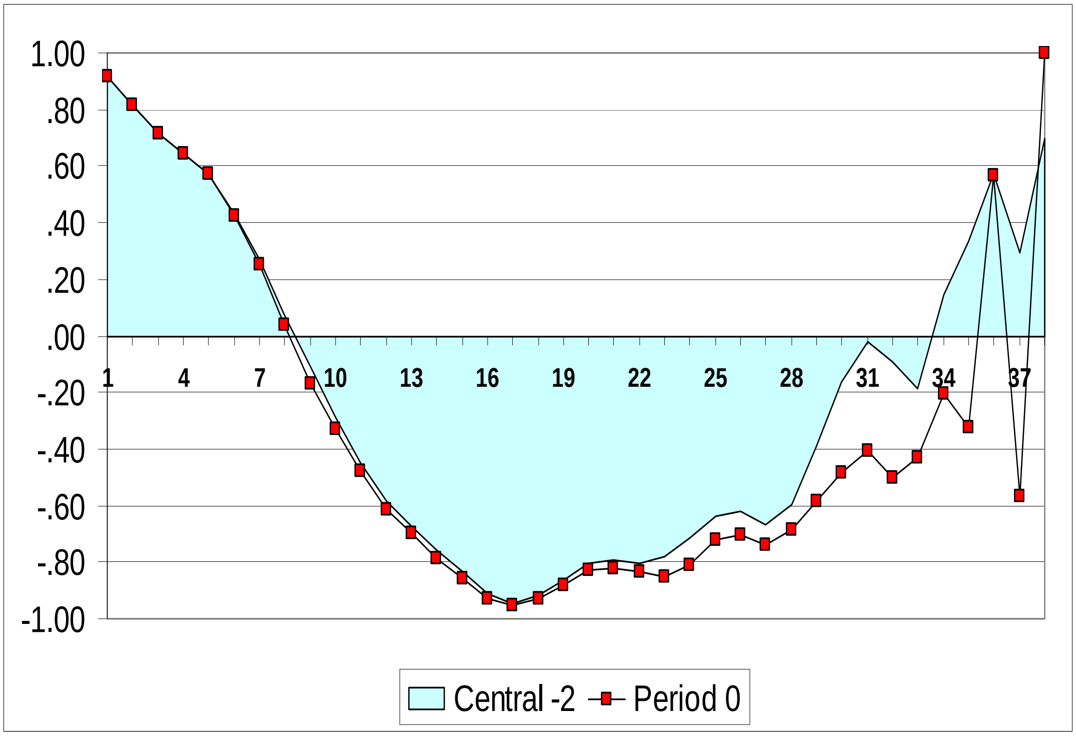



**Table 14.3 Bond auto-correlations over 40 years**

| Period: | 4 | 3 | 2 | 1 | 0 | -1 | -2 | -3 | -4 | -2 |
|---|---|---|---|---|---|---|---|---|---|---|
| Lag | | | | | | | | | | central |
| 1 | .92 | .92 | .92 | .92 | .92 | .92 | .91 | .91 | .91 | .91 |
| 2 | .83 | .83 | .82 | .82 | .82 | .82 | .81 | .81 | .81 | .81 |
| 3 | .74 | .74 | .73 | .72 | .72 | .71 | .71 | .71 | .72 | .71 |
| 4 | .68 | .67 | .66 | .65 | .64 | .64 | .64 | .65 | .65 | .64 |
| 5 | .62 | .60 | .58 | .58 | .57 | .57 | .57 | .57 | .58 | .57 |
| 6 | .52 | .48 | .44 | .42 | .43 | .43 | .42 | .43 | .46 | .43 |
| 7 | .37 | .34 | .29 | .26 | .25 | .25 | .25 | .28 | .32 | .27 |
| 8 | .19 | .16 | .13 | .08 | .04 | .02 | .06 | .11 | .15 | .08 |
| 9 | .01 | -.04 | -.06 | -.11 | -.17 | -.17 | -.13 | -.07 | -.01 | -.11 |
| 10 | -.16 | -.23 | -.29 | -.32 | -.33 | -.33 | -.31 | -.26 | -.21 | -.29 |
| 11 | -.29 | -.40 | -.49 | -.50 | -.48 | -.46 | -.45 | -.45 | -.42 | -.45 |
| 12 | -.42 | -.53 | -.59 | -.62 | -.61 | -.58 | -.58 | -.59 | -.55 | -.58 |
| 13 | -.53 | -.58 | -.63 | -.68 | -.70 | -.71 | -.69 | -.66 | -.61 | -.67 |
| 14 | -.60 | -.65 | -.69 | -.73 | -.79 | -.82 | -.80 | -.73 | -.65 | -.76 |
| 15 | -.68 | -.72 | -.76 | -.81 | -.86 | -.88 | -.87 | -.82 | -.73 | -.83 |
| 16 | -.77 | -.81 | -.86 | -.91 | -.93 | -.93 | -.93 | -.91 | -.86 | -.91 |
| 17 | -.83 | -.89 | -.93 | -.95 | -.95 | -.95 | -.95 | -.95 | -.93 | -.95 |
| 18 | -.86 | -.90 | -.92 | -.93 | -.93 | -.93 | -.92 | -.91 | -.91 | -.92 |
| 19 | -.86 | -.87 | -.87 | -.88 | -.88 | -.87 | -.87 | -.85 | -.83 | -.86 |
| 20 | -.84 | -.85 | -.83 | -.83 | -.83 | -.82 | -.81 | -.80 | -.77 | -.81 |

