EMPIRICAL ANALYSIS OF INVESTOR UTILITIES IN INVESTMENT CHOICE

INSTITUTE OF ACTUARIES RESEARCH CONFERENCE

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Investor Utilities

- What is the form of the investor utility function? How do investor utilities combine to form an aggregate investor utility function, and does this create a mean-variance optimized universe?
- What are the factors that describe Investor utility? Are there differences according to personality or gender or education?
- How does investor utility change? Is there a way of describing the inertia of choice? What happens when an event triggers choice?

Paper

- Section 1 proposed an experimental model that operates during instantaneous time and forced choice to estimate the *E(U)* for groups of investors.
- Section 2 presents the empirical results of aggregate *E(U)* for the experiment of forced choice and makes a surprising discovery.
- Section 3 extends analysis to test whether information is correctly interpreted whether *E(r)* is consistent, and whether we can identify sub-groups.
- Section 4 reviews the outcomes of a first-choice event.
- Section 5 analyses data of choices actually made over a 6 year period.
- Section 6 concludes.

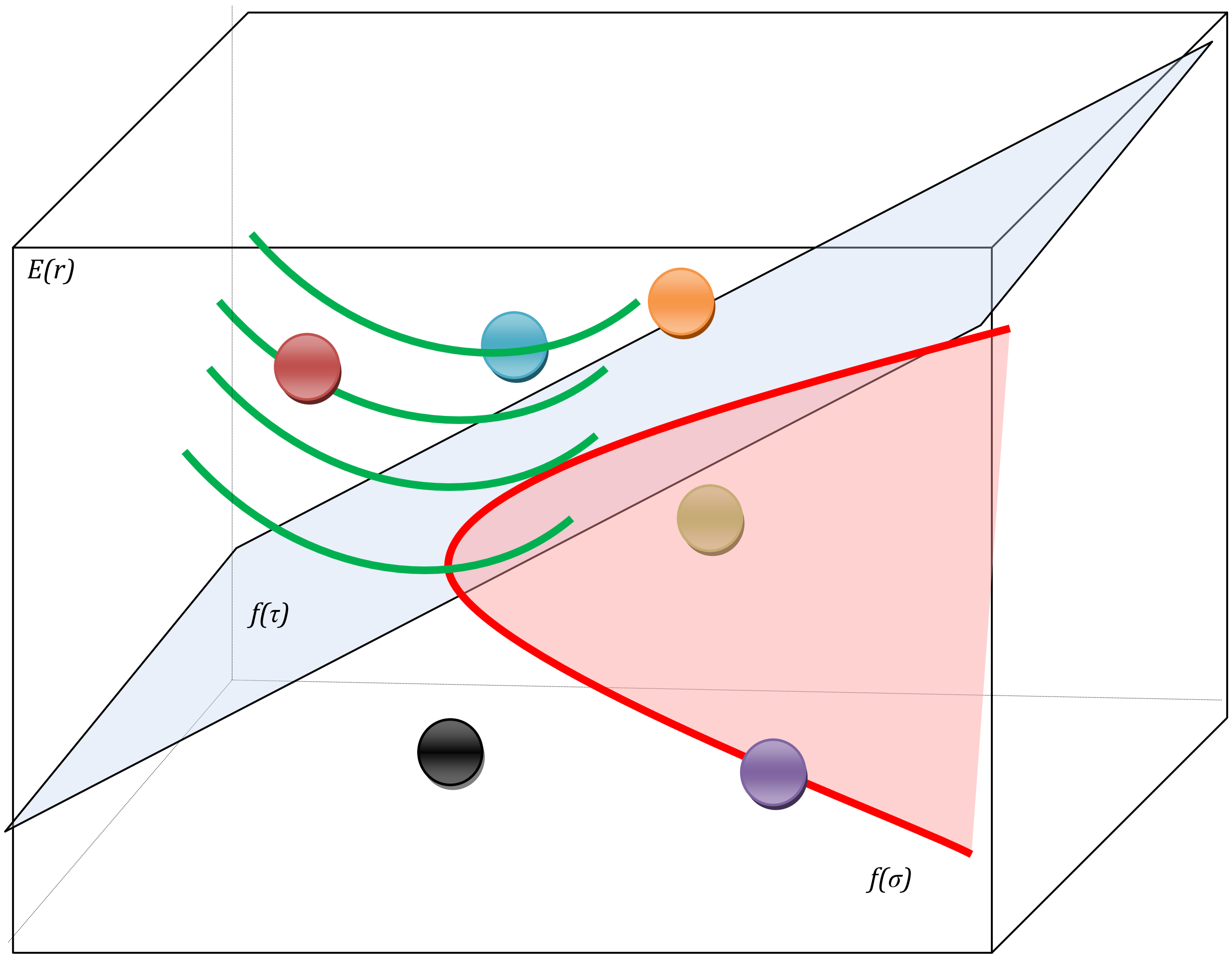
Central Concept

- Investor Utilities drive the market equilibrium
  - Switching between portfolios with risk return characteristics
  - Attitudes and Beliefs to their behaviour
- Approach
  - Experiment of 236 Investors’ behaviour and attitudes
  - Event Analysis
  - Quantitative Analysis of 4,000 investment decisions from 2002 to 2006

Section 1: The derivation of Investor E(U) and their aggregation.

- *E(U)* defines as some form of mean-variance optimality in MPT, as the interaction of investor utility with the tangent to the efficient frontier.
  - The optimization : $L = E(U_i)(q_1, q_2, \ldots q_n) - T(q_1, q_2, \ldots q_n)$
- Define *E(U) = f[E(r), f(σ), f(τ)]*
- Create a set of attributes with values:
  - $q_i \in$ Expected Return *E(r)*, Risk f*(σ)*, Time Horizon *f(τ)*},
- A portfolio is created by a random draw from each of the three attribute sets: $P_x(q_1,q_2,q_3)$:$\{x_1 \ldots x_n\} \cap \{y_1 \ldots . y_n\} \cap \{z_1 \ldots . z_n\}$
- $[E(U) \mid \Phi, \Omega] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$

Stylised figure of experiment



Section 2: Experimental Construction

Return $E(r)$	Risk (Annualised Chance of a Negative Return) $f(\sigma)$	Time Horizon $f(\tau)$
3.9%	no chance	1 year
6.0 - 6.3%	13% chance	3 year
6.5 - 7.2%	20% chance	5 year
7.2 - 8.1%	25% chance	10 year
8.0 - 9.0%	33% chance	

	Numbers
Investors who had recently made a change in investment portfolio (Switchers)	186
Investors who had not made a change in investment portfolio (Non-Switchers)	50
TOTAL	236

