Institute of Actuaries of Australia

Australian Investment Performance 1960 to 2007 (and Investment Assumptions for Stochastic Models)

Prepared by Colin R. Grenfell FIA, FIAA, FASFA

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The Institute of Actuaries of Australia
Level 7 Challis House 4 Martin Place
Sydney NSW Australia 2000
Telephone: +61 2 9233 3466 Facsimile: +61 2 9233 3446
Email: insact@actuaries.asn.au Website: www.actuaries.asn.au

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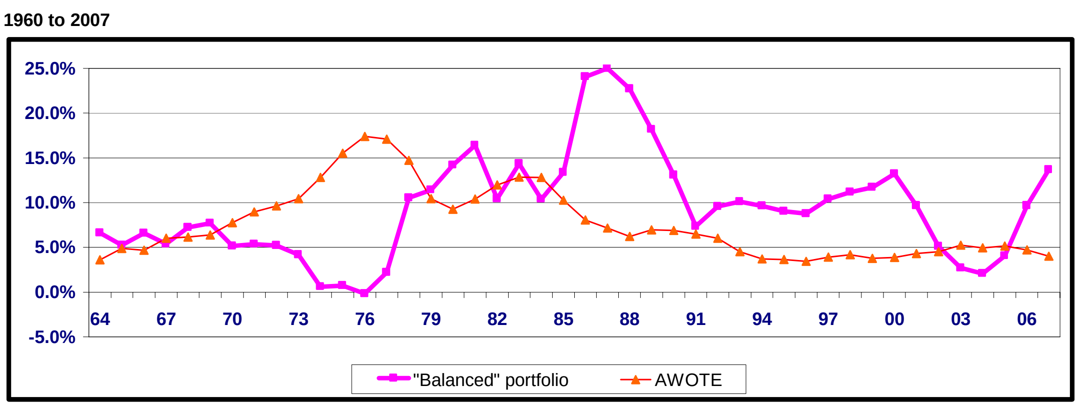
1960 to 2007



Historical 4-year (ending 30/6/64 to 30/6/07) compound average annual returns

Source: *Austmod,* net of tax and fees

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Abstract

This paper analyses 47 years, or 188 quarters, of Australian investment performance for the period from 30 June 1960 (and earlier for some sectors) to 30 June 2007. The paper updates and expands on a similar paper presented at the May 2005 Institute of Actuaries of Australia Biennial Convention. It aims to test whether the methodology for determining assumptions in the original paper is robust enough to still produce reasonable financial assumptions now that a further two years and one quarter of financial data is being examined.

The analysis covers eleven investment classes plus four financial indicators:

Growth Securities	Interest Income	Financial Indicators
Australian shares	Australian fixed interest	CPI
Int'l shares (unhedged)	Int'l fixed interest (hedged)	AWOTE
Property trusts	Government semis (0-3yrs)	90 day bill rates
Direct property	Inflation linked bonds	10 year bond rates
	Loans (floating rate)	
	Mortgage trust	
	Cash	

For each of these 15 "sectors" the annualised average results are tabulated and summarised for:

- risk margins (over 10 year bond rates),
- coefficients of variation,
- skewness,
- kurtosis,
- cross-correlations, and
- auto-correlations.

From these results, some assumptions are developed for the mean, standard deviation, skewness, kurtosis, cross-correlations and auto-correlations for each sector. These assumptions are primarily designed for use in stochastic investment and asset/liability modelling conducted in the period from 2007 to say 2009, after which they should be updated. The assumptions are intended for both medium-term (say 3 to 10 year) and long-term (say 10 to 40 year) modelling.

Section 5 of the paper analyses economic cycles using a 'sum of two sine curves' technique.

The assumptions, except those for correlations, are derived by examination of 38 year periods (and 24 year periods for risk margins and coefficients of variation), fitting a quadratic EXCEL trend function to the annualised average results and extrapolating forward these results by a couple of years. The assumptions for cross-correlations are derived using a similar approach. The assumptions for auto-correlations, in particular those for Australian Shares and 10 year bond rates, are derived using the same approach as for cross-correlations, but based on 40 year (rather than 38 year) periods.

Much of the content of Sections 2 to 4 of the paper is unchanged from the original 2005 paper but it has been included here so that this paper is complete and self-contained. Section 8 and Sections 15 to 17 include some results for Balanced and Capital Stable portfolios and compound (as well as arithmetic) means which were not in the 2005 paper.

Key words: investment assumptions, stochastic, skewness, kurtosis, cross-correlations, auto-correlations, cycles

1 Introduction

1.1 **Demand:** Most actuarial work requires assumptions about future investment returns. Often assumptions are also explicitly or implicitly made about future rates of inflation and/or salary increases.

1.2 **Supply:** However, often for commercial reasons, little information is publicly available about the data from which investment return assumptions can be derived.

1.3 This paper attempts to bridge, at least partly, the gap between demand and supply.

2 Methodology

2.1 The one methodology was used, with only slight variations, for calculating assumptions for risk margins, coefficients of variation, skewness, kurtosis and correlations.

2.2 **Step 1:** annual rates of return and forces of return for the 15 sectors were sorted by years ending every quarter 31 March, 31 December, 30 September and 30 June.

2.3 **Step 2:** determined the periods over which statistics are calculated. Periods of 38 years and 24 years were chosen for the reasons explained in Section 5 (in particular sections 5.18 and 5.19).

2.4 **Step 3:** 38-year risk margins, coefficients of variation, skewness, kurtosis and 105 ranked and unranked cross-correlations were calculated and tabulated for each of the thirty-eight year running periods for each quarter ending in the nine years between 30/9/98 and 30/6/07 inclusive. Risk margins and coefficients of variation were also calculated and tabulated for 24-year periods.

2.5 **Step 4:** the four quarter-ending results were averaged to give sets of nine 38-year running period results. That is:

Table 2.1 Periods for 38 year calculations

Period	Average of four periods ending:
8	30/9/98, 31/12/98, 31/3/99 and 30/6/99
7	30/9/99, 31/12/99, 31/3/00 and 30/6/00
6	30/9/00, 31/12/00, 31/3/01 and 30/6/01
5	30/9/01, 31/12/01, 31/3/02 and 30/6/02
4	30/9/02, 31/12/02, 31/3/03 and 30/6/03
3	30/9/03, 31/12/03, 31/3/04 and 30/6/04
2	30/9/04, 31/12/04, 31/3/05 and 30/6/05
1	30/9/05, 31/12/05, 31/3/06 and 30/6/06
0	30/9/06, 31/12/06, 31/3/07 and 30/6/07

Thus for example, as at 30 June 2007:

0 represents periods ending between 0 and 0.75 years ago, and
8 represents periods ending between 8 and 8.75 years ago.

2.6 Additional periods (9 to 17) were used for the 24 year calculations. Thus for these calculations the earliest period, period 17, relates to average results for the four 24 year periods ending 30/9/89, 31/12/89, 31/3/90 and 30/6/90, and for these calculations data before 30/9/65 was disregarded.

2.7 **Step 5:** a weighted quadratic EXCEL trend function was fitted to each of the 9 results and projected forward six years. A quadratic basis was chosen for the fitting to avoid spurious turning-points. For the cross and auto correlations the approach was slightly varied (see Sections 12 and 14). The number of fittings and projections was:

Table 2.2 Number of fittings and projections

Projection	Fittings & projections	Number	Bases
Risk margins	Two per sector except bonds	28	24 & 38 yrs
Coeff. of variation	Two per sector	30	24 & 38 yrs
Skewness	One per sector	15	38 years
Kurtosis	One per sector	15	38 years
Cross correlations	One per combination	105	38 years
Rank cross correlns.	One per combination	105	38 years
Auto correlations	38 each for bonds and shares	76	40 years

2.8 **Step 6:** recommendations of medium to long-term assumptions were made based primarily, but not totally, on the two-year projected results. The rationale for this is that these can be considered as a 'best estimate' of the 38 year (or 24 or 40 year) four-quarter averages expected in two years time. The two years might also be considered as the possible lifetime of these assumptions until they are subsequently updated. This weighted trend plus two year approach has been designed to give greatest weight to the latest period with recognition of past trends assumed to continue for at least the next two years. The projected results for year 2 were used as a guide (or "pointer") to set assumptions for the full term of the stochastic (or deterministic if applicable) modelling.

2.9 Pragmatically, the trend plus two year approach should also provide a reasonable response to the critic who argues that, because of such-and-such recent changes, use of (unadjusted) historical results is not appropriate for setting financial assumptions. The validity of the methodology increases if there are no discernible significant data changes over the past 40 years that are "outside the norm".

2.10 The projected results for years 3 to 6 of the projection were used:

- to check that the results are not trending to ridiculous levels such as negative coefficients of variation or correlations greater than 0.99, and
- to get 'smooth' correlation assumptions at 2 years by averaging the results for years 0 to 4.

2.11 Consistent with accepted practice for input to stochastic projection models the following were calculated and tabulated:

(a) risk margins and means based on **arithmetic** averages of rates

(b) coefficients of variation and standard deviations based on **rates**

(c) skewness, kurtosis and correlations based on **forces**.

The possibility of inconsistencies arising from (a), (b) or (c) is discussed in Section 10.

2.12 Once assumptions relating to the results in section 2.11 (a), (b) and (c) above have been set, it is of course possible to calculate equivalent compound average rates (and forces) for each sector.

2.13 A deliberate consequence of section 2.5 above is that all individual results quoted in Sections 6 to 14 are the average of 4 results, **not** individual results for 38 or 24 years (or 40 years) ending on one date.

3 The 15 Sectors

3.1 The 15 "sectors" included in the analysis are:

Table 3.1 Sector descriptions

Sector	Description
B	Bill rate (90 day bank) in middle of year
C	Cash and short term fixed interest sector
D	10 year bonD rate in middle of year
F	Fixed interest sector
G	Government semis 0-3 years (SBC/UBS Warburg index SSG03)
I	International shares sector (MSCIAI prior 30/6/88)
J	International bonds sector
L	Loans sector (market valued, mainly floating rate)
M	Mortgage trust (valued on a hold to maturity basis)
N	Inflation linked bonds (all maturities) UBS index
P	Direct Property (one third NM/AXA, two-thirds AMP)
Q	Property trust accumulation index (from 31/1/01 S&P/ASX 300, from 30/6/02 GICS)
S	Shares sector (All Ordinaries accumulation index prior 31/3/65)
W	AWOTE by quarter (= average 1.5 months lag), not seasonally adjusted. Full-time adults (post 9/81), Males original (pre 9/81), AWE Males (pre 1/75)
X	CPI indeX by quarter

3.2 The codes B to X above are used throughout the paper to denote each of the sectors.

3.3 The 15 sectors comprise 4 growth (or "equity") sectors (I, P, Q and S), 7 interest income sectors (C, F, G, J, L, M and N) and 4 financial indicators (B, D, W and X). The main database contains **annual forces at quarterly intervals**.

3.4 The financial indicator data has been collected at the same intervals and for the same periods as the investment data because they are useful in both deterministic and stochastic models for valuing, projecting and/or illustrating liabilities.

3.5 Bills and bonds are taken at mid-year because this gives a far better correlation against annual cash, loan and mortgage rates. The lead between bond rates and annual investment returns for the F sector (Australian fixed interest) is considered in Section 6. The lag between bond rates and CPI and AWOTE is considered in Section 13.

3.6 At the time this paper was finalised the May 2007 AWOTE, due 16/8/07, had not been published. An estimate of 1078.0 has been used in all calculations. Due to round-off, this should have no or only minimal effect on the results tabulated in pages 8 to 39.

4 "Backdating"

4.1 The available data for the various sectors commence from different dates. The following is a summary of the historical start dates for **quarterly** data (for bonds and CPI, yearly data are available from earlier dates, and for some other sectors, one or two month's data is available prior the date shown).

Table 4.1 Historical start dates for data series

Series Start Date	Sector	Data series
30/9/41	W	AWE all males, total earnings
30/9/48	X	CPI (by quarter end)
1950	S	All ordinaries unweighted average dividend yield
31/3/58	D	Bond rate
31/3/58	S	All ordinaries price index
31/12/59	B	13 week treasury notes (see section 4.6)
31/3/65	S	Australian equities (EFG system)
31/3/65	F	Australian government securities (EFG system "G" sector)
30/9/69	B	90 day bank bills
30/6/71	P	AMP property sector
31/12/71	P	Property (EFG system)
31/12/71	S	Australian shares (EFG system)
1974	S	All ordinaries weighted average dividend yield
31/12/74	W	AWOTE males, ordinary time earnings
31/3/77	Q	Listed property trust accumulation index
30/9/79	C	Cash (EFG system)
30/9/81	W	AWOTE full-time adults, ordinary time, not seasonally adjusted
30/9/82	I	International shares (EFG system, see section 4.6)
30/9/85	F	Australian fixed interest (EFG system, previously "G" sector)
30/6/86	J	International bonds (ceased 30/6/87, recommenced 30/6/92)
30/6/86	L	Loans (market valued)
30/6/88	M	Mortgage trust (gross of MER's)
31/12/89	G	Semi-government bonds SBC index (0 to 3 years)
31/3/91	N	Inflation-linked bonds WDR index (all maturities)

4.2 It is evident from the above summary that data definitions for six sectors, i.e. W, S, B, F, P and I have changed over time, though most of these changes relate to prior 1972.

4.3 It is also evident from the above summary that the National Mutual EFG investment unitisation system started on 31 March 1965. Sales brochures in the 1970's described it in these terms:

> *"Of major significance was the introduction in 1965 of a selective investment facility known as the EFG system. Evidence of the success and wide acceptance of this concept, which was pioneered by National Mutual in Australia, may now be seen in the fact that it has since been adopted by a number of other financial institutions as a medium for superannuation investment."*

4.4 There can be little doubt that the EFG system provoked the rapid development of the now enormous managed fund industry and the extensive use of unitisation throughout the superannuation industry.

4.5 It is desirable with stochastic investment models to have consistency between assumptions both within sectors (e.g. risk margins and standard deviations) and across sectors (e.g. cross correlations). An efficient way to gain such consistency is to have a 'complete' database for all sectors back to the one date, hence the term "backdating".

4.6 The chosen common start date for the complete database was 30 June 1960. The backdating was achieved primarily but not solely by the method of least squares, based on fitted parameters determined from data **after** the respective start dates and applying them to known data for other 'like' sectors **before** these start dates.

4.7 The formula used to backdate the annual forces for each sector, prior the start dates in Table 4.1 were:

Q = 52.06%F + 30.42%S + 18.59%M

P = 88.58%C + 50.02%X - 23.89%F

L = 89.90%C + 21.23%F + 1.50%P

M = 77.48%C + 34.49%L

G = 74.84%C + 37.27%F

J = 76.74%F + 19.25%C

C = 22.68%B_{-2} + 27.44%B_{-1} + 22.82%B + 25.76%B_{+1}
where B_t = B in t quarter's time

N = 71.38%X + 62.99%F - 195.05%d
where d = delta D force

B = in nominal terms, 13 week Treasury Note rate + 1.37%
where 1.37% = median excess of 90 day bank bill rate over 13 week Treasury Note rate 30/9/69 to 30/9/79

F = prior 31/3/65,
87.09%D + 14.33%B - 673.02%d

S = approximate all ordinaries accumulation index prior 31/3/65. June values as published by the RBA. September, December and March values based on all ordinaries price indices plus unweighted average dividend yields less 1.75% per annum (being the average difference between unweighted and weighted dividend yields between 1974 and 1984).

I = MSCI accumulation index from 30/6/70 to 30/6/88. Prior to 30/6/70 the S&P500 series plus an assumed 3% per annum average dividend yield was used. All returns were then adjusted for $AU/$US exchange rate movements.

4.8 The process of creating historical data by regression analysis means that several series must be correlated, at least for the period over which the data is created. The correlation coefficients between the series for any 38-year period that extends back into the "backdated" period must therefore be illusionary in part. It is very difficult to unravel this effect. An examination was made to try and judge whether any significant bias was introduced. For the seven most effected sectors (i.e. G, J, L, M, N, P and Q) 16 cross correlations were calculated over the full period since 31/3/1965 to the calculation date and over the shorter periods from when each of these sectors commenced up to the calculation date. Each of these was compared with the relevant backdating parameter used to calculate the "backdated data". The analysis indicated that the parameters introduced very little, if any, bias overall.

4.9 Any bias introduced will also 'wear off' over time due to the trend-fitting, weighting and extrapolation inherent in the methodology described in Section 2. Nevertheless, users of the *Austmod* investment simulation model (refer Section 20) and users of the cross-correlations assumptions tabulated in Sections 13 and 15, need to be aware that the "backdating" may have introduced some slight bias for those sectors where actual historical data started at later dates (see Table 4.1).

5 Economic Cycles

5.1 This section considers economic cycles. Though closely related, matters relating to auto-correlations are considered later, in Section 14.

5.2 Dwonczyk (1993) identified 33 year cycles in Australian CPI inflation data. "A sine wave was fitted to the data which best estimated the actual [yearly rates] time series for the period 1956 to 1992". He graphed the actuals, seven year moving averages and time series estimates over the 91-year period from 1 July 1901 to 30 June 1992.

5.3 For comparison, using the data described in Section 3 above with annual forces at quarterly intervals, the following curve-fitting results were obtained. Each combined curve is **the sum of two sine curves**. The period of each sine curve is shown in Table 5.1 and only optimum results (with some minor rounding) are tabulated. The start points for Bonds (31 March 1958) and CPI (year-ending September 1949) correspond with when quarterly data first became available.

5.4 To give greater significance to more recent data, the data behind Table 5.1 was discounted on a compound basis for each quarter prior 30 June 2007. Thus, for example, with a data discount of 0.5% per quarter, data before December 1961 was given a weight of less than 40% and data before March 1986 was given a weight of less than 65%. By changing the data discount rate it is possible to get an indication of whether or not results are period dependent.

5.5 Table 5.1 on the next page expands and updates the corresponding table in the original 2005 paper. Economic cycles relating to Australian Shares have been added and results are now included for each short cycle and each long cycle as well as for the combined cycles.

5.6 The Institute of Actuaries of Australia Taskforce (2005) which was established to provide a tenth anniversary review of the Resilience Reserves used in the determination of statutory solvency and capital adequacy requirements of life insurance companies in Australia, stated in Section 4 of their report:

"Actuarial intuition, reflected in almost all actuarial models, is that dividend yields, real interest rates and inflation show a tendency to revert to some longer term mean. If true, this means that the probability of a future adverse shock depends, at least in part upon the current state of the market relative to its mean position."

In this context, it is relevant to note that sine curves (plus random noise) are equivalent to mean reversion.

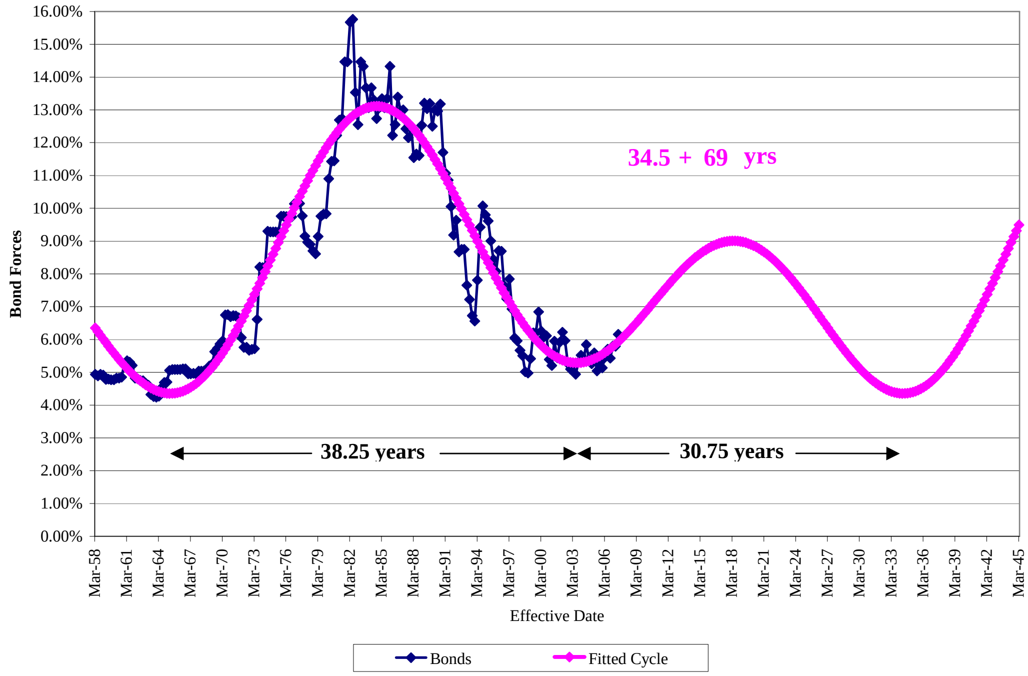
Table 5.1 Economic cycles

Period of Cycles (years)	**Explained % of sum of squared residuals**				**Fitted curve minimum date**	**Fitted curve next minimum**	**Period between dates (years)**	**Refer Para.**
	Short cycle only	Long cycle only	**Both cycles**	Both cycles				
	Data discount per quarter							
	0.5%			0%				
BONDS (Data from 31 March 1958)								
23.5 + 49	7.6	84.2	**91.7**	92.1	(very flat at minimums)			5.7
34.5 + 69	74.9	53.4	**90.9**	91.0	Mar 65	Jun 03	38.25	5.8 5.9
CPI (Data from year-ending 30 September 1949)								
12.25 + 35.25	8.3	54.8	**63.7**	60.1	Sep 58	Mar 95	36.5	5.10
24.5 + 35.25	4.1	54.8	**60.1**	56.2	Sep 62	Dec 99	37.25	5.11
6.25 + 35.75	5.8	54.8	**59.9**	55.8	Mar 61	Jun 98	37.25	5.12
AWOTE (Data from year-ending 30 September 1949)								
7.25 + 36.25	7.2	43.4	**51.3**	46.1	Jun 63	Sep99	36.25	5.13
22.75 + 36	3.8	43.4	**51.3**	46.4	Mar 62	Dec 00	38.75	5.14
SHARES (Data from year-ending 30 June 1961)								
4.096 + 9.343	12.3	11.3	**24.4**	26.4	Mar 74	Mar 11	37	5.15
4 + 9 1/3 rd	9.7	11.3	**22.2**	24.6	Mar 74	Mar 02	28	5.15
1.977 + 4.096	5.8	12.3	**17.9**	20.6	Sep 98	Mar 52	53.5	5.16
2 + 4	5.1	9.7	**14.8**	17.6	(flat)			5.16

5.7 For bonds, the 23.5 plus 49 year curve explained a very high 91.7% of the sum of squared residuals from the mean. However the curve is a strange shape with very flat values at the minimums. For example, the curve has a low point with annual forces of between 5.1% and 4.5% for 10.5 years from December 2007 to June 2018. The curve then cycles to a maximum force of 13.3% in 2032. This is consistent with a **high** yield/ high inflationary future period similar to the past (where the fitted curve reaches a maximum force of 13.4% in December 1984).

5.8 For bonds, the 34.5 plus 69 year curve explained about 91% of the sum of squared residuals for both data discount rates, indicating that it is less period dependent than the previous curve. Unlike the previous curve, it is smooth over time (and its first differences are also smooth) and it is consistent with a **low** yield / low inflationary future period. The period between the last two minimums of the fitted curve is 38.25 years. Figure 5.1 below graphs this curve and the actual data.