14.7 Tables 14.2 and 14.3 include only the first 20 year lags because beyond the half-way point the auto-correlations often fluctuate.

## 15. Assumptions

15.1 The following assumptions, developed as explained in previous sections, are gross of (i.e. before) tax, imputation credits and fees. They have not yet been "road-tested", but are not significantly different from the previously tested ("OLD") assumptions.

**Table 15.1 Investment assumptions**

| Sector | Risk margin (arithmetic average) | Mean rate (arithmetic average) | Coefficient of variation | Standard deviation of rates | Skewness | Kurtosis |
|---|---|---|---|---|---|---|
| S | 4.0% | 10.0% | 1.600 | 16.0% | -19% | 50% |
| I | 4.0% | 10.0% | 1.600 | 16.0% | -28% | 53% |
| Q | 3.5% | 9.5% | 1.221 | 11.6% | -23% | 93% |
| P | 1.2% | 7.2% | 1.000 | 7.2% | -128% | 326% |
| L | 1.0% | 7.0% | 0.472 | 3.3% | 56% | -28% |
| M | 1.0% | 7.0% | 0.472 | 3.3% | 69% | -73% |
| F | 1.0% | 7.0% | 0.700 | 4.9% | -93% | 242% |
| G | 0.5% | 6.5% | 0.600 | 3.9% | 9% | -33% |
| J | 0.8% | 6.8% | 0.691 | 4.7% | -95% | 237% |
| C | -0.5% | 5.5% | 0.490 | 2.7% | 70% | -77% |
| N | 1.0% | 7.0% | 0.800 | 5.6% | -57% | 71% |
| B | -0.50% | 5.50% | 0.527 | 2.90% | 81% | -38% |
| D | | 6.00% | 0.384 | 2.30% | 38% | -118% |
| W | -2.25% | 3.75% | 0.600 | 2.25% | 162% | 324% |
| X | -3.50% | 2.50% | 0.700 | 1.75% | 58% | -47% |

**Table 15.2 40 year auto-correlation assumptions**

| Lag (years) | S sector | D sector | Lag (years) | S sector | D sector |
|---|---|---|---|---|---|
| 1 | -.08 | .91 | 21 | .25 | -.79 |
| 2 | -.17 | .81 | 22 | -.37 | -.80 |
| 3 | .02 | .71 | 23 | -.43 | -.78 |
| 4 | .06 | .64 | 24 | .20 | -.72 |
| 5 | -.11 | .57 | 25 | .12 | -.64 |
| 6 | -.11 | .43 | 26 | -.12 | -.62 |
| 7 | .07 | .27 | 27 | -.08 | -.67 |
| 8 | .30 | .08 | 28 | .49 | -.60 |
| 9 | -.21 | -.11 | 29 | .32 | -.39 |
| 10 | -.12 | -.29 | 30 | -.42 | -.16 |
| 11 | .01 | -.45 | 31 | -.19 | -.02 |
| 12 | -.07 | -.58 | 32 | .37 | -.09 |
| 13 | -.04 | -.67 | 33 | .07 | -.19 |
| 14 | -.06 | -.76 | 34 | -.52 | .15 |
| 15 | -.13 | -.83 | 35 | -.42 | .33 |
| 16 | -.11 | -.91 | 36 | .54 | .57 |
| 17 | -.08 | -.95 | 37 | .24 | .30 |
| 18 | .09 | -.92 | 38 | -.50 | .70 |
| 19 | -.02 | -.86 | | | |
| 20 | .11 | -.81 | | | |

