A ‘Simple’ Stochastic Model for Longevity Risk revisited through Bootstrap

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Projecting Australian Mortality using the CMI Mortality Projections Model

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Results - Males
Future work

Major limitation: deterministic
Next step: range of outcomes
Why does the issue arise?

• UK’s CMI method is deterministic
• How to determine percentiles for
  – Pricing
  – Reserving
  – Internal capital modelling
  – Of longevity risk in annuity portfolio?
• How to incorporate prudence or conservatism with quantification?
The Original Model (Koller 2011)

Sample paths of $C_t$

Sample paths of annuity payments
The Original Model (Koller 2011)

\[ \hat{q}_{x,t} = q_{x,t} \times C_t + \varepsilon_{x,t} \]

\[ C_t = e^{X_t} \times C_{t-1} \]

\[ C_0 = 1 \]
The Original Model (Koller 2011)

$$(X_t)_{t \in \mathbb{N}_0}, \text{iid } N(\mu, \sigma^2)$$

$$(e^{X_t})_{t \in \mathbb{N}_0}, \text{iid lognormal}$$

$$\mathbb{E}[e^{X_t}] = 1$$, is imposed, so that best estimate is followed

$$\mathbb{E}[e^{X_t}] = e^{\mu + 0.5\sigma^2}$$, follows from above
The Original Model (Koller 2011)

Sample paths of $C_t$

Sample paths of annuity payments
Modifications

• What if $X_t$ is not normal?
• Transformation - rejected
• Non-parametric approach – bootstrap
• Improved model selection
• Consider X rather than $X_t$
A numerical example

- Australian Females
- Human Mortality Database
- Ages 55 to 89 inclusive
- 1954 to 2008
- Surface of 35 ages and 55 cal. years
Crude Mortality Improvement Rates
Histogram of Crude MI Rates
Model assessment

• Use smoothing models only to help determine our best estimate of future variability

• M1-M8 from LifeMetrics (JP Morgan)
  – Standardised residuals and BIC
  – Forecasting properties
QQ Plots
<table>
<thead>
<tr>
<th>Model</th>
<th>Mean</th>
<th>Variance</th>
<th>Skewness</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.000606</td>
<td>1.489723</td>
<td>-0.087587</td>
<td>-10 152</td>
</tr>
<tr>
<td>2</td>
<td>0.007536</td>
<td>1.088392</td>
<td>0.134849</td>
<td>-10 192</td>
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<tr>
<td>3</td>
<td>-0.003997</td>
<td>2.028962</td>
<td>-0.085894</td>
<td>- 10 917</td>
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<tr>
<td>4</td>
<td>-0.019290</td>
<td>2.078517</td>
<td>0.172425</td>
<td>2 136</td>
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<td>5</td>
<td>0.166069</td>
<td>3.876925</td>
<td>0.139640</td>
<td>-12 399</td>
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<tr>
<td>6</td>
<td>0.034808</td>
<td>1.842268</td>
<td>2.350169</td>
<td>-10 309</td>
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<tr>
<td>7</td>
<td>-0.005507</td>
<td>1.496240</td>
<td>-1.015056</td>
<td>-10 397</td>
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<tr>
<td>8</td>
<td>0.020308</td>
<td>1.582045</td>
<td>1.035231</td>
<td>-10 297</td>
</tr>
</tbody>
</table>
Forecasting properties

- Now choosing between M2 and M3
- M2 unstable
- Backtesting for M3
Prediction Intervals I
Prediction Intervals II
QQ Plot of $X_t$ for M3 Australian Females
Age dependence or independence?