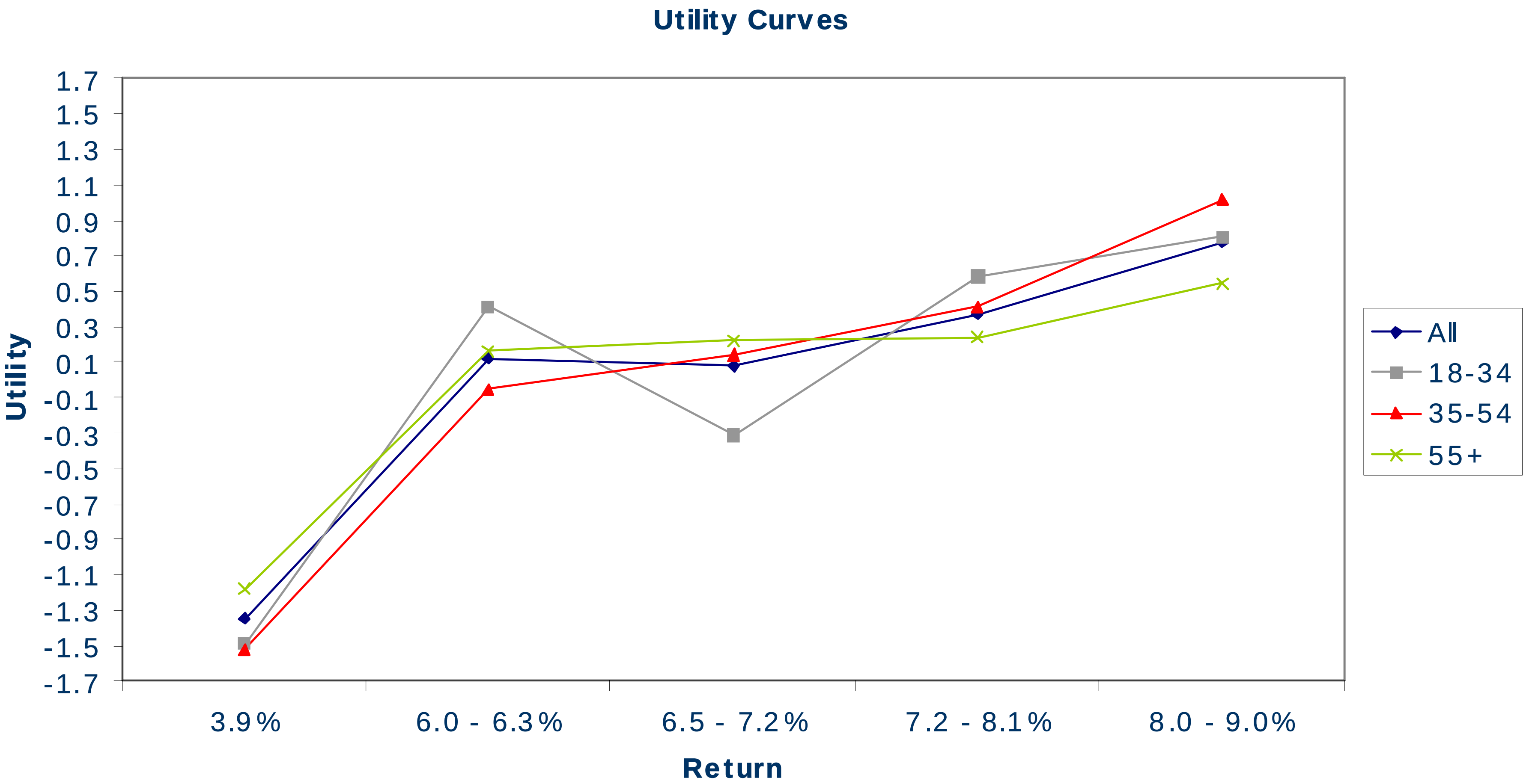
- Choice-based conjoint analysis
  - State preference tasks
- 236 respondents
  - 16 Portfolio Pairs
- E.g.: Choose between
  - A – 3.9%, 13% chance of a negative return, 1 year time horizon
  - B – 8.0% to 9.0% return, 20% chance of a negative return, 2 year time horizon
- Random picks
  - Non optimal portfolios

Utilities generated using MLN

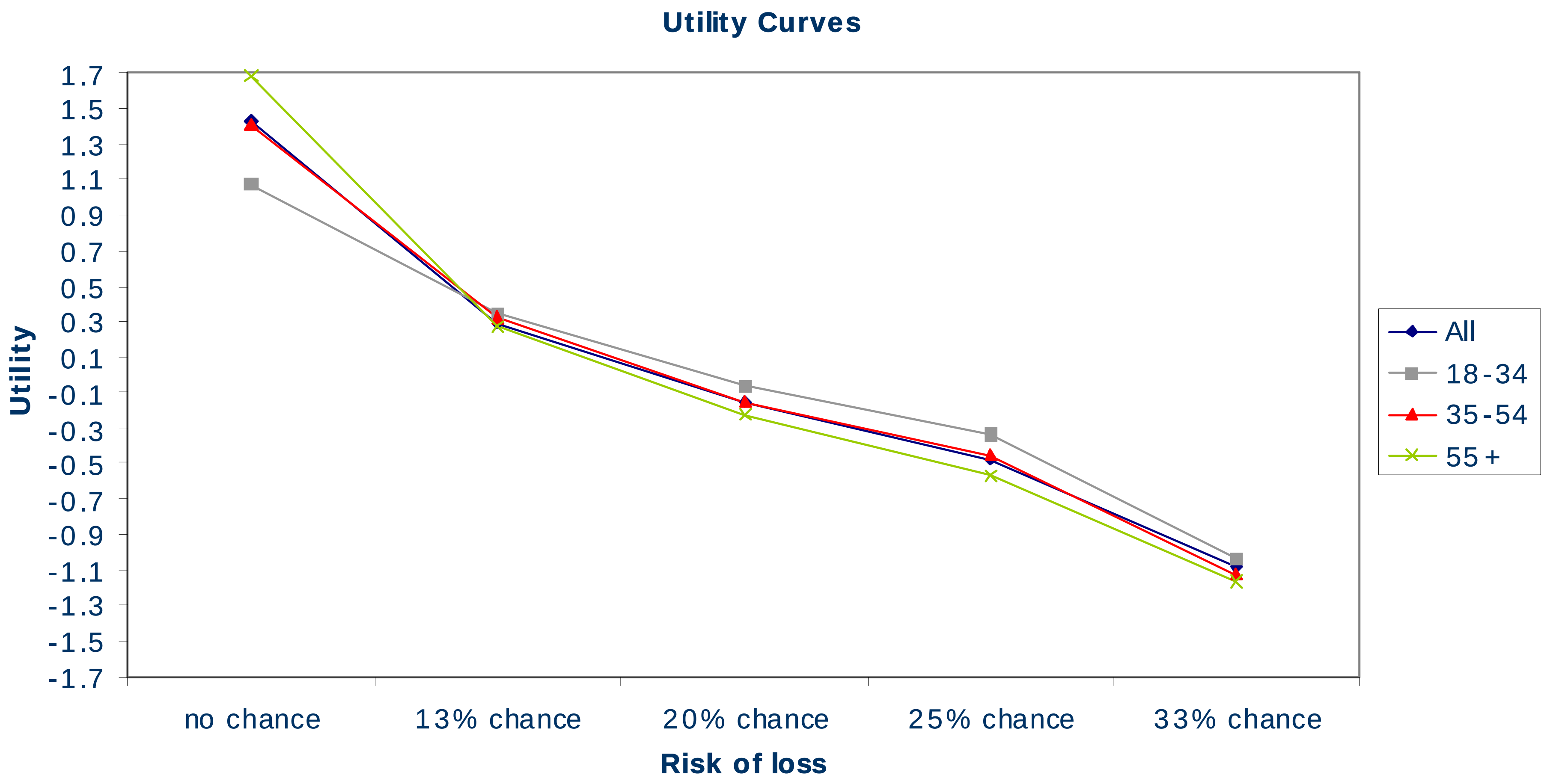
Variables			Utilities by Respondent Segment			
Attribute	Value	Description	All	18-34	35-54	55+
E(r)						
1	1	3.9%	-1.352	-1.491	-1.522	-1.172
1	2	6.0 - 6.3%	0.121	0.415	-0.052	0.165
1	3	6.5 - 7.2%	0.083	-0.308	0.142	0.221
1	4	7.2 - 8.1%	0.375	0.579	0.417	0.242
1	5	8.0 - 9.0%	0.774	0.805	1.016	0.544
$f(\tau)$						
2	1	1 year	-0.008	-0.411	0.194	-0.072
2	2	3 year	0.009	0.013	0.140	-0.118
2	3	5 year	0.177	0.327	0.083	0.235
2	4	10 year	-0.178	0.071	-0.417	-0.045
$f(\sigma)$						
3	1	no chance	1.425	1.074	1.403	1.681
3	2	13% chance	0.295	0.347	0.323	0.269
3	3	20% chance	-0.153	-0.060	-0.147	-0.223
3	4	25% chance	-0.479	-0.331	-0.460	-0.563
3	5	33% chance	-1.087	-1.031	-1.120	-1.164
4	1	Neither	-1.36218	-2.13867	-1.45955	-1.03366
Respondents			236	42	101	93

