
# Volatilities, Correlations and Risk Capital Allocation in the US PC Insurance Industry, parts of a cross-industry study with recommendations.

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"Consciousness [is] only a tool possessed by a unit mammal which found itself in need of some half-decent predictive capability." - Don Paterson

- Risk management = "half-decent predictive capability" (Half-decent, but a small improvement goes a long way to improve survival.)
- = awareness and communication
- = something we've always been doing but only recently been able to do scientifically

- “The unreasonable effectiveness of mathematics in the natural sciences.”
- - E. Wigner

- Not true of financial mathematics.
- “Reasonable effectiveness of mathematics.”
- Goal is not truth but best use of available information.
- What is best? What is information?

Definition of Risk

1. The possibility of suffering harm or loss; danger.
2. A factor, thing, element, or course involving uncertain danger; a hazard.
3. a. The danger or probability of loss to an insurer.
   b. The amount that an insurance company stands to lose.
4. a. The variability of returns from an investment.
   b. The chance of nonpayment of a debt.
5. Probability and severity of loss linked to hazards.

- Move towards making definition of risk more objective and quantitative
- Risk like "utility" in Economics needs to retain some subjective elements to accurately capture what we actually do.

RISK

| | |
|---|---|
| danger | probability |
| possibility | expectation |
| hazard | variance |
| uncertainty | probability+severity |
| | distribution |

Qualitative ⟶ Quantitative

| RISK | LUCK |
| --- | --- |
| Objective | Subjective |
| General | Individual |
| Quantitative | Magical, Mercurial |
| “Value at Risk” | “Lady Luck” |
| “High-Risk/Low Risk” | “Winning Streak” |
| “Risk-taker” | Charismatic, Lucky |
| “Risk-averse” | Confidence/Bubble |
| Casino Owner | Casino Patron |
| Statistician/Actuary | Tipster/ Astrologer |
| Risk Expert | Winners & Losers |
| Fundamentals | “Blink” |

## RISK

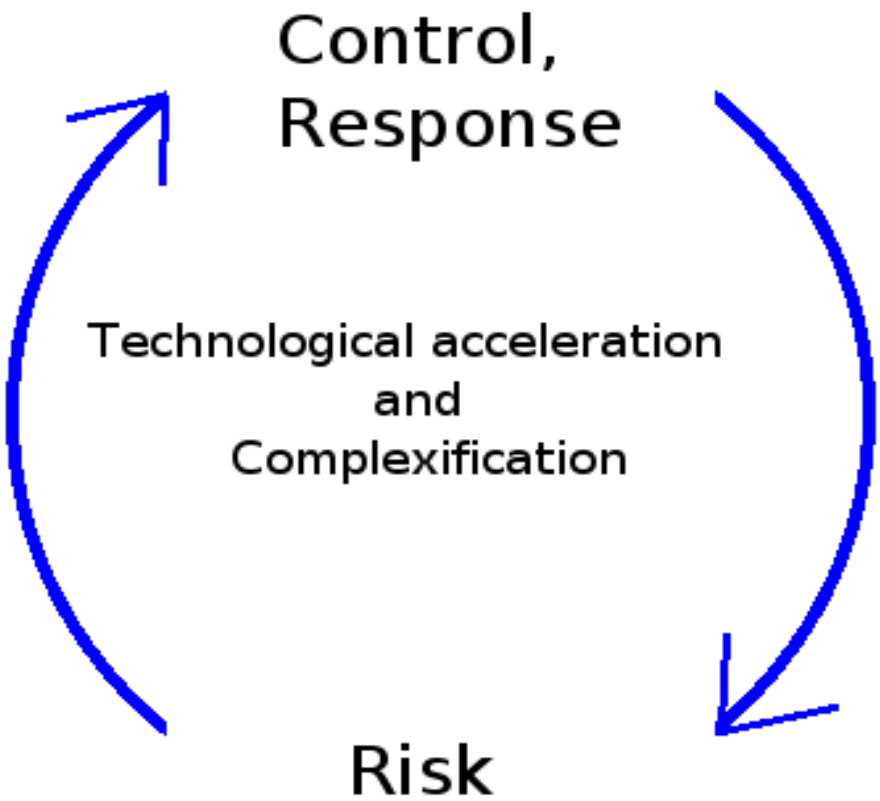
**Loss Distribution**

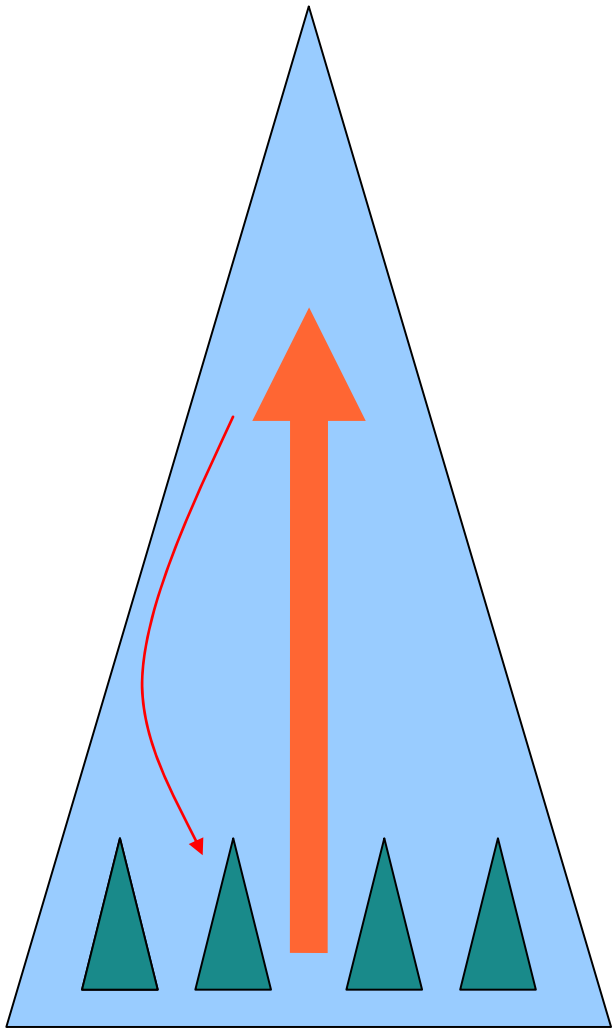
**Confirmed or falsified only in the long run. (Long term, corporate survival time, especially in Insurance)**

## LUCK

**Point Estimate**

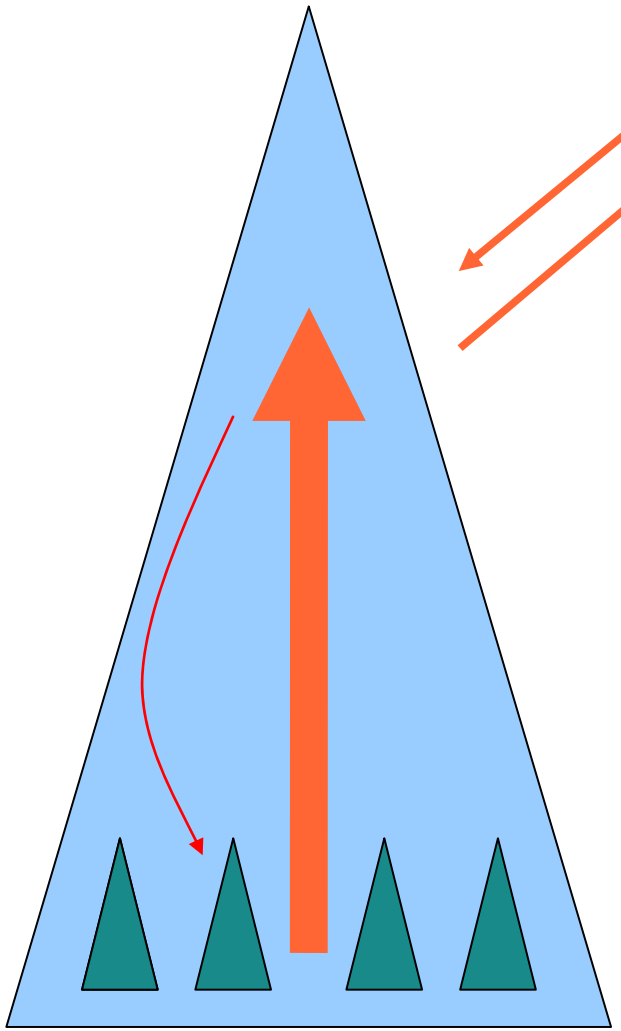
**Confirmed or falsified immediately! (Operational, or careerwise time (?)) May be catastrophic in long term.**


Control,
Response
Technological acceleration
and
Complexification
Risk



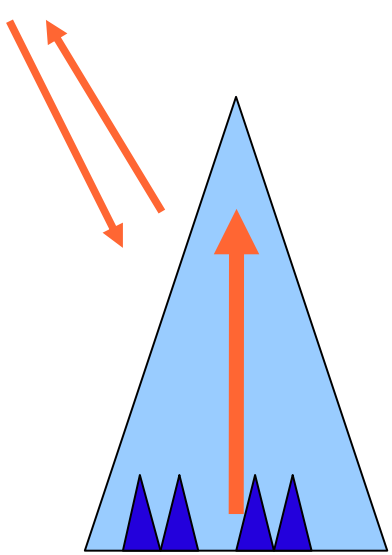
Information about risk within a corporation moves from individual divisions up to higher levels of management.