Figure 5.1 Ten year bonds – economic cycle



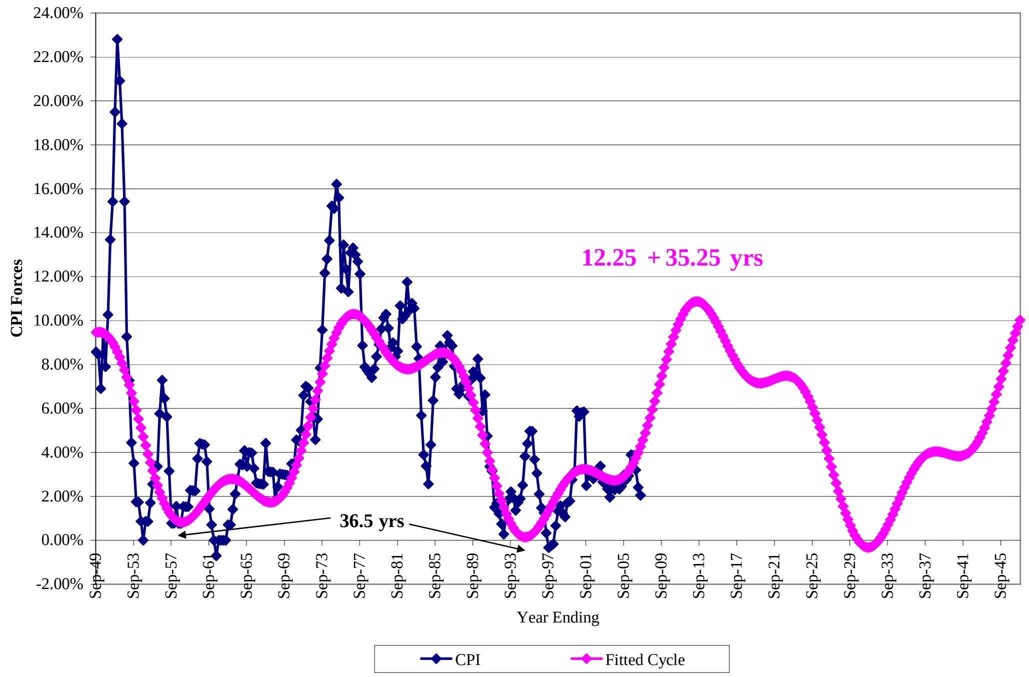
5.9 The 34.5 plus 69 year bond curve and the corresponding 35 plus 70 year curve in Grenfell (2005) have been good predictors of ten year bond rates in the last three years. Consider the following:

Date	Actual Force	Fitted April 05 (35 + 70)	Actual Force	Fitted July 07 (34.5 + 69)
30 Sep 2004	5.40%	5.44%	5.40%	5.35%
31 Dec 2004	5.26%	5.47%	5.26%	5.38%
31 Mar 2005	5.59%	5.51%	5.59%	5.41%
30 June 2005		5.56%	5.04%	5.45%
30 Sep 2005		5.61%	5.29%	5.50%
31 Dec 2005		5.66%	5.13%	5.55%
31 Mar 2006		5.72%	5.45%	5.60%
30 June 2006		5.78%	5.70%	5.66%
30 Sep 2006		5.85%	5.43%	5.72%
31 Dec 2006		5.92%	5.80%	5.78%
31 Mar 2007		6.00%	5.80%	5.85%
30 June 2007		6.08%	6.16%	5.93%
30 Sep 2007		6.16%		6.00%

A bond force of 6.00% is equivalent to a nominal annual rate of 6.09%.

5.10 For CPI, the 12.25 plus 35.25 year curve explained 63.7% of the sum of squared residuals. This was the highest percentage explained of any one curve or two-curve combination; the CPI curve-fitting is significantly less close to the data than for bonds and is more period dependent. This curve is consistent with a **high** yield/ high inflationary future period similar to the past (the fitted curve reaches a maximum force of 10.3% in December 1976 and 10.9% in 2013). The period between the last two minimums of the fitted curve is 36.5 years. Figure 5.2 below graphs this curve and the actual data.

Figure 5.2 CPI – economic cycles

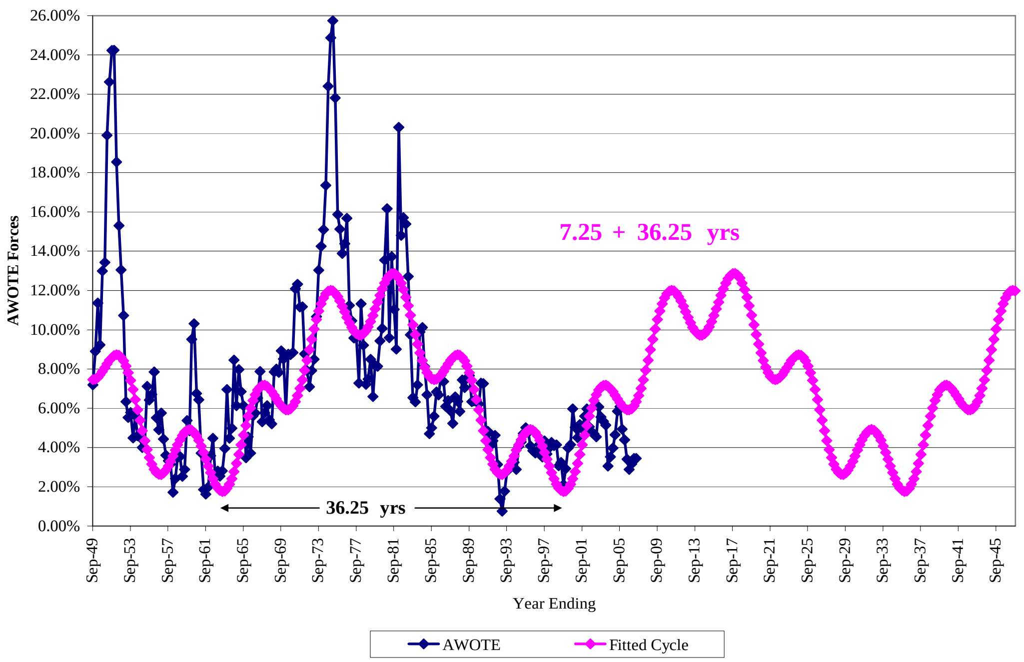


5.11 For CPI, the 24.5 plus 35.25 year curve explained 60.1% of the sum of squared residuals. This curve is consistent with a **high** yield/ high inflationary future period similar to the past (the fitted curve reaches a maximum force of 10.0% in September 1978 and 10.4% in 2018). Unlike the previous curve and the next curve, it is smooth over time (and its first differences are also smooth). The period between the last two minimums of the fitted curve is 37.25 years.

5.12 For CPI, the 6.25 plus 35.75 year curve explained 59.9% of the sum of squared residuals. This curve is consistent with a **high** yield/ high inflationary future period very similar to the past (the fitted curve reaches a maximum force of 10.6% in September 1982 and in 2014). The period between the last two minimums of the fitted curve is also 37.25 years.

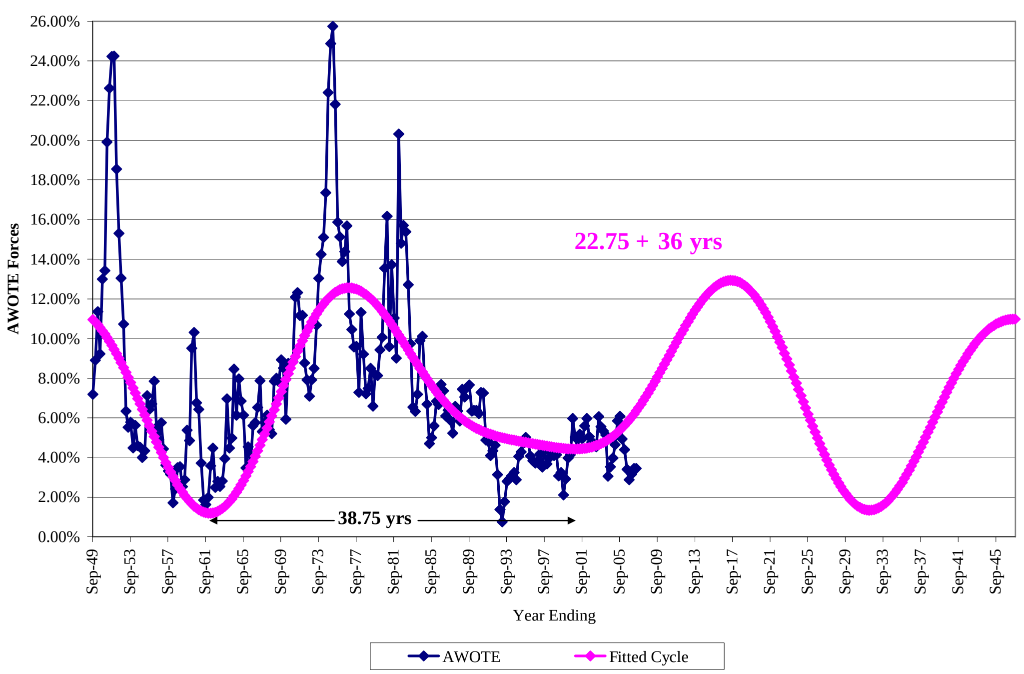
5.13 For AWOTE, the 7.25 plus 36.25 year curve explained 51.3% of the sum of squared residuals. The AWOTE curve-fitting is also significantly less close to the data than for bonds and is more period dependent. This curve is consistent with a **high** yield/ high inflationary future period very similar to the past (the fitted curve reaches a maximum force of about 12.9% in September 1981 and in 2017. The period between the last two minimums of the fitted curve is 36.25 years. Figure 5.3 on the next page graphs this curve and the actual data.

Figure 5.3 AWOTE – economic cycles



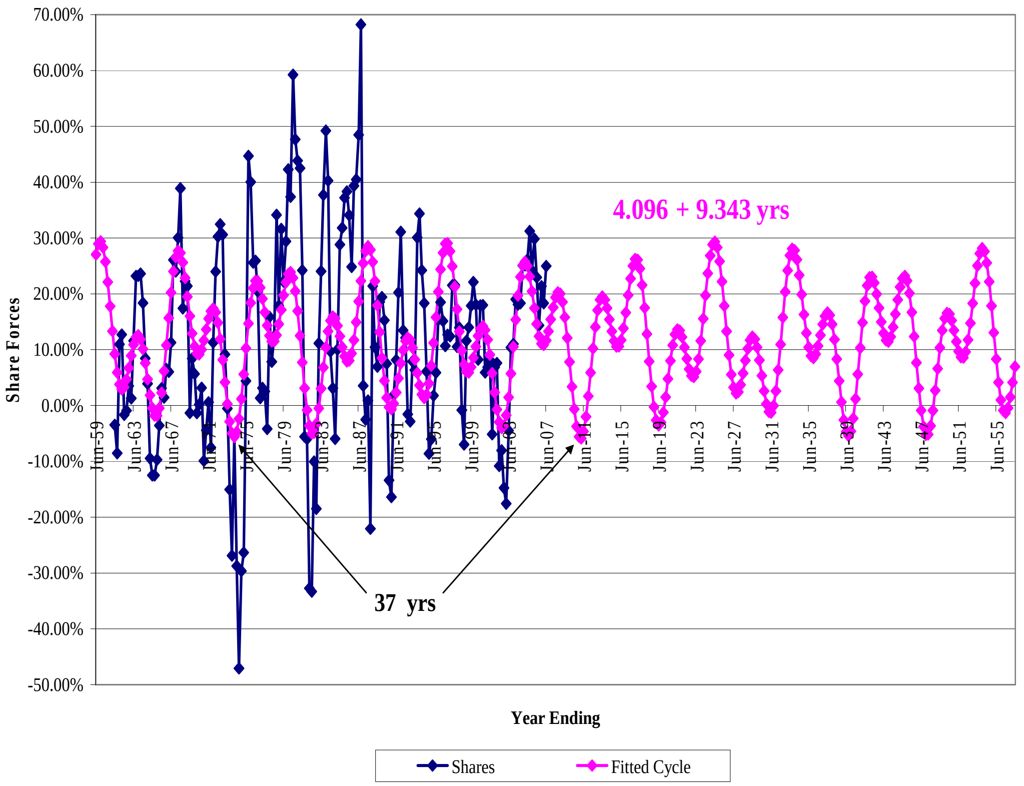
5.14 For AWOTE, the 22.75 plus 36 year curve also explained 51.3% of the sum of squared residuals. This curve is smooth and is consistent with a **high** yield/ high inflationary future period similar to the past but with a shorter wave length period (the fitted curve reaches a minimum force of 4.4% in December 2000 and a maximum force of 12.9% in June 1917, just 16.5 years later). The period between the last two minimums of the fitted curve is 38.75 years. Figure 5.4 below graphs this curve and the actual data.

Figure 5.4 AWOTE – economic cycle



5.15 For Shares, the 4.096 plus 9.343 year curve explained 24.4% of the sum of squared residuals from the mean. The Shares curve-fitting is significantly less close to the data than for bonds, CPI and AWOTE. This is primarily due to the more volatile share returns. The periods of 4.096 and 9.343 years are equal to 16 quarters plus 35 days and 37 quarters plus 34 days respectively. The period between the last two minimums of the fitted curve is 37 years. Table 5.1 also includes results for a 4 year plus 9 1/3 rd year curve. Figure 5.5 below graphs the 4.096 plus 9.343 year curve and the actual data.

Figure 5.5 Shares – economic cycles



5.16 For Shares, the 1.977 plus 4.096 year curve explained 17.9% of the sum of squared residuals from the mean. The 1.977 year period is equal to 7 quarters plus 83 days. Table 5.1 also includes results for a 2 year plus 4 year curve.

5.17 The 4 or 4.096 year cycle, evident in each of the Share curves, can perhaps be attributed to United States of America Presidential elections. This possibility is analysed in Nickles (2004) which shows that during the period 1942 to 2002 USA stock market cycles averaged 4.02 years. During this period, bull markets averaged about three years, while bear markets averaged less than a year. A more detailed investigation that includes presidential election cycles for the period from 1941 through 2000 reveals some interesting findings. Stock market lows have occurred surprisingly close to mid-year congressional elections, or approximately two years before presidential elections.

5.18 Consideration of the second last column of Table 5.1 above shows that the periods between **past** minimums have ranged from 36.25 to 38.75 years (disregarding the last three, sub-optimum, Share curves) depending on the curve-fitting and whether one's focus is on bonds, CPI, AWOTE or Shares. Much of the work in the following sections is therefore based on 38 year past periods because:

- this is close to mid-way (within 6 months) in the 36.25 to 38.75 years range,
- **where-ever possible it is desirable for the setting of long-term assumptions to analyse results over a full economic cycle**,
- the curve-fitting for bonds is clearly superior (in terms of the percentage of residuals explained and period dependency) to that for CPI, AWOTE and Shares,
- the risk margins in Section 6 are defined relative to bonds, and
- for bonds, the 34.5 plus 69 year curve (which is consistent with a low inflationary/low yield future period) had a period between past minimums of 38.25 years.

5.19 However, for some sectors (e.g. those denoted G, J, L, M and N) the 38 years chosen in section 5.15 will often extend well back into the "back-dated data" periods described in Section 4. Thus it was considered desirable, for comparison, to also consider a shorter period, at least for risk margins and coefficients of variation. A period of 24 years was chosen for this purpose because of the evidence of 23.5, 24.5 and 22.75 year cycle influences for bonds, CPI and AWOTE in the first column of Table 5.1 (and recognising that **past** cycle periods have generally been longer than the dominant cycle in the first column of that table).

6 Risk Margins

6.1 For the purposes of this paper, risk margins are calculated relative to the ten year bond rate. The risk margin is the excess of the sector annual investment return over the annualised effective point-in-time bond rate (but see paragraph 6.4 on the next page).

6.2 For sector D the point-in-time bond rate is taken at mid-year, that is 6 months prior to the end of the period over which the annual investment returns are determined (but see paragraph 6.4).

6.3 Now consider the correlation between the F sector and D sector for various lags:

Table 6.1 D and F sector correlations

24 yr Correlation of F against D lagged by Y years:										
Y	**0**	**0.25**	**0.5**	**0.75**	**1**	**1.25**	**1.5**	**1.75**	**2**
End date									
30/06/1984	.333	.402	.462	.477	.482	.484	.461	.435	.402
30/06/1985	.370	.435	.492	.507	.511	.510	.490	.466	.432
30/06/1986	.416	.474	.528	.541	.541	.5410	.525	.500	.468
30/06/1987	.468	.524	.580	.588	.589	.588	.570	.544	.515
30/06/1988	.487	.544	.602	.609	.608	.611	.594	.571	.545
30/06/1989	.477	.538	.597	.603	.599	.602	.581	.555	.528
30/06/1990	.493	.555	.616	.620	.615	.615	.595	.570	.542
30/06/1991	.519	.585	.645	.648	.642	.644	.622	.594	.562
30/06/1992	.513	.593	.667	.677	.675	.677	.655	.628	.596
30/06/1993	.452	.539	.627	.643	.643	.650	.628	.598	.566
30/06/1994	.393	.489	.588	.604	.604	.614	.591	.559	.527
30/06/1995	.441	.552	.655	.666	.660	.669	.650	.618	.584
30/06/1996	.514	.630	.735	.739	.718	.708	.673	.642	.616
30/06/1997	.362	.487	.629	.643	.625	.629	.605	.583	.556
30/06/1998	.274	.405	.539	.545	.521	.523	.498	.476	.455
30/06/1999	.295	.412	.534	.542	.521	.521	.496	.474	.453
30/06/2000	.356	.468	.578	.584	.563	.561	.539	.515	.491
30/06/2001	.366	.470	.577	.584	.562	.556	.527	.501	.481
30/06/2002	.414	.511	.610	.613	.593	.592	.565	.540	.520
30/06/2003	.430	.522	.617	.6169	.597	.598	.576	.556	.542
30/06/2004	.500	.580	.664	.656	.631	.625	.599	.580	.568
30/06/2005	.586	.648	.723	.707	.671	.658	.627	.602	.586
30/06/2006	.593	.657	.728	.713	.680	.667	.640	.619	.605
30/06/2007	.597	.660	.732	.723	.691	.675	.647	.625	.612

6.4 Table 6.1 shows that when the D sector forces are lagged by say a further 9 months (giving a total of 15 months) the correlation between D and F sector forces increases to a maximum, or close to the maximum. Further, this relationship appears to hold reasonably consistently over time. Therefore, to give greater stability to F sector risk margins, and to a lesser extent other sector risk margins, **all the following risk margins are defined as**:

(annual investment return) less (bond rate lagged 15 months)

6.5 The choice in the previous paragraph of a further 9 months (giving a total of 15 months) has no significant effect on the general level of the following margins, other than introducing some stability to the results. This greater stability of the risk margins improves the accuracy of the quadratic trend fitting and the two-year projections in steps 5 and 6 of the methodology (see Section 2). The 9 month adjustment has no practical impact on the application of the resultant margins or on their use in stochastic or other models.

6.6 Figure 6.1 and Table 6.4 show the risk margin results.

EXPLANATION

6.7 In the following figures and tables the meaning of "Period" is:

Table 6.2 Statistics for each "period"

Period	Average statistic of four periods ending:
8	30/9/98, 31/12/98, 31/3/99 and 30/6/99
7	30/9/99, 31/12/99, 31/3/00 and 30/6/00
6	30/9/00, 31/12/00, 31/3/01 and 30/6/01
5	30/9/01, 31/12/01, 31/3/02 and 30/6/02
4	30/9/02, 31/12/02, 31/3/03 and 30/6/03
3	30/9/03, 31/12/03, 31/3/04 and 30/6/04
2	30/9/04, 31/12/04, 31/3/05 and 30/6/05
1	30/9/05, 31/12/05, 31/3/06 and 30/6/06
0	30/9/06, 31/12/06, 31/3/07 and 30/6/07
-1	30/9/07, 31/12/07, 31/3/08 and 30/6/08 (projections)
-2	30/9/08, 31/12/08, 31/3/09 and 30/6/09 (projections)
-3	30/9/09, 31/12/09, 31/3/10 and 30/6/10 (projections)
-4	30/9/10, 31/12/10, 31/3/11 and 30/6/11 (projections)
-5 & -6	calculated as above, but not tabulated

6.8 The above 'projections' (or extrapolations) are based on weighted quadratic EXCEL trend functions (refer section 2.7). The Period is indicated on the x-axis of all figures.

6.9 The following codes are used:

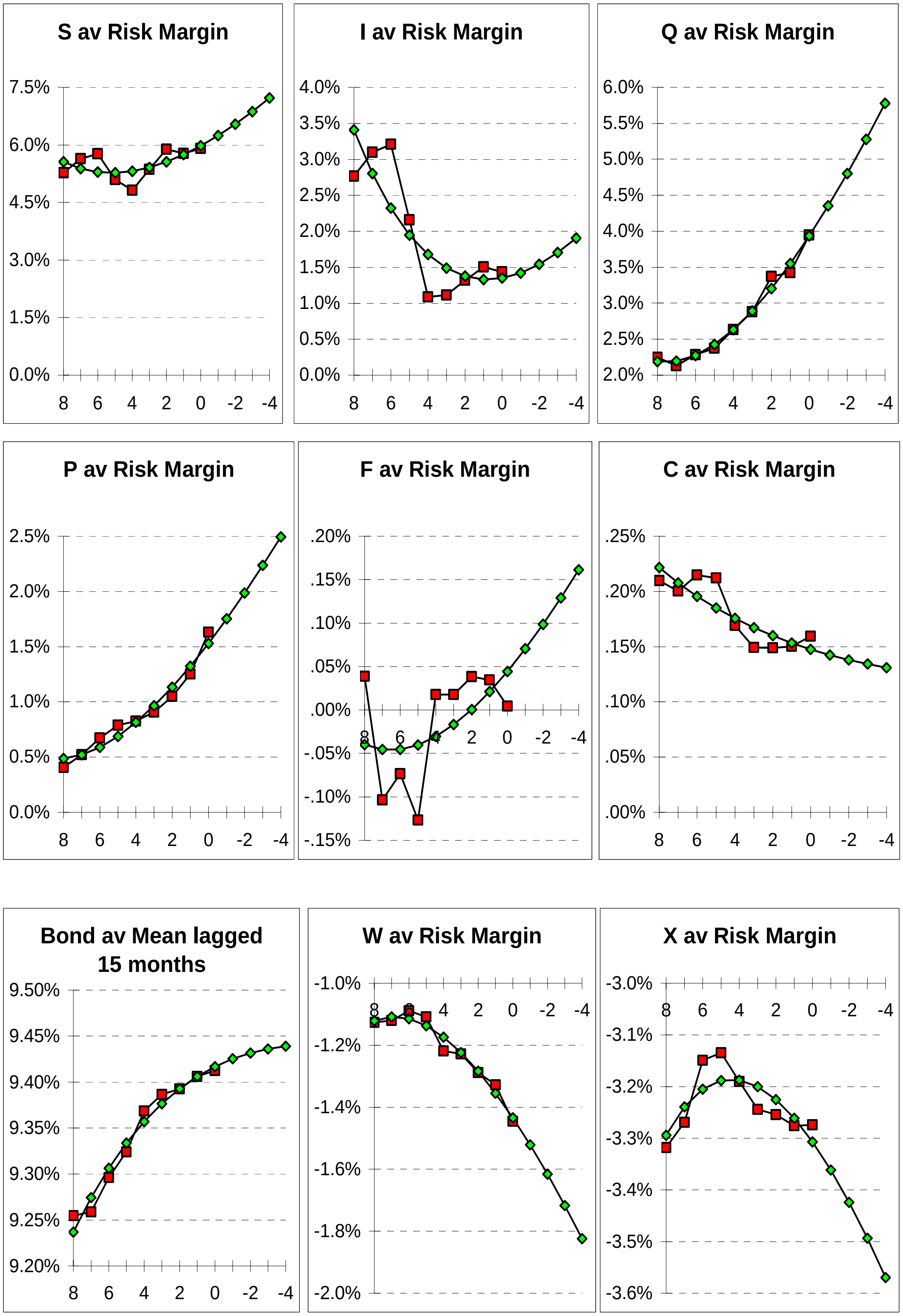
<u>Sector</u>	<u>Relates to</u>
D	ten year Bon<u>d</u> Rate in the middle of the year (i.e. lagged by 6 months)
D-9	ten year Bond Rate lagged by 15 months (i.e. as for D but lagged by a further 9 months)
X	annual increase in the Consumer Price Inde<u>x</u>
X-8	annual increase in the Consumer Price Index lagged by 8 years
W	annual increase in A<u>W</u>OTE
W-8	annual increase in AWOTE lagged by 8 years
[square]	actual results
[diamond]	trend results

6.10 The following "pointers" (refer section 2.8) are referred to:

Table 6.3 "Pointers"

Pointer	Refers to:
38 – 2	Average 38-year statistic for "Period -2"
24 – 2	Average 24-year statistic for "Period -2"
OLD	Previous assumptions based on data up to 31/3/2005
NEW	Assumptions now proposed based on the preceding three "pointers".