- No significant difference for age
- Utilities can be added
  - Conjoint choice

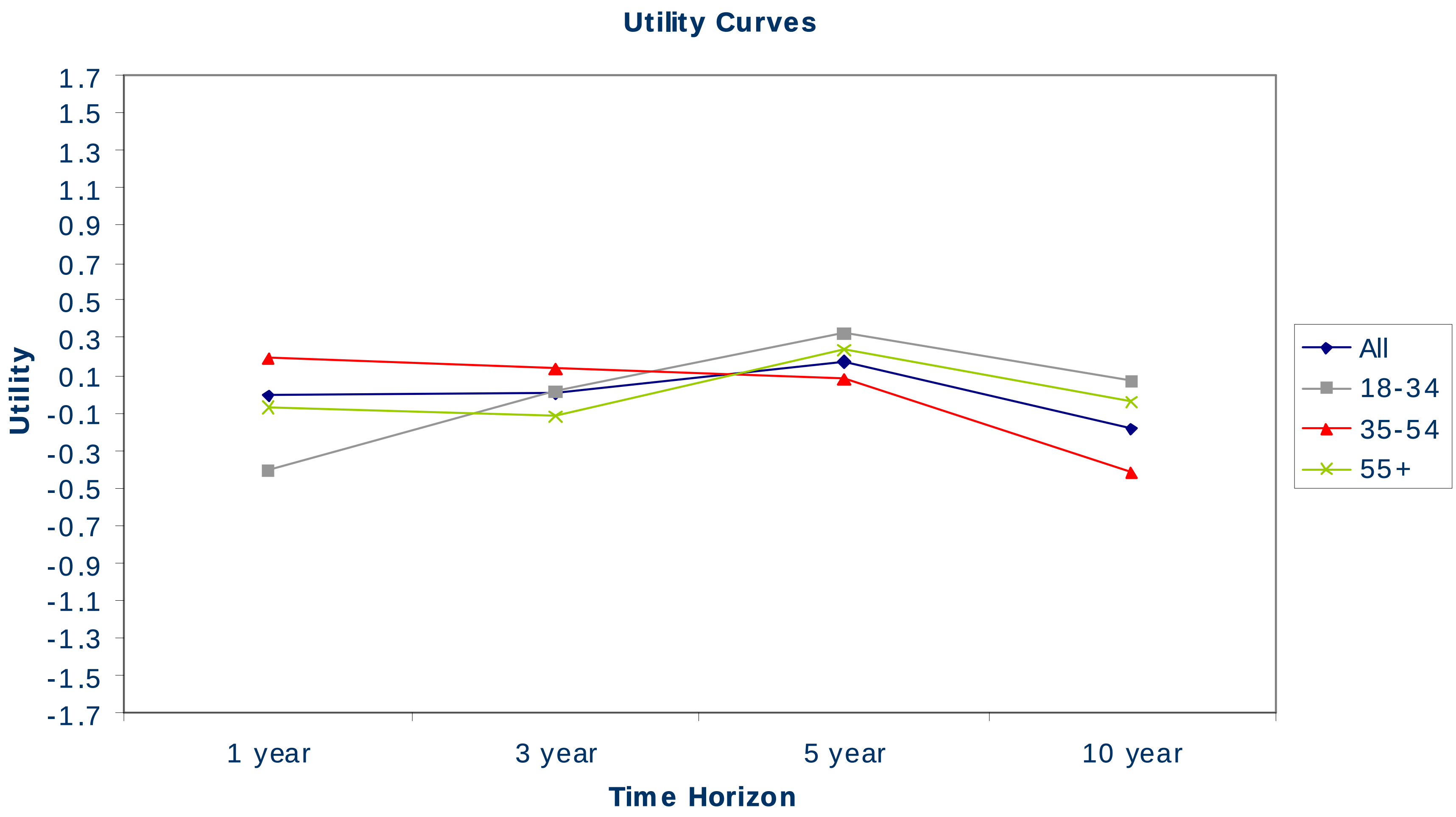
Choice Modelling of Utilities: Return, Risk



Choice Modelling of Utilities: Return, Risk



Choice Modelling of Utilities: Time Horizon



PORTFOLIO INDIFFERENCE CURVES – ISOUTILITIES

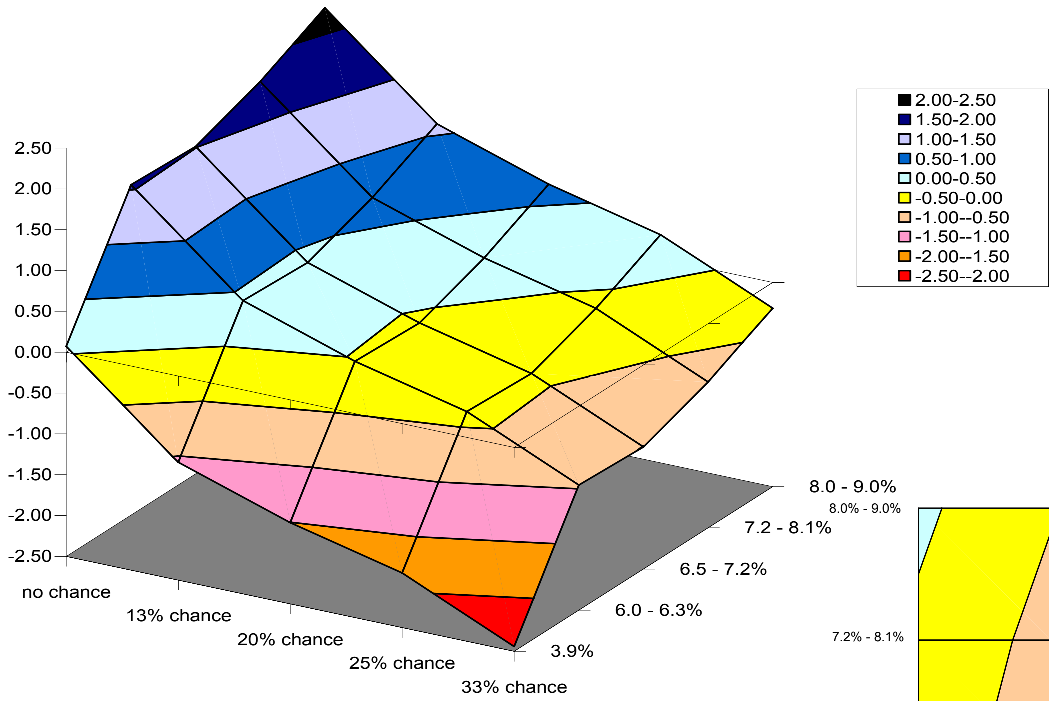
	pU of *f(σ)*	1.425	0.295	-0.153	-0.479	-1.087
pU of *E(r)*		0	13%	20%	25%	33%
-1.352	3.9%	0.07	-1.06	-1.51	-1.83	-2.44
0.121	6.0% - 6.3%	1.55	0.42	-0.03	-0.36	-0.97
0.083	6.5% - 7.2%	1.51	0.38	-0.07	-0.40	-1.00
0.375	7.3% - 8.0%	1.80	0.67	0.22	-0.10	-0.71
0.774	8.0% - 9.0%	2.20	1.07	0.62	0.29	-0.31

