

Fair Value of Liabilities

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How Do We Define “Closest” Asset Match?

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- **Assets are OK**
 - Normally a market
- **No market for long tail claims liabilities**
- **Approach requires the “closest match” asset portfolio**
- **Use the discount rate implied by those assets to discount liability cashflows**



Ultimate Surplus

- Project asset and liability cashflows
- When all liabilities extinguished what assets remain are the “ultimate surplus”
- “Closest match” defined by reference to this ultimate surplus
- Most important risk for insurance company is insolvency
- Insolvency = negative ultimate surplus



“Closest Match”

- With stochastic cashflows the probability of insolvency is always non-zero [unless infinite initial asset amount]
- As initial assets \uparrow
probability of insolvency \downarrow
- Two variables
 - Probability of insolvency
 - Initial asset amount
- Need to fix one of these to produce a unique solution



Closest Match is

The asset portfolio which, for a given probability of negative ultimate surplus, requires the lowest initial asset amount.



Liabilities

- Long tail outstanding claims
- Runs off in 10 years
- Quarterly time intervals
- Experience run-off triangle shown in Appendix A



Liability Model

- Stochastic chain ladder (Renshaw & Verrall)
- Modelled as a GLM
- Log link function
- Gamma error distribution
- Linear predictor
 - $\theta + \beta_i + \delta_i$



- **Asset Classes**
 - 90 day bank bills
 - 10 year government bonds
 - Australian equities : All Ordinaries Index
- **Jon Carter's model**
 - An expanded Wilkie type cascade structure
 - Fitted to the Australian markets



Asset Portfolios

- **100% cash**
- **100% bonds**
- **100% equities**
- **Balanced – 30% cash, 30% bonds, 40% equities**

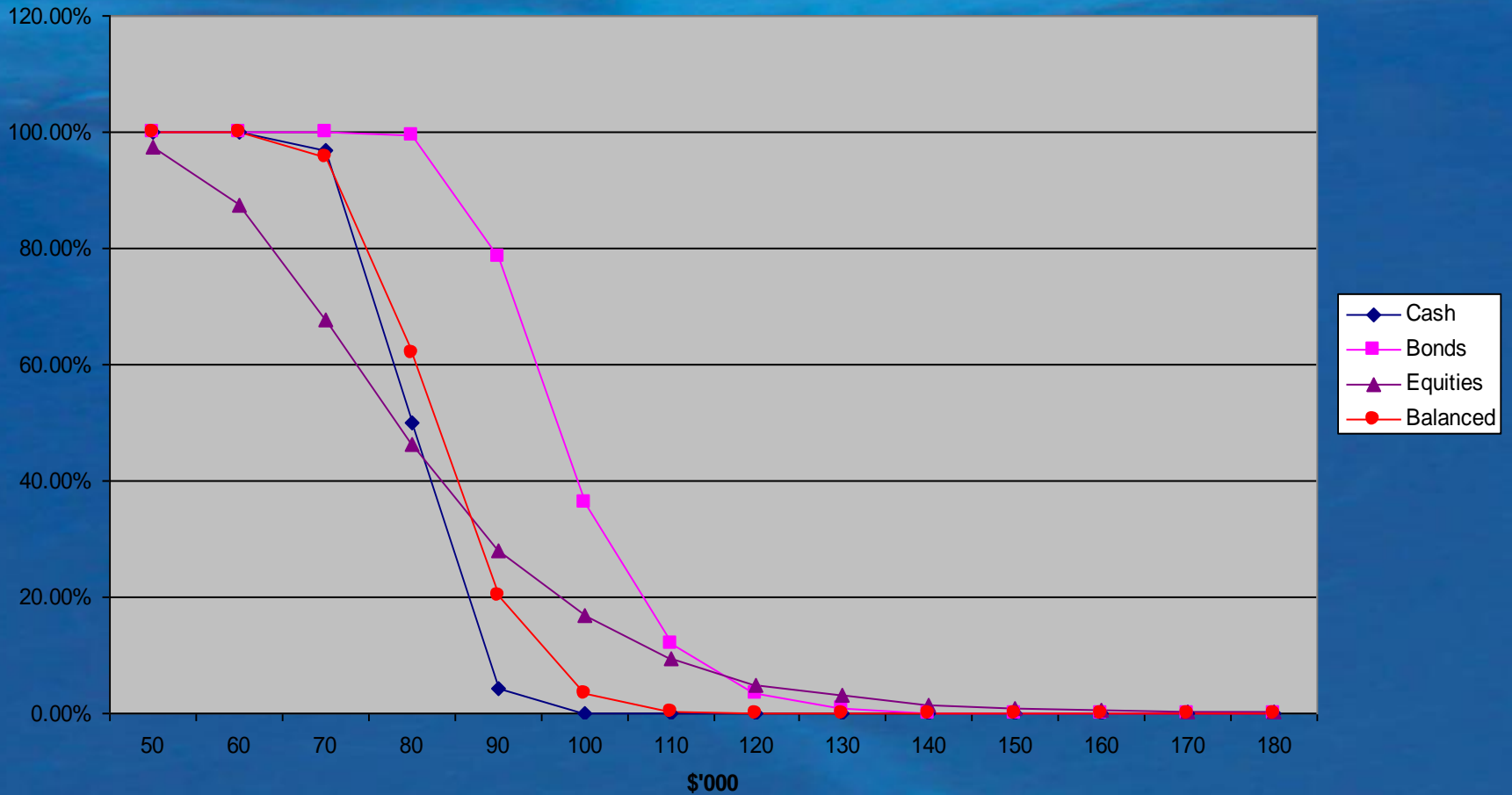


Probabilities of Insolvency

Initial Asset Amount	All Cash	All Bond	All Equity	Balanced
50000	100.00%	100.00%	97.33%	100.00%
60000	99.99%	100.00%	87.57%	99.96%
70000	96.90%	100.00%	67.78%	95.68%
80000	50.08%	99.37%	46.15%	62.09%
90000	4.27%	78.44%	27.98%	20.16%
100000	0.06%	36.22%	16.99%	3.42%
110000	0.02%	12.06%	9.48%	0.33%
120000	0.03%	3.42%	4.88%	0.02%
130000	0.01%	0.83%	3.02%	0.05%
140000	0.01%	0.10%	1.50%	0.01%
150000	0.04%	0.03%	0.93%	0.03%
160000	0.05%	0.02%	0.43%	0.04%
170000	0.05%	0.04%	0.21%	0.03%
180000	0.01%	0.00%	0.16%	0.01%



Probabilities of Insolvency



Further Research

- **Different asset models**
- **Different liability portfolios**
- **Complete interactions between asset and liability models**