Figure 6.1 Risk margins over 38 years



X-axis: Period 8 = average 38 years ending 30/9/98, 31/12/98, 31/3/99 and 30/6/99
Period 0 = average 38 years ending 30/9/06, 31/12/06, 31/3/07 and 30/6/07 (see section 6.7)

Table 6.4 Risk margins over 38 years

Actual

Period	S	I	Q	P	L	M	F	G
8	5.3%	2.8%	2.2%	0.4%	1.4%	1.9%	0.0%	1.2%
7	5.6%	3.1%	2.1%	0.5%	1.4%	1.9%	-0.1%	1.1%
6	5.8%	3.2%	2.3%	0.7%	1.4%	1.9%	-0.1%	1.1%
5	5.1%	2.2%	2.4%	0.8%	1.5%	1.9%	-0.1%	1.1%
4	4.8%	1.1%	2.6%	0.8%	1.5%	1.8%	0.0%	1.1%
3	5.4%	1.1%	2.9%	0.9%	1.4%	1.8%	0.0%	1.0%
2	5.9%	1.3%	3.4%	1.0%	1.4%	1.8%	0.0%	1.0%
1	5.8%	1.5%	3.4%	1.3%	1.4%	1.8%	0.0%	0.9%
0	5.9%	1.4%	3.9%	1.6%	1.4%	1.8%	0.0%	0.9%

J	C	N	B	D-9	W	X	Period
-0.4%	0.2%	0.9%	0.3%	9.25%	-1.1%	-3.3%	8
-0.5%	0.2%	0.9%	0.3%	9.26%	-1.1%	-3.3%	7
-0.5%	0.2%	1.0%	0.3%	9.30%	-1.1%	-3.1%	6
-0.5%	0.2%	1.0%	0.3%	9.32%	-1.1%	-3.1%	5
-0.3%	0.2%	1.2%	0.2%	9.37%	-1.2%	-3.2%	4
-0.3%	0.1%	1.2%	0.2%	9.39%	-1.2%	-3.2%	3
-0.2%	0.1%	1.3%	0.2%	9.39%	-1.3%	-3.3%	2
-0.2%	0.2%	1.3%	0.2%	9.41%	-1.3%	-3.3%	1
-0.2%	0.2%	1.3%	0.2%	9.41%	-1.4%	-3.3%	0

Quadratic Trend

Period	S	I	Q	P	L	M	F	G
8	5.6%	3.4%	2.2%	0.5%	1.4%	1.9%	0.0%	1.2%
7	5.4%	2.8%	2.2%	0.5%	1.4%	1.9%	0.0%	1.1%
6	5.3%	2.3%	2.3%	0.6%	1.4%	1.9%	0.0%	1.1%
5	5.3%	1.9%	2.4%	0.7%	1.4%	1.9%	0.0%	1.1%
4	5.3%	1.7%	2.6%	0.8%	1.4%	1.9%	0.0%	1.0%
3	5.4%	1.5%	2.9%	1.0%	1.4%	1.8%	0.0%	1.0%
2	5.6%	1.4%	3.2%	1.1%	1.4%	1.8%	0.0%	1.0%
1	5.7%	1.3%	3.5%	1.3%	1.4%	1.8%	0.0%	0.9%
0	6.0%	1.3%	3.9%	1.5%	1.4%	1.8%	0.0%	0.9%
-1	6.2%	1.4%	4.4%	1.8%	1.4%	1.8%	0.1%	0.9%
-2	6.5%	1.5%	4.8%	2.0%	1.4%	1.8%	0.1%	0.8%
-3	6.9%	1.7%	5.3%	2.2%	1.4%	1.8%	0.1%	0.8%
-4	7.2%	1.9%	5.8%	2.5%	1.4%	1.8%	0.2%	0.8%

J	C	N	B	D-9	W	X	Period
-0.5%	0.2%	0.9%	0.3%	9.24%	-1.1%	-3.3%	8
-0.5%	0.2%	1.0%	0.3%	9.27%	-1.1%	-3.2%	7
-0.4%	0.2%	1.0%	0.3%	9.31%	-1.1%	-3.2%	6
-0.4%	0.2%	1.1%	0.3%	9.33%	-1.1%	-3.2%	5
-0.3%	0.2%	1.1%	0.2%	9.36%	-1.2%	-3.2%	4
-0.3%	0.2%	1.2%	0.2%	9.38%	-1.2%	-3.2%	3
-0.2%	0.2%	1.3%	0.2%	9.39%	-1.3%	-3.2%	2
-0.2%	0.2%	1.3%	0.2%	9.41%	-1.4%	-3.3%	1
-0.1%	0.1%	1.4%	0.2%	9.42%	-1.4%	-3.3%	0
-0.1%	0.1%	1.4%	0.1%	9.43%	-1.5%	-3.4%	-1
0.0%	0.1%	1.5%	0.1%	9.43%	-1.6%	-3.4%	-2
0.0%	0.1%	1.5%	0.1%	9.44%	-1.7%	-3.5%	-3
0.1%	0.1%	1.6%	0.1%	9.44%	-1.8%	-3.6%	-4

Pointers (risk margins)

	S	I	Q	P	L	M	F	G
38 -2	6.5%	1.5%	4.8%	2.0%	1.4%	1.8%	0.1%	0.8%
24 -2	5.6%	4.2%	5.8%	0.9%	1.4%	1.3%	2.4%	1.0%
OLD	4.0%	4.0%	3.5%	1.2%	1.0%	1.0%	1.0%	0.5%
NEW	4.5%	4.2%	3.7%	2.0%	1.0%	1.0%	0.5%	0.1%

J	C	N	B	D-9	W	X	
0.0%	0.1%	1.5%	0.1%	9.4%	-1.62%	-3.42%	38 -2
1.9%	-0.4%	2.0%	-0.5%	8.9%	-4.70%	-6.02%	24 -2
0.8%	-0.5%	1.0%	-0.5%	6.0%	-2.25%	-3.50%	OLD
0.3%	-0.4%	0.5%	-0.4%	6.0%	-2.20%	-3.50%	NEW

6.11 In all cases the “NEW” risk margin assumptions (except for sectors G and N – see below) are set in the range from:

a) the lowest of the first 3 pointers, to
b) the highest of the first 3 pointers.

However in many cases this is a wide range. Considerable weight was given to the “OLD” (see page 15) risk margin assumptions. The following further brief explanations should give some indication of the setting process:

S Combined with I, the average of (6.5%, 5.6%, 1.5% and 4.2%) is 4.48%

I Combined with S (because of the divergent results) but, since the average of 4.48% exceeds both “38 - 2” and “24 - 2”, the maximum (4.2%) was chosen

Q The average difference between S and Q was (6.5% - 4.8% + 5.6% - 5.8%)/2 equals 0.79%. Hence, after rounding, the Q assumption was set .8% below S, that is 4.5% less .8% = 3.7%. Also it was noted that this falls 68% of the way between the “NEW” S and P risk margins

P Set equal to “38 – 2”

F Set midway between “38 – 2” and OLD = 0.55% rounded down to 0.5%. Also noted that the 24-year average risk margin for “Period” 10.25 was 0.46%. This “Period” ends 12 years after the maximum of the fitted curve in section 5.8, thus it reflects 12 years of increasing bond rates and 12 years of decreasing bond rates

J Set equal to F less 0.2% to cover the cost of hedging (including the direct transaction costs and indirect transaction costs arising from the re-investment of cash flows from expiry of forward foreign exchange contracts). Also noted that average difference between F and J was (0.1% - 0.0% + 2.4% - 1.9%)/2 = 0.27%

C Set equal to the minimum of “38 – 2” and “24 - 2” (and also equal to B)

N Set equal to F because of the short data term (16.25 years)

D The base was set equal to 6%

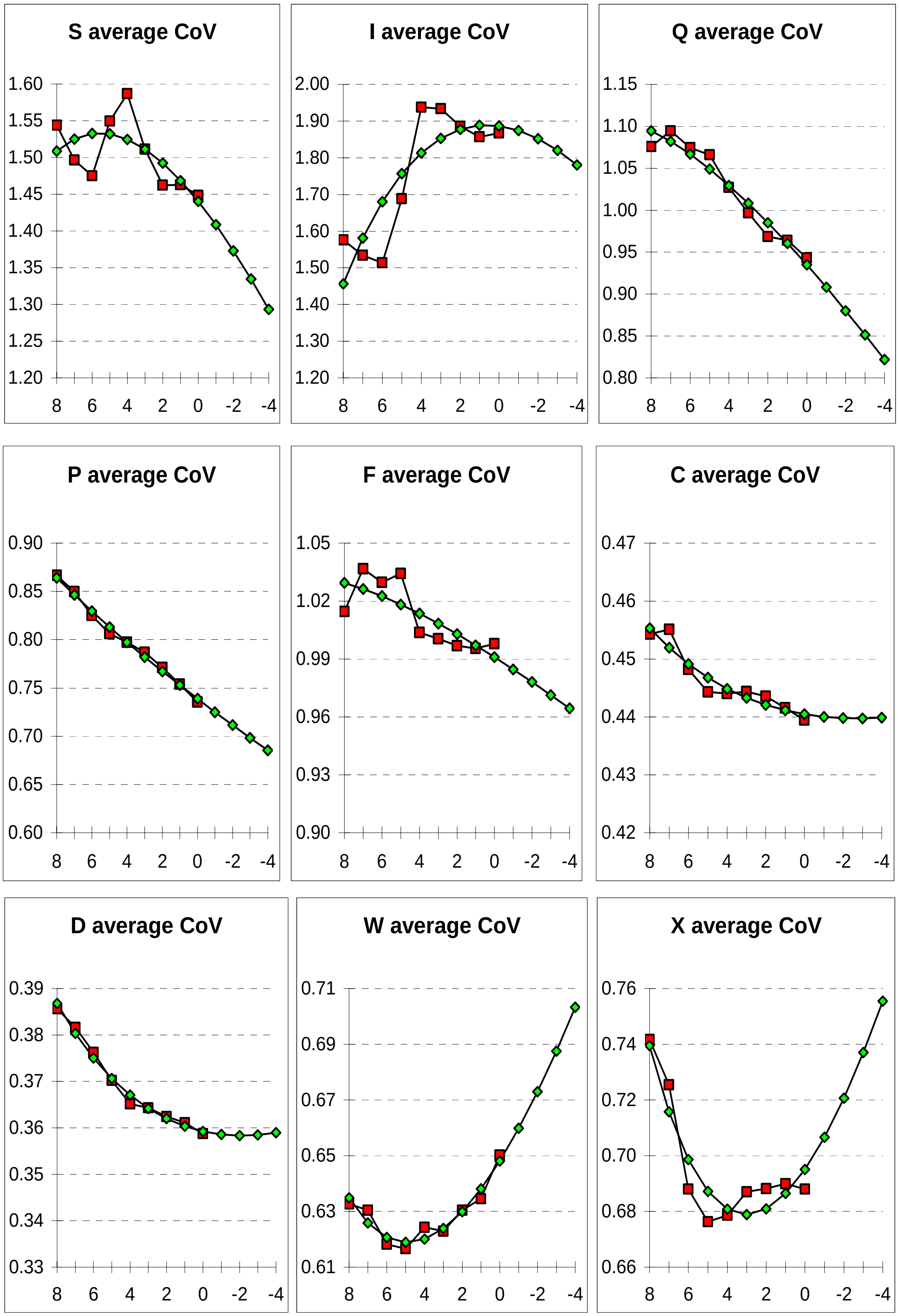
X Set equal to “38 – 2” rounded to -3.5%

W The “productivity” difference between W and X for “24 – 2” was (6.02% - 4.70%) equals 1.32%. Hence the W assumption was set 1.3% above X.

7 Coefficients of Variation

7.1 The coefficient of variation is equal to the standard deviation divided by the mean. Figure 7.1 and Table 7.1 show the results.

Figure 7.1 Coefficients of variation over 38 years



X-axis: Period 8 = average 38 years ending 30/9/98, 31/12/98, 31/3/99 and 30/6/99
Period 0 = average 38 years ending 30/9/06, 31/12/06, 31/3/07 and 30/6/07 (see section 6.7)

Table 7.1 Coefficients of variation over 38 years

Actual

Period	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
8	1.54	1.58	1.08	0.87	0.48	0.44	1.01	0.56	0.87	0.45	0.83	0.48	0.39	0.63	0.74
7	1.50	1.53	1.09	0.85	0.48	0.44	1.04	0.57	0.89	0.46	0.83	0.48	0.38	0.63	0.73
6	1.48	1.51	1.07	0.83	0.48	0.44	1.03	0.56	0.88	0.45	0.82	0.47	0.38	0.62	0.69
5	1.55	1.69	1.07	0.81	0.47	0.44	1.03	0.56	0.87	0.44	0.81	0.47	0.37	0.62	0.68
4	1.59	1.94	1.03	0.80	0.47	0.43	1.00	0.56	0.84	0.44	0.79	0.47	0.37	0.62	0.68
3	1.51	1.93	1.00	0.79	0.47	0.43	1.00	0.56	0.83	0.44	0.79	0.47	0.36	0.62	0.69
2	1.46	1.89	0.97	0.77	0.47	0.43	1.00	0.57	0.82	0.44	0.77	0.48	0.36	0.63	0.69
1	1.46	1.86	0.96	0.75	0.47	0.43	1.00	0.57	0.82	0.44	0.77	0.48	0.36	0.63	0.69
0	1.45	1.87	0.94	0.74	0.47	0.43	1.00	0.57	0.82	0.44	0.77	0.47	0.36	0.65	0.69

Quadratic Trend

Period	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
8	1.51	1.46	1.09	0.86	0.48	0.44	1.03	0.56	0.89	0.46	0.84	0.47	0.39	0.63	0.74
7	1.53	1.58	1.08	0.85	0.48	0.44	1.03	0.56	0.88	0.45	0.82	0.47	0.38	0.63	0.72
6	1.53	1.68	1.07	0.83	0.47	0.44	1.02	0.56	0.87	0.45	0.81	0.47	0.38	0.62	0.70
5	1.53	1.76	1.05	0.81	0.47	0.44	1.02	0.56	0.86	0.45	0.80	0.47	0.37	0.62	0.69
4	1.52	1.81	1.03	0.80	0.47	0.43	1.01	0.56	0.85	0.44	0.79	0.47	0.37	0.62	0.68
3	1.51	1.85	1.01	0.78	0.47	0.43	1.01	0.56	0.84	0.44	0.79	0.47	0.36	0.62	0.68
2	1.49	1.88	0.99	0.77	0.47	0.43	1.00	0.57	0.83	0.44	0.78	0.47	0.36	0.63	0.68
1	1.47	1.89	0.96	0.75	0.47	0.43	1.00	0.57	0.82	0.44	0.77	0.48	0.36	0.64	0.69
0	1.44	1.89	0.93	0.74	0.47	0.43	0.99	0.57	0.82	0.44	0.76	0.48	0.36	0.65	0.70
-1	1.41	1.87	0.91	0.72	0.47	0.43	0.98	0.58	0.81	0.44	0.76	0.48	0.36	0.66	0.71
-2	1.37	1.85	0.88	0.71	0.47	0.43	0.98	0.58	0.80	0.44	0.75	0.48	0.36	0.67	0.72
-3	1.33	1.82	0.85	0.70	0.47	0.43	0.97	0.58	0.79	0.44	0.74	0.48	0.36	0.69	0.74
-4	1.29	1.78	0.82	0.69	0.47	0.43	0.96	0.59	0.78	0.44	0.74	0.48	0.36	0.70	0.76

Pointers (CoV)

	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
38 -2	1.37	1.85	0.88	0.71	0.47	0.43	0.98	0.58	0.80	0.44	0.75	0.48	0.36	0.67	0.72
24 -2	1.22	1.69	0.76	0.89	0.54	0.55	0.62	0.62	0.52	0.56	0.65	0.61	0.48	0.34	0.74
OLD	1.600	1.600	1.221	1.000	0.472	0.472	0.700	0.600	0.691	0.490	0.800	0.527	0.384	0.600	0.700
NEW	1.533	1.539	1.299	0.900	0.500	0.500	0.723	0.607	0.698	0.500	0.800	0.536	0.417	0.552	0.720

7.2 In all cases the "NEW" coefficient of variation assumptions (except those for Property Trusts – see sector Q below) are set in the range from:

a) the lowest of the first 3 pointers, to
b) the highest of the first 3 pointers.

Considerable weight was given to the "OLD" (see page 15) coefficient of variation assumptions. The following brief explanations should give some indication of the setting process:

S Combined with I, the average of (1.37, 1.22, 1.85 and 1.69) is 1.534, rounded to 1.533 for S and 1.539 for I so that that the standard deviations for each are integral multiples of 0.1%

I Combined with S (as for risk margins)

Q The risk margin for Q was set 68% of the way between the "NEW" S and P risk margins. The CoV for Q was set 63% of the way between the "NEW" S and P CoV's because this results in a standard deviation for Q of 12.6% which is the maximum (and the latest) standard deviation recorded for periods 8 to 0 in section 8.5

P Set approximately equal to "24 – 2"

F Set equal to the average of (0.98, 0.62, 0.80 and 0.52) = 0.730, rounded to 0.723 so that that the standard deviation is an integral multiple of 0.1%

J Set equal to "OLD" (which is between "24 – 2" and "38 – 2"), rounded to 0.698 so that that standard the deviation is an integral multiple of 0.1%

C Set equal to the average of "38 – 2" and "24 – 2"

N Set equal to "OLD" (which, consistent with "24 – 2" exceeds the CoV for F)

B Consistent with C, set equal to the average of "38 – 2" and "24 – 2" (equals 0.547, rounded to 0.536 so that that the standard deviation is an integral multiple of 0.1%)

D Approximately the average of (0.36 and 0.48) =.420, rounded down to 0.417

W The average of "24 – 2" and "38 – 2" is (0.67 and 0.34) =0.508. Set equal to the average of this result and "OLD", i.e. (0.508 + 0.600)/2 =0.554 rounded to 0.552 so that that the standard deviation is an integral multiple of 0.1%

X Set equal to "38 – 2".

8 Means and Standard Deviations

8.1 When formulating assumptions for stochastic models, risk margins and coefficients of variation from the previous two sections were used to determine corresponding means and standard deviations. The resulting assumptions are summarised in Section 15. In **this section** the 38-year means and 38-year standard deviations are tabulated, **purely for historical interest.**

8.2 It should be noted that all the results in this section, and all results up to and including Section 15, are **gross of tax and gross of fees**.

8.3 Results are also included for "Balanced" and "Capital Stable" portfolios based on the following fixed asset allocations:

Sectors	S	I	Q	P	F	J	C	N
Balanced	36%	25%	7%	2%	16%	7%	5%	2%
Capital Stable	15%	9%	3%	2%	32%	12%	25%	2%

These asset allocations proportions are equal to the Mercer Pooled Fund asset-weighted average benchmarks at **30 September 2006** with some minor rounding. No allowance has been made for changes in asset allocations over time. Except for increased allocations to International Shares and reduced allocations to Direct Property, typical "Balanced" asset allocations have not changed markedly over recent decades. For example, the Mercer Pooled Fund asset-weighted average benchmarks at **28 February 1997** which were used in Grenfell (1997) were:

Sectors	S	I	Q	P	F	J	C	N
Balanced	35%	20%	4%	7%	20%	6%	8%	0%

8.4 In the following three tabulations the sectors denoted G, J, L, M and N have been omitted because they started after 1985. The 38-year statistics for these sectors are dependent on the backdating described in Section 4 and might be misleading.