## Table 15.3 cross-correlation assumptions

**RANK** CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

| | S | I | Q | P | L | M | F | G | J | C | N | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **S** | 1 | .61 | .54 | .03 | .21 | .22 | .17 | .23 | .16 | .12 | .18 | .11 | .09 | -.13 | .00 |
| **I** | .61 | 1 | .37 | .08 | .24 | .31 | .32 | .32 | .31 | .17 | .36 | .20 | .26 | -.10 | .01 |
| **Q** | .54 | .37 | 1 | .06 | .27 | .17 | .48 | .36 | .50 | .08 | .49 | .04 | .09 | -.12 | -.10 |
| **P** | .03 | .08 | .06 | 1 | .27 | .33 | .01 | .24 | .12 | .41 | .04 | .40 | .41 | .49 | .59 |
| **L** | .21 | .24 | .27 | .27 | 1 | .92 | .58 | .90 | .61 | .81 | .60 | .78 | .84 | .28 | .42 |
| **M** | .22 | .31 | .17 | .33 | .92 | 1 | .49 | .88 | .51 | .90 | .52 | .90 | .90 | .36 | .51 |
| **F** | .17 | .32 | .48 | .01 | .58 | .49 | 1 | .77 | .96 | .37 | .90 | .35 | .41 | -.13 | -.05 |
| **G** | .23 | .32 | .36 | .24 | .90 | .88 | .77 | 1 | .78 | .81 | .76 | .79 | .77 | .28 | .39 |
| **J** | .16 | .31 | .50 | .12 | .61 | .51 | .96 | .78 | 1 | .41 | .88 | .38 | .45 | -.07 | .02 |
| **C** | .12 | .17 | .08 | .41 | .81 | .90 | .37 | .81 | .41 | 1 | .42 | .97 | .85 | .53 | .66 |
| **N** | .18 | .36 | .49 | .04 | .60 | .52 | .90 | .76 | .88 | .42 | 1 | .39 | .42 | .02 | .14 |
| **B** | .11 | .20 | .04 | .40 | .78 | .90 | .35 | .79 | .38 | .97 | .39 | 1 | .85 | .53 | .66 |
| **D** | .09 | .26 | .09 | .41 | .84 | .90 | .41 | .77 | .45 | .85 | .42 | .85 | 1 | .37 | .54 |
| **W** | -.13 | -.10 | -.12 | .49 | .28 | .36 | -.13 | .28 | -.07 | .53 | .02 | .53 | .37 | 1 | .85 |
| **X** | .00 | .01 | -.10 | .59 | .42 | .51 | -.05 | .39 | .02 | .66 | .14 | .66 | .54 | .85 | 1 |
| | Average | .441422 | | | | | | | | | | | | | |