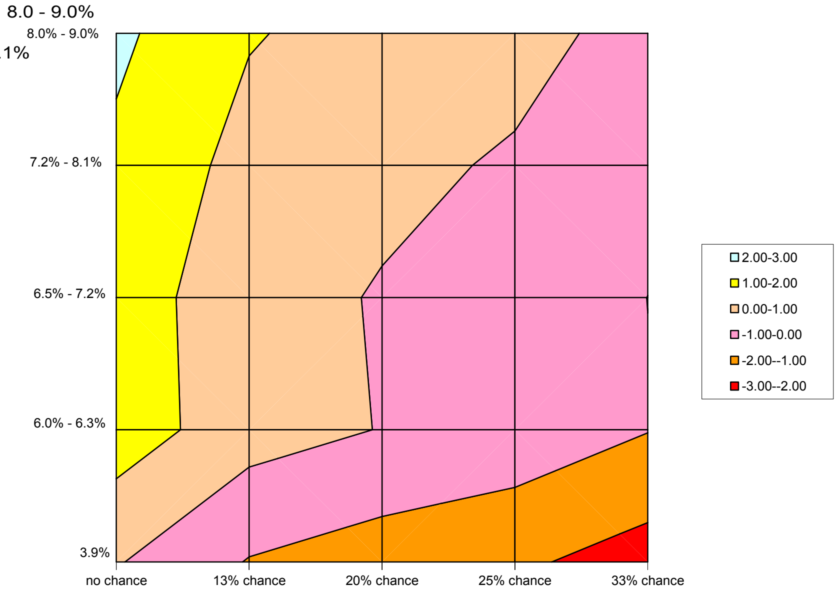
Arithmetic

$$E(U) = 2.6296 \ln(E(r)) + 3.2612\, f(\sigma)^2 - 8.5644\, f(\sigma) + 8.6409$$

	pU of *f(σ)*	1.425	0.295	-0.153	-0.479	-1.087
pU of *E(r)*		0	13%	20%	25%	33%
-1.352	3.9%	0.11	-0.95	-1.47	-1.83	-2.36
0.121	6.0% - 6.3%	1.31	0.25	-0.27	-0.63	-1.16
0.083	6.5% - 7.2%	1.59	0.53	0.01	-0.35	-0.88
0.375	7.3% - 8.0%	1.88	0.82	0.30	-0.06	-0.59
0.774	8.0% - 9.0%	2.16	1.10	0.58	0.22	-0.31

Function


2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
no chance
13% chance
20% chance
25% chance
33% chance
3.9%
6.0 - 6.3%
6.5 - 7.2%
7.2 - 8.1%
8.0 - 9.0%
2.00-2.50
1.50-2.00
1.00-1.50
0.50-1.00
0.00-0.50
-0.50-0.00
-1.00--0.50
-1.50--1.00
-2.00--1.50
-2.50--2.00


8.0% - 9.0%
7.2% - 8.1%
6.5% - 7.2%
6.0% - 6.3%
3.9%
no chance
13% chance
20% chance
25% chance
33% chance
2.00-3.00
1.00-2.00
0.00-1.00
-1.00-0.00
-2.00--1.00
-3.00--2.00

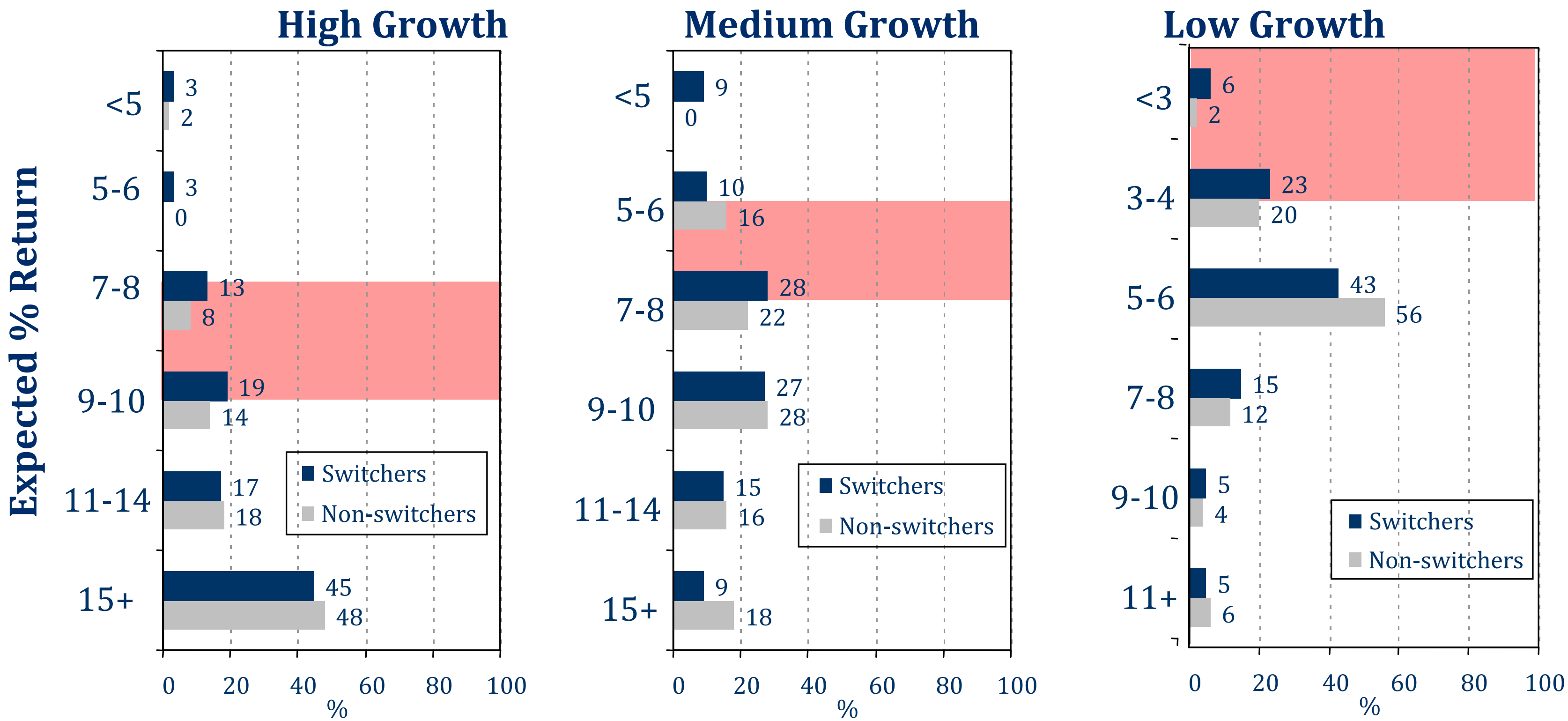
Implications

- Monotonic pU's for *E(r)* and *f(σ)* that hold for MRRT.
- Portfolios not necessarily Efficient
- Mechanism to drive market equilibrium