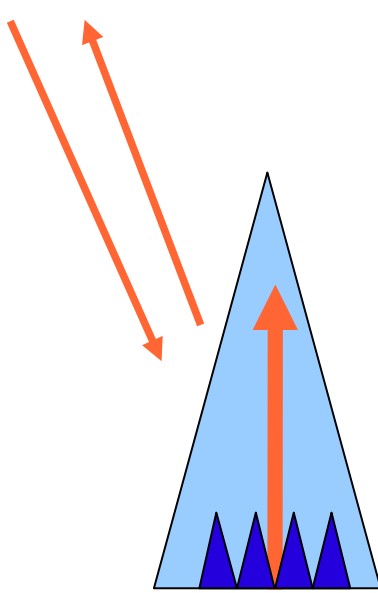
Information from separate units must be comparable. Good risk management prompts more data from divisions.



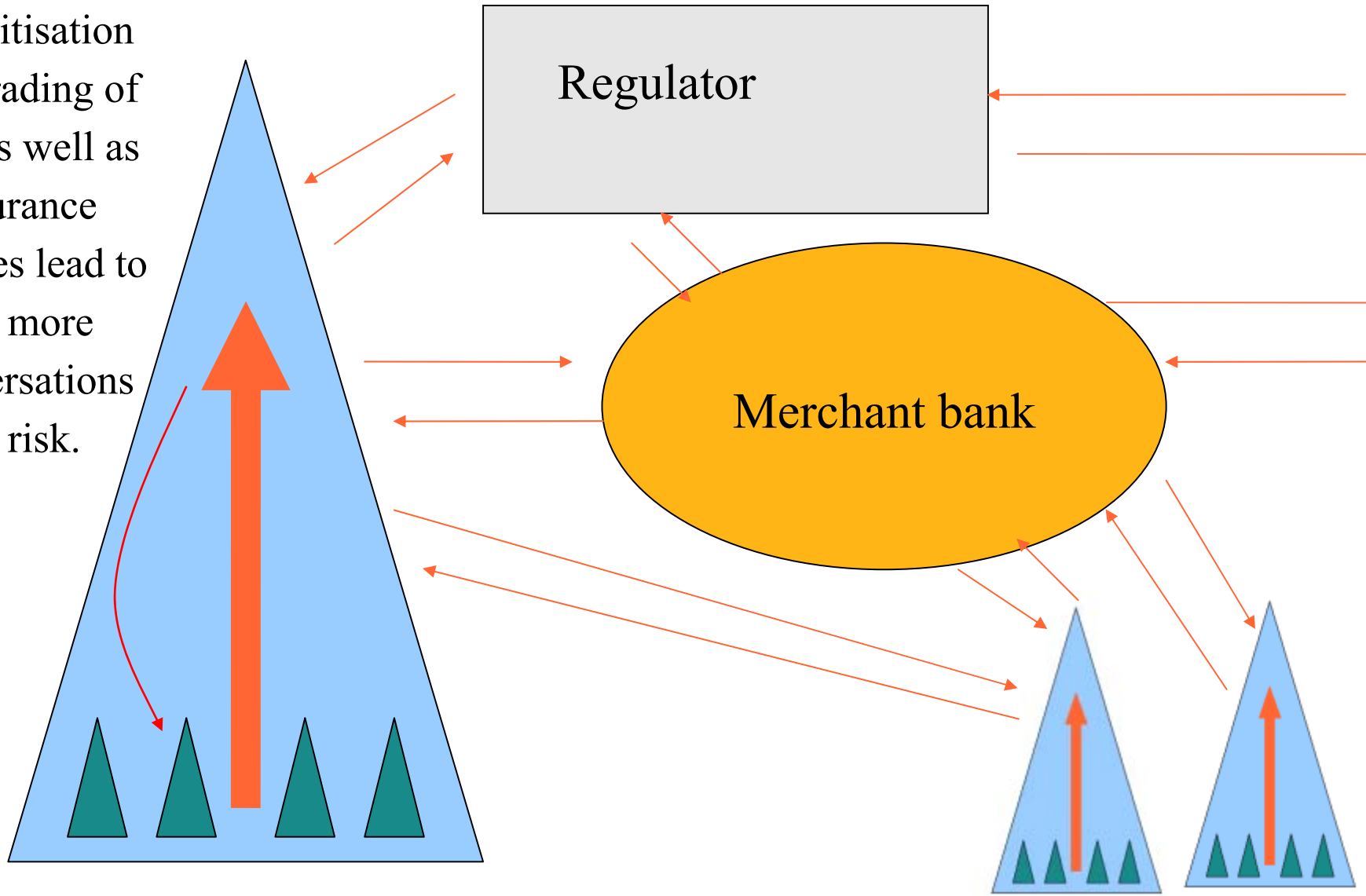
Industry regulator & Auditor

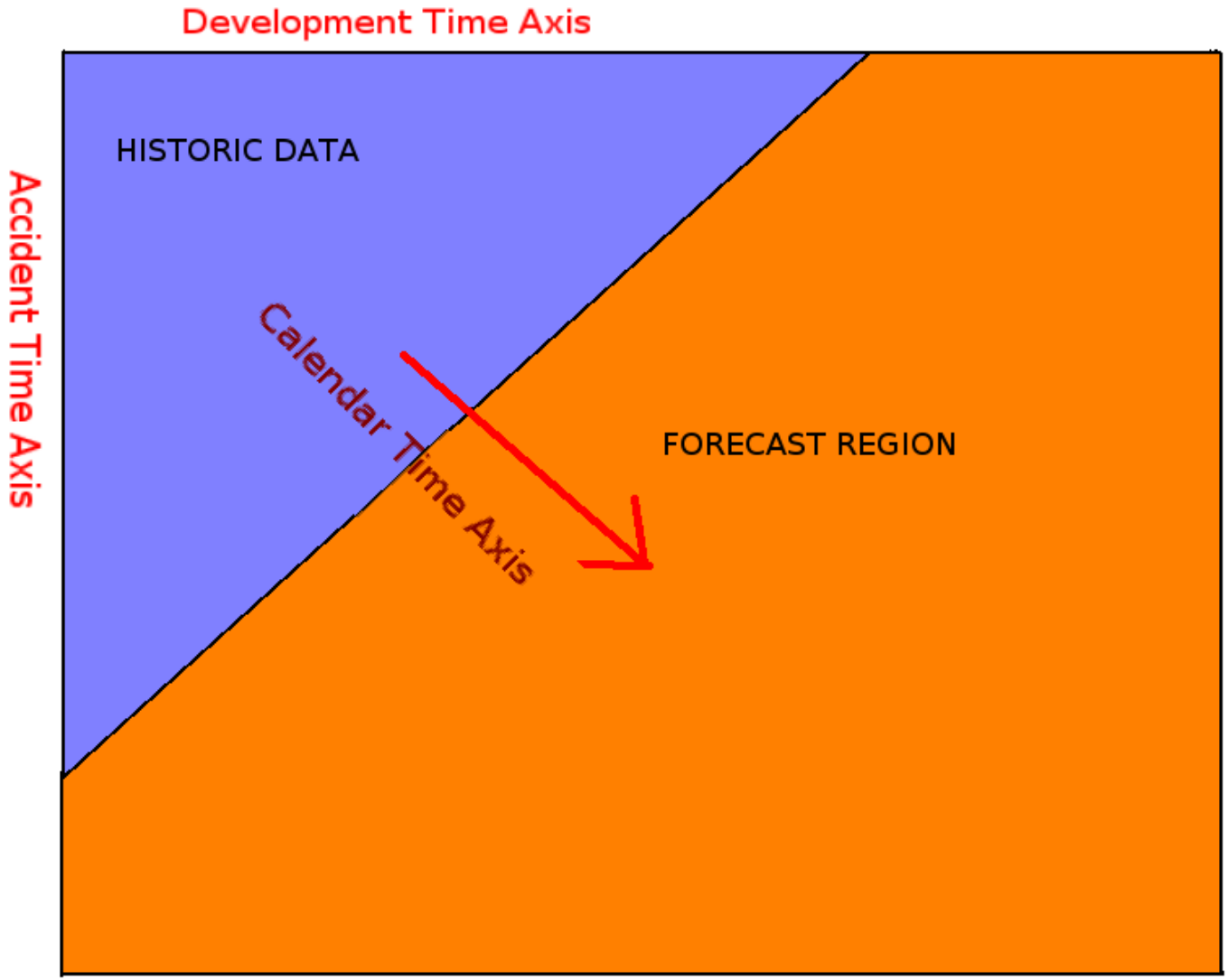
Two-way conversation about risk management with each corporate entity.





Securitisation and trading of risk as well as reinsurance treaties lead to many more conversations about risk.