**STANDARD** CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

| | S | I | Q | P | L | M | F | G | J | C | N | B | D | W | X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **S** | 1 | .62 | .62 | .02 | .16 | .13 | .30 | .24 | .28 | .10 | .25 | .08 | .11 | -.29 | -.05 |
| **I** | .62 | 1 | .44 | .06 | .33 | .34 | .42 | .40 | .40 | .29 | .42 | .26 | .31 | -.18 | .00 |
| **Q** | .62 | .44 | 1 | .00 | .34 | .24 | .62 | .47 | .61 | .17 | .57 | .11 | .18 | -.28 | -.14 |
| **P** | .02 | .06 | .00 | 1 | .27 | .39 | -.12 | .16 | -.06 | .44 | -.04 | .41 | .36 | .30 | .48 |
| **L** | .16 | .33 | .34 | .27 | 1 | .94 | .66 | .93 | .71 | .88 | .66 | .82 | .87 | .16 | .36 |
| **M** | .13 | .34 | .24 | .39 | .94 | 1 | .49 | .88 | .55 | .98 | .53 | .94 | .93 | .31 | .51 |
| **F** | .30 | .42 | .62 | -.12 | .66 | .49 | 1 | .83 | .98 | .39 | .92 | .33 | .42 | -.29 | -.16 |
| **G** | .24 | .40 | .47 | .16 | .93 | .88 | .83 | 1 | .86 | .82 | .81 | .77 | .81 | .06 | .26 |
| **J** | .28 | .40 | .61 | -.06 | .71 | .55 | .98 | .86 | 1 | .45 | .93 | .40 | .47 | -.24 | -.10 |
| **C** | .10 | .29 | .17 | .44 | .88 | .98 | .39 | .82 | .45 | 1 | .44 | .97 | .92 | .39 | .59 |
| **N** | .25 | .42 | .57 | -.04 | .66 | .53 | .92 | .81 | .93 | .44 | 1 | .38 | .44 | -.07 | .09 |
| **B** | .08 | .26 | .11 | .41 | .82 | .94 | .33 | .77 | .40 | .97 | .38 | 1 | .90 | .39 | .58 |
| **D** | .11 | .31 | .18 | .36 | .87 | .93 | .42 | .81 | .47 | .92 | .44 | .90 | 1 | .28 | .49 |
| **W** | -.29 | -.18 | -.28 | .30 | .16 | .31 | -.29 | .06 | -.24 | .39 | -.07 | .39 | .28 | 1 | .82 |
| **X** | -.05 | .00 | -.14 | .48 | .36 | .51 | -.16 | .26 | -.10 | .59 | .09 | .58 | .49 | .82 | 1 |
| | Average | .439822 | | | | | | | | | | | | | |

## 16. Gross/Net of Tax

16.1 All results in previous sections are gross (i.e. before) tax and imputation credits. The following results illustrate the impact of tax and imputation credits on the Section 15 assumptions for mean rates for superannuation in the accumulation stage.

**Table 16.1 Gross/net of tax**

| | **Mean rate (arithmetic average)** | | |
|---|---|---|---|
| | Before tax | After tax and imputation credits | Average tax rate |
| **S** | 10.00% | 9.65% | 3.5% |
| **I** | 10.00% | 9.08% | 9.2% |
| **Q** | 9.50% | 8.48% | 10.7% |
| **P** | 7.20% | 6.14% | 14.8% |
| **L** | 7.00% | 5.95% | 15.0% |
| **M** | 7.00% | 5.95% | 15.0% |
| **F** | 7.00% | 5.95% | 15.0% |
| **G** | 6.50% | 5.53% | 15.0% |
| **J** | 6.80% | 5.78% | 15.0% |
| **C** | 5.50% | 4.68% | 15.0% |
| **N** | 7.00% | 6.01% | 14.1% |
| **B** | 5.50% | 4.68% | 15.0% |
| **D** | 6.00% | 5.10% | 15.0% |

16.2 The above table allows for income tax at the 15% superannuation rate and, on an approximate basis, for imputation credits and the lower rates of tax on realised capital gains.

## 17. Gross/Net of Fees

17.1 All results in previous sections are gross (i.e. before) fees. The following results illustrate the impact of wholesale passive investment fees on the Section 15 and 16 assumptions.

**Table 17.1 Gross/net of fees**

| Mean rate (arithmetic average) | | | |
|---|---|---|---|
| | Before tax<br>Before fees | Before tax<br>After fees | After tax & IC's<br>After fees |
| **S** | 10.00% | 9.74% | 9.43% |
| **I** | 10.00% | 9.71% | 8.84% |
| **Q** | 9.50% | 9.21% | 8.23% |
| **P** | 7.20% | 6.38% | 5.44% |
| **L** | 7.00% | 6.71% | 5.70% |
| **M** | 7.00% | 6.71% | 5.70% |
| **F** | 7.00% | 6.82% | 5.80% |
| **G** | 6.50% | 6.32% | 5.37% |
| **J** | 6.80% | 6.62% | 5.63% |
| **C** | 5.50% | 5.31% | 4.51% |
| **N** | 7.00% | 6.81% | 5.85% |
| **B** | 5.50% | 5.50% | 4.68% |
| **D** | 6.00% | 6.00% | 5.10% |

## 18. Concluding Remarks

18.1 The objective of this paper was to help bridge the gap between the demand from actuaries (and consumers) for robust assumptions in respect of future investment returns across a broad range of investment sectors, and the limited supply of data readily available for a the range of investment sectors.