8.5 For sectors which started before 1985 (and for the section 8.3 "Balanced" and "Capital Stable" portfolios) the 38-year average arithmetic means have been:

Table 8.1 Arithmetic means over 38 years

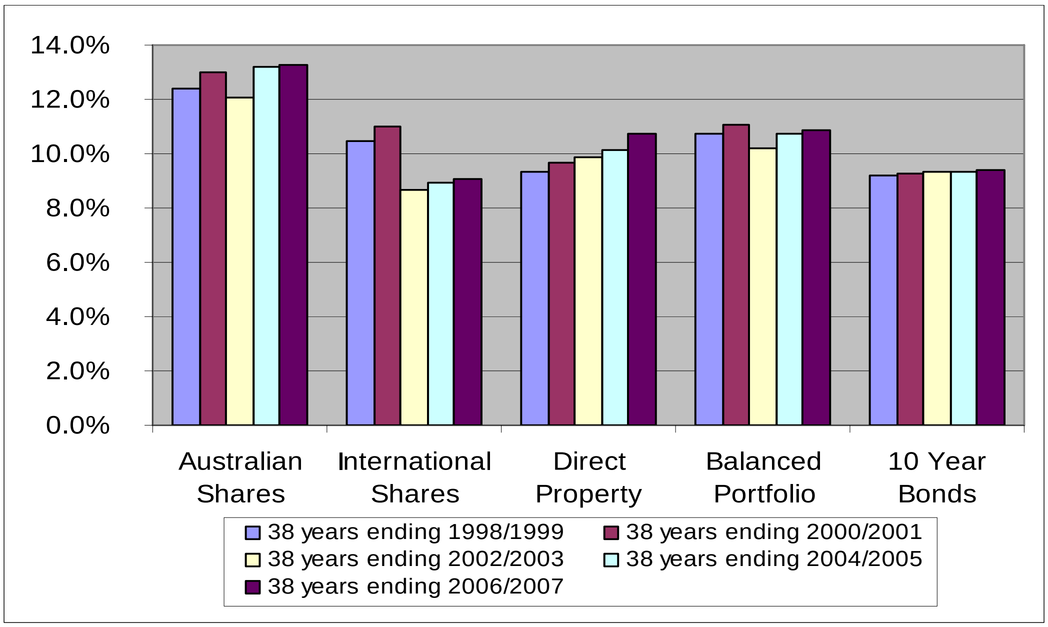
Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb
8	14.5%	12.0%	11.5%	9.7%	9.3%	9.5%	9.6%	9.3%	8.1%	5.9%	11.5%	10.0%
7	14.9%	12.4%	11.4%	9.8%	9.2%	9.5%	9.6%	9.3%	8.1%	6.0%	11.6%	10.0%
6	15.1%	12.5%	11.6%	10.0%	9.2%	9.5%	9.6%	9.3%	8.2%	6.1%	11.8%	10.1%
5	14.4%	11.5%	11.7%	10.1%	9.2%	9.5%	9.6%	9.4%	8.2%	6.2%	11.3%	9.9%
4	14.2%	10.5%	12.0%	10.2%	9.4%	9.5%	9.6%	9.4%	8.2%	6.2%	11.0%	9.9%
3	14.7%	10.5%	12.3%	10.3%	9.4%	9.5%	9.6%	9.4%	8.2%	6.1%	11.2%	10.0%
2	15.3%	10.7%	12.8%	10.4%	9.4%	9.5%	9.6%	9.4%	8.1%	6.1%	11.5%	10.1%
1	15.2%	10.9%	12.8%	10.7%	9.4%	9.6%	9.6%	9.4%	8.1%	6.1%	11.6%	10.2%
0	15.3%	10.8%	13.4%	11.0%	9.4%	9.6%	9.6%	9.4%	8.0%	6.1%	11.6%	10.2%

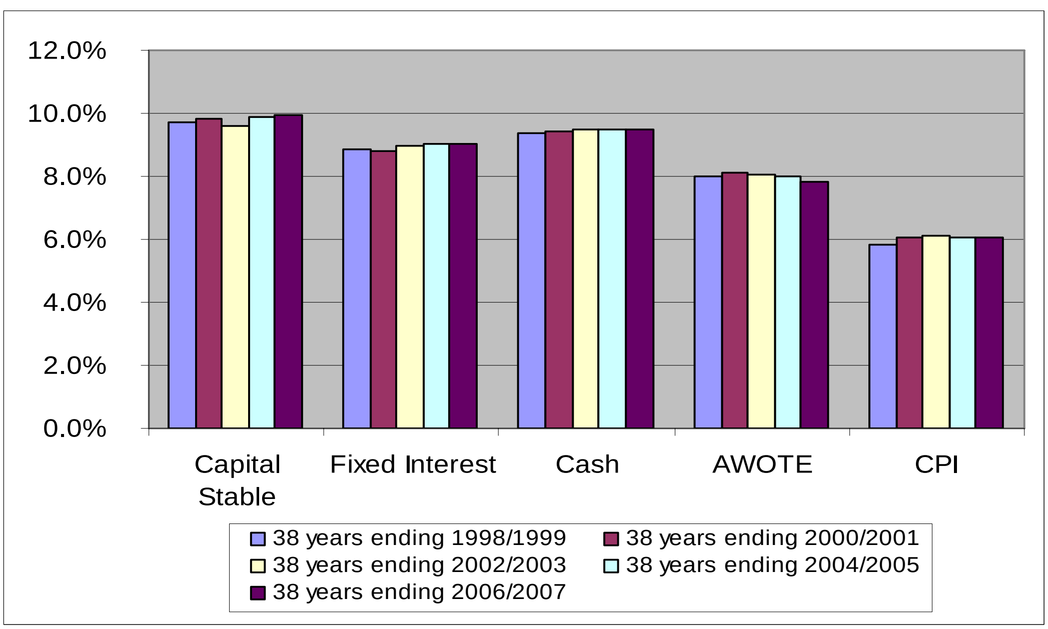
8.6 The 38-year average compound means have been:

Table 8.2 Compound means over 38 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb
8	12.4%	10.5%	10.8%	9.3%	8.9%	9.4%	9.5%	9.2%	8.0%	5.8%	10.7%	9.7%
7	12.8%	10.8%	10.7%	9.5%	8.7%	9.4%	9.5%	9.2%	8.0%	5.9%	10.9%	9.8%
6	13.0%	11.0%	10.9%	9.7%	8.8%	9.4%	9.5%	9.3%	8.1%	6.1%	11.0%	9.8%
5	12.3%	9.9%	11.0%	9.8%	8.8%	9.5%	9.5%	9.3%	8.1%	6.1%	10.5%	9.7%
4	12.1%	8.7%	11.3%	9.9%	9.0%	9.5%	9.5%	9.3%	8.0%	6.1%	10.2%	9.6%
3	12.7%	8.7%	11.6%	10.0%	9.0%	9.5%	9.5%	9.3%	8.0%	6.1%	10.5%	9.7%
2	13.2%	8.9%	12.1%	10.1%	9.0%	9.5%	9.5%	9.4%	8.0%	6.1%	10.8%	9.9%
1	13.1%	9.1%	12.2%	10.4%	9.0%	9.5%	9.5%	9.4%	8.0%	6.0%	10.8%	9.9%
0	13.2%	9.1%	12.7%	10.7%	9.0%	9.5%	9.5%	9.4%	7.9%	6.1%	10.9%	9.9%

Figure 8.1 Compound means over 38 years





8.7 The 38-year average standard deviations have been:

Table 8.3 Standard deviations over 38 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb
8	22.4%	19.0%	12.4%	8.4%	9.4%	4.3%	4.6%	3.6%	5.1%	4.4%	12.8%	7.9%
7	22.3%	18.9%	12.5%	8.3%	9.5%	4.3%	4.6%	3.5%	5.1%	4.3%	12.8%	7.8%
6	22.3%	18.9%	12.4%	8.2%	9.5%	4.3%	4.5%	3.5%	5.1%	4.2%	12.7%	7.8%
5	22.4%	19.4%	12.5%	8.2%	9.5%	4.2%	4.5%	3.5%	5.1%	4.2%	12.9%	7.9%
4	22.6%	20.3%	12.3%	8.1%	9.4%	4.2%	4.5%	3.4%	5.1%	4.2%	13.2%	7.9%
3	22.3%	20.3%	12.2%	8.1%	9.4%	4.2%	4.6%	3.4%	5.1%	4.2%	13.1%	7.8%
2	22.4%	20.2%	12.4%	8.1%	9.4%	4.2%	4.6%	3.4%	5.1%	4.2%	13.0%	7.8%
1	22.3%	20.3%	12.4%	8.0%	9.4%	4.2%	4.6%	3.4%	5.1%	4.2%	13.0%	7.8%
0	22.2%	20.3%	12.6%	8.1%	9.4%	4.2%	4.6%	3.4%	5.2%	4.2%	13.0%	7.8%

8.8 For comparison with Table 8.2, the 24-year average compound means **for all sectors** have been:

Table 8.4 Compound means over 24 years

Period	S	I	Q	P	L	M	F	G	J
8	17.3%	15.0%	14.9%	11.0%	13.1%	13.4%	12.5%	13.0%	11.6%
7	16.5%	15.1%	14.3%	10.8%	12.9%	13.2%	12.4%	12.8%	11.5%
6	16.6%	14.7%	14.5%	10.5%	12.8%	13.1%	12.4%	12.6%	11.6%
5	16.2%	13.8%	14.3%	10.3%	12.6%	12.8%	11.9%	12.3%	11.2%
4	14.4%	11.8%	13.8%	10.1%	12.3%	12.5%	11.8%	12.0%	11.2%
3	13.1%	11.9%	13.7%	9.9%	12.2%	12.3%	12.0%	11.9%	11.4%
2	12.3%	11.6%	14.1%	9.7%	11.9%	12.0%	12.1%	11.6%	11.5%
1	14.5%	13.0%	14.2%	9.5%	11.5%	11.5%	12.1%	11.3%	11.4%
0	15.4%	11.8%	14.7%	9.5%	10.9%	10.9%	11.4%	10.6%	10.8%

C	N	B	D	W	X	Balncd	CapStb	Period
11.2%	13.0%	11.3%	11.2%	7.6%	6.5%	14.9%	13.1%	8
11.0%	12.7%	11.1%	11.0%	7.1%	6.1%	14.5%	12.8%	7
10.9%	12.5%	11.0%	10.9%	6.8%	5.7%	14.4%	12.8%	6
10.7%	12.0%	10.8%	10.7%	6.6%	5.5%	13.9%	12.4%	5
10.5%	11.8%	10.5%	10.5%	6.5%	5.3%	12.7%	11.8%	4
10.3%	11.8%	10.3%	10.3%	6.3%	4.9%	12.3%	11.6%	3
10.0%	12.1%	10.0%	10.1%	5.9%	4.6%	12.0%	11.5%	2
9.6%	12.2%	9.5%	9.7%	5.5%	4.4%	13.1%	11.8%	1
9.1%	11.3%	9.1%	9.3%	5.1%	4.0%	13.0%	11.4%	0

8.9 For each of the 12 sectors in Table 8.2, the average 38-year returns are more stable than the average 24-year returns in Table 8.4. This is partly due to the longer averaging periods underlying Table 8.2 but, importantly, it is also due to Table 8.4 comprising primarily the down-slope of the economic cycles identified in Section 5.

8.10 For the reasons explained in the previous paragraph, the 38-year statistics form better indicators of long-term returns. Where-ever possible it is desirable for the setting of long-term assumptions to analyse results over a full economic cycle

8.11 It should be noted that all individual results quoted in Sections 6 to 14, including those in this section, are the average of 4 annual results at quarterly intervals, **not individual results** for 38 or 24 years ending on one date.

9 Outliers

9.1 Outliers can introduce bias into the sample means and standard deviations and in particular can have a very significant affect on skewness and kurtosis. To illustrate the sensitivity of results to one extreme outlier, consider the following 38-year annual forces for years ending 30 September, 31 December, 31 March and 30 June for the F sector (Australian fixed interest securities). The period chosen corresponds to "Period 0" in section 2.5.

Table 9.1 F sector annual forces

Year Ending	30-Sep	31-Dec	31-Mar	30-Jun	Year Ending	30-Sep	31-Dec	31-Mar	30-Jun
1969/70	-1.3%	-3.2%	-7.1%	-4.3%	1988/89	12.7%	10.5%	5.8%	5.5%
1970/71	-3.5%	-3.0%	1.5%	6.7%	1989/90	6.9%	13.0%	15.8%	16.1%
1971/72	7.0%	12.7%	20.4%	20.2%	1990/91	15.0%	18.0%	22.1%	22.5%
1972/73	19.9%	14.0%	6.1%	-6.2%	1991/92	27.2%	26.2%	18.6%	21.6%
1973/74	-22.5%	-21.9%	-21.4%	-18.7%	1992/93	15.9%	9.5%	16.7%	13.4%
1974/75	-1.3%	-1.3%	-0.7%	9.0%	1993/94	15.5%	15.3%	6.1%	-1.7%
1975/76	4.7%	5.7%	4.8%	5.7%	1994/95	-5.3%	-5.3%	2.0%	11.2%
1976/77	10.0%	7.6%	7.3%	7.3%	1995/96	15.4%	17.7%	12.9%	9.9%
1977/78	7.4%	16.8%	20.2%	20.6%	1996/97	11.9%	12.0%	12.3%	16.2%
1978/79	21.0%	13.5%	6.6%	1.0%	1997/98	14.7%	11.5%	14.8%	10.6%
1979/80	1.2%	-0.4%	-1.2%	3.2%	1998/99	8.9%	9.5%	6.6%	3.5%
1980/81	1.1%	0.4%	5.9%	5.5%	1999/00	1.6%	-0.8%	1.9%	6.4%
1981/82	3.2%	7.5%	4.5%	7.9%	2000/01	6.3%	11.5%	11.9%	7.5%
1982/83	20.0%	21.2%	18.8%	18.6%	2001/02	9.7%	4.8%	1.8%	4.7%
1983/84	15.0%	15.1%	18.7%	18.3%	2002/03	4.8%	7.9%	8.9%	9.5%
1984/85	15.2%	11.4%	11.9%	12.0%	2003/04	6.2%	3.9%	4.9%	3.2%
1985/86	10.6%	7.9%	17.6%	17.5%	2004/05	5.3%	6.9%	5.0%	7.5%
1986/87	14.3%	22.2%	14.0%	15.3%	2005/06	5.9%	5.9%	6.8%	3.7%
1987/88	22.8%	18.0%	19.0%	17.4%	2006/07	4.5%	3.0%	3.7%	4.0%

Statistics for all 38 years:					Average across
8.63%	8.56%	8.57%	8.74%	mu	8.63%
9.17%	9.03%	8.74%	8.50%	sigma	8.86%
-81%	-91%	-93%	-76%	skewness	-85%
240%	228%	230%	162%	kurtosis	215%

Modified statistics for all 38 years:					Average across
8.63%	8.56%	8.57%	8.74%	mu	8.63%
8.70%	9.03%	8.74%	8.50%	sigma	8.75%
-55%	-91%	-93%	-76%	skewness	-79%
143%	228%	230%	162%	kurtosis	191%

9.2 The annual force for the year ending 30 September 1973 was –22.5% (boxed). This represents 3.394 standard deviations below the mean. If the F sector had a normal distribution this value has about 1 chance in 2900 of occurring. It compares with the most extreme outliers for Australian Shares, in September 1974, of 2.688 standard deviations below the mean and, in September 1987, of 2.533 standard deviations above the mean. The F sector annual forces for the years ending 31 December 1973, 31 March 1974 and 30 June 1974 all represents more than 3 standard deviations below the mean.

9.3 The F sector skewness and kurtosis for the 38 years ending 2006/2007 averaged **-85% and 215%** respectively. However the outlier of -22.5% (for the year ending 30 September 1973) significantly affects these results.

9.4 To indicate the sensitivity of the skewness and kurtosis to the 30 September 1973 outlier, if this force had been 4% greater (i.e. –18.5%) and the force for the previous year had been 4% less (i.e. 15.9%), then the modified average skewness and kurtosis for the 38 years ending 2006/2007 would have been **-79% and 191%** respectively.

9.5 In the previous paragraph, the F sector average skewness for "Period 0" increased from -85% to -79% (and the average kurtosis reduced from 215% to 191%) with only a 4% change in one outlier. These reductions have **not** been reflected in the following two sections, or in subsequent sections, because the unmodified results do not seem to be abnormal. For example, the unmodified skewness for the 38 years ending 30 September 2006 is –81% which is within the range of the –93% to –76% results for other 38-year periods ending 2006/2007. Even the unmodified kurtosis for the 38 years ending 30 September 2006 is only marginally outside (by 10%) the range of results for other 38-year periods ending 2006/2007.

9.6 It can also be noted from Table 9.1 that the average sigma reduced from 8.86% to 8.75% with only a 4% change in the one outlier. This change (of about 0.1%) was not considered significant since in equates to the usual round-off for quoted annual standard deviations.

9.7 The conclusion drawn from paragraphs 9.1 to 9.6 above is that the analysis has sufficient data and the methodology is sufficiently robust, to not be overly concerned about outliers.

9.8 Chan, Ng and Tong (2006) make the following relevant observation:

"Whether or not it is appropriate to adjust the data for the outliers depends on the purpose to which the model so derived will be used. If the model is to be used in an application for which extreme stochastic fluctuations are less important (e.g. to ensure that premiums are adequate in most, but not extreme scenarios), then it may be preferable to use a model based on risk-adjusted data. If, however, the model is to be used in an application for which extreme stochastic fluctuations are important (such as pricing catastrophe risks or ensuring that investment guarantee reserves are sufficient to keep an insurance company solvent in all but the most extreme scenarios), then a model which is sympathetic to outliers in the data ought to be used."

The conclusion in paragraph 9.7 above reflects the author's view that most stochastic modelling applications will be closer to the latter than the former.

10 Skewness

10.1 Skewness characterises the degree of asymmetry of a distribution around its mean. Positive skewness indicates a distribution with an asymmetric tail extending towards more positive values. Negative skewness indicates a distribution with an asymmetric tail extending towards more negative values.

10.2 As explained in section 2.11, the skewness results shown in this section, and the kurtosis, cross-correlation and auto-correlation results shown in Sections 11 to 14 are based on annual forces.

10.3 When using stochastic models it is often desired to convert assumptions and/or model results from rates to forces and vice versa and to combine the results from a number of sectors to form or approximate results for a portfolio or composite sector. Such changes are often based on the assumption, or an implied assumption, of a normal distribution for the forces and a lognormal distribution for the rates. However if non-zero values for skewness and kurtosis are used as input to a stochastic model, then formulae (based on a normal distribution for the forces and a lognormal distribution for the rates) will not hold.

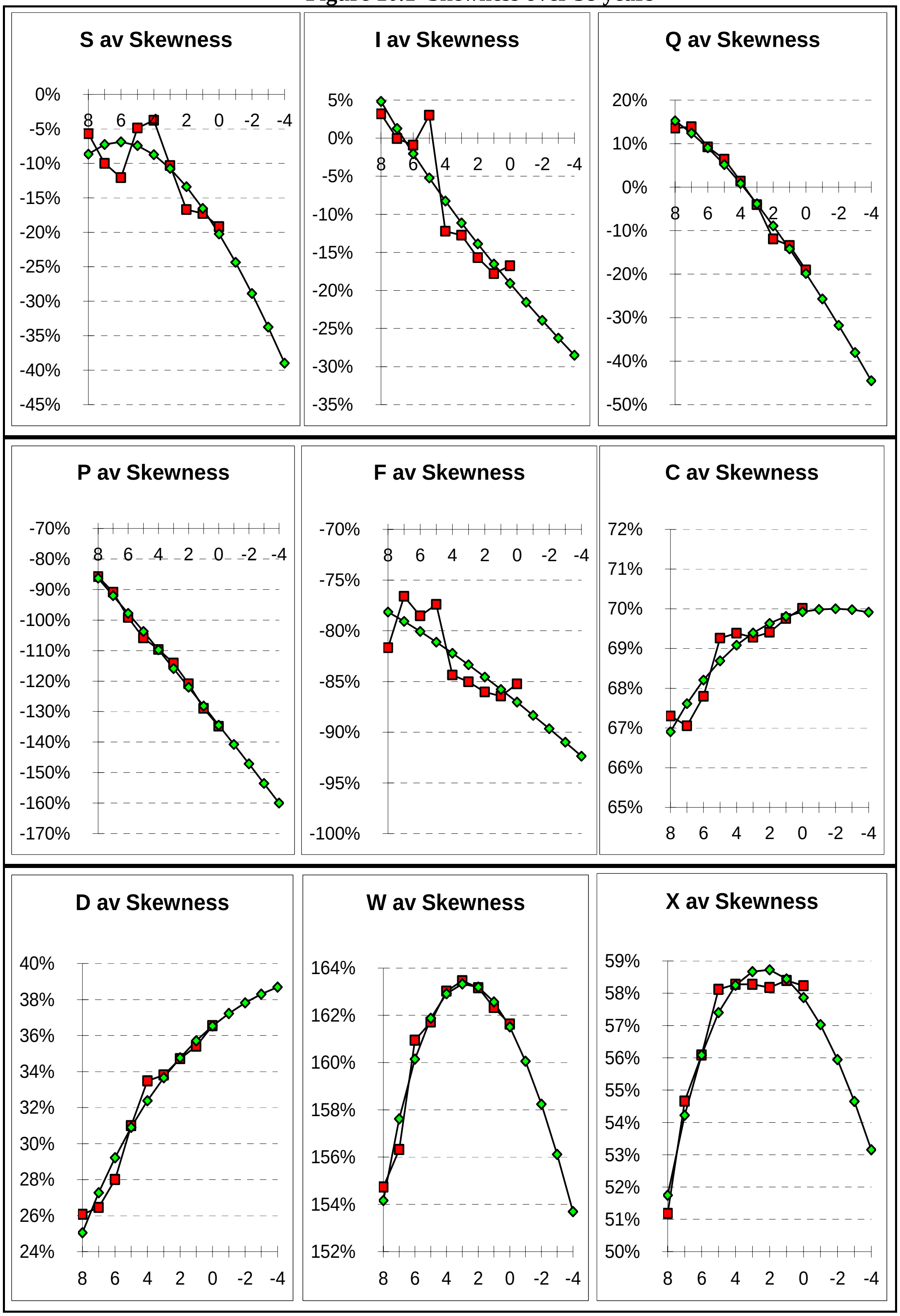
10.4 For example, when using dependent variables and non-zero values for skewness and kurtosis, it is not easy to determine:

- Long-term estimates of standard deviations of **portfolio or composite sector** returns from long-term estimates of standard deviations of returns for **each individual sector**, or
- Long-term estimates of means and/or standard deviations of **forces** for each individual sector from long-term estimates of means and/or standard deviations of **rates** for each individual sector.

10.5 However, under suitable assumptions, formulae can be determined to resolve these issues. For example, Appendix A of Grenfell (2005) contains formulae for statistical conversion between rates and forces of return for use when skewness and/or kurtosis are not zero.

10.6 The following Figure 10.1 and Table 10.1 illustrate the skewness results for each sector.

Figure 10.1 Skewness over 38 years



X-axis: Period 8 = average 38 years ending 30/9/98, 31/12/98, 31/3/99 and 30/6/99
Period 0 = average 38 years ending 30/9/06, 31/12/06, 31/3/07 and 30/6/07 (see section 6.7)

Table 10.1 Skewness over 38 years

Actual

Period	S	I	Q	P	L	M	F	G
8	-6%	3%	14%	-86%	53%	65%	-82%	9%
7	-10%	0%	14%	-91%	53%	65%	-77%	10%
6	-12%	-1%	9%	-99%	53%	66%	-79%	9%
5	-5%	3%	6%	-106%	53%	67%	-77%	9%
4	-4%	-12%	1%	-110%	55%	68%	-84%	9%
3	-10%	-13%	-4%	-114%	55%	68%	-85%	8%
2	-17%	-16%	-12%	-121%	55%	68%	-86%	9%
1	-17%	-18%	-13%	-129%	54%	68%	-86%	9%
0	-19%	-17%	-19%	-135%	54%	68%	-85%	10%

J	C	N	B	D	W	X	Period
-68%	67%	-33%	83%	26%	155%	51%	8
-64%	67%	-33%	82%	26%	156%	55%	7
-68%	68%	-38%	83%	28%	161%	56%	6
-69%	69%	-39%	83%	31%	162%	58%	5
-78%	69%	-45%	83%	33%	163%	58%	4
-81%	69%	-45%	82%	34%	163%	58%	3
-84%	69%	-50%	82%	35%	163%	58%	2
-85%	70%	-53%	82%	35%	162%	58%	1
-85%	70%	-53%	82%	37%	162%	58%	0

Trend

Period	S	I	Q	P	L	M	F	G
8	-9%	5%	15%	-86%	53%	65%	-78%	10%
7	-7%	1%	12%	-92%	53%	65%	-79%	9%
6	-7%	-2%	9%	-98%	54%	66%	-80%	9%
5	-7%	-5%	5%	-104%	54%	67%	-81%	9%
4	-9%	-8%	1%	-110%	54%	67%	-82%	9%
3	-11%	-11%	-4%	-116%	54%	68%	-83%	9%
2	-13%	-14%	-9%	-122%	54%	68%	-85%	9%
1	-17%	-17%	-14%	-128%	54%	68%	-86%	9%
0	-20%	-19%	-20%	-134%	54%	68%	-87%	10%
-1	-24%	-22%	-26%	-141%	54%	68%	-88%	11%
-2	-29%	-24%	-32%	-147%	54%	68%	-90%	11%
-3	-34%	-26%	-38%	-154%	54%	68%	-91%	12%
-4	-39%	-28%	-45%	-160%	53%	68%	-92%	13%

J	C	N	B	D	W	X	Period
-64%	67%	-31%	83%	25%	154%	52%	8
-67%	68%	-34%	83%	27%	158%	54%	7
-70%	68%	-38%	83%	29%	160%	56%	6
-73%	69%	-41%	83%	31%	162%	57%	5
-76%	69%	-44%	83%	32%	163%	58%	4
-79%	69%	-46%	82%	34%	163%	59%	3
-82%	70%	-49%	82%	35%	163%	59%	2
-84%	70%	-52%	82%	36%	163%	58%	1
-87%	70%	-54%	81%	37%	162%	58%	0
-90%	70%	-57%	81%	37%	160%	57%	-1
-92%	70%	-59%	81%	38%	158%	56%	-2
-95%	70%	-61%	80%	38%	156%	55%	-3
-97%	70%	-64%	80%	39%	154%	53%	-4

10.7 The quadratic trend Period –2 results are the skewness assumptions that are tabulated in Section 15.

11 Kurtosis

11.1 Kurtosis characterizes the relative peakedness or flatness of a distribution compared to the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution.