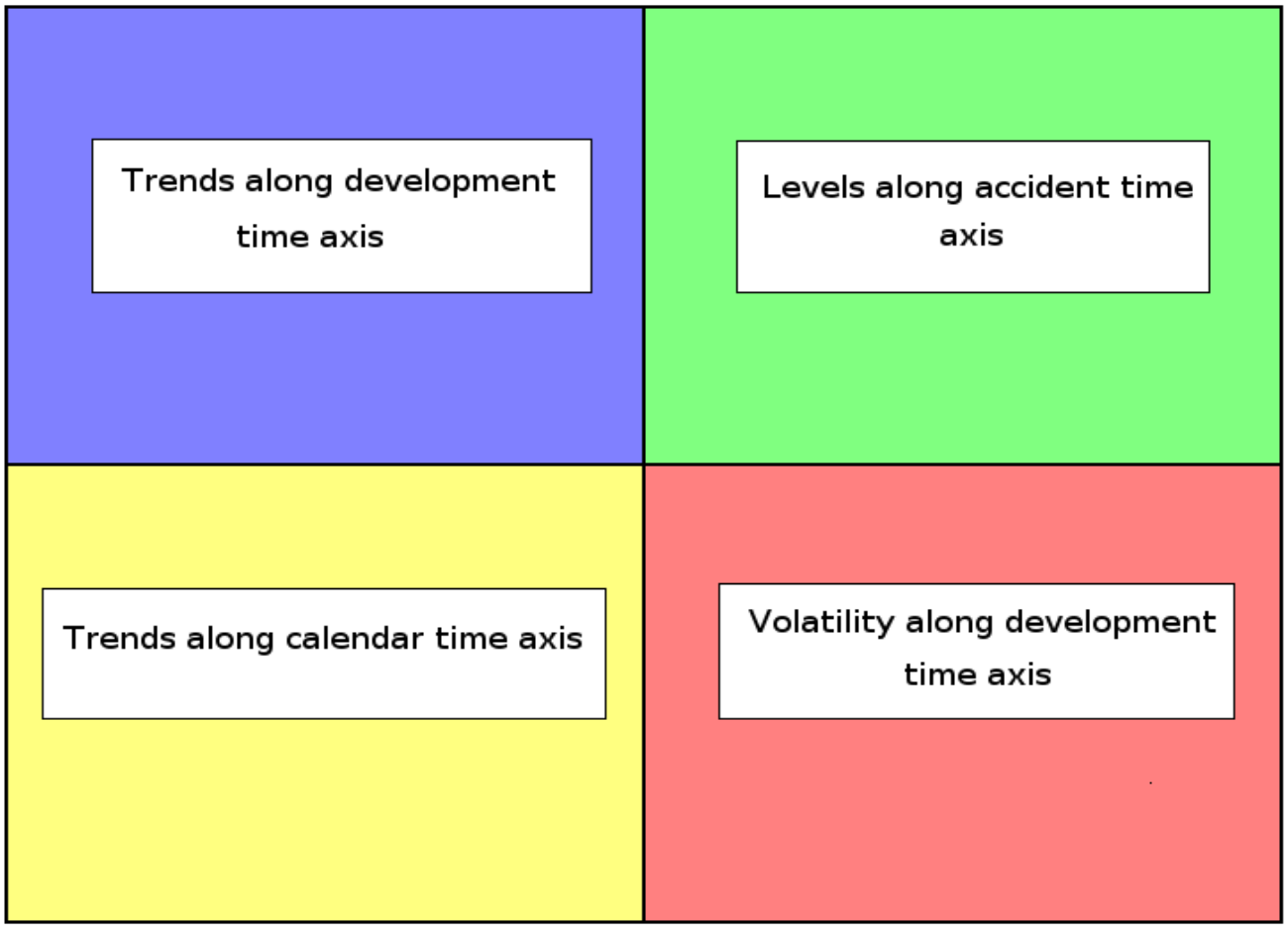



Long-tail lines in general (P&C) insurance are amenable to mathematical modelling. Probability distributions and correlations can be estimated for optimum risk measurement.

# Desiderata for modelling long-tail loss distributions

- Model the cashflows i.e. the incrementals – also claim counts etc.
- Express models in ordinary language: trends, levels, volatilities etc.
- Models should be consistent and smooth so that they can be used to compare businesses.
- Modelling procedures must incorporate diagnostics.
- Models should suggest questions for department executives.
- Models should be seamless with forecast scenarios.
- Models should enable measurement of correlations in volatility component of forward estimates.

# Probability Trend Family (PTF) schema

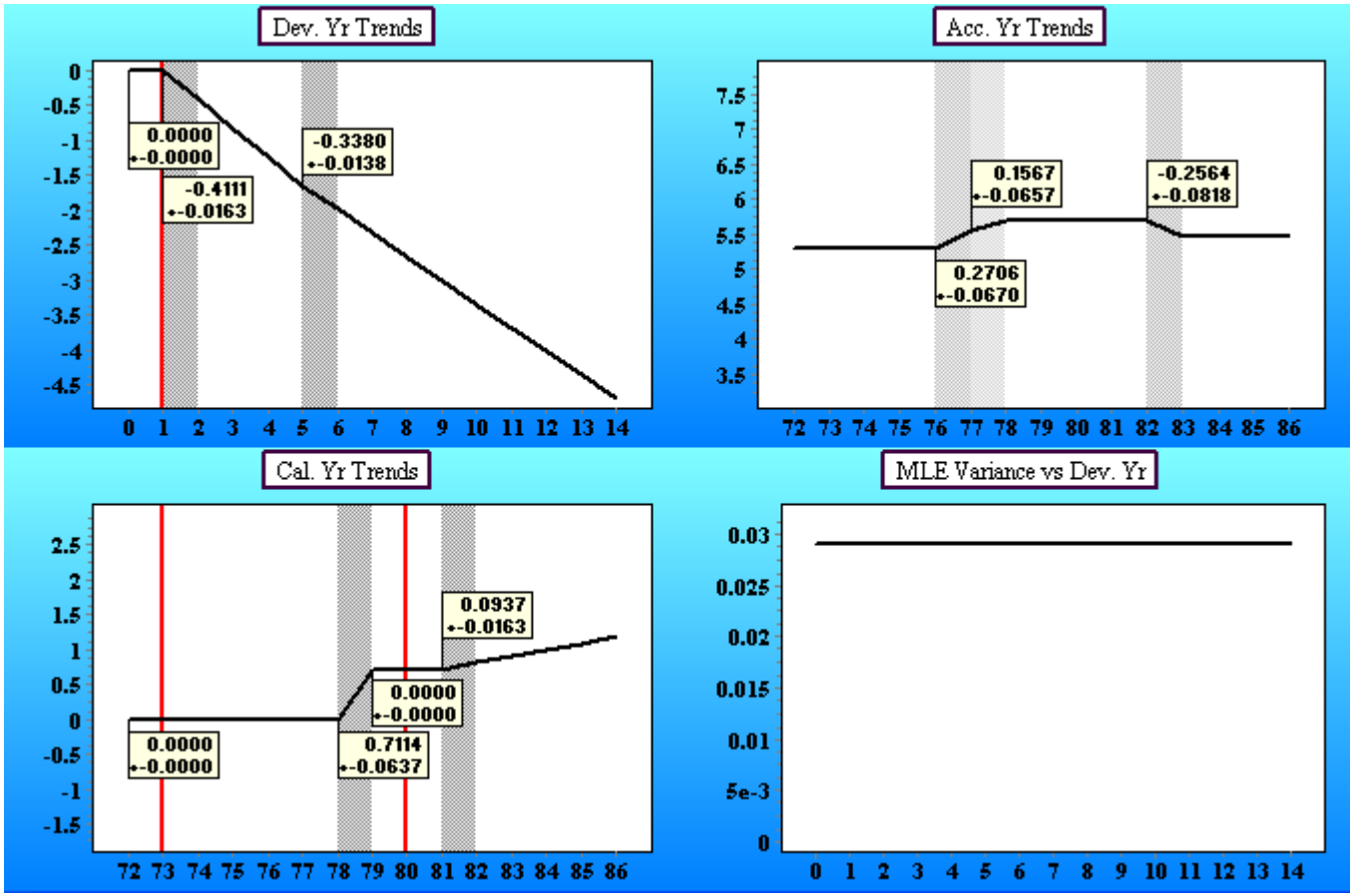


# Probability Trend Family (PTF) schema

| | |
|---|---|
| Development decay pattern characteristic of the line of business.<br><br>Inherent to type of insurances written, mix of business | Accident period levels.<br><br>Effectivess of exposure vector, market share changes by period, changes in underwriting rules, sales policies etc.<br><br>Reflects inter- and intra business changes. |
| Calendar period trends relate performance to economic indicators, inflation and social inflation. Legislative changes etc.<br>Also rates of closure, rules for dealing with claims.<br>Economic environment & intra-business. | Volatilities depend on inherent properties of line of business, size of portfolio (market share) and ability of underwriters to identify "good" risks. |