• P-values of two-variable regression of X against age (x) and year (t)

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Variable</th>
<th>ANOVA p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous Case</td>
<td>Year</td>
<td>0.1079</td>
</tr>
<tr>
<td></td>
<td>Age</td>
<td>0.8350</td>
</tr>
<tr>
<td>Discrete Case</td>
<td>Year</td>
<td>0.9965</td>
</tr>
<tr>
<td></td>
<td>Age</td>
<td>0.9999</td>
</tr>
</tbody>
</table>
Paths of $C_t, q_{65,t}$ for Australian Females
Fan plots of Ct, q65,t for Australian Females
Aust Females
(Age Independent Case, X)

Histogram (LHS) of $a_{65 \mid 31}$ (at 4% pa) and cohort survival curves (RHS)
Aust Females
(Age Dependent Case, $X_t$)

Histogram (LHS) of $a_{65:31}$ (at 4% pa) and cohort survival curves (RHS)
Paths of cohort survival curves for a 65:[31] for Australian Females (Two Cases)
Age independence => coherence?

- Example of crossover in Crude Mortality Rates for Australian Females (red cell indicates mortality in year $t$ is greater for age $x$ than for age $x+1$)
Age independence => coherence?

- Example of crossover in one simulation for Australian Females
  (red cell indicates mortality in year t is greater for age x than for age x+1)
Comparison with other methods

• Tickle and Booth (2014) evaluated and updated a forecast of mortality for Australian seniors using the Booth-Maindonald-Smith variant of Lee-Carter (generously shared underlying work)
  – Central estimate and 80% PI for $q_{x,t}$

• LPS 115 Section 38 (APRA, 2013, p7)
  – 20% drop in $q_{x,t}$
  – $< 0.5\%$ probability actual claims exceed
Forecast mortality rates for Australian Females aged 65, central estimate & selected percentiles
Paths of cohort survival curves (ie cashflows) for Australian Females, central estimate, APRA basis and 99.5\textsuperscript{th} percentiles for both cases
### Comparison of Cohort Life Expectancy & Annuity Values (at 4% pa) for Aust. Females aged 65 in 2010, central estimate & selected percentiles

<table>
<thead>
<tr>
<th></th>
<th>e65[31],2010</th>
<th>% diff</th>
<th>a65[31],2010</th>
<th>% diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central estimate (here, Tickle &amp; Booth)</td>
<td>22.668</td>
<td></td>
<td>14.166</td>
<td></td>
</tr>
<tr>
<td>APRA 20% drop (99.5th percentile)</td>
<td>23.655</td>
<td>4.35%</td>
<td>14.610</td>
<td>3.14%</td>
</tr>
<tr>
<td>Bootstrap, Age independent, X (10th percentile)</td>
<td>22.287</td>
<td>-1.68%</td>
<td>14.019</td>
<td>-1.03%</td>
</tr>
<tr>
<td>Bootstrap, Age independent, X (90th percentile)</td>
<td>22.845</td>
<td>0.78%</td>
<td>14.254</td>
<td>0.62%</td>
</tr>
<tr>
<td>Bootstrap, Age independent, X (99.5th percentile)</td>
<td>23.102</td>
<td>1.91%</td>
<td>14.325</td>
<td>1.12%</td>
</tr>
<tr>
<td>Bootstrap, Age dependent, Xt (10th percentile)</td>
<td>21.415</td>
<td>-5.53%</td>
<td>13.636</td>
<td>-3.74%</td>
</tr>
<tr>
<td>Bootstrap, Age dependent, Xt (90th percentile)</td>
<td>23.817</td>
<td>5.07%</td>
<td>14.645</td>
<td>3.39%</td>
</tr>
<tr>
<td>Bootstrap, Age dependent, Xt (99.5th percentile)</td>
<td>24.912</td>
<td>9.90%</td>
<td>15.038</td>
<td>6.16%</td>
</tr>
</tbody>
</table>
Conclusion

• CMI method increasingly widely applied - UK, USA, Canada, Australia and China at least
• Some national statistics bodies also deterministic
  (ABS, 2013, ONS, 2013)
• This method allows the user to pragmatically add stochastic variation to a deterministic model
• Bootstrapping improves tail modelling
• Help to inform understanding of longevity risk
References