11.2 The Institute's Guidance Note 552, "Economic Valuations" (July 2004), states at paragraph 5.3:

"Stochastic valuation techniques

In applying stochastic valuation techniques, the Member should be satisfied that the economic and probability models are reasonable and make appropriate allowance for the impact of correlations and that the tails of distributions are suitably modelled."

11.3 The tails of a distribution are significantly influenced by the kurtosis, and to a lesser extent the skewness.

11.4 Figure 11.1 and Table 11.1, on the next two pages, illustrate the kurtosis results for each sector.

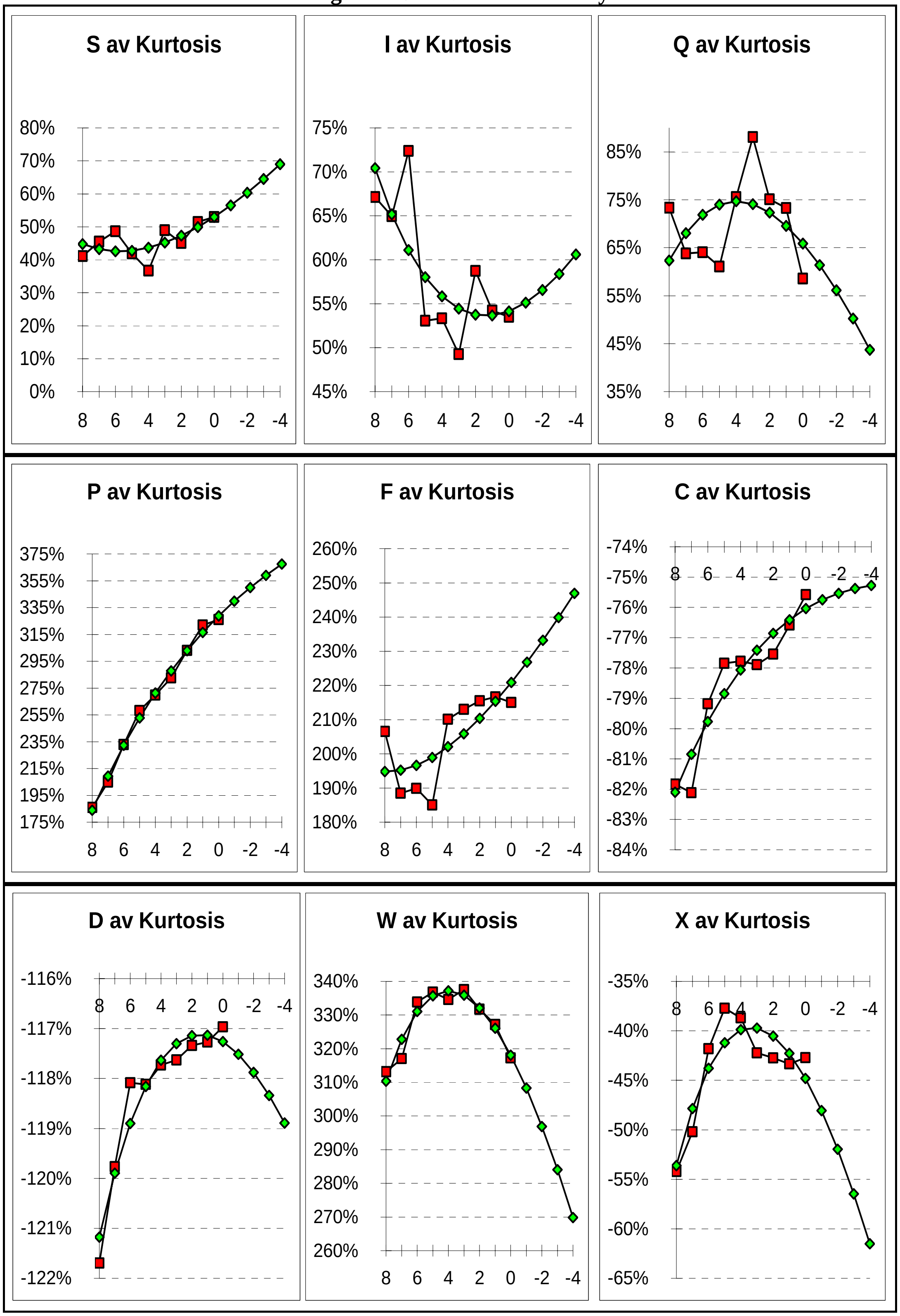
11.5 For illustrative purposes it is informative to classify and rank the skewness and kurtosis assumptions (tabulated in Section 15) as follows:

(a) **minimal** between -60% and +60% skewness or kurtosis.
(b) **moderate** between -61% and -200% skewness or kurtosis, or between -61% and -200% skewness or kurtosis.
(c) **extreme** over 200% skewness or kurtosis.

Sector		Skewness	Kurtosis	Classification
Q	Property Trusts	-32%	56%	Minimal
S	Australian Shares	-29%	60%	Minimal
I	International Shares	-24%	57%	Minimal
G	Semi govt (0 to 3 yrs)	11%	-46%	Minimal
L	Loans (floating rate)	54%	-35%	Minimal
X	CPI	56%	-52%	Minimal
CapStb	Capital Stable	-76%	183%	Moderate
Balncd	Balanced	-63%	73%	Moderate
N	Inflation Linked Bonds	-59%	71%	Moderate
D	10 Year Bonds	38%	-118%	Moderate
M	Mortgages	68%	-72%	Moderate
C	Cash	70%	-76%	Moderate
B	90 Day Bills	81%	-38%	Moderate
P	Direct Property	-147%	350%	Extreme
J	Int'l Fixed Interest	-92%	229%	Extreme
F	Fixed Interest	-90%	233%	Extreme
W	AWOTE	158%	297%	Extreme

Although this classification is only illustrative, the number of assumptions that fall outside the minimal classification sounds a warning for those who use normal or lognormal models in relation to investment performance.

Figure 11.1 Kurtosis over 38 years



X-axis: Period 8 = average 38 years ending 30/9/98, 31/12/98, 31/3/99 and 30/6/99
Period 0 = average 38 years ending 30/9/06, 31/12/06, 31/3/07 and 30/6/07 (see section 6.7)

Table 11.1 Kurtosis over 38 years

Actual

Period	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
8	41%	67%	73%	186%	-44%	-76%	207%	-25%	165%	-82%	16%	-36%	-122%	313%	-54%
7	46%	65%	64%	205%	-44%	-76%	188%	-34%	149%	-82%	12%	-36%	-120%	317%	-50%
6	49%	72%	64%	233%	-43%	-74%	190%	-33%	153%	-79%	19%	-33%	-118%	334%	-42%
5	42%	53%	61%	258%	-38%	-73%	185%	-33%	155%	-78%	22%	-33%	-118%	337%	-38%
4	37%	53%	76%	270%	-34%	-73%	210%	-27%	184%	-78%	41%	-34%	-118%	335%	-39%
3	49%	49%	88%	283%	-33%	-73%	213%	-32%	193%	-78%	41%	-35%	-118%	338%	-42%
2	45%	59%	75%	303%	-33%	-73%	216%	-35%	201%	-78%	51%	-36%	-117%	332%	-43%
1	51%	54%	73%	322%	-36%	-72%	217%	-37%	201%	-77%	57%	-36%	-117%	327%	-43%
0	53%	53%	59%	326%	-33%	-72%	215%	-41%	203%	-76%	55%	-35%	-117%	317%	-43%

Trend

Period	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
8	45%	70%	62%	184%	-46%	-76%	195%	-30%	151%	-82%	9%	-35%	-121%	310%	-54%
7	43%	65%	68%	209%	-42%	-75%	195%	-30%	156%	-81%	16%	-35%	-120%	323%	-48%
6	43%	61%	72%	232%	-40%	-74%	197%	-30%	162%	-80%	23%	-34%	-119%	331%	-44%
5	43%	58%	74%	253%	-37%	-74%	199%	-30%	169%	-79%	29%	-34%	-118%	336%	-41%
4	44%	56%	75%	271%	-36%	-73%	202%	-32%	176%	-78%	35%	-34%	-118%	337%	-40%
3	45%	54%	74%	288%	-35%	-73%	206%	-33%	184%	-77%	41%	-34%	-117%	336%	-40%
2	47%	54%	72%	303%	-34%	-72%	210%	-35%	192%	-77%	47%	-35%	-117%	332%	-41%
1	50%	54%	70%	316%	-34%	-72%	215%	-37%	201%	-76%	53%	-35%	-117%	326%	-42%
0	53%	54%	66%	329%	-34%	-72%	221%	-40%	210%	-76%	59%	-36%	-117%	318%	-45%
-1	56%	55%	61%	340%	-34%	-72%	227%	-43%	219%	-76%	65%	-37%	-118%	308%	-48%
-2	60%	57%	56%	350%	-35%	-72%	233%	-46%	229%	-76%	71%	-38%	-118%	297%	-52%
-3	65%	58%	50%	359%	-36%	-72%	240%	-49%	238%	-75%	76%	-39%	-118%	284%	-56%
-4	69%	61%	44%	367%	-37%	-72%	247%	-53%	248%	-75%	82%	-40%	-119%	270%	-62%

11.6 The quadratic trend Period –2 results are the kurtosis assumptions that are tabulated in Section 15.

12 Cross-correlations

12.1 The methodology used for risk margins, CoV's, skewness and kurtosis is detailed in Section 2. However for correlations it is essential that projected statistics are entirely consistent with each other over sectors (for cross-correlations) or over time (for auto-correlations). To obtain this greater consistency, the methodology was therefore slightly varied for correlations.

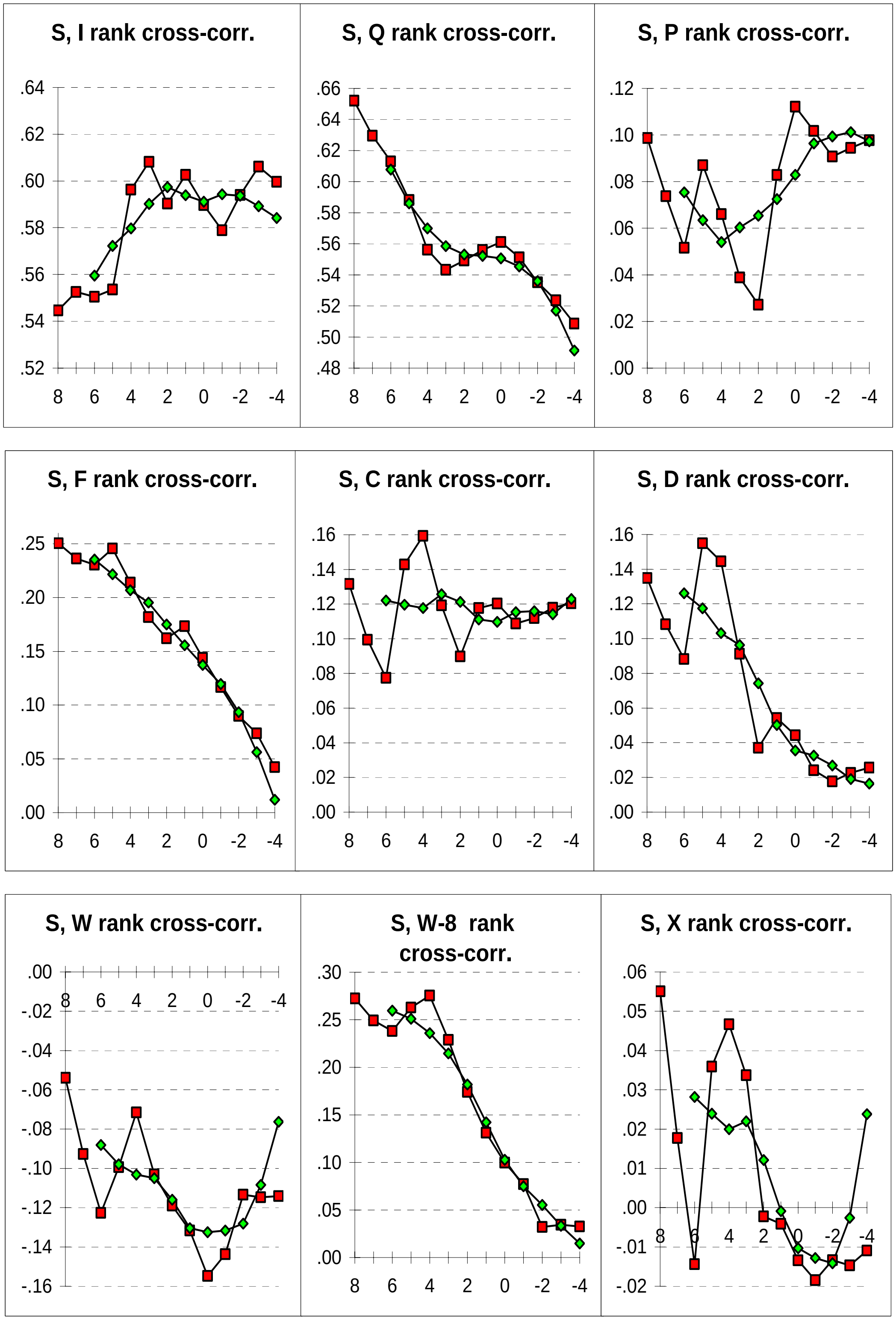
12.2 After step 2 in Section 2 the database was extended by adding on 6 years of "projected" annual forces for each quarter date. For this purpose the "projected" values were obtained from the projected trend risk margins (for example see Table 6.4) by adding back the fixed Bond base value (6% per annum) and converting to forces.

12.3 After the adjustments in 12.2 above, the extended database of each sector covered 52 years for each of the four quarter end-dates. Steps 3 and 4 in Section 2 were then applied and step 5 was replaced by central 5-year running averages.

12.4 It is well documented that sample correlations can be distorted by outliers and non-zero skewness and kurtosis. To overcome this problem, for cross-correlations, 52-year ranks of the extended database forces were used as well as the actual forces. Thus two sets of cross-correlations were obtained, one based on ranks and one based on actual (i.e. actual plus backdated plus projected) forces. Correlations based on the former will be referred to as "rank" correlations, and correlations based on the latter will be referred to as "standard" correlations. In the *Austmod* investment simulation model, the rank cross-correlations are used because they are not distorted by outliers or by non-zero skewness or kurtosis (because the ranks of the forces are uniformly distributed) and the impacts of skewness and kurtosis are separately allowed for in the model.

12.5 Figure 12.1 and Table 12.1 illustrate the above process and show the results obtained for the S (Shares) sector rank cross-correlations.

Figure 12.1 Cross-correlations over 38 years



X-axis: Period 8 = average 38 years ending 30/9/98, 31/12/98, 31/3/99 and 30/6/99
Period 0 = average 38 years ending 30/9/06, 31/12/06, 31/3/07 and 30/6/07 (see section 6.7)

Table 12.1 Rank cross-correlations over 38 years

Actual

Period	S I	S Q	S P	S L	S M	S F	S G	S J
8	.54	.65	.10	.27	.23	.25	.31	.26
7	.55	.63	.07	.24	.20	.24	.28	.24
6	.55	.61	.05	.23	.18	.23	.27	.23
5	.55	.59	.09	.27	.26	.25	.31	.25
4	.60	.56	.07	.27	.27	.21	.30	.20
3	.61	.54	.04	.23	.22	.18	.26	.17
2	.59	.55	.03	.18	.18	.16	.22	.15
1	.60	.56	.08	.17	.20	.17	.21	.17
0	.59	.56	.11	.16	.18	.14	.18	.14

S C	S N	S B	S D	S W	S X	S W-8	S X-8	Period
.13	.25	.12	.13	-.05	.06	.27	.20	8
.10	.23	.09	.11	-.09	.02	.25	.18	7
.08	.21	.06	.09	-.12	-.01	.24	.16	6
.14	.24	.14	.15	-.10	.04	.26	.19	5
.16	.20	.16	.14	-.07	.05	.28	.21	4
.12	.17	.13	.09	-.10	.03	.23	.17	3
.09	.16	.09	.04	-.12	.00	.17	.11	2
.12	.19	.09	.05	-.13	.00	.13	.07	1
.12	.16	.09	.04	-.15	-.01	.10	.05	0

Five-year central averages

Period	S I	S Q	S P	S L	S M	S F	S G	S J
6	.56	.61	.08	.26	.23	.24	.29	.24
5	.57	.59	.06	.25	.23	.22	.28	.22
4	.58	.57	.05	.24	.22	.21	.27	.20
3	.59	.56	.06	.23	.22	.20	.26	.19
2	.60	.55	.07	.20	.21	.18	.23	.17
1	.59	.55	.07	.18	.19	.16	.20	.15
0	.59	.55	.08	.15	.17	.14	.18	.13
-1	.59	.55	.10	.14	.17	.12	.16	.12
-2	.59	.54	.10	.12	.16	.09	.14	.09
-3	.59	.52	.10	.09	.14	.06	.10	.05
-4	.58	.49	.10	.06	.13	.01	.08	.00

S C	S N	S B	S D	S W	S X	S W-8	S X-8	Period
.12	.23	.11	.13	-.09	.03	.26	.19	6
.12	.21	.12	.12	-.10	.02	.25	.18	5
.12	.20	.12	.10	-.10	.02	.24	.17	4
.13	.19	.12	.10	-.10	.02	.21	.15	3
.12	.18	.11	.07	-.12	.01	.18	.12	2
.11	.16	.10	.05	-.13	.00	.14	.08	1
.11	.15	.09	.04	-.13	-.01	.10	.04	0
.12	.14	.09	.03	-.13	-.01	.07	.01	-1
.12	.11	.09	.03	-.13	-.01	.06	-.01	-2
.11	.08	.08	.02	-.11	.00	.03	-.03	-3
.12	.05	.09	.02	-.08	.02	.01	-.05	-4

12.6 The five year central average Period –2 results are the S sector cross-correlation assumptions that are tabulated in the first of the two half-page matrices in Table 15.3.

12.7 Table 15.3 contains both the **rank** cross-correlation assumptions and the **standard** cross-correlation assumptions (defined in paragraph 12.4). Both of these were calculated using the methodology described in paragraphs 12.2 and 12.3.

12.8 Sweeting (2007) states,

"… Correlation gives one measure of the linkages, but assumes that the relationships between the marginal distributions are constant, whatever the levels of those distributions. It is only appropriate if the marginal distributions are jointly elliptical. Other measures such as Kendall's tau and Spearman's rho, which do not depend on the marginal distributions, might be more appropriate. …

If the degree of association is greater at extreme values of the marginal distributions (as it often is), then the above approaches understate the tail risk in the aggregate distribution. The solution is to use copulas. … However, although there have been a number of papers on the subject of copulas, their use still appears to be limited. This is not least because the dependency structure is not straightforward. Since the dependency structure is largely defined by what happens at the tails, and as always, the tail of the distribution contains only a few observations, the form of the copula functions is not always clear. …"

12.9 To give some insight into the extent to which the degree of association is greater at extreme values of the marginal distributions and when this might occur, Table 12.2 tabulates the **difference** between the standard cross-correlation assumptions and the rank cross-correlation assumptions.

Table 12.2 Standard less rank cross-correlations

	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
S	0	.03	.08	-.05	-.01	-.07	.17	.04	.15	-.05	.10	-.04	.03	-.17	-.07
I	.03	0	.09	-.06	.14	.04	.12	.11	.12	.11	.08	.07	.05	-.10	-.04
Q	.08	.09	0	-.07	.11	.09	.18	.14	.15	.06	.09	.06	.13	-.17	-.06
P	-.05	-.06	-.07	0	.01	.06	-.12	-.08	-.15	-.02	-.06	-.02	.00	-.20	-.14
L	-.01	.14	.11	.01	0	.02	.13	.05	.13	.09	.12	.03	.04	-.13	-.05
M	-.07	.04	.09	.06	.02	0	.01	.01	.07	.09	.04	.04	.03	-.08	.02
F	.17	.12	.18	-.12	.13	.01		.08	.03	.04	.02	-.02	.02	-.21	-.13
G	.04	.11	.14	-.08	.05	.01	.08	0	.10	.04	.07	-.02	.03	-.25	-.14
J	.15	.12	.15	-.15	.13	.07	.03	.10	0	.07	.06	.02	.04	-.23	-.12
C	-.05	.11	.06	-.02	.09	.09	.04	.04	.07	0	.04	.01	.10	-.14	-.07
N	.10	.08	.09	-.06	.12	.04	.02	.07	.06	.04	0	.00	.03	-.14	-.05
B	-.04	.07	.06	-.02	.03	.04	-.02	-.02	.02	.01	.00	0	.06	-.14	-.08
D	.03	.05	.13	.00	.04	.03	.02	.03	.04	.10	.03	.06	0	-.13	-.04
W	-.17	-.10	-.17	-.20	-.13	-.08	-.21	-.25	-.23	-.14	-.14	-.14	-.13	0	-.01
X	-.07	-.04	-.06	-.14	-.05	.02	-.13	-.14	-.12	-.07	-.05	-.08	-.04	-.01	0

12.10 The boxed cells in the upper right triangular matrix of Table 12.2 are all the differences greater than or equal to 0.15. Only 4 of the 105 cells are boxed **-** these indicate where the degree of association is likely to be greater at extreme values of the marginal distributions.

12.11 The boxed cells in the lower left triangular matrix of Table 12.2 are all the differences less than or equal to **-**0.15. Only 7 of the 105 cells are boxed **-** these indicate where the degree of association is likely to be less at extreme values of the marginal distributions. Expressed another way, these indicate where diversification benefits may exist at the tails.

12.12 94 of the 105 cells in each triangular matrix are not boxed (in either triangle) **-** these indicate where the degree of association at the tails is likely to be similar to that for the central values.