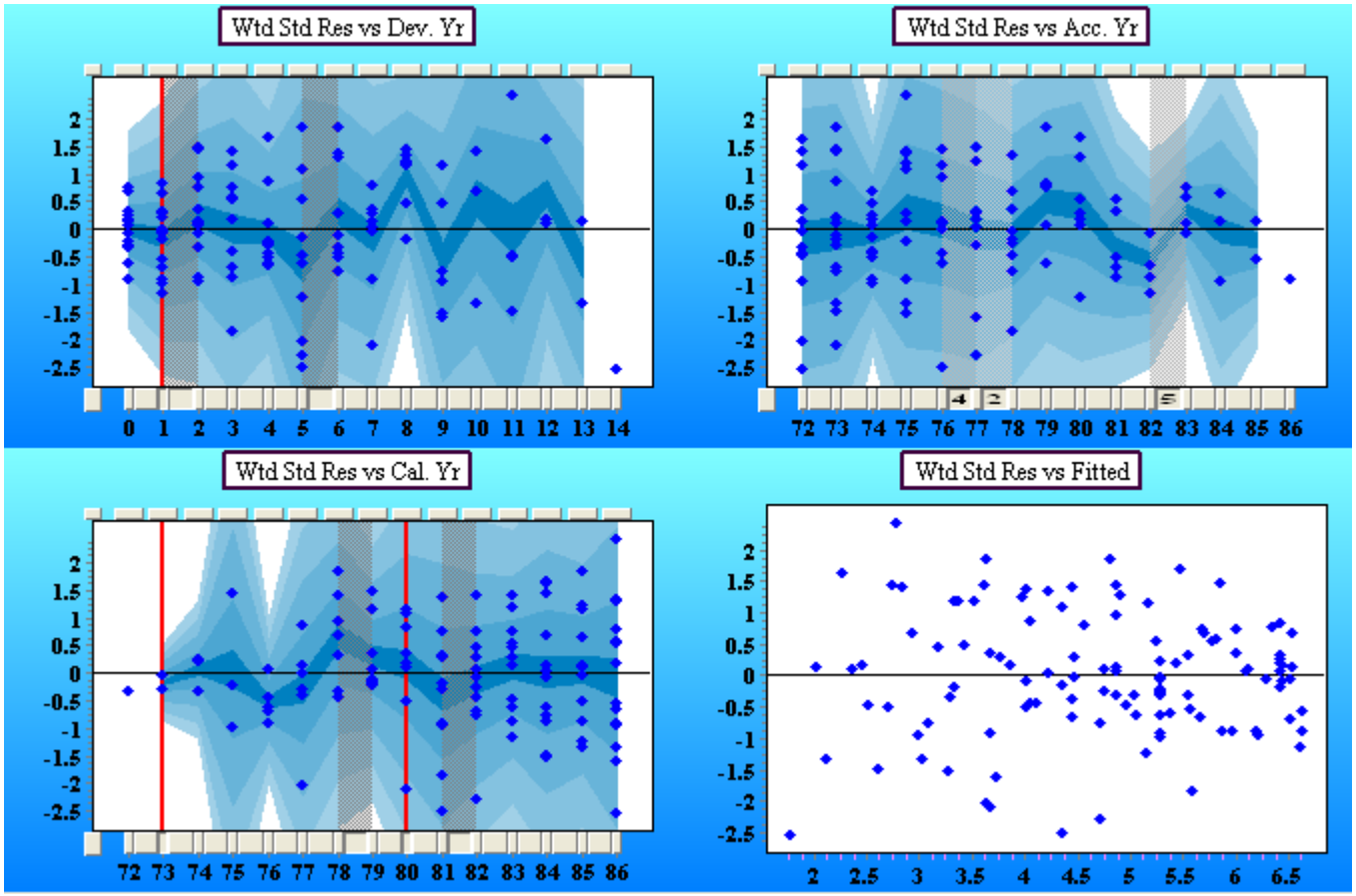
# Probability Trend Family (PTF) schema

Model Display



# Probability Trend Family (PTF) schema

Residual Display



# Multiple Probability Trend Family (MPTF)

**Final Weighted Residual Correlations Between Datasets**

| Datasets oName | PL0(I) | PL1(I) | PL2(I) | PL3(I) | PL4(I) | PL5(I) | PL6(I) | PL7(I) |
|---|---|---|---|---|---|---|---|---|
| PL0(I) | 1 | -0.001453 | -0.663271 | -0.261735 | 0.145582 | 0.096887 | 0.115298 | 0.040202 |
| PL1(I) | -0.001453 | 1 | -0.233650 | -0.416273 | 0.101330 | 0.053068 | -0.040581 | -0.075512 |
| PL2(I) | -0.663271 | -0.233650 | 1 | 0.760055 | -0.229879 | -0.183267 | -0.038093 | -0.039726 |
| PL3(I) | -0.261735 | -0.416273 | 0.760055 | 1 | -0.140173 | -0.189140 | 0.093515 | 0.005829 |
| PL4(I) | 0.145582 | 0.101330 | -0.229879 | -0.140173 | 1 | 0.444232 | -0.221013 | -0.049324 |
| PL5(I) | 0.096887 | 0.053068 | -0.183267 | -0.189140 | 0.444232 | 1 | -0.056441 | -0.031059 |
| PL6(I) | 0.115298 | -0.040581 | -0.038093 | 0.093515 | -0.221013 | -0.056441 | 1 | 0.420219 |
| PL7(I) | 0.040202 | -0.075512 | -0.039726 | 0.005829 | -0.049324 | -0.031059 | 0.420219 | 1 |

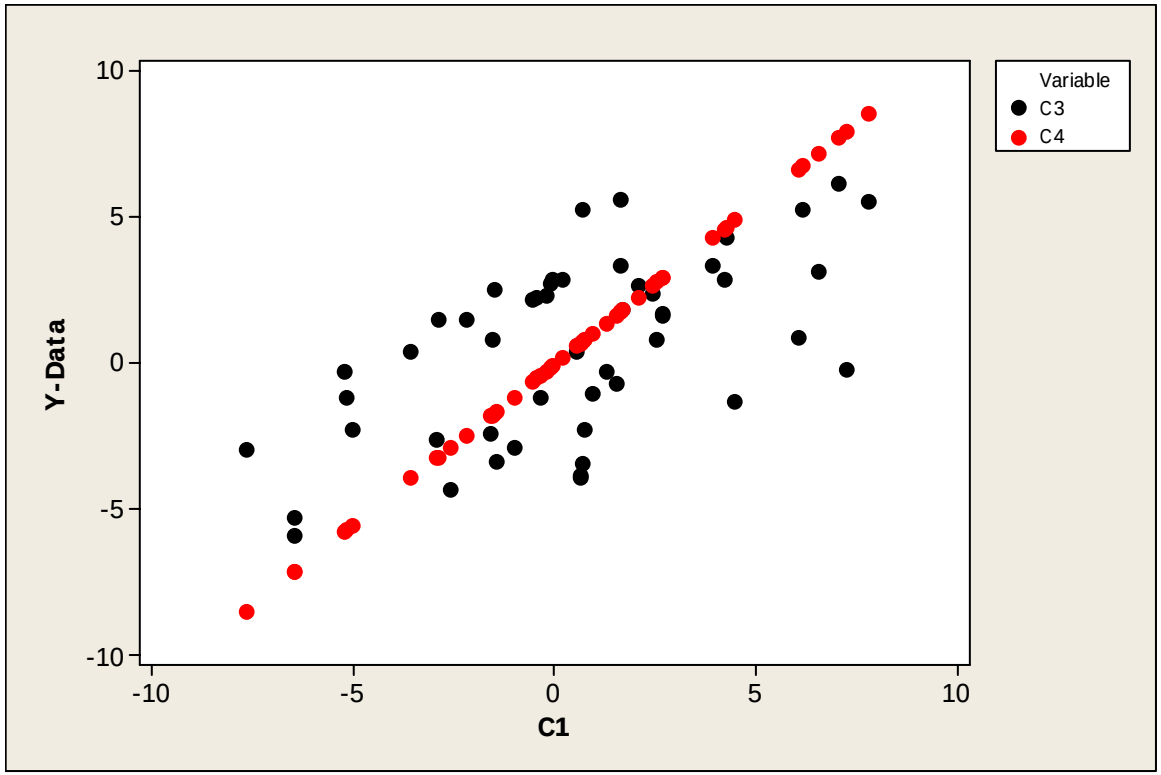
11 iterations were executed

# Multiple Probability Trend Family (MPTF)

| Reserve Forecast Correlations Between Datasets (Totals) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | PL0(I) | PL1(I) | PL2(I) | PL3(I) | PL4(I) | PL5(I) | PL6(I) | PL7(I) |
| PL0(I) | 1 | 0.001855 | -0.339170 | 0.080258 | 0.066008 | 0.029733 | 0.088957 | 0.014461 |
| PL1(I) | 0.001855 | 1 | -0.087193 | -0.139630 | 0.042163 | 0.016841 | -0.004793 | -0.016585 |
| PL2(I) | -0.339170 | -0.087193 | 1 | -0.009125 | -0.141686 | -0.104765 | -0.042180 | -0.023351 |
| PL3(I) | 0.080258 | -0.139630 | -0.009125 | 1 | -0.050955 | -0.000655 | 0.022626 | -0.001060 |
| PL4(I) | 0.066008 | 0.042163 | -0.141686 | -0.050955 | 1 | 0.242877 | -0.129163 | -0.034638 |
| PL5(I) | 0.029733 | 0.016841 | -0.104765 | -0.000655 | 0.242877 | 1 | 0.006952 | -0.012772 |
| PL6(I) | 0.088957 | -0.004793 | -0.042180 | 0.022626 | -0.129163 | 0.006952 | 1 | 0.195034 |
| PL7(I) | 0.014461 | -0.016585 | -0.023351 | -0.001060 | -0.034638 | -0.012772 | 0.195034 | 1 |

# What is correlation?

- Correlation refers to the strength of *linear association* between two variables.
- Correlation, linearity, normality, weighted least squares, and linear regression are closely related concepts.
- $Y=aX+b+\varepsilon$ where $\varepsilon$ is a *random variable*, known as *error* or *residual* or *noise* and is (ideally) assigned to a *distribution* with mean zero and finite variance independent of X.




**The black dots represent the equation Y=aX +b + ε**

**The red dots represent the equation Y=aX +b**

In the equation $Y=aX+b+\varepsilon$, the $\varepsilon$ represents a random error or noise component.

$a$ and $b$ are parameters to be estimated from the data.

The distribution of $\varepsilon$ matters! It affects the way that $a$ and $b$ should be estimated in order to make use of all available information in the best way.

If $\varepsilon$ is distributed Normally the least-squares method is best. If $\varepsilon$ is not distributed Normally (but is unimodal with mean zero) least-squares is pretty good.