Section 3: Tests of Efficient Interpretation of Information Tests of the Influence of Demographics

- Section 2 did not prove investors make efficient decisions.
  - Investors, in making state preference choices, interpret the information of E(r); f(σ) and f(τ) correctly; or
  - Only portfolios that exist on the Efficient Frontier are available in the market
- First test is a test of Φ: Investors expectations consistent with professionals who construct Efficient Portfolios
  - $[E(U) \mid \Phi, \Omega] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$
- Assume Φ as the unfiltered and non-transformed information, we define Φ* as the probability density function of a group of investors such that:
  - Φi*=Pr[KTi. Φ | Ω] then
  - $[E(U) \mid \Phi^*, \Omega] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$

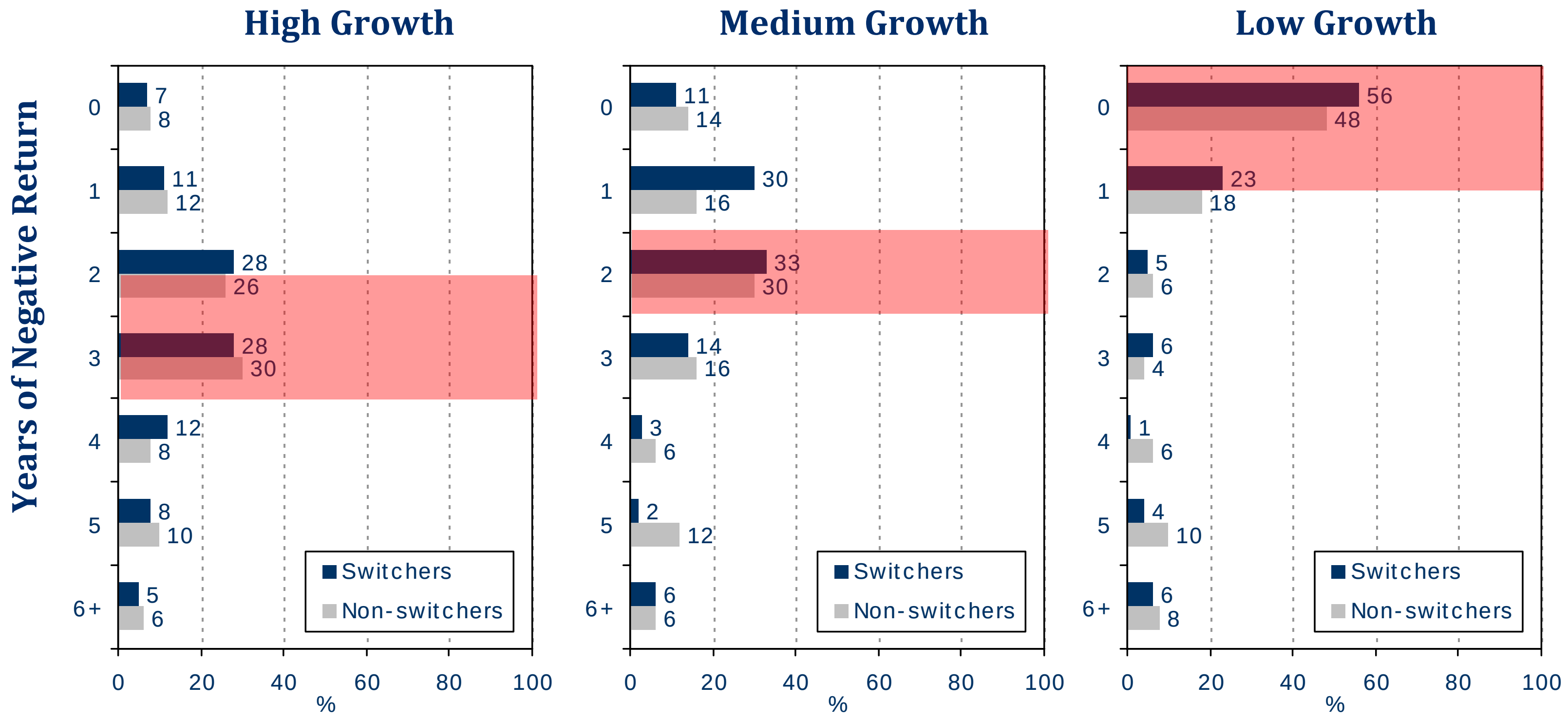
EXPECTED RETURNS



Investors tend to overestimate Returns

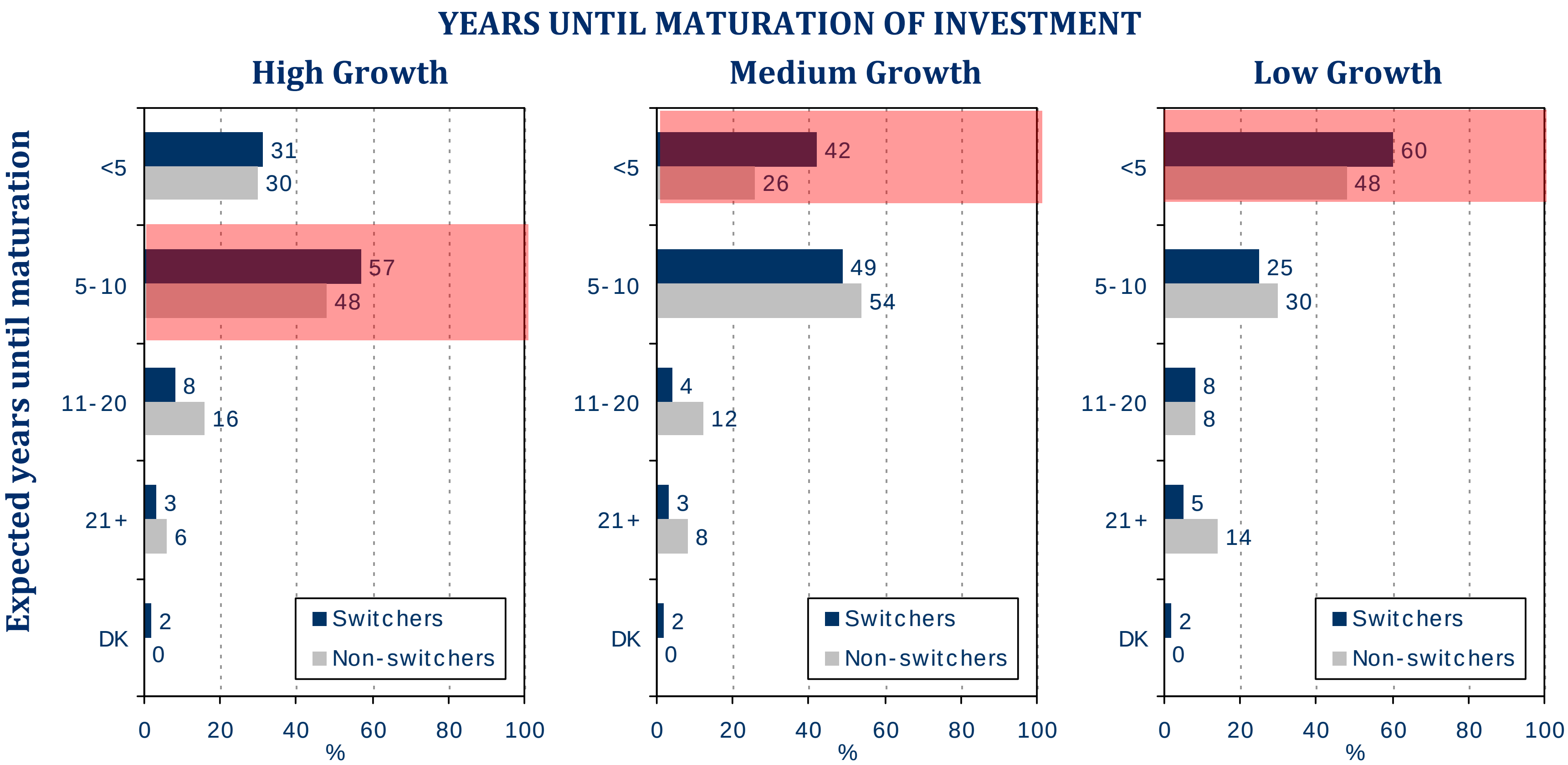
Investor Expectations of Returns

NUMBER OF NEGATIVE RETURN YEARS OUT OF 10 YEARS



Base: All members (n=236)

Investor Expectations of Risk



Base: All members (n=236)

Investor Expectations of Time Horizon

SWITCHERS

	S	S	N	N	
I	13%	3%	3%	6%	J