18.2 The advent of unit price data in Australia 40 years ago, through National Mutual's "EFG" system, provided both the inspiration for this paper and a substantial database of investment performance statistics, across a broad range of investment sectors.

18.3 The paper developed and summarized assumptions sets for medium to long-term use in stochastic investment models, across the range of investment sectors and economic indicators that were examined. In determining those assumptions, the paper has illustrated techniques for 'backdating' incomplete data series for use, particularly, with cross-correlation and autocorrelation analyses.

18.4 In addition, the paper adopted the technique used by Dwonczyk in fitting sine waves to CPI data, and applied an extended version of this technique to 10 year bonds (and to CPI and AWOTE). This analysis appears to provide support for the existence of 'long-term' economic cycles extending over 38 years. This, together with auto-correlations, appears to be an area which warrants further research.

## 19. Acknowledgments

19.1 While I take full responsibility for the content of this paper, I would like to sincerely thank the following:

- Alan Brown who has graciously assisted me for many years with guidance on statistics, stochastic modelling, Excel programming and related topics.

- Cary Helenius for peer-reviewing this paper and for encouraging me to give it greater emphasis on investment assumptions for stochastic models.

- Clive Amery for peer-reviewing this paper and for his painstaking assistance with the automatic formatting, numbering and layout of the paper's contents, figures, tables, and appendices.

- The Institute of Actuaries of Australia for granting me some special leeway with the timing of this paper because of the impossibility of getting investment performance information up to 31 March 2005 within the Conference timetable for submission of papers.

- AXA Australia and National Mutual for the regular provision of unit price and index data for many of the 11 investment sectors.

- The designers, in 1965, of the National Mutual EFG investment system.

19.2 Finally, I would like to express my gratitude to my wife Barbara for her patience while I laboured for such a long time with drafts of this paper and with many updates to the *Austmod* stochastic and historical investment simulation model.

## 20. References

*Austmod* investment simulation model, *Appendix A: Uses of S.I.S. (Superannuation Investment Simulations)*, The Institute of Actuaries of Australia, 1997 Transactions (pp. 439 to 446)

Dwonczyk D, 1993, *A Model for Long Term Forecasting of Consumer Prices Inflation*, The Institute of Actuaries of Australia, June 1993 Quarterly Journal.

Grenfell C R, 1997, *Uses of S.I.S. (Superannuation Investment Simulations)*, The Institute of Actuaries of Australia, 1997 Transactions.

## Appendix A: Statistical Conversion between Rates and Forces of Return

Rates of return, r, and forces of return, f, are linked at all times by the relationship

$$e^f = 1 + r$$

Given a time series of data, suppose for the rate of return that,

m = mean

s = standard deviation

and for the force of return that,

mu = mean

sigma = standard deviation

g = skewness

k = kurtosis.

One way to find the relationship between these statistics is to consider the moment generating function of the force of return. This can be expressed in the form

$$E(e^{ft}) = 1 + \text{mu } t + 1/2 \text{ sigma}^2 t^2 + 1/6 \text{ g sigma}^3 t^3 + 1/24 \text{ k sigma}^4 t^4 + \ldots$$

By putting the auxiliary parameter t =1, and ignoring higher moments the following approximation is obtained.

$$E(1 + r) = 1 + \text{mu} + 1/2 \text{ sigma}^2 + 1/6 \text{ g sigma}^3 + 1/24 \text{ k sigma}^4$$

so that

$$m = \text{mu} + 1/2 \text{ sigma}^2 + 1/6 \text{ g sigma}^3 + 1/24 \text{ k sigma}^4 \qquad [1]$$

Likewise when t =2, a second approximation is obtained.

$$E(1 + r)^2 = 1 + 2 \text{ mu} + 2 \text{ sigma}^2 + 4/3 \text{ g sigma}^3 + 2/3 \text{ k sigma}^4$$

which leads to

$$E(r)^2 = \text{sigma}^2 + \text{g sigma}^3 + 7/12 \text{ k sigma}^4 \qquad [2]$$

Since

$$s^2 = E(r)^2 - m^2 \qquad [3]$$

the two approximations and the relationship [3] enable both m and s to be calculated given mu, sigma, g and k.