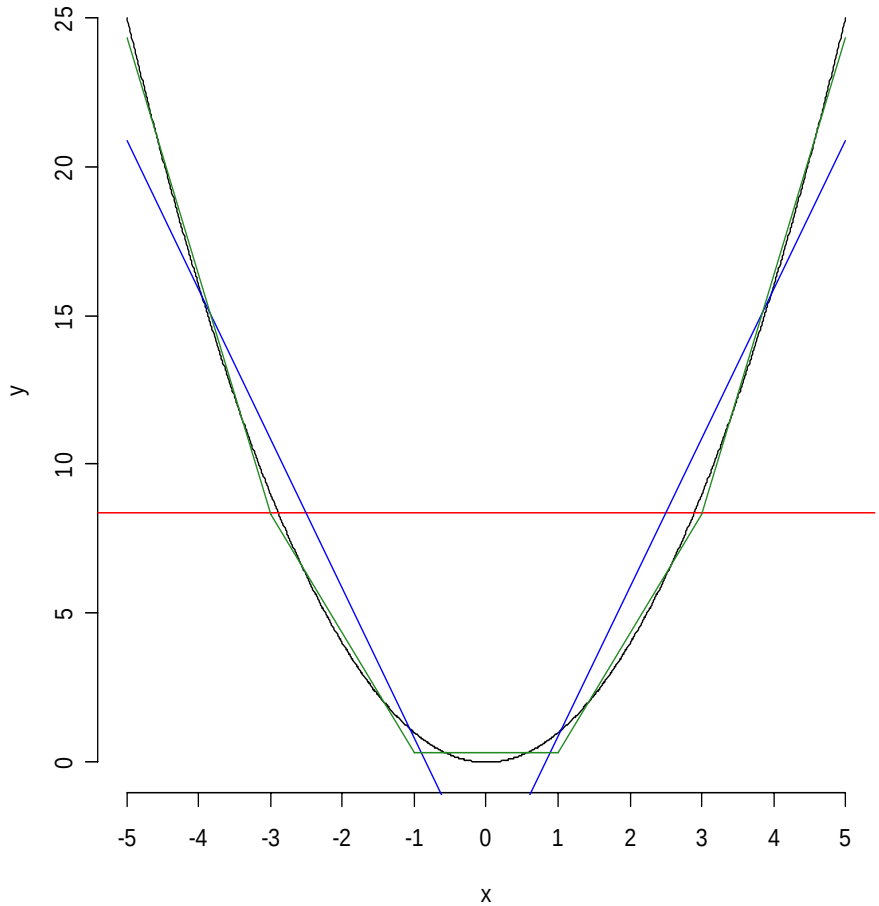
# Correlation and non-linear relationship

$Y=X^2$

X and Y are uncorrelated over the interval [-5,5]

Over [-5,0] they have a correlation of -0.96

Over [0,5] they have a correlation of 0.96



# Correlation and non-linear relationship

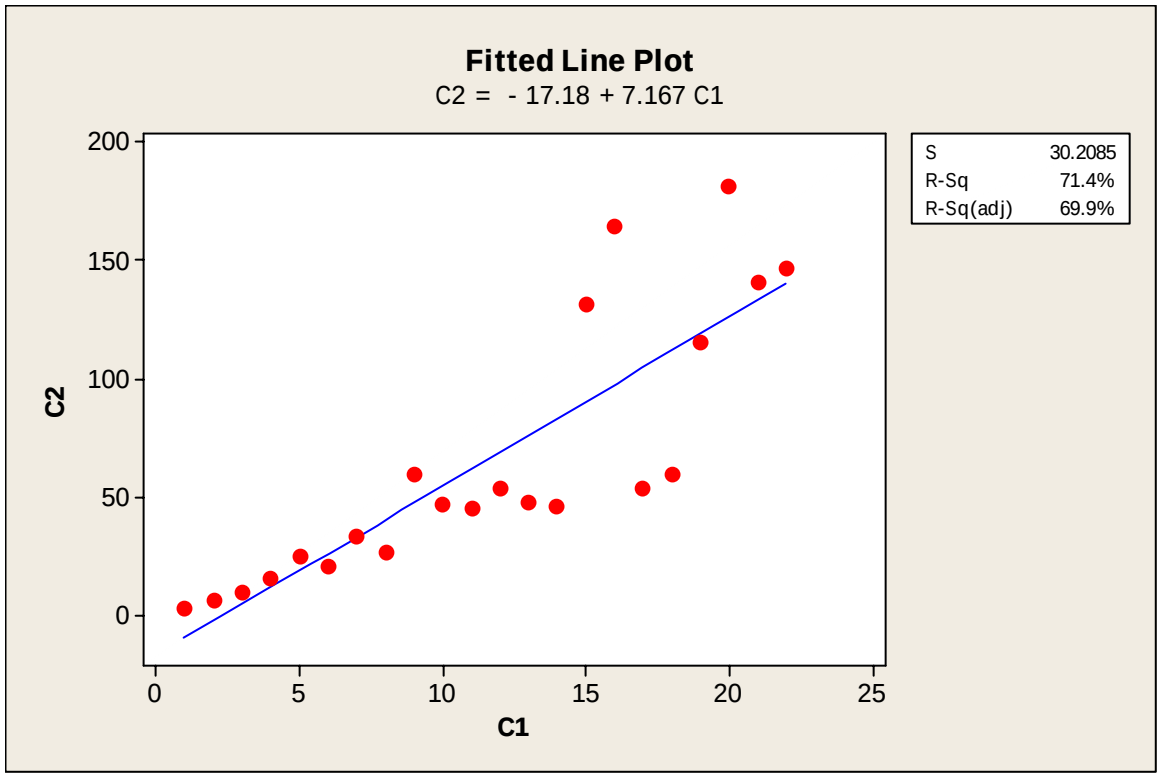
Power relationship:

$Y=bX^{a}(1+\varepsilon)$

Where the error term ε is now multiplicative.

Typically: two dollar axes.

Correlation is 0.85 but it clear that a curve would fit the relationship better.



# Correlation and non-linear relationship

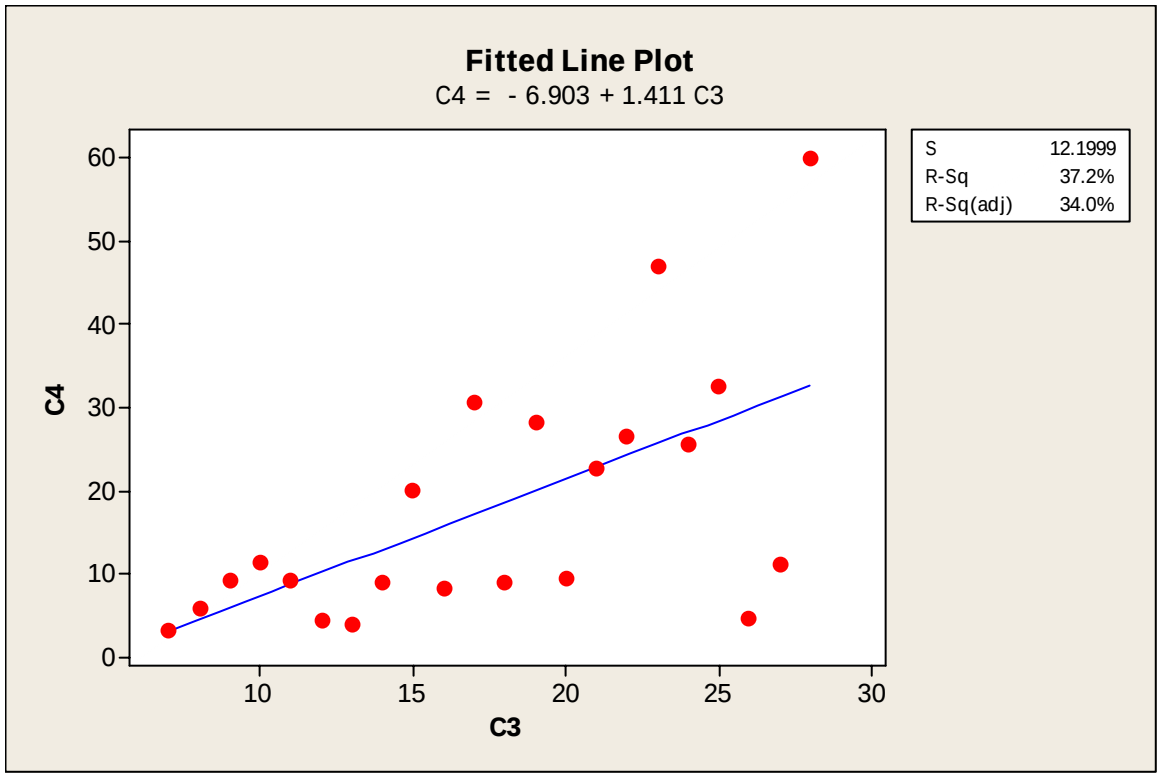
Power relationship.

$Y=ba^{X}(1+\varepsilon)$

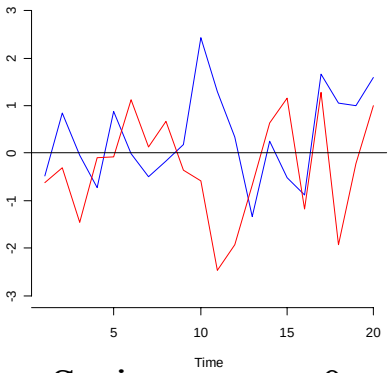
Typically: dollars vs. time.

Correlation is 0.61

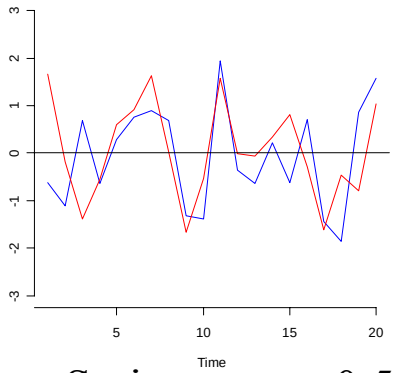
Clearly a curve would be better fit and the error is not additive and has a variance increasing with X.