I	7%	3%	2%	2%	P
E	11%	3%	1%	3%	P
E	19%	5%	4%	16%	J
	T	F	F	T	

NON-SWITCHERS

	S	S	N	N	
I	18%	4%	12%	4%	J
I	8%	4%	2%	2%	P
E	10%	2%	2%	4%	P
E	16%	2%	4%	6%	J
	T	F	F	T	

Education Levels	Switchers %	Non-Switchers %
Some secondary school	4	8
Intermediate/School Certificate	11	16
Leaving Certificate/HSC	14	18
Trade qualification/Diploma	35	40
University Undergraduate Degree	19	12

Personality and Demographics may matter

Conclusion

- Expectations showed a dispersion $\Pr(\Phi^*)$
- Ω, the conditioning of E(U) based on demographics or other investor characteristics, debatable whether in aggregate, this characterizes the effects of the transform of Φ.
- $[E(U)\,|\,\Pr[\Phi^*]] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$

Section 4: Event Studies

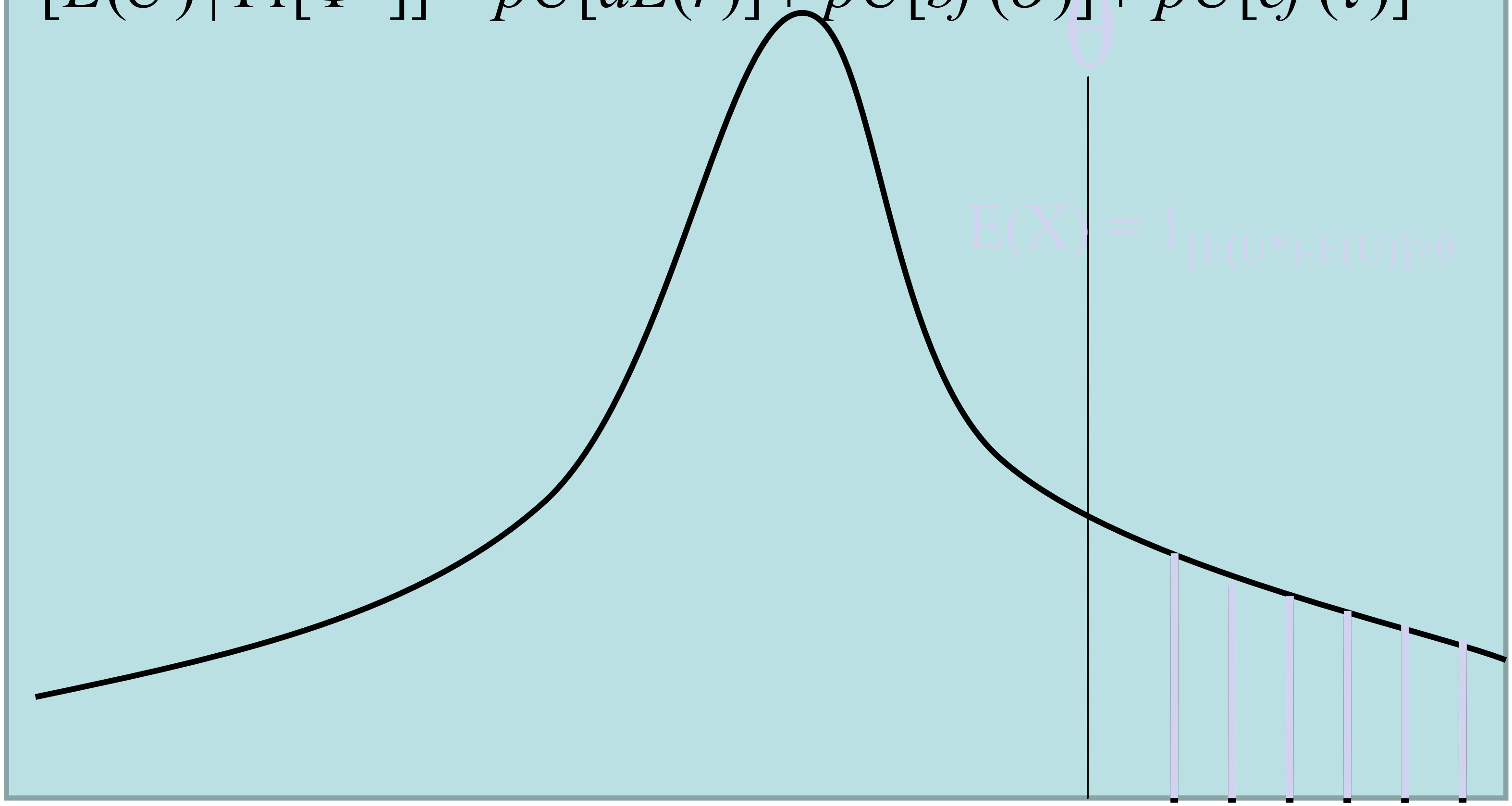
- $[E(U)\,|\,\Pr[\Phi^*]] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$
- For trade: $E(U_i^*) > E(U_i)$
- But require information change:
  - $E(U_1) - E(U_i^*) = \frac{\partial E(U_1)}{\partial \Phi} = \frac{\partial E(U_1)}{\partial (E(r),\sigma,\tau)} \cdot \frac{\partial (E(r),\sigma,\tau)}{\partial \Phi}$
  - New orthogonal constraint B
- Is an event proof of change of information or some other constraint?