13 Leads and Lags

13.1 Consider the correlation between the D sector (Bonds) and CPI for various lags:

Table 13.1 D sector and CPI correlation

24 yr Correlation of D against CPI lagged by Y years:

Y End date	0	6.25	6.75	7	7.5	7.75	8	8.75	9	10
30/6/84	.687	.804	.816	.813	.801	.790	.771			
30/6/85	.591	.790	.805	.812	.815	.812	.799	.766	.744	
30/6/86	.550	.820	.808	.804	.801	.800	.794	.789	.776	.704
30/6/87	.513	.828	.848	.852	.832	.815	.797	.783	.771	.729
30/6/88	.459	.822	.843	.853	.861	.862	.852	.801	.778	.726
30/6/89	.414	.818	.838	.848	.8571	.857	.851	.842	.827	.732
30/6/90	.347	.817	.841	.850	.858	.856	.848	.837	.828	.790
30/6/91	.238	.763	.828	.852	.868	.863	.850	.834	.824	.791
30/6/92	.124	.726	.797	.832	.877	.885	.874	.825	.802	.764
30/6/93	.060	.667	.751	.792	.853	.869	.871	.846	.814	.708
30/6/94	.081	.607	.666	.704	.772	.794	.799	.810	.797	.685
30/6/95	.090	.579	.641	.675	.732	.751	.756	.770	.758	.665
30/6/96	.080	.523	.609	.652	.713	.734	.732	.711	.690	.587
30/6/97	.227	.478	.551	.593	.678	.702	.705	.678	.647	.479
30/6/98	.441	.521	.563	.590	.634	.652	.643	.624	.598	.429
30/6/99	.579	.603	.639	.656	.677	.667	.645	.576	.537	.389
30/6/00	.675	.653	.694	.713	.732	.726	.702	.604	.558	.367
30/6/01	.674	.686	.736	.756	.782	.779	.761	.679	.634	.408
30/6/02	.703	.693	.748	.773	.805	.804	.788	.721	.682	.471
30/6/03	.742	.728	.764	.784	.809	.810	.800	.752	.719	.536
30/6/04	.765	.783	.794	.806	.828	.832	.822	.771	.747	.610
30/6/05	.758	.803	.816	.820	.832	.835	.830	.823	.806	.666
30/6/06	.713	.791	.807	.814	.819	.824	.819	.828	.829	.773
30/6/07	.688	.758	.786	.799	.812	.815	.815	.817	.814	.782

13.2 Also consider the correlation between the D sector and AWOTE for various lags:

Table 13.2 D sector and AWOTE correlation

24 yr Correlation of D against AWOTE lagged by Y years:

Y End date	0	6.75	7	7.5	7.75	8	8.5	8.75	9	10
30/6/84	.527	.786	.811	.835	.822	.789				
30/6/85	.445	.751	.779	.806	.806	.789	.756	.747	.728	
30/6/86	.330	.716	.743	.777	.773	.762	.732	.737	.730	.690
30/6/87	.242	.707	.729	.761	.759	.750	.719	.721	.716	.698
30/6/88	.169	.695	.722	.758	.753	.741	.713	.718	.714	.688
30/6/89	.096	.674	.700	.741	.738	.730	.711	.715	.711	.692
30/6/90	.003	.682	.707	.737	.722	.712	.690	.703	.702	.683
30/6/91	-.085	.647	.687	.746	.742	.722	.679	.681	.679	.670
30/6/92	-.147	.605	.649	.726	.730	.734	.710	.706	.672	.611
30/6/93	-.137	.561	.617	.705	.707	.704	.685	.695	.690	.571
30/6/94	-.057	.550	.597	.673	.677	.683	.668	.673	.652	.571
30/6/95	-.008	.554	.597	.669	.661	.663	.639	.643	.625	.522
30/6/96	.006	.520	.581	.686	.688	.684	.649	.635	.607	.475
30/6/97	.130	.520	.568	.664	.669	.680	.664	.662	.623	.446
30/6/98	.357	.526	.568	.656	.663	.667	.642	.637	.608	.458
30/6/99	.512	.566	.598	.665	.660	.658	.633	.632	.600	.456
30/6/00	.600	.615	.641	.692	.687	.683	.653	.644	.610	.471
30/6/01	.609	.651	.673	.724	.721	.718	.685	.676	.644	.496
30/6/02	.611	.671	.703	.757	.754	.747	.716	.709	.679	.530
30/6/03	.613	.686	.714	.768	.768	.768	.742	.735	.706	.577
30/6/04	.635	.707	.728	.777	.779	.781	.757	.749	.723	.611
30/6/05	.645	.783	.777	.779	.776	.782	.775	.779	.761	.636
30/6/06	.607	.765	.766	.782	.781	.783	.767	.775	.770	.710
30/6/07	.605	.762	.757	.770	.767	.787	.793	.801	.777	.711

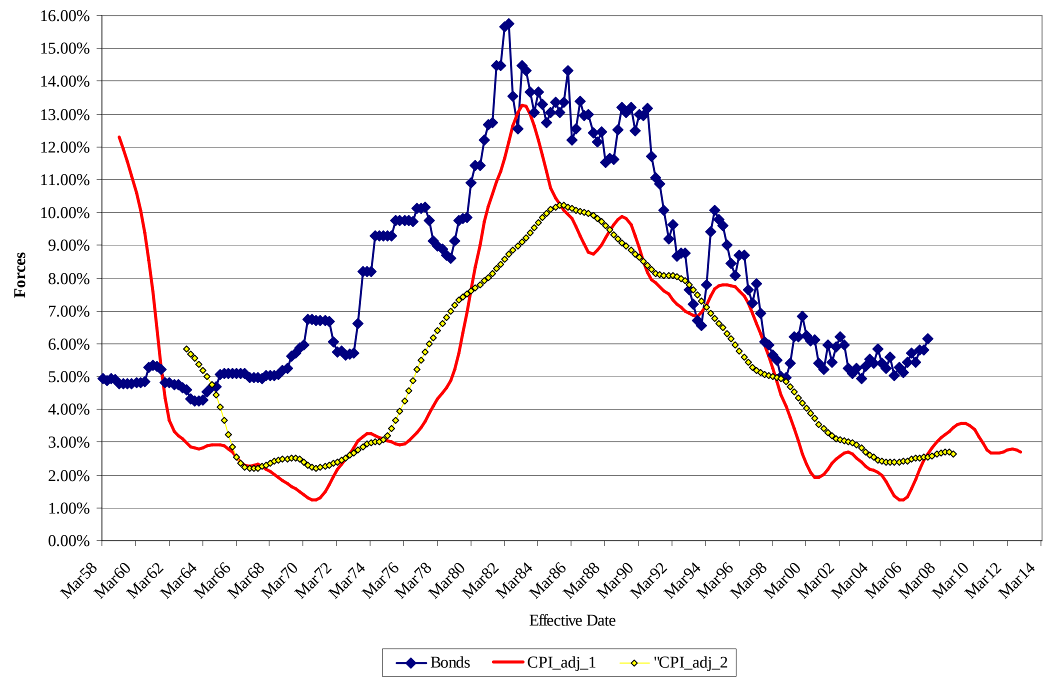
13.3 Tables 13.1 and 13.2 have some hidden columns but the hidden columns do not contain any maximums. Both tables are calculated from the main database, which contains annual forces at quarterly intervals (refer section 3.3).

13.4 Table 13.1 and Table 13.2 show that when CPI and AWOTE forces are lagged by about 7.5 to 8.5 years their correlation with D forces increases to a maximum. Further, these relationships appear to hold reasonably consistently over time. It should be noted that the 8-year lagged cross-correlations are also far more stable than the non-lagged results.

13.5 Sections 3.1 and 6.9 explain that the "D sector" is by definition equal to the 10-year bond rate lagged by 6 months. Hence the 8-year lagged cross-correlations in Table 13.1 correspond to CPI lagged 7.5 years relative to the actual bond rate.

13.6 It is difficult to find an economic justification as to why changes in CPI and AWOTE appear to lead changes in bond rates by about 7.5 years (with a correlation of between 64% and 87%). Figure 13.1 below explores this relationship for CPI. It includes the actual bond forces from Figure 5.1 together with adjusted CPI forces. If the actual CPI forces are averaged over 4 years they will be lagged on average by 2 years. Thus the first adjusted CPI curve (labelled "CPI_adj_1") is CPI forces averaged over 4 years and then advanced by 5.5 years, giving a total lead averaging 7.5 years. If the actual CPI forces are averaged over 12 years they will be lagged on average by 6 years. Thus the second adjusted CPI curve (labelled "CPI_adj_2") is CPI forces averaged over 12 years and then advanced by 1.5 years, again giving a total lead averaging 7.5 years. Both the adjusted CPI force curves visually show a reasonably close relationship with the bond forces.

Figure 13.1 Bond and CPI Forces



13.7 The increase in correlation with the D sector as the lag changes from zero to say 8 years is significant for both CPI and AWOTE. The W-8 and X-8 columns of Table 13.3 show that a similar significant increase occurs when lagged CPI (sector X) and lagged AWOTE (sector W) are correlated against the forces of most other sectors.

13.8 The following table compares a wider range of correlations both with and without the 8 year lag.

Table 13.3 Alternative cross-correlation assumptions

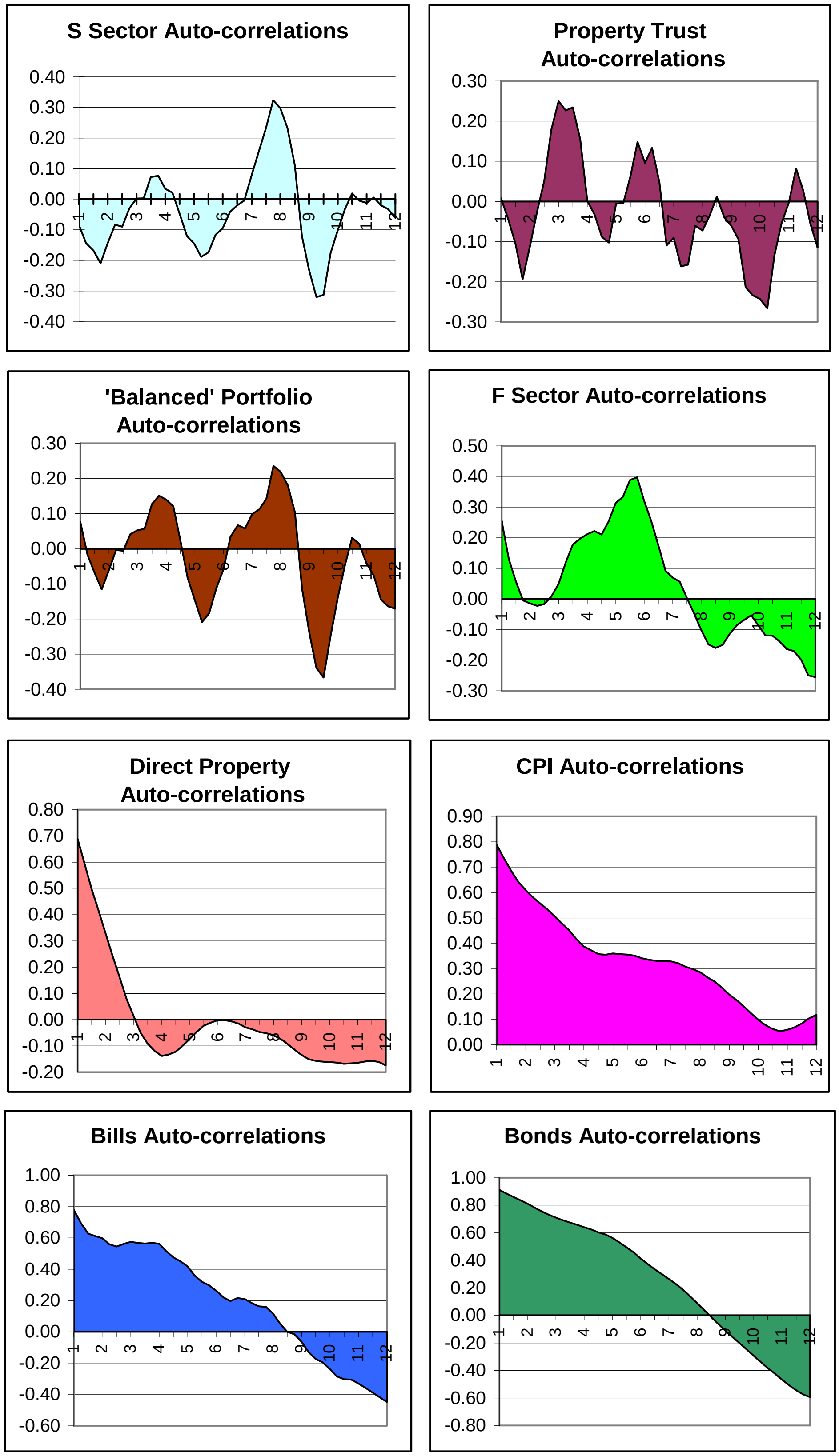
	RANK CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)					**STANDARD** CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)			
	W	X	W-8	X-8		W	X	W-8	X-8
S	-.13	-.01	.06	-.01	S	-.30	-.08	-.05	-.01
I	-.09	.02	.18	.15	I	-.19	-.02	.21	.25
Q	-.13	-.13	.00	-.04	Q	-.30	-.19	.10	.09
P	.46	.56	.30	.15	P	.26	.42	.29	.17
L	.33	.42	.73	.62	L	.20	.37	.73	.70
M	.43	.51	.76	.64	M	.35	.53	.77	.72
F	-.05	-.04	.41	.41	F	-.26	-.17	.38	.40
G	.36	.41	.70	.59	G	.11	.27	.68	.65
J	.02	.01	.41	.41	J	-.21	-.11	.41	.41
C	.57	.67	.61	.45	C	.43	.60	.74	.67
N	.10	.14	.36	.26	N	-.04	.09	.39	.32
B	.56	.67	.64	.49	B	.42	.59	.70	.63
D	.47	.54	.76	.69	D	.34	.50	.78	.79
W	1	.85	.36	.13	W	1	.84	.26	.06
X	.85	1	.35	.14	X	.84	1	.35	.14
W-8	.36	.35	1	.80	W-8	.26	.35	1	.80
X-8	.13	.14	.80	1	X-8	.06	.14	.80	1

13.9 The results in this section suggest that stochastic investment models might be more realistic if allowance for say an eight year lag is built in for CPI and AWOTE. Some further comments on this are included in Appendix B.

14 Auto-correlations

14.1 Auto-correlations vary significantly between the 15 sectors examined in this paper. To produce the following auto-correlation charts the annual forces for each sector were tabulated at quarterly intervals for the forty year period from 30/6/1967 to 30/6/2007. From this, forty year or 160 quarter database, auto-correlations were calculated for lags of 4 to 48 quarters for seven of the sectors plus the "Balanced" portfolio composite sector. Figure 14.1 shows the results. The x-axis of these charts is the lag in years.

Figure 14.1 Auto-correlations over 40 years



14.2 Auto-correlations are also dependent on the period over which they are measured. For some distributions this applies even if the underlying distribution has been consistent over time. For example, if annual forces follow say an exact 35 year sine-curve over an infinite number of years, then auto-correlations measured over any say 30, 35 or 40 year periods will produce different auto-correlations for any given lag.

14.3 When the *Austmod* investment simulation model was first developed in the early 1990's, it was based on 32 year projections partly because at that time less than 30 years' investment performance data was available for the main sectors. In view of the longer data period now available the model now produces 40 year projections.

14.4 For these reasons it was decided to measure auto-correlations over the following 40 year periods:

Table 14.1 Periods for auto-correlations

Period	Average of four periods ending:
6	30/9/00, 31/12/00, 31/3/01 and 30/6/01
5	30/9/01, 31/12/01, 31/3/02 and 30/6/02
4	30/9/02, 31/12/02, 31/3/03 and 30/6/03
3	30/9/03, 31/12/03, 31/3/04 and 30/6/04
2	30/9/04, 31/12/04, 31/3/05 and 30/6/05
1	30/9/05, 31/12/05, 31/3/06 and 30/6/06
0	30/9/06, 31/12/06, 31/3/07 and 30/6/07

14.5 The results were then projected forward using the same methodology (but with two less periods) to that used for cross-correlations in Section 12.

14.6 Appendix B explains that the auto-correlations for Australian Shares and 10 Year Bond Rates "form two extremes". The following Figures 14.2 and 14.3 and Tables 14.2 and 14.3 therefore illustrate the auto-correlations results for only the S and D sectors.

Figure 14.2 S sector auto-correlations over 40 years

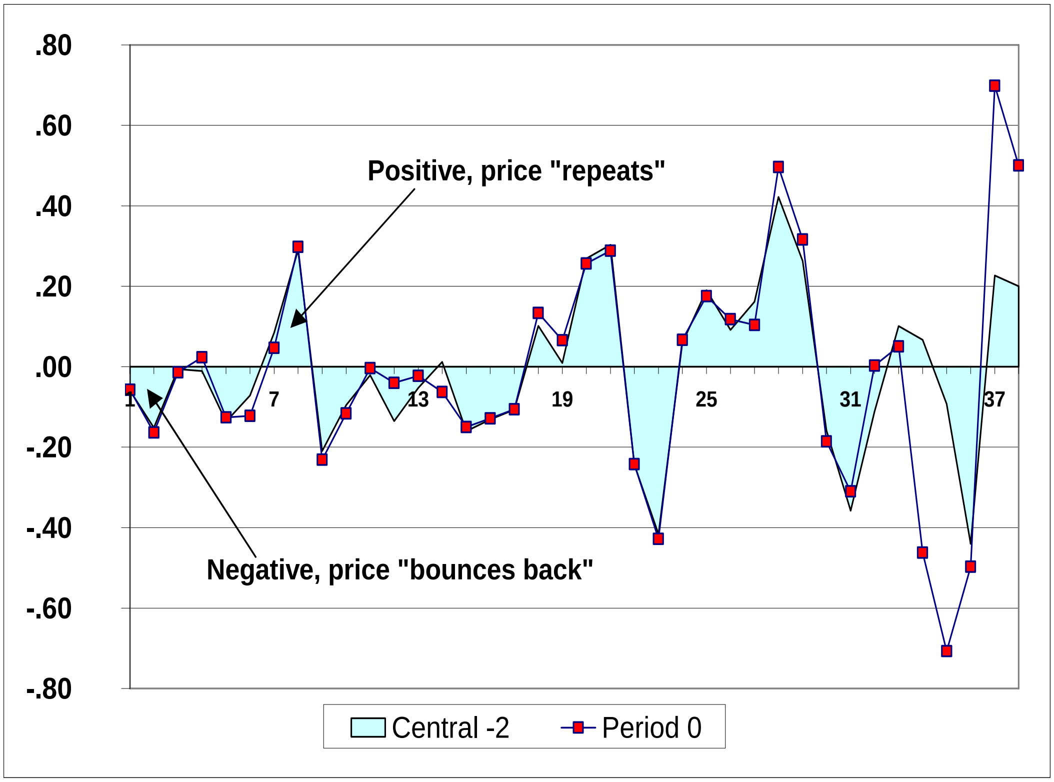


Table 14.2 S sector auto-correlations over 40 years

Period:	6	5	4	3	2	1	0	-1	-2	-3	-4	-2
Lag												central
1	-.09	-.09	-.07	-.06	-.07	-.07	-.06	-.06	-.06	-.07	-.06	-.06
2	-.18	-.18	-.18	-.17	-.19	-.17	-.16	-.15	-.15	-.15	-.14	-.15
3	.01	.01	-.01	-.01	.01	-.01	-.01	.01	.01	.01	-.04	-.01
4	.09	.09	.08	.07	.07	.04	.02	.01	.00	-.02	-.07	-.01
5	-.08	-.09	-.08	-.08	-.09	-.12	-.13	-.12	-.13	-.16	-.15	-.14
6	-.12	-.11	-.11	-.10	-.12	-.12	-.12	-.08	-.05	-.04	-.06	-.07
7	.03	.04	.03	.04	.05	.06	.05	.10	.10	.09	.09	.08
8	.33	.31	.32	.32	.32	.30	.30	.28	.28	.28	.30	.29
9	-.08	-.08	-.10	-.11	-.16	-.23	-.23	-.22	-.22	-.21	-.17	-.21
10	-.13	-.13	-.11	-.09	-.15	-.11	-.12	-.11	-.11	-.09	-.05	-.10
11	-.02	-.01	-.04	-.01	.03	.00	.00	-.02	-.02	.00	-.06	-.02
12	.02	-.01	.00	-.02	-.05	-.04	-.04	-.10	-.12	-.19	-.22	-.14
13	-.05	-.07	-.06	-.05	-.05	-.04	-.02	-.06	-.07	-.08	-.03	-.05
14	-.15	-.14	-.13	-.12	-.12	-.07	-.06	.01	.03	.05	.04	.01
15	-.15	-.19	-.18	-.19	-.15	-.12	-.15	-.15	-.18	-.18	-.14	-.16
16	.08	.03	-.03	-.06	-.03	-.11	-.13	-.17	-.15	-.13	-.08	-.13
17	.09	.09	.04	.03	-.07	-.12	-.11	-.11	-.12	-.09	-.10	-.11
18	-.05	-.06	-.02	.03	.07	.14	.13	.09	.10	.10	.09	.10
19	.02	.03	-.06	-.04	.02	.05	.07	.00	.00	-.01	.00	.01
20	.02	.06	.07	.03	.05	.16	.26	.28	.28	.29	.24	.27

Period 0: average 40 years ending 30/9/06, 31/12/06, 31/3/07 and 30/6/07 (see section 6.7)