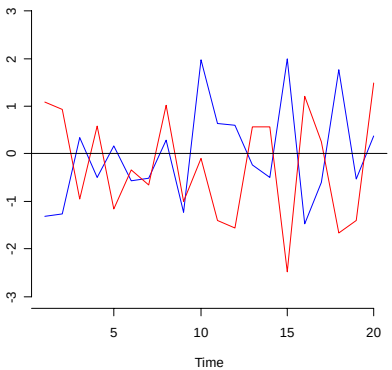
# Correlation in time-series



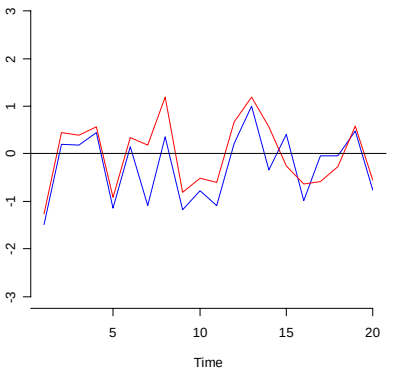

Series corr. = 0




Series corr. = 0.5

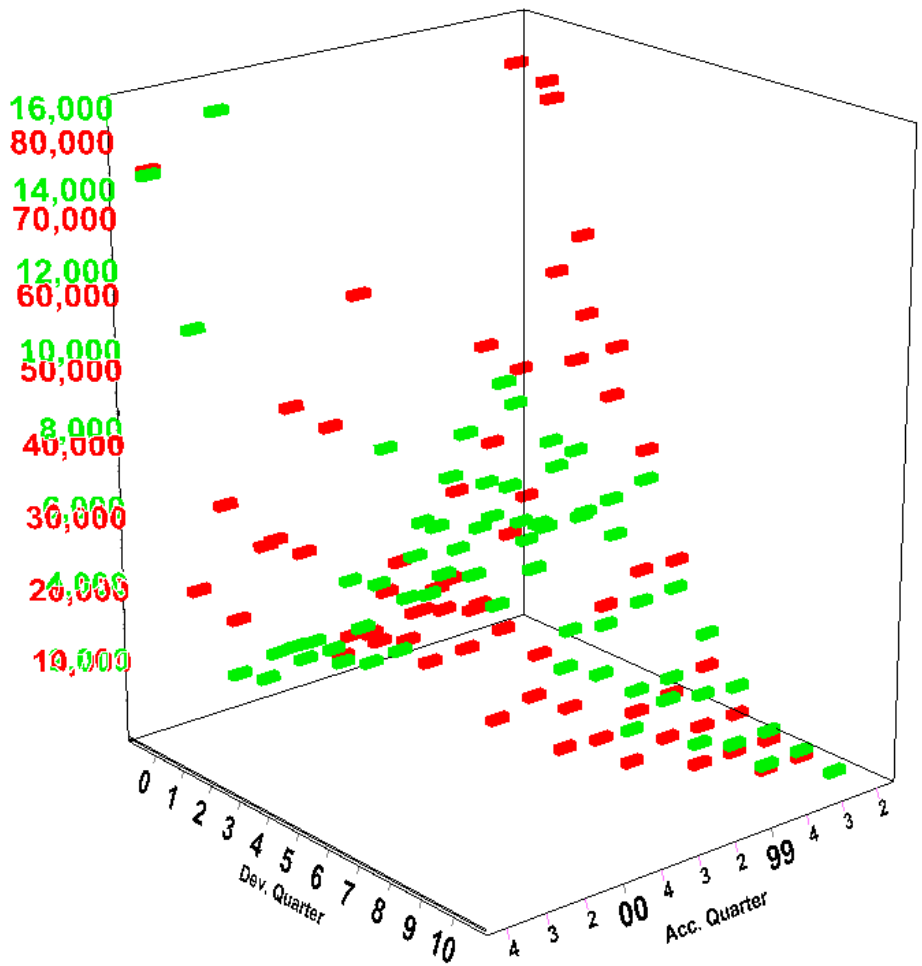



Series corr. = -0.5



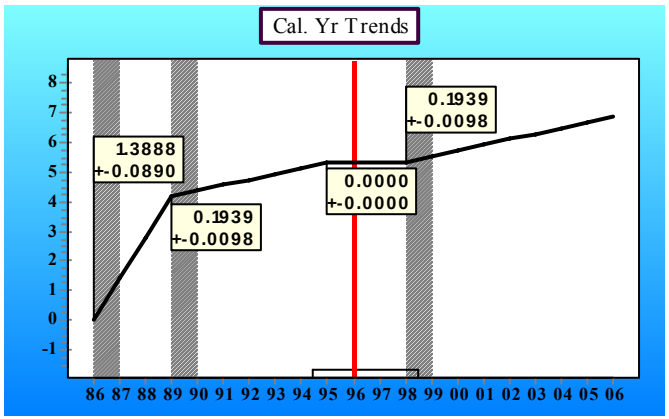

Series corr. = 0.8

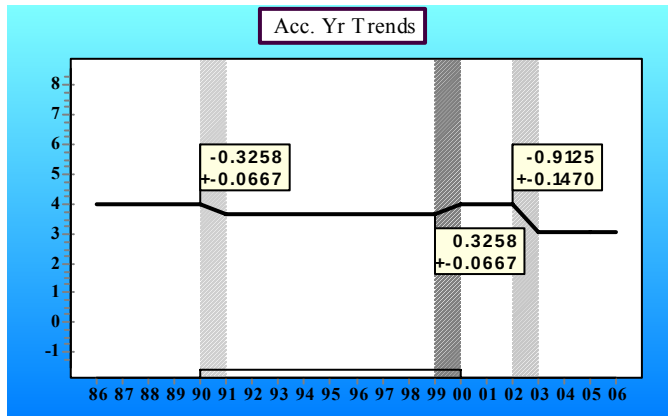
We call the correlation of the random component (after modelling) of two loss arrays: **process correlation**.

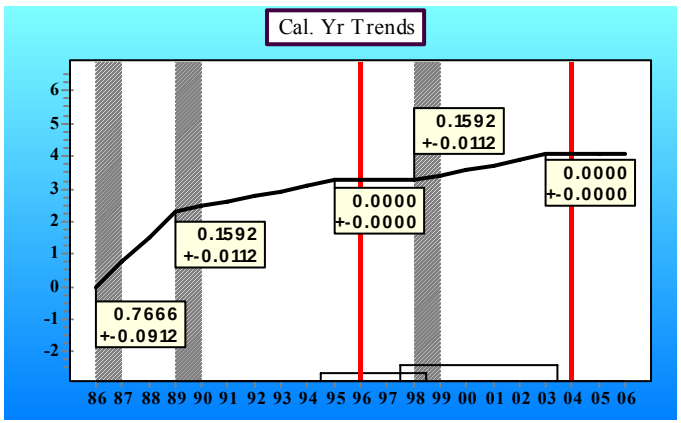


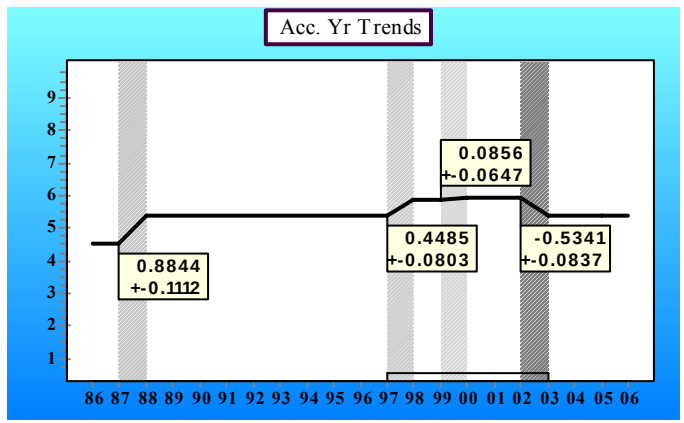

These two triangular loss arrays have corr. = 0.9 after modelling

# Common Drivers: Gross vs. Net

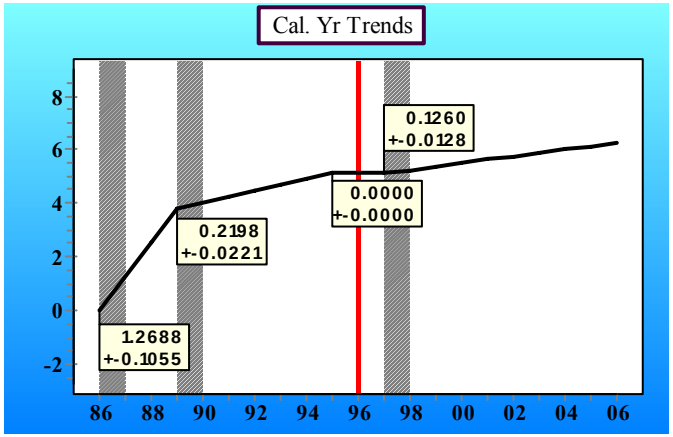


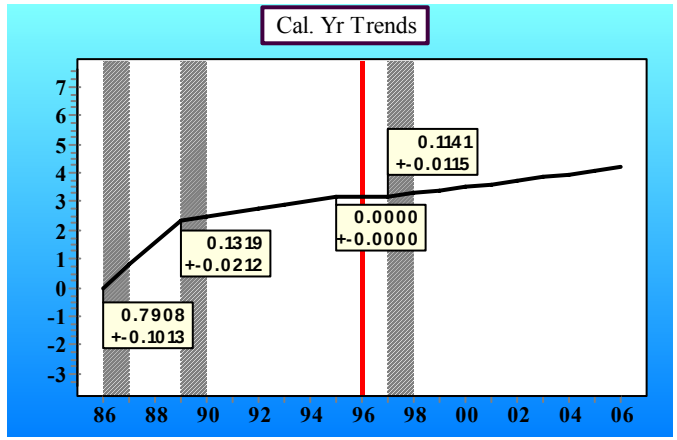




PTF Calendar trends and Accident levels, Gross (above) Net (below)