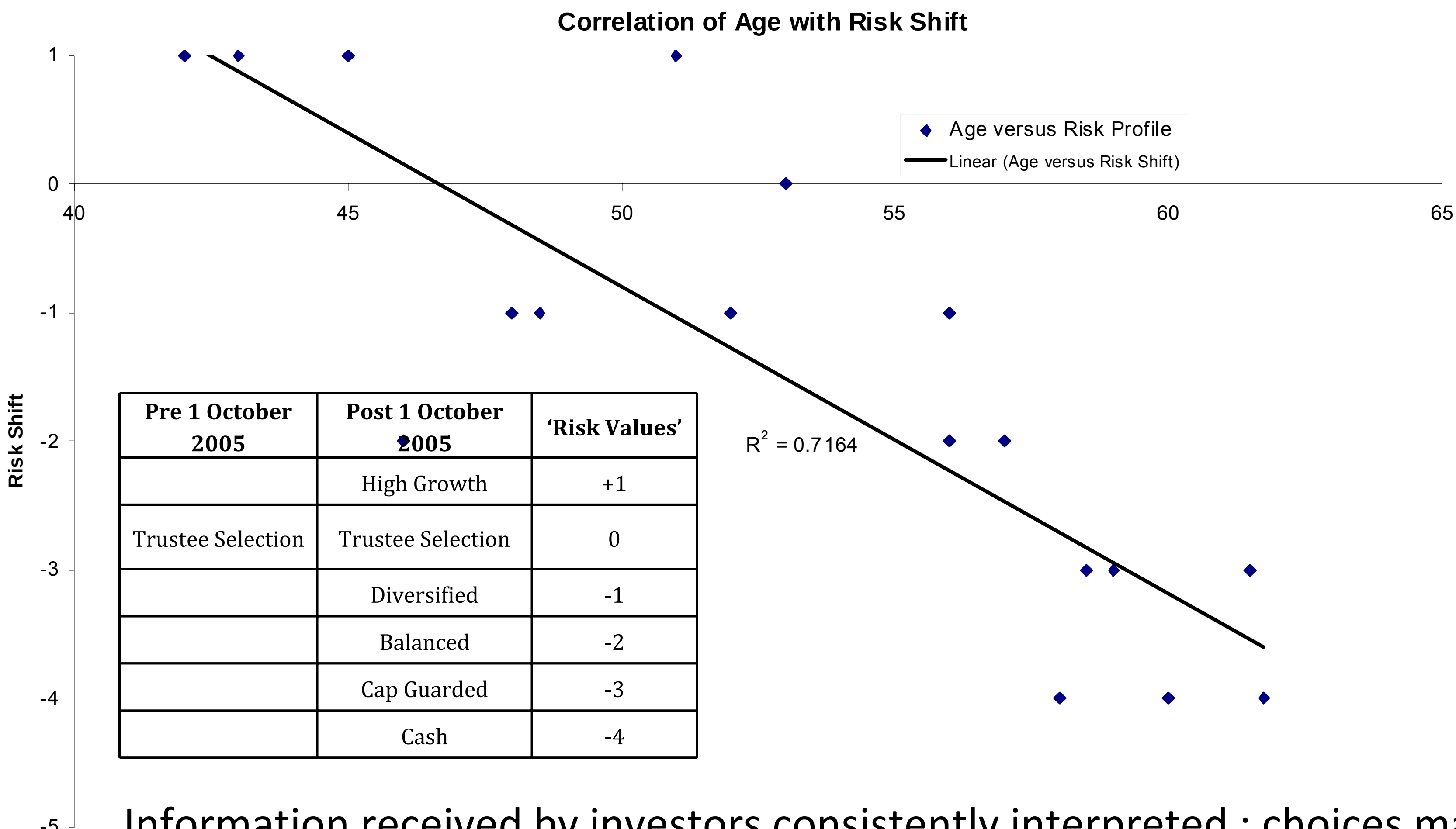
$$L = E(U_i)(q_1, q_2, ..q_n) - \lambda T(q_1, q_2, ..q_n) - \mu B(q_1, q_2, ...q_n)$$

- Inertia
  - Bernoulli variable *X*, where
  - $E(X) = 1_{[E(U^*)-E(U)]>\theta}$ for trade to occur
  - Threshold θ that is set endogenously by each investor:

Dispersion of Pr[Φ] and Event study explain why only some choose

$$[E(U)\,|\,\Pr[\Phi^*]] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$$





Pre 1 October 2005	Post 1 October 2005	'Risk Values'
	High Growth	+1
Trustee Selection	Trustee Selection	0
	Diversified	-1
	Balanced	-2
	Cap Guarded	-3
	Cash	-4

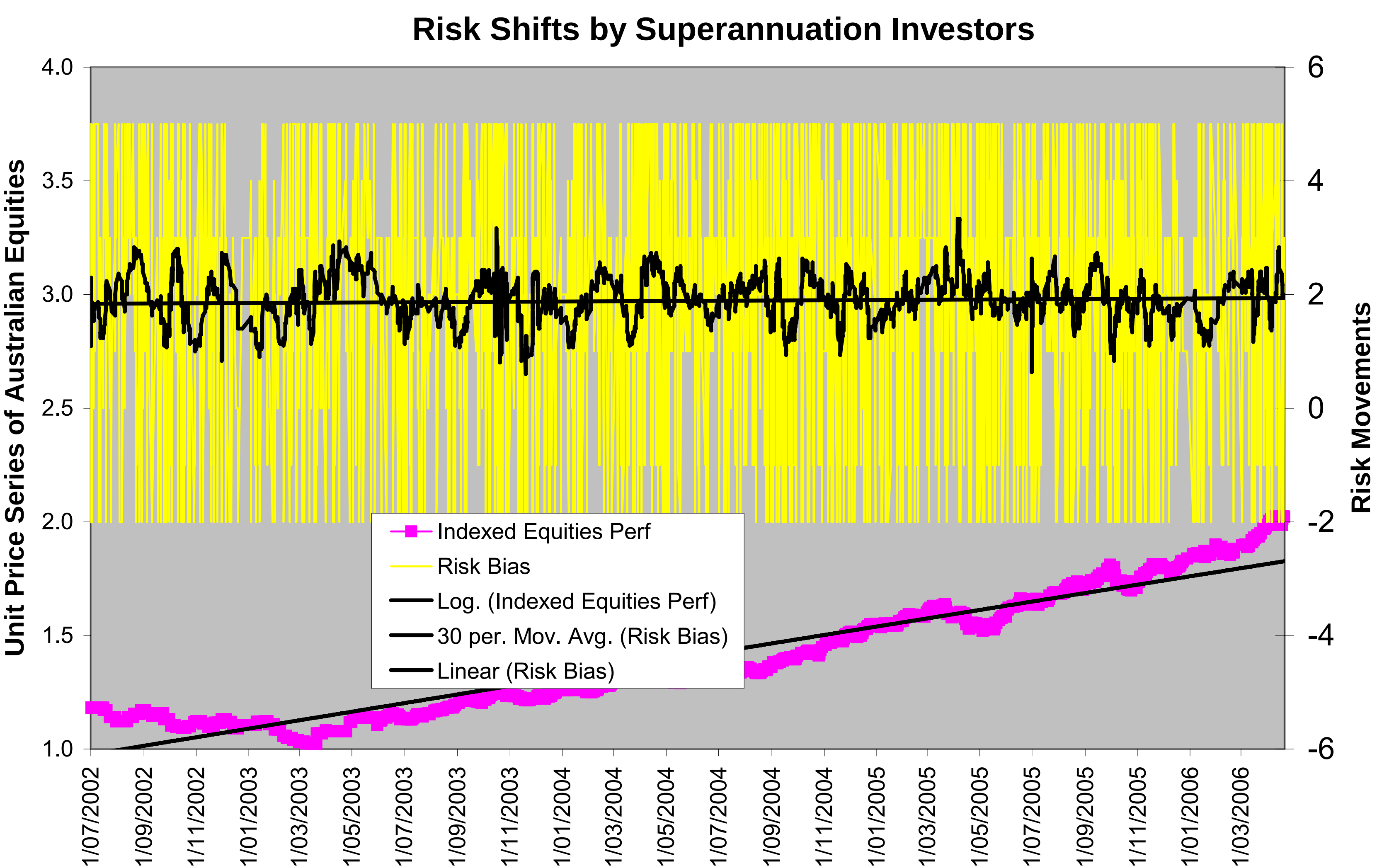
Information received by investors consistently interpreted ; choices made were entirely reliant on removal of constraint (B).