Figure 14.3 Bond auto-correlations over 40 years

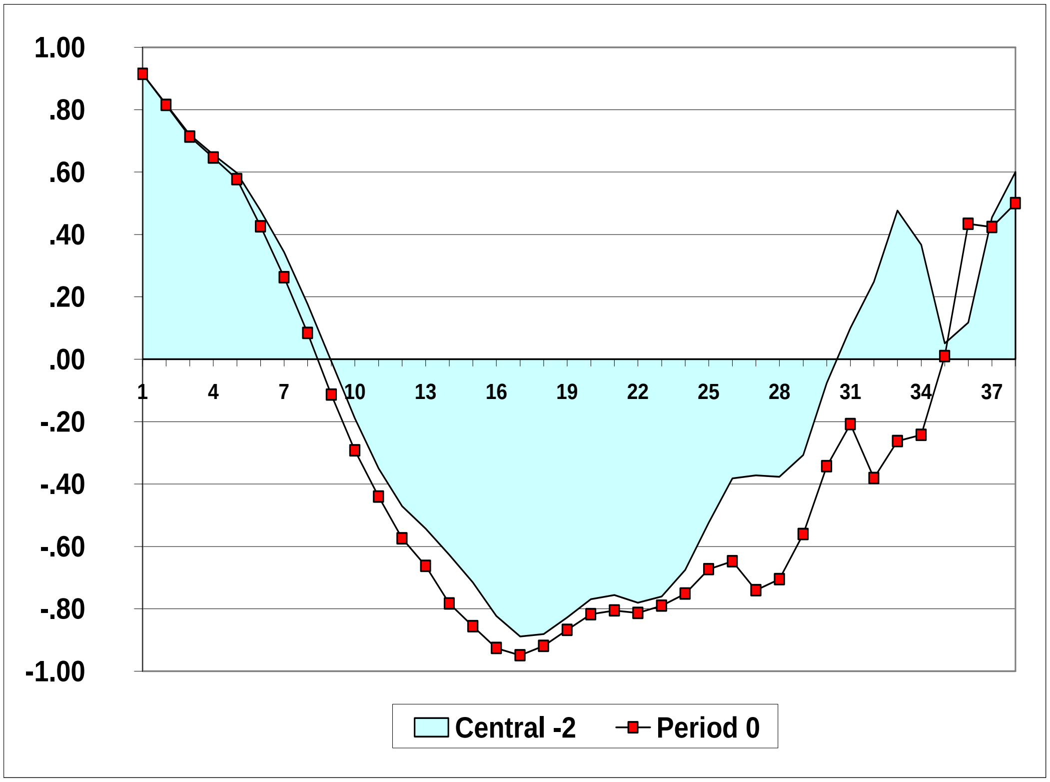


Table 14.3 Bond auto-correlations over 40 years

Period:	6	5	4	3	2	1	0	-1	-2	-3	-4	-2
Lag												central
1	.92	.92	.92	.92	.92	.92	.91	.91	.91	.91	.92	.91
2	.83	.83	.82	.82	.82	.82	.82	.81	.82	.82	.82	.82
3	.74	.73	.73	.72	.72	.71	.71	.72	.72	.72	.73	.72
4	.67	.67	.66	.65	.64	.64	.65	.65	.65	.66	.68	.66
5	.62	.60	.58	.58	.57	.57	.58	.57	.59	.61	.64	.60
6	.50	.47	.44	.42	.43	.43	.43	.44	.47	.51	.54	.48
7	.37	.32	.28	.26	.25	.25	.26	.30	.34	.39	.43	.34
8	.18	.16	.11	.07	.03	.04	.08	.13	.18	.23	.26	.18
9	-.01	-.05	-.06	-.13	-.17	-.16	-.11	-.05	.00	.04	.09	-.01
10	-.18	-.24	-.31	-.32	-.33	-.32	-.29	-.25	-.19	-.14	-.07	-.19
11	-.31	-.43	-.49	-.50	-.46	-.45	-.44	-.44	-.39	-.29	-.19	-.35
12	-.45	-.54	-.60	-.62	-.61	-.56	-.57	-.57	-.52	-.41	-.28	-.47
13	-.54	-.59	-.64	-.68	-.70	-.70	-.66	-.64	-.57	-.48	-.36	-.54
14	-.62	-.66	-.69	-.74	-.80	-.81	-.78	-.70	-.63	-.56	-.47	-.63
15	-.69	-.73	-.77	-.82	-.87	-.88	-.86	-.81	-.70	-.64	-.57	-.71
16	-.78	-.82	-.87	-.92	-.93	-.93	-.93	-.90	-.85	-.75	-.69	-.82
17	-.85	-.90	-.94	-.95	-.95	-.95	-.95	-.94	-.92	-.87	-.76	-.89
18	-.88	-.91	-.93	-.93	-.93	-.93	-.92	-.91	-.90	-.87	-.81	-.88
19	-.87	-.87	-.87	-.88	-.88	-.87	-.87	-.85	-.83	-.81	-.77	-.83
20	-.84	-.84	-.83	-.83	-.83	-.83	-.82	-.80	-.77	-.74	-.72	-.77

Period 0: average 40 years ending 30/9/06, 31/12/06, 31/3/07 and 30/6/07 (see section 6.7)

14.7 Tables 14.2 and 14.3 include only the first 20 year lags because beyond the half-way point the auto-correlations often fluctuate.

15 Assumptions

15.1 The following assumptions, developed as explained in previous sections, are gross of (i.e. before) tax and fees and are before additions for imputation credits. They have not yet been "road-tested", but are not significantly different from the previously tested ("OLD") assumptions.

Table 15.1 Investment assumptions

Sector	Risk margin (arithmetic average)	Mean rate (arithmetic average)	Compound average	Coefficient of variation	Standard deviation of rates	Skewness	Kurtosis
S	4.5%	10.5%	9.3%	1.533	16.1%	-29%	60%
I	4.2%	10.2%	9.1%	1.539	15.7%	-24%	57%
Q	3.7%	9.7%	9.0%	1.299	12.6%	-32%	56%
P	2.0%	8.0%	7.7%	0.900	7.2%	-147%	350%
L	1.0%	7.0%	6.9%	0.500	3.5%	54%	-35%
M	1.0%	7.0%	6.9%	0.500	3.5%	68%	-72%
F	0.5%	6.5%	6.4%	0.723	4.7%	-90%	233%
G	0.1%	6.1%	6.0%	0.607	3.7%	11%	-46%
J	0.3%	6.3%	6.2%	0.698	4.4%	-92%	229%
C	-0.4%	5.6%	5.6%	0.500	2.8%	70%	-76%
N	0.5%	6.5%	6.4%	0.800	5.2%	-59%	71%
Balncd	2.7%	8.7%	8.3%	1.082	9.4%	-63%	73%
CapStb	1.1%	7.1%	6.9%	0.673	4.7%	-76%	183%
B	-0.40%	5.60%	5.56%	0.536	3.00%	81%	-38%
D		6.00%	5.97%	0.417	2.50%	38%	-118%
W	-2.20%	3.80%	3.78%	0.552	2.10%	158%	297%
X	-3.50%	2.50%	2.48%	0.720	1.80%	56%	-52%

Table 15.2 40 year auto-correlation assumptions

Lag (years)	S sector	D sector	Lag (years)	S sector	D sector
1	-.06	.91	21	.30	-.76
2	-.15	.82	22	-.24	-.78
3	-.01	.72	23	-.42	-.76
4	-.01	.66	24	.06	-.68
5	-.14	.60	25	.19	-.52
6	-.07	.48	26	.09	-.38
7	.08	.34	27	.16	-.37
8	.29	.18	28	.42	-.38
9	-.21	-.01	29	.26	-.31
10	-.10	-.19	30	-.16	-.08
11	-.02	-.35	31	-.36	.10
12	-.14	-.47	32	-.11	.25
13	-.05	-.54	33	.10	.48
14	.01	-.63	34	.07	.37
15	-.16	-.71	35	-.09	.05
16	-.13	-.82	36	-.44	.12
17	-.11	-.89	37	.23	.45
18	.10	-.88	38	.20	.60
19	.01	-.83			
20	.27	-.77			

Table 15.3 cross-correlation assumptions

RANK CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
S	1	.59	.54	.10	.12	.16	.09	.14	.09	.12	.11	.09	.03	-.13	-.01
I	.59	1	.33	.12	.16	.27	.27	.26	.24	.15	.32	.16	.22	-.09	.02
Q	.54	.33	1	.11	.15	.08	.38	.24	.40	.05	.40	-.01	-.03	-.13	-.13
P	.10	.12	.11	1	.21	.27	-.04	.18	.06	.40	-.02	.38	.28	.46	.56
L	.12	.16	.15	.21	1	.92	.53	.88	.56	.79	.53	.79	.84	.33	.42
M	.16	.27	.08	.27	.92	1	.48	.88	.48	.89	.49	.90	.90	.43	.51
F	.09	.27	.38	-.04	.53	.48	1	.75	.95	.35	.89	.35	.41	-.05	-.04
G	.14	.26	.24	.18	.88	.88	.75	1	.75	.79	.73	.80	.79	.36	.41
J	.09	.24	.40	.06	.56	.48	.95	.75	1	.39	.86	.38	.43	.02	.01
C	.12	.15	.05	.40	.79	.89	.35	.79	.39	1	.40	.96	.82	.57	.67
N	.11	.32	.40	-.02	.53	.49	.89	.73	.86	.40	1	.38	.40	.10	.14
B	.09	.16	-.01	.38	.79	.90	.35	.80	.38	.96	.38	1	.85	.56	.67
D	.03	.22	-.03	.28	.84	.90	.41	.79	.43	.82	.40	.85	1	.47	.54
W	-.13	-.09	-.13	.46	.33	.43	-.05	.36	.02	.57	.10	.56	.47	1	.85
X	-.01	.02	-.13	.56	.42	.51	-.04	.41	.01	.67	.14	.67	.54	.85	1

Average .422044

STANDARD CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

	S	I	Q	P	L	M	F	G	J	C	N	B	D	W	X
S	1	.62	.62	.05	.11	.09	.26	.18	.24	.07	.21	.05	.06	-.30	-.08
I	.62	1	.42	.06	.30	.31	.39	.37	.36	.26	.40	.23	.27	-.19	-.02
Q	.62	.42	1	.04	.26	.17	.56	.38	.55	.11	.49	.05	.10	-.30	-.19
P	.05	.06	.04	1	.22	.33	-.16	.10	-.09	.38	-.08	.36	.28	.26	.42
L	.11	.30	.26	.22	1	.94	.66	.93	.69	.88	.65	.82	.88	.20	.37
M	.09	.31	.17	.33	.94	1	.49	.89	.55	.98	.53	.94	.93	.35	.53
F	.26	.39	.56	-.16	.66	.49	1	.83	.98	.39	.91	.33	.43	-.26	-.17
G	.18	.37	.38	.10	.93	.89	.83	1	.85	.83	.80	.78	.82	.11	.27
J	.24	.36	.55	-.09	.69	.55	.98	.85	1	.46	.92	.40	.47	-.21	-.11
C	.07	.26	.11	.38	.88	.98	.39	.83	.46	1	.44	.97	.92	.43	.60
N	.21	.40	.49	-.08	.65	.53	.91	.80	.92	.44	1	.38	.43	-.04	.09
B	.05	.23	.05	.36	.82	.94	.33	.78	.40	.97	.38	1	.91	.42	.59
D	.06	.27	.10	.28	.88	.93	.43	.82	.47	.92	.43	.91	1	.34	.50
W	-.30	-.19	-.30	.26	.20	.35	-.26	.11	-.21	.43	-.04	.42	.34	1	.84
X	-.08	-.02	-.19	.42	.37	.53	-.17	.27	-.11	.60	.09	.59	.50	.84	1

Average .426133

16 Gross/Net of Tax

16.1 All results in previous sections are gross (i.e. before) tax and imputation credits. The following results illustrate the impact of tax and imputation credits on the Section 15 assumptions for mean rates for superannuation in the accumulation stage.

Table 16.1 Gross/net of tax

Sector	Mean rate (arithmetic average) Before tax	After tax and imputation credits	Average tax rate	Compound Average rate After tax and imputation credits
S	10.50%	10.10%	3.8%	9.15%
I	10.20%	9.26%	9.2%	8.37%
Q	9.70%	8.66%	10.7%	8.04%
P	8.00%	6.84%	14.5%	6.64%
L	7.00%	5.95%	15.0%	5.91%
M	7.00%	5.95%	15.0%	5.91%
F	6.50%	5.53%	15.0%	5.44%
G	6.10%	5.19%	15.0%	5.14%
J	6.30%	5.36%	15.0%	5.28%
C	5.60%	4.76%	15.0%	4.73%
N	6.50%	5.59%	14.0%	5.49%
Balncd	8.68%	7.99%	8.0%	7.66%
CapStb	7.05%	6.24%	11.5%	6.16%
B	5.60%	4.76%	15.0%	4.73%
D	6.00%	5.10%	15.0%	5.08%

16.2 The above table allows for income tax at the 15% superannuation rate and, on an approximate basis, for imputation credits and the lower rates of tax on realised capital gains.

17 Gross/Net of Fees

17.1 All results in previous sections are gross (i.e. before) fees. The following results illustrate the impact of wholesale passive investment fees on the Section 15 and 16 assumptions.

Table 17.1 Gross/net of fees

Sector	Mean rate (arithmetic average)			Compound Average rate
	Before tax Before fees	Before tax After fees	After tax & IC's After fees	After tax & IC's After fees
S	10.50%	10.24%	9.88%	8.92%
I	10.20%	9.91%	9.01%	8.12%
Q	9.70%	9.41%	8.42%	7.80%
P	8.00%	7.18%	6.14%	5.94%
L	7.00%	6.71%	5.70%	5.66%
M	7.00%	6.71%	5.70%	5.66%
F	6.50%	6.32%	5.37%	5.28%
G	6.10%	5.92%	5.03%	4.98%
J	6.30%	6.12%	5.20%	5.13%
C	5.60%	5.41%	4.60%	4.57%
N	6.50%	6.31%	5.43%	5.33%
Balncd	8.68%	8.42%	7.77%	7.44%
CapStb	7.05%	6.83%	6.05%	5.97%
B	5.60%	5.60%	4.76%	4.73%
D	6.00%	6.00%	5.10%	5.08%

17.2 The "balanced" portfolio compound average rate in the right hand column of the above table and the compound average rates for AWOTE and CPI in Table 15.1 are directly comparable with the actuarial projections for long-term investment returns, wage growth and growth in the Consumer Price Index (CPI) reported by superannuation funds and published in APRA (2007). Averaging over the three years to June 2006, June 2005 and June 2004, Table B of the APRA Bulletin shows:

421 defined benefit and hybrid plans	Average	Median
Actuarial projection for investment return	6.94%	7.00%
Actuarial projection for wage growth	4.51%	4.50%
Actuarial projection for CPI growth	3.18%	3.00%

386 non-public sector plans	Average	Median
Actuarial projection for investment return	6.97%	7.00%
Actuarial projection for wage growth	4.53%	4.67%
Actuarial projection for CPI growth	3.20%	3.00%

35 public sector plans	Average	Median
Actuarial projection for investment return	6.72%	7.00%
Actuarial projection for wage growth	4.25%	4.00%
Actuarial projection for CPI growth	2.93%	3.00%

18 Concluding Remarks

18.1 An objective of this paper was to help bridge the gap between the demand from actuaries (and consumers) for robust assumptions in respect of future investment returns across a broad range of investment sectors, and the limited supply of data readily available for a range of investment sectors.

18.2 The advent of unit price data in Australia 42 years ago, through National Mutual's "EFG" system, supplemented with other published indices and rates, provided a substantial database of investment performance statistics, across a broad range of investment sectors.

18.3 The paper developed and summarized assumptions sets for medium to long-term use in stochastic investment models, across the range of investment sectors and economic indicators that were examined. In determining those assumptions, the paper has illustrated techniques for 'backdating' incomplete data series for use, particularly, with cross-correlation and autocorrelation analyses.

18.4 In addition, the paper adopted the technique used by Dwonczyk in fitting sine waves to CPI data, and applied an extended version of this technique to 10 year bonds (and to CPI, AWOTE and Australian Shares). This analysis appears to provide support for the existence of 'long-term' economic cycles extending over 36.25 to 38.75 years. This, together with auto-correlations, appears to be an area which warrants further research.

18.5 The paper emphasises that where-ever possible it is desirable for the setting of long-term assumptions to analyse results over a full economic cycle.

18.6 Significant differences in the assumptions now recommended, as compared to those recommended 2 years and 3 months ago, include:

		Now Recommended	**Previously Recommended**
Risk Margin	Sector S	4.5%	4.0%
	P	2.0%	1.2%
	F	0.5%	1.0%
Coefficient of Variation	Sector Q	1.299	1.221
Standard Deviation	Q	12.6%	11.6%
Skewness	Sector P	-147%	-128%
Kurtosis	Sector Q	56%	93%
	W	297%	326%
Rank Cross corr.	Sectors S & F	9%	17%
	S & P	10%	3%
	F & W	-5%	-14%
Auto-correlation	S @ lag 4 yrs	-1%	6%
	D @ lag 9 yrs	-1%	-11%

Often there were no or insignificant changes. For example, the number of assumptions for which the absolute change was zero or between -5% and 5% inclusive was 12 (of 15) for skewness, 7 (of 15) for kurtosis and 73 (of 105) for cross-correlations.

18.7 Though of less importance, the following is a summary comparison of:

(a) the projected 38-year trend result for Period -2.25 years from the April 2005 calculations for Grenfell (2005), with

(b) the 38-year trend results for Period 0 years from this current investigation.

Both (a) and (b) above relate to **the average of results for 38 years ending 30/9/2006, 31/12/2006, 31/3/2007 and 30/6/2007.** Only the results with the largest absolute differences between (a) and (b) are tabulated. These extremes arise primarily from differences in the shape and/or turning-points of the April 2005 quadratic trends as compared with the shape and/or turning-points of the current quadratic trends. Recognising that Table 2.2 includes 374 projections it is to be expected that some projection fittings will fail. These wide discrepancies are partially due to the 38-year averages in the April 2005 calculations being based on 7 data points (i.e. Periods 0 to 6 inclusive). Fewer wide discrepancies should be expected from the corresponding current calculations since they each have 9 data points.

	Sector/s	**Current (b)**	**Previous (a)**
Risk Margin	I	1.3%	0.1%
Coefficient of Variation	I	1.89	2.14
Skewness	I	-19%	-29%
Kurtosis	J	210%	242%
Cross correlation	P & D	33%	41%

19 Acknowledgments

19.1 While I take full responsibility for the content of this paper, I would like to sincerely thank the following:

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- AXA Australia and National Mutual for the regular provision of unit price and index data for many of the 11 investment sectors.

- The designers, in 1965, of the National Mutual EFG investment system.

19.2 Finally, I would like to express my gratitude to my wife Barbara for her patience while I laboured for such a long time with drafts of this paper and with many updates to the *Austmod* stochastic and historical investment simulation model.

20 References

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20.8 Resilience Reserves Taskforce Report, 2005, The Institute of Actuaries of Australia, 2005 Biennial Convention, Cairns.

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Appendix A: Modelling Skewness and Kurtosis

A1 This appendix describes a method for adding skewness (denoted g) and kurtosis (denoted k) to a 40 year stochastic projection.

A2 Let x represent a uniform random variable (or "step") with the values 0.5, 1.5, 2.5, 3.5 … 39.5.

A3 Let z represent a standardised normal random variable. For example, using an EXCEL function, z = NORMSINV(x/40).

A4 Applying the Normal Power Approximation the variable y given by:

y = z + g*(z^2-1)/6 + k*(z^3-3*z)/24 – (g^2)*(2*z^3-5*z)/36 [1]

will be found over the 40 steps from 0.5 to 39.5 inclusive to have a mean of approximately 0, standard deviation of approximately 1, skewness of approximately g and kurtosis of approximately k. By trial-and-error (or by using EXCEL Solver) it is possible to change the value of g to *g* and the value of k to *k* so that the variable *y* (note *italics*) given by:

y = z + *g**(z^2-1)/6 + *k**(z^3-3*z)/24 – (*g*^2)*(2*z^3-5*z)/36 {2]

has, over the 40 steps from 0.5 to 39.5, skewness of exactly g and kurtosis of exactly k. When the variable *y* is standardised it will still have skewness of g and kurtosis of k.

A5 The Normal Power Approximation, applied as described above works well for those sectors with negative skewness and positive kurtosis and for those with low skewness and kurtosis. This group includes all sectors **where modest to significant negative returns are expected**, that is:

S, I, Q, P, F, J and N

A6 However the above method is unsatisfactory (because the resultant variable does not increase for all steps from 0 to 40) for those sectors **where negative returns are not expected or where occasional small negative returns are expected.** This group includes:

L, M, C, B, D and X

A7 For this latter group it was found that instead of [1] above the following gamma exponential variable, y, expressed in EXCEL functions, could be used:

y = EXP(1-GAMMINV(1-x/40,alpha,beta)) [3]

A8 Again, by trial-and-error (or by using EXCEL Solver) it is possible to change the value of alpha to *alpha* and the value of beta to *beta* so that the variable *y* given by:

y = EXP(1-GAMMINV(1-x/40,*alpha*,*beta*)) [4] has, over the 40 steps from 0.5 to 39.5, skewness of exactly g and kurtosis of exactly k. When the variable *y* is standardised it will still have skewness of g and kurtosis of k. The resultant variable also increases for all steps from 0 to 40.

A9 Sector W (AWOTE) does not fit naturally into either of the two groups described on the previous page. It has high positive skewness and high positive kurtosis and (depending on the coefficient of variation assumed) can be expected to have occasional modest negative values. Interestingly, it was found that relationships [2] and [4] both produce satisfactory and almost identical values of *y* for this sector.

A10 Sector G (Government semis 0-3 years) also does not fit naturally into either of the two groups described on the previous page. For this Sector it was found that relationship [2] produced satisfactory values of *y* for the "OLD" (April 2005) calculations but not for the "NEW" (July 2007) calculations. For the latter, relationship [4] produced satisfactory values of *y*.

A11 The process described above produces standardised variables for all the 15 sectors with the desired skewness and kurtosis. However this alone does not mean that the entire **shape of the distribution** of returns is realistic. To test this, the following analysis was performed:

(a) Forty years of forces for each sector ending 31/12/05, 31/3/06, 30/6/06 and 30/9/06 were each **sorted and standardised –** giving four datasets.

(b) The skewness and kurtosis of each sector and dataset was calculated.

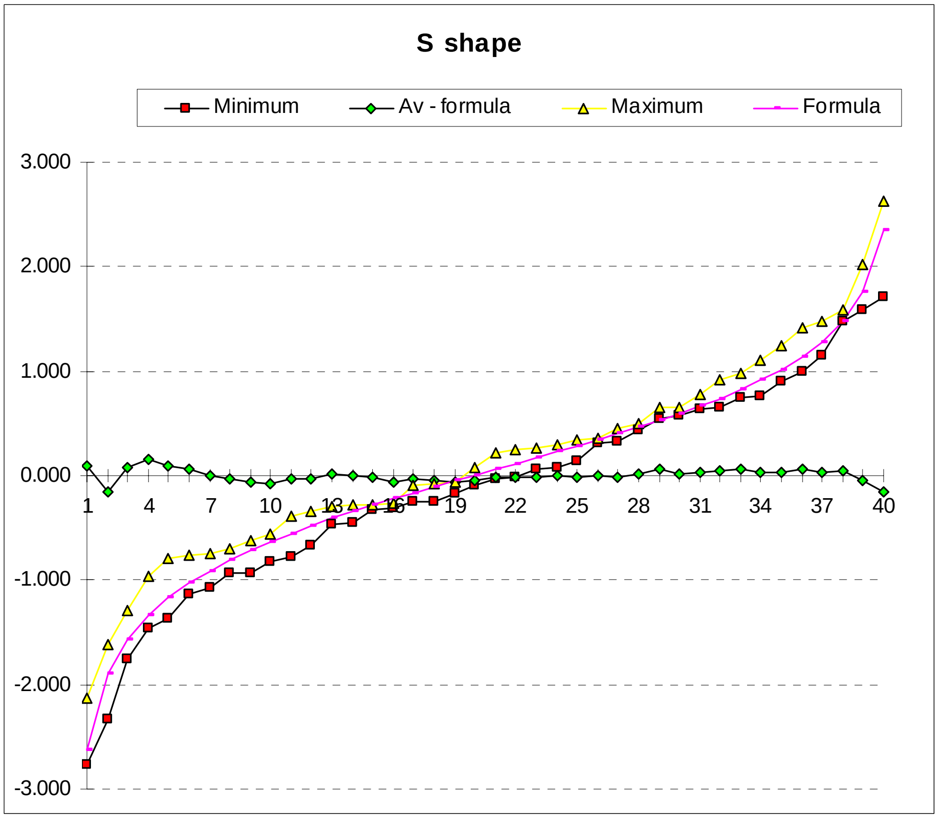
(c} The average skewness and kurtosis of each sector was calculated.