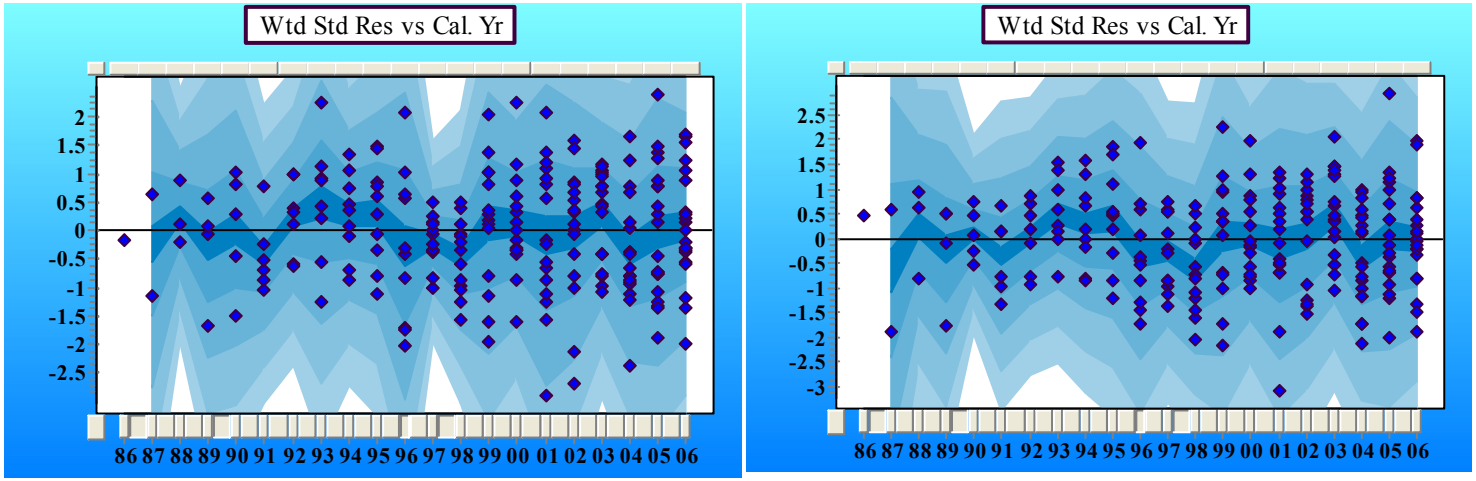


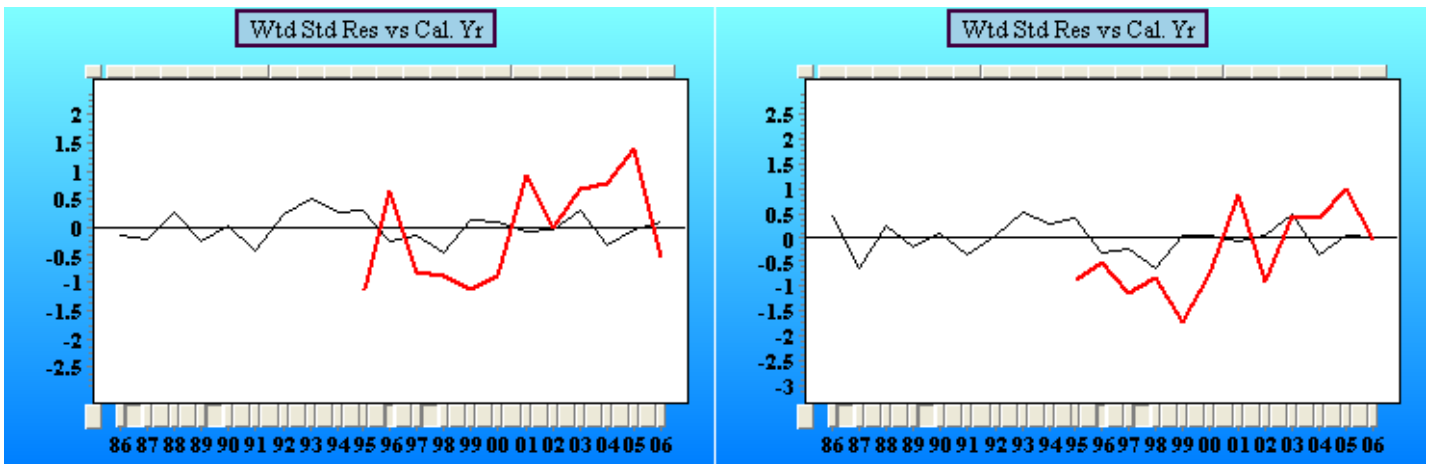

MPTF model calendar trends for Gross (left) and Net (right).

Note the difference in the final trend in the Net data. In PTF this was zero, in MPTF it is 11.4% +_1.1%. Pooling the information by way of the high correlation coefficient, enabled us to give a positive value to this trend. It was seen in PTF but judged to be statistically insignificant.

Correlation and “credibility” are closely related notions.



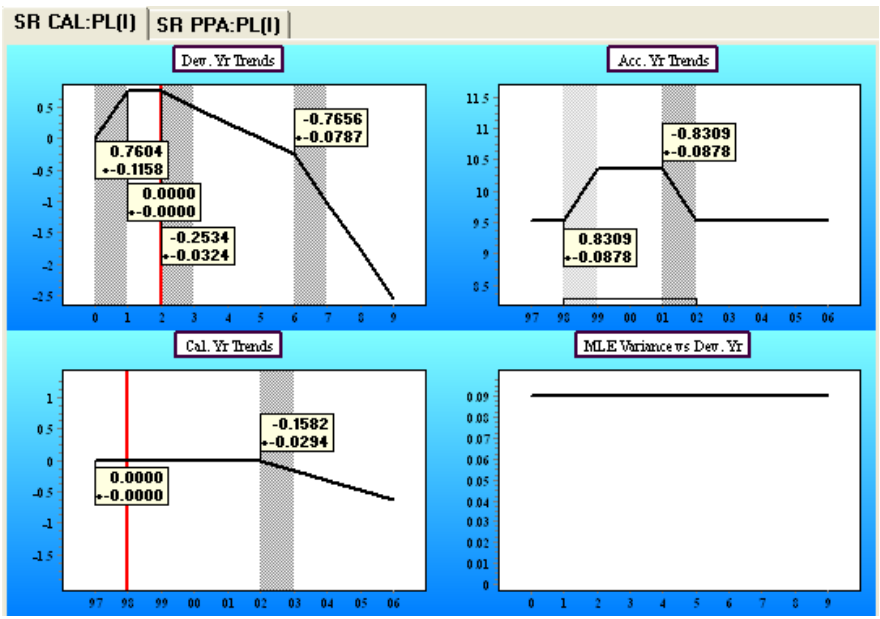

Residual plots by calendar year gross (left) net (right)

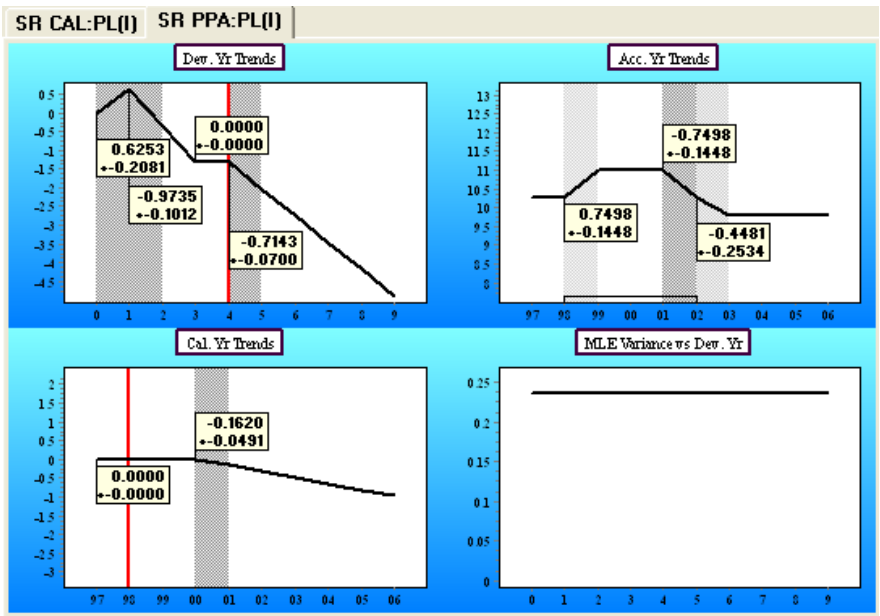



Red line is single accident year (1995)

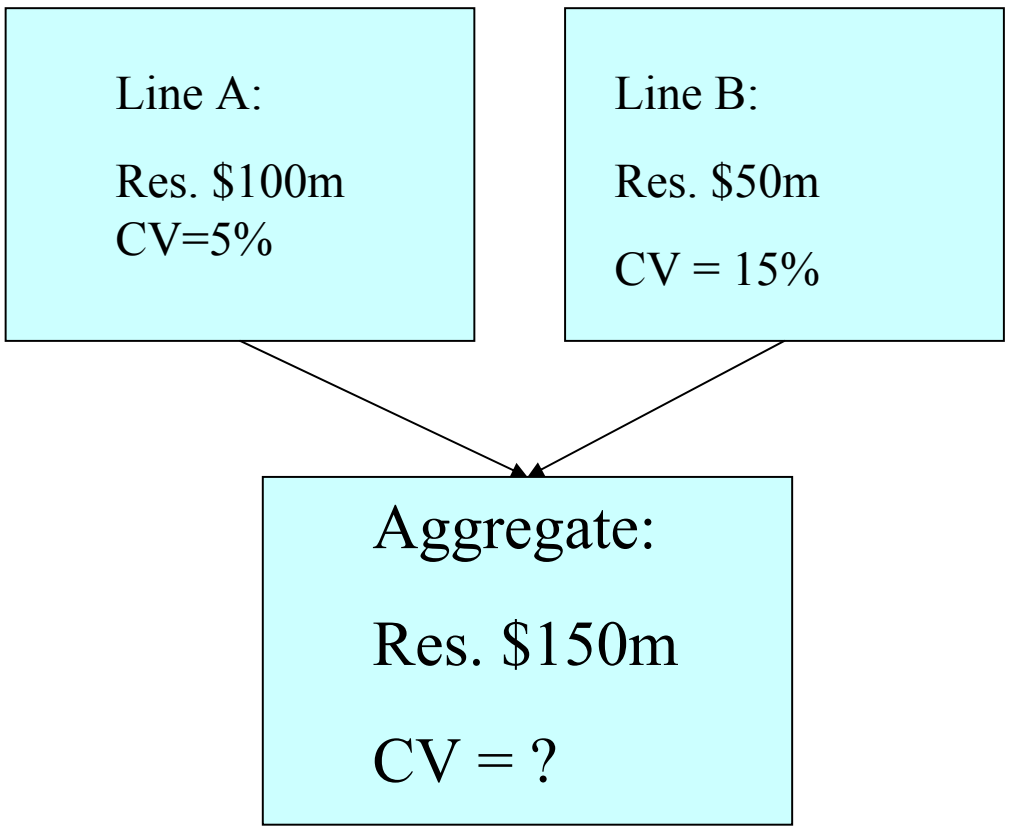
# Swiss Re. CAL (left) and PPA (right)

Commercial Auto Liability and Private Passenger Auto at Swiss Re. have a 0.56 process correlation. Common drivers? The negative calendar trend appears in different years, the common pattern in accident years suggests common effects of internal company policies.