$$L = E(U_i)(q_1, q_2, \ldots q_n) - \lambda T(q_1, q_2, \ldots q_n) - \mu B(q_1, q_2, \ldots q_n)$$ holds

Section 5: Continuous time analysis

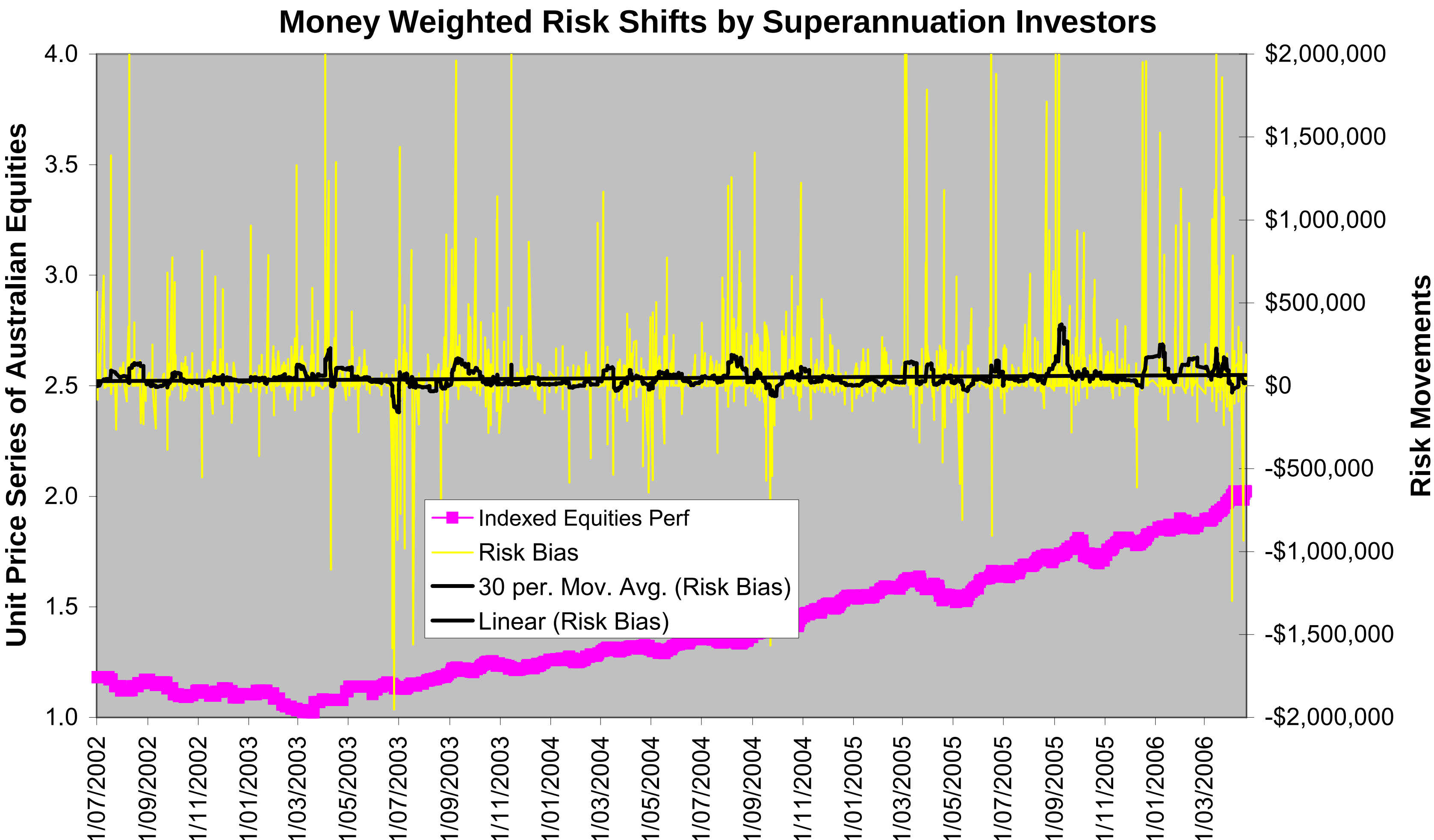
- Consistent Method of assigning values to 'Riskiness' for quantitative analysis

Portfolio Names	**'High Growth'**	**'Trustee Selection'**	**'Divers-ified'**	**'Bal-anced'**	**'Capital Guarded'**	**'Cash'**
Typical Assets held	*85-90% Equities, Property*	*75% - 85% Equities, Property*	*65-70% Equities, Property*	*45-55% Equities, Property, with the remainder in Bonds, Cash*	*<15% Equities, Property, with the remainder in Bonds, Cash*	*Largely Cash with possibly some short-dated Bonds*
Relative Risk 'Value'	**+1**	**0**	**-1**	**-2**	**-3**	**-4**



Risk Shifts and Market Direction

Money Weighted Risk Shifts and Market Direction



Correlation between Aggregate Risk Shift Series and Australian Equities Unit Price Series	0.01
Correlation between Aggregate ***Weighted*** Risk Shift Series and Australian Equities Unit	0.06

Wealth doesn’t seem to change correlation.

Conclusion : Generalised Utility

- Firstly, the utility function of the aggregation of investors can be written in the form:
- $[E(U) \mid \Pr[\Phi^*]] = pU[aE(r)] + pU[bf(\sigma)] + pU[cf(\tau)]$
  - Where: Φ_i*=Pr[KT_i. Φ | Ω]
  - Investors optimise to MRRT (Market Risk / Reward Theorem), don't necessarily choose efficient portfolios.
- Secondly, no evidence that demographic factors are conditions on aggregate utility.
- Thirdly, event studies show that trade occurs for reasons other than changes in information
  - $L = E(U_i)(q_1, q_2, .. q_n) - \lambda T(q_1, q_2, .. q_n) - \mu B(q_1, q_2, ... q_n)$
- $E(X) = 1_{[E(U^*)-E(U)]>\theta}$ presents inertia