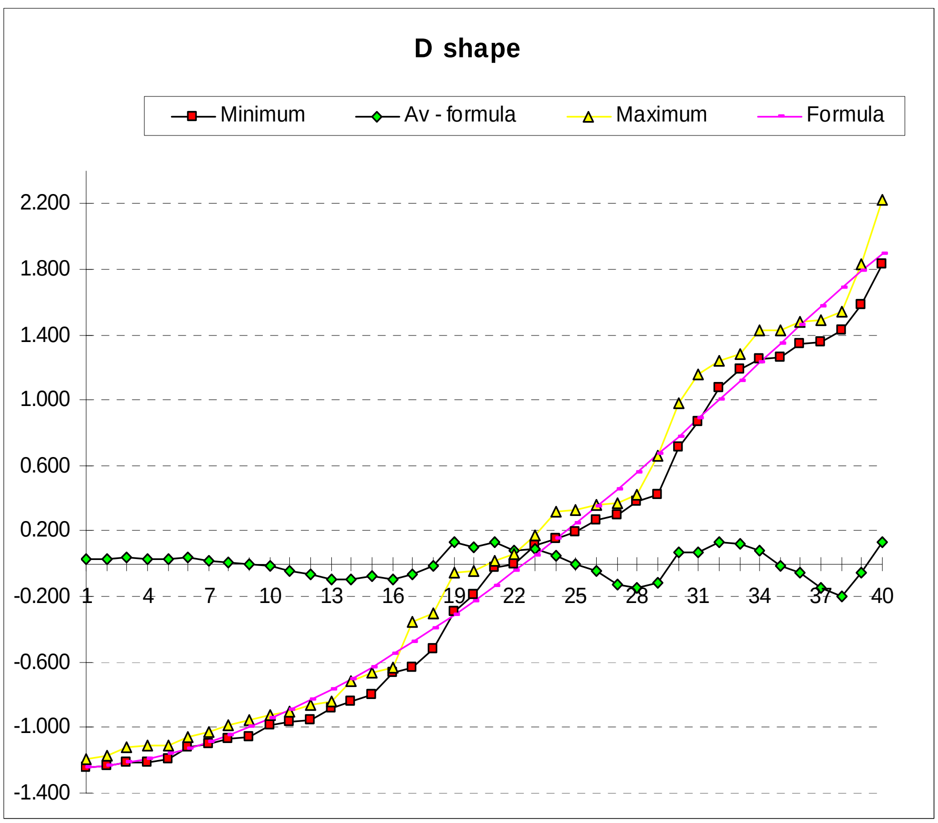
(d) Relationships [2] and [4] were applied based on the average skewness and kurtosis of each sector from (c) above, **not** based on long-term estimates of g and k.

(e) The resultant variables from (d) above were compared with the four datasets from (a) above. The comparison focused, in particular, on whether the tail values for the resultant variables were:

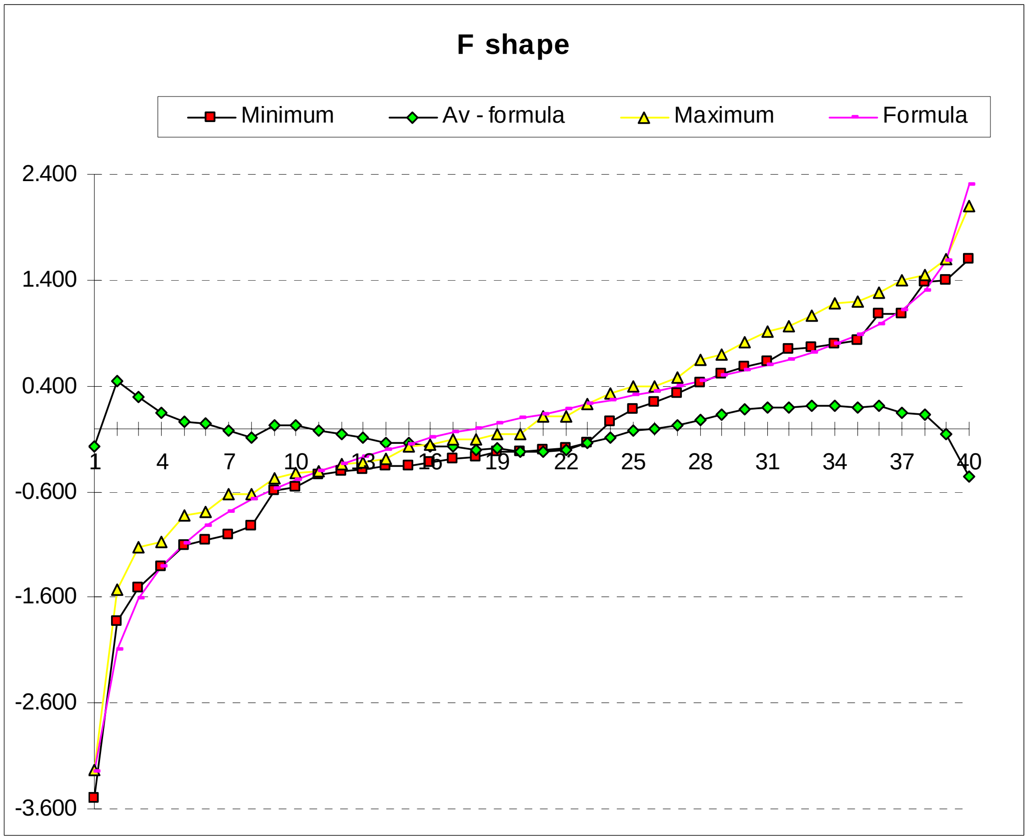
- greater than the minimum of the tail values from the four datasets in (a) above, and
- less than the maximum of the tail values from the four datasets in (a) above.

(f) Two examples of where (e) was satisfied are shown in the following charts, one for the S sector based on relationship [2], and one for the D sector based on relationship [4]. Each chart contains the maximum and minimum sorted standardised forces from the four datasets and also includes a graph (denoted "Av – formula") of the difference between the average of the four datasets and the formula result.


S shape
Minimum
Av - formula
Maximum
Formula
3.000
2.000
1.000
0.000
-1.000
-2.000
-3.000
1
4
7
10
13
16
19
22
25
28
31
34
37
40


D shape
Minimum
Av - formula
Maximum
Formula
2.200
1.800
1.400
1.000
0.600
0.200
-0.200
-0.600
-1.000
-1.400
1
4
7
10
13
16
19
22
25
28
31
34
37
40

(g) An example of where (e) was not satisfied is shown in the following F sector chart.



A12 The chart in (g) above shows that relationship [2] has satisfactorily captured the bottom left hand tail values for the F sector. This same outlier is referred to in section 9.2. It relates to the year ending 30 September 1973. However relationship [2] has not satisfactorily captured the top right hand tail values for this sector since the formula result at point 40 exceeds the maximum. Because of the "backdating" formulae in Section 4, a similar problem occurred for the G, J, L and N sectors.

A13 To rectify the problem identified in the previous paragraph, it was found that an effective pragmatic solution was to reduce **both** the skewness and kurtosis for the F, G, J, L and N sectors by 0.15, 0.17, 0.17, 0.30 and 0.15 respectively. With these reductions, relationships [2] and [4] satisfactorily captured both the bottom left hand and top right hand tail values for the five sectors and the shape for the non-tail values was insignificantly changed.

A14 The skewness and kurtosis assumptions tabulated in Table 15.1 are **gross** of (i.e. before) the reductions in the previous paragraph.

Appendix B: Modelling Auto-correlations

B1 An examination of Figure 14.1 shows that auto-correlations are of three broad types:

(a) S sector, Property Trusts and the "Balanced" portfolio

(b) F sector, Direct Property and CPI (and AWOTE)

(c) Bills and Bonds.

B2 Also of interest is the similarity between the auto-correlations of the S sector and that of the "Balanced" portfolio. This is primarily due to the greater volatility of the S sector which dominates the shape of the "Balanced" portfolio auto-correlations. Though not included in Figure 14.1, the auto-correlations of a "Capital Stable" portfolio are quite different from those for the "Balanced" portfolio. The "Capital Stable" portfolio auto-correlations are similar to those for the F sector.

B3 A closer examination of Figure 14.1 reveals that there is a gradual progression from the **S sector**, to Property Trusts, to the "Balanced" portfolio, to the F sector, to Direct Property, to CPI, to Bills and then to **Bonds**.

B4 **The auto-correlations of the S sector and Bonds form two extremes**. This hypothesis is supported by the following analysis of the auto-correlations underlying Figure 14.1:

i) for lags of 1 to 2.5 years, the S sector has the lowest auto-correlations of all the 15 sectors. In stochastic modelling these shorter term lags are most important because there are far more observations at the short end than at the long end. Further, the longer results are to some extent repeats of the shorter results.

ii) for lags of 1 to 5.75 years, Bonds (the D sector) have the highest auto-correlations of all the 15 sectors.

iii) for lags of 6 to 6.5 years, the S sector again has the lowest auto-correlations of all the 15 sectors.

iv) for lags of 10.5 to 12 years and 13.75 to 30 years, Bonds (the D sector) have the lowest auto-correlations of all the 15 sectors.

B5 Intuitively, the S sector is significant because of its higher volatility and expected investment returns. In addition, the D sector is significant because of its influence on current market investment yields.

B6 The above analysis forms the background as to why the *Austmod* stochastic investment simulation model focuses on the S and D sector autocorrelations.

B7 Briefly, the *Austmod* model sets D sector auto-correlations by sorting results into 32-year "bumpy" cycles (but note paragraph B12 on the next page) as follows:

- subdividing (126 times) D sector zero auto-correlated stochastic results for 16 years out of 96 into six groups of 16 years,
- sorting each group of 16 year D sector results,
- positioning each D sector result, first of first 16 to first of 32, first of second 16 to last of 32, second of second 16 to second of 32, etc, and
- retaining each result for the 14 other sectors with their original D sector value (thus leaving cross-correlations undisturbed).

B8 Briefly, the *Austmod* model then sets S sector autocorrelations by discarding all but one of the above D sector combinations, and retaining the D sector combination which is associated with the S sector auto-correlation closest to that desired. The method of least squares is used to determine the closest S sector auto-correlation, with the auto-correlations weighted towards the shorter lags (which have the greatest number of observations). The weight used is the square of (39 less the lag in years).

B9 Having obtained the S and D sector auto-correlations as above the (retained) cross-correlations carry through these auto-correlation distributions to the other sectors. The Cholesky decomposition formula is used to create the cross-correlation distributions.

B10 Without lagging CPI (X) and AWOTE (W) values, the process described in the previous paragraph works reasonably well for all sectors except X and W, where only about one-third of the desired auto-correlation distributions was obtained for X and significantly less than this for W. In view of the D and X (.54) and D and W (.47) cross correlations in Figure 15.3, this result is not surprising.

B11 The much higher D and X-8 (.69) and D and W-8 (.76) cross correlations in Figure 13.3 have been found to produce much better results if lagged X and W forces were used throughout the modelling. By increasing these two lagged cross-correlations by a further 0.13 (beyond which the Cholesky decomposition fails) it was found that about 75% of the desired auto-correlation distributions were obtained for both X and W.

B12 The results in Table 15.2 indicate that the D sector auto-correlation is zero at a lag of about 8.95 years whereas model results currently indicate a D sector auto-correlation of zero at a lag of about 7.85 years. This analysis indicates that the 32-year cycle referred to on the previous page should be increased by about 4 times 1.1 or say 4 years (to 36 years).

B13 For further information about the appendices, the historical database or *Austmod*, the author can be contacted at colin.grenfell@supereasy.com.au or colnbarb@hotmail.com.

Appendix C: *Austmod* Investment Simulation Model - Inputs

C1 *Austmod* is a stochastic and historical investment simulation model. The model is an EXCEL workbook that displays up to 47 years of historical and 40 years of simulated investment performance for the 15 "sectors" described in Section 3.

C2 Appendix A of Grenfell (1997) has a specification of the then version O of the model.

C3. This appendix describes the input for the latest versions of the model (version T based on the "OLD" assumptions and version U based on the "NEW" assumptions).

C4 The 26 inputs to the model are:

		Input
TAX	**0** = Nil **1** = Ordinary **2** = Superannuation **3** = Exempt **4** = other	[1]
MODEL	Type : 0 = stochastic 1 = historical random start date	[2]
	Start seed : Enter a number from 1 to 8000 (or enter R for random)	[3]
PPNS	**Usually** enter 0% : Bill Rate, Bond Rate, AWOTE, CPI [4] **Composite** portfolio (enter 0, 1 or 2) : [5] Press F9 if 'manual' **0** = Input primes **1** = Balanced **2** = Capital stable **Balanced >>** 36% Shares; 25% International Shares; 7% Property Trust; 2% Property Direct; 0% Loans; 0% Mortgage; 16% Fixed Interest; 0% Semi Government; 7% International Bonds; 5% Cash; 2% Inflation Linked	[6]
PLAN	Assets at start $	[7]
	Liabilities (or accumulated retirement benefits) at start $	[8]
	Annual net Enter the net cash flow per annum at start (as a number)	[9]
	cash flow : ~ varies W = ~ AWOTE X = ~ CPI $ = dollar P = % assets	[10]
	Driver: A=3yr geo av B=bills C=cash D=bonds E=earnings W=AWOTE	[11]
	Crediting rate formula: 1 = standard 2 = driver 3 = other	[12]
INVEST	**Means:** 1 = based on standard 2 = other	[13]
	Additional return: (all investment sectors, input additional rate before tax)	[14]
	Net of fees: 0 = no 1 = standard 2 = other	[15]
	Standard deviations: 0 = nil 1 = standard 2 = other	[16]
	Skew & kurtosis: 0 = nil 1 = std (skew only) 2 = std (both)	[17]
	Cross correlations: 0 = nil 1 = standard 2 = std+WX 8 yr lags 3 = other	[18]
	Auto correlations: 0 = nil 1 = C cycles 2 = S+D trends	[19]
	Seasons start: D =downwds B =bottom U =upwards T =top R =random	[20]
	Historical results: Enter number of years (between 1 and 48)	[21]
	Historical results ending: Enter 31/3, 30/6, 30/9 or 31/12 dates only	[22]
INITIAL	Fund earning rate in the year prior 30/6/07 (= assumed start date)	[23]
	Fund earning rate in 2nd year prior 30/6/07 (needed when driver =A)	[24]
	Nominal bill rate at start (if Driver = B and end or start year based)	[25]
	Nominal bond rate at start (if Driver = D and end or start year based)	[26]

C5 Inputs [1] to [26] are briefly explained on the next three pages.

TAX (Taxation)

[1] Enter 0 for no taxation and no imputation credits.
Enter 1 for life company "Ordinary" taxation.
Enter 2 for superannuation tax (in the accumulation stage).
Enter 3 for exempt from taxation but with imputation credits (for example, pensions and annuities).
Enter 4 for another taxation basis (the "watch" column, not illustrated above, will then indicate where the new taxation basis is entered).

MODEL (Modelling basis)

[2] Enter 0 for stochastic modelling.
Enter 1 for "historical random start" modelling. With this basis, historical data for the period specified in [21] and [22] on page 60 will be stacked end-on-end to form a recurrent sequence and then 1, 8, 40, 200 or 1000 40-year scenarios will be reported from this sequence, each based on a random start date. This is a form of bootstrapping – the following extract from Sweeting (2007) explains:

"In stochastic modelling the first distinction is between bootstrapping and forward-looking approaches. For bootstrapping, all that is needed is a set of historical data for the asset classes being modelled. For example, monthly market returns for the last 20 years could be used. However rather than simply using this as a single 'run' of data, modelling is carried out by selecting a slice of data randomly, in this case the results from a particular month. This forms the first observation. This observation is then 'replaced' and another month is chosen randomly. This means that a relatively small data set can be used to generate a large number of random observations.

The main advantage of bootstrapping is that the underlying characteristics of the data and linkages between data series are captured without having to resort to parameterisation."

Unlike bootstrapping, "historical random start" modelling captures most of the auto-correlation of the historical data as well as most of the cross-correlation. For the chosen period (specified in [21] and [22]) and for all sectors, all of the historical auto-correlation and cross-correlation is captured except for when one sequence stops and another starts.

[3] Enter a "seed" number between 1 and 8000 or enter "R" for a random series.

PPNS (Proportions)

[4] For bills, bonds, AWOTE and CPI the default 0% proportion will normally apply since it is not possible to "invest" in these sectors. However for model-testing it may be desired to enter 100% in one of these cells.

[5] Enter 0 if proportions for "prime" investment sectors are being entered in [6] below.
Enter 1 if the default "Balanced" portfolio ppns (illustrated in C4 above) are to be used.
Enter 2 if the default "Capital Stable" portfolio proportions are to be used.

[6] If the default composite portfolio proportions are not being used, enter the desired proportions for each sector. These proportions must total 100%.

PLAN (Plan details]

[7] Enter the market value of assets at the projection start date (i.e. at the date [22]).
[8] Enter the liabilities or the accumulated retirement benefits at the projection start date.

[9] Enter the annual net cash flow at the projection start date.

[10] Enter "W" if the annual net cash flow varies proportional to AWOTE changes.
Enter "X" if the annual net cash flow varies proportional to CPI changes.
Enter "$" if the annual net cash flow is a constant dollar amount.
Enter "P" if the annual net cash flow is a proportion of plan assets (the "watch" column, not illustrated, will then indicate where the required proportion/s is/are entered).

[11] The "driver" is a component of the crediting rate basis or of the growth in liabilities.
Enter "A" if the driver is the geometric 3 year average of past plan investment returns.
Enter "B" if the driver is based on the Bill rate.
Enter "C" if the driver is based on the Cash investment return.
Enter "D" if the driver is based on the Bond rate.
Enter "E" if the driver is the plan investment return (earnings).
Enter "W" if the driver is based on changes in AWOTE.

[12] Enter 1 if the crediting rate formula is "standard" (this is unlikely to apply but a standard formula is built into the model which can be used, varied or ignored).
Enter 2 if the crediting rate equals the driver.
Enter 3 if neither 1 nor 2 applies (the "watch" column, not illustrated, will then indicate where the "other" basis is entered – similarly, when "other" is chosen for [13], [15], 16] or [18] below, the "watch" column will indicate where the basis is entered).

INVEST (Investment assumptions)

[13] Enter 1 if the mean is "standard" (see Table 15.1).
Enter 2 if the assumption for the mean is "other" (note the "watch" column).

[14] Enter any "additional return" (before tax) to be added to [13] above (for all investment sectors).

[15] Enter 0 if investment returns are to be before investment fees.
Enter 1 if investment returns are to be net of "standard" investment fees.
Enter 2 if investment returns are to be net of "other" investment fees.

[16] Enter 0 if the standard deviation of all returns is zero (in which event all projections will be "deterministic").
Enter 1 if the standard deviation is "standard" (see Table 15.1).
Enter 2 if the standard deviation is "other".

[17] Enter 0 if the skewness and kurtosis of all returns is zero (in which event all projections will be based on log-normal distributions).
Enter 1 if L, M, C, B, D, W and X sector returns (only) are to be skewed on a "standard" basis so as avoid negative returns (or significantly negative returns for W and X sectors) and for the 8 other sectors log-normal distributions are required.
Enter 2 if the skewness and kurtosis of all returns is "standard" (see Table 15.1 and Appendix A).

[18] Enter 0 if the cross-correlations of returns are zero (i.e. independent).
Enter 1 if the cross-correlations of returns are "standard" (see Table 15.3 and section 12.4).
Enter 2 if the cross-correlations of returns are "standard" with an 8-year lag for W and X sectors (see Table 13.3 and Section 13).
Enter 3 if the cross-correlations of returns are "other".

[19] Enter 0 if the auto-correlations of returns are zero (i.e. serially independent).
Enter 1 if the auto-correlations of C sector returns are based on a fixed 4-year cycle.
Enter 2 if the auto-correlations of S and D sector returns are based on the method explained in Appendix B.

[20] When 1 or 2 is entered in [19] above, the returns for the C or D sector will be based on cycles (also referred to as "seasons"). The code entered in [20] then determines when these cycles start.
Enter D if the cycle starts with a downward movement (half-way from top to bottom).
Enter B if the cycle starts at the bottom.
Enter U if the cycle starts with an upward movement (half-way from bottom to top).
Enter T if the cycle starts at the top.
Enter R if the cycle starts from a random position.

[21] Enter the number of years (between 1 and 48) over which historical returns are required.

[22] Enter the end-date (31 March, 30 June, 30 September or 31 December dates only) for historical returns. This date is also the start date for projections.

INITIAL (Initial information)

If further information is not necessary, the description for [23] to [26] will be blank and any input will be irrelevant (as explained, but not illustrated, in the "watch" column). In most situations the input entered in [11] will **not** require further "INITIAL" information.

[23] Enter the fund earning rate in the year prior [22].
[24] Enter the fund earning rate in the year prior to the year prior [22].
[25] Enter the nominal Bill rate at the date in [22].
[26] Enter the nominal Bond rate at the date in [22].

C6 For further information about the *Austmod* investment simulation model, see www.supereasy.com.au/welcome.asp?pg=investmentsimulation or www.supereasy.biz/investment_simulator/investment_simulator.htm

UPDATE FOR MAY 2007 AWOTE

Section 3.6 explains that at the time this paper was finalised the May 2007 AWOTE had not been published. An estimate of 1078.0 has been used in all calculations. The May 2007 AWOTE was published 16/8/07 and was 1090.0. The effect of this 1.1% increase on the results tabulated in the paper, is:

			Estimate	**Actual**
Section 3.6	May 2007 AWOTE		1078.0	1090.0
Table 5.1	AWOTE 7.25 + 36.25			
		Short cycle explained	7.2%	7.1%
		Long cycle explained	43.4%	43.6%
		Both cycles explained	**51.3%**	**51.4%**
	AWOTE 22.75 + 36			
		Short cycle explained	3.8%	3.7%
		Long cycle explained	43.4%	43.6%
		Both cycles explained	**51.3%**	**51.4%**
		Curve minimum	Dec 00	Sep 00
		Period (years)	38.75	38.5
Table 6.4	Pointers (risk margin for W)			
		38 -2	-1.62%	-1.61%
		24 -2	-4.70%	-4.69%
Section 6.11	"Productivity" (W less X)			
		24 -2	1.32%	1.33%
Table 10.1	Skewness, trend at Period -2 (W)		158%	159%
Table 11.1	Kurtosis, trend at Period -2 (W)		297%	299%
Table 12.1	Rank cross-correlations at Period -2			
		S and W	-.13	-.12
Table 12.2	Standard less rank cross-correlations. Small changes directly refecting the nine changes in Table 13.3 and Table 15.3 below.			
Table 13.3 and Table 15.3	Rank cross-correlation assumptions			
		S and W	-.13	-.12
		I and W	-.09	-10
		P and W	.46	.47
		M and W	.43	.42
		F and W	-.05	-.06
		J and W	.02	.01
		N and W	.10	.09
		B and W	.56	.57
		D and W	.47	.46
Table 13.3	Standard cross-correlation assumptions			
		X-8 and W	.06	.05

All other results were unchanged. The AWOTE, sector W, assumptions in Table 15.1 and section 11.5 change to reflect the above changes to Table 10.1 and Table 11.1.