Numerical methods can be used to go in the reverse direction, i.e. to calculate mu and sigma given m, s, g and k.

## Appendix B: Modelling Auto-correlations

An examination of Figure 14.1 shows that auto-correlations are of three broad types:

(a) S sector, Property Trusts and the "Balanced" portfolio

(b) F sector, Direct Property and CPI (and AWOTE)

(c) Bills and Bonds.

Also of interest is the similarity between the auto-correlations of the S sector and that of the "Balanced" portfolio. This is primarily due to the greater volatility of the S sector which dominates the shape of the "Balanced" portfolio auto-correlations. Though not included in Figure 14.1, the auto-correlations of a "Capital Stable" portfolio are quite different from those for the "Balanced" portfolio. The "Capital Stable" portfolio auto-correlations are similar to those for the F sector.

A closer examination of Figure 14.1 reveals that there is a gradual progression from the **S sector**, to Property Trusts, to the "Balanced" portfolio, to the F sector, to Direct Property, to CPI, to Bills and then to **Bonds**.

**The auto-correlations of the S sector and Bonds form two extremes**. This hypothesis is supported by the following analysis of the auto-correlations underlying Figure 14.1:

i) for lags of 1 to 2.5 years, the S sector has the lowest auto-correlations of all the 15 sectors. In stochastic modelling these shorter term auto-correlations are most important because there are far more observations at the short end than at the long end. Further, the longer results are to some extent repeats of the shorter results.

ii) for lags of 1 to 6 years, Bonds (the D sector) have the highest auto-correlations of all the 15 sectors.

iii) for lags of 6 to 6.5 years, the S sector again has the lowest auto-correlations of all the 15 sectors.

iv) for lags of 13.75 to 29.75 years, Bonds (the D sector) have the lowest auto-correlations of all the 15 sectors.

Intuitively, the share sector and Bonds are also significant because the former's higher volatility and expected investment returns and the latter's influence on current market investment yields.

The above analysis forms the background as to why the *Austmod* stochastic investment simulation model focuses on the S and D sector autocorrelations.

Briefly, the *Austmod* model sets D sector auto-correlations by:

- subdividing (many times over) D sector zero auto-correlated results into two groups of 16 years,
- ranking each group of 16 year D sector results
- positioning each D sector result, first of first 16 to first of 32, first of second 16 to last of 32, second of second 16 to second of 32, etc etc.
- retaining each result for the 14 other sectors with their original D sector value (thus leaving cross-correlations undisturbed).

Briefly, the *Austmod* model then sets S sector autocorrelations by discarding all but one of the above D sector combinations, and retaining the D sector combination which is associated with the S sector auto-correlation closest to that desired. The method of least squares is used to determine the closest S sector auto-correlation, with the auto-correlations linearly weighted towards the shorter lags (which have the greatest number of observations).

Having obtained the S and D sector auto-correlations as above the (retained) cross-correlations carry through these auto-correlation distributions to the other sectors. The Cholesky decomposition formula is used to create the cross-correlation distributions.

The process described in the previous paragraph works reasonably well for all sectors except X and W where only about one-third of the desired auto-correlation distributions are obtained. Given the D and X (.54) and D and W (.37) cross correlations in Figure 15.3, this result is not surprising. The much higher D and X-7 (.77) and D and W-7 (.82) cross correlations in Figure 13.3 should produce better results if lagged X and W forces were used throughout the modelling. The results in Table 15.2 also indicate that the 32 year cycle referred to on the previous page should be increased to about 40 years.

Much of this appendix is "work-in-progress". If an update is required, the author can be contacted on colnbarb@hotmail.com.