# Risk capital Allocation



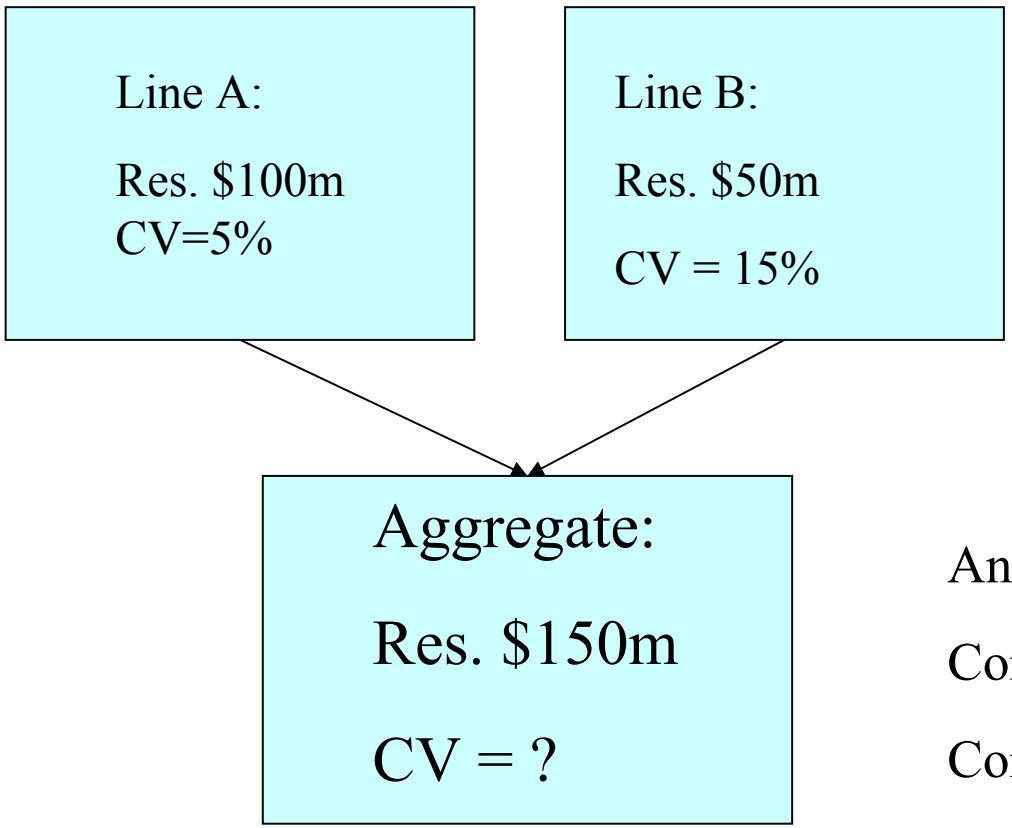

Risk Capital at 98th percentile = 2 Standard Deviations

Line A: $10m

Line B: $15m

Aggregate Risk Capital = $25m ?

# Risk capital Allocation




Risk Capital at 98th percentile = 2 Standard Deviations

Line A: $10m

Line B: $15m

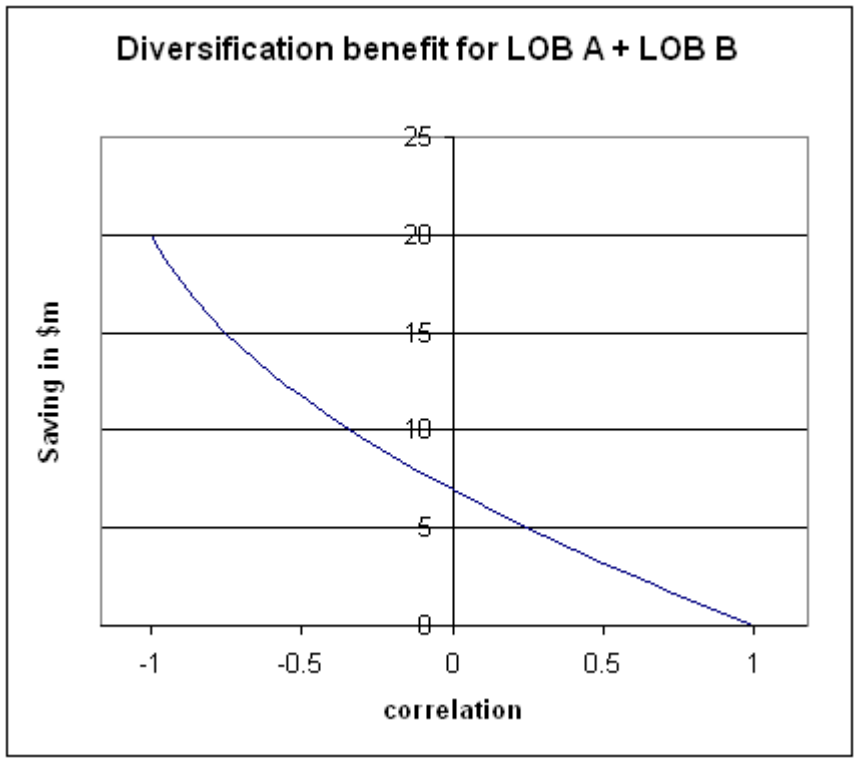
Aggregate Risk Capital = $25m ?

Answer depends on correlation.

Corr = +1.0 ARC = $25m

Corr = 0.0 ARC = $18m

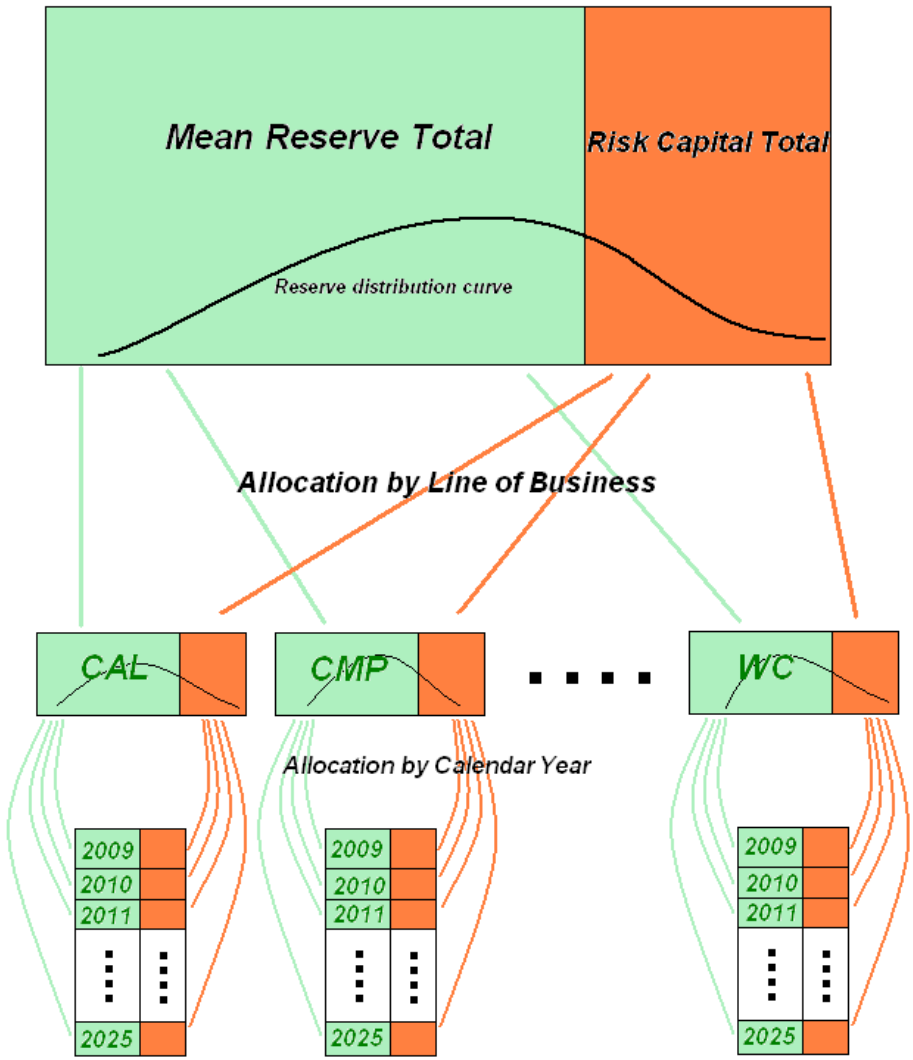
# Risk capital allocation: Diversification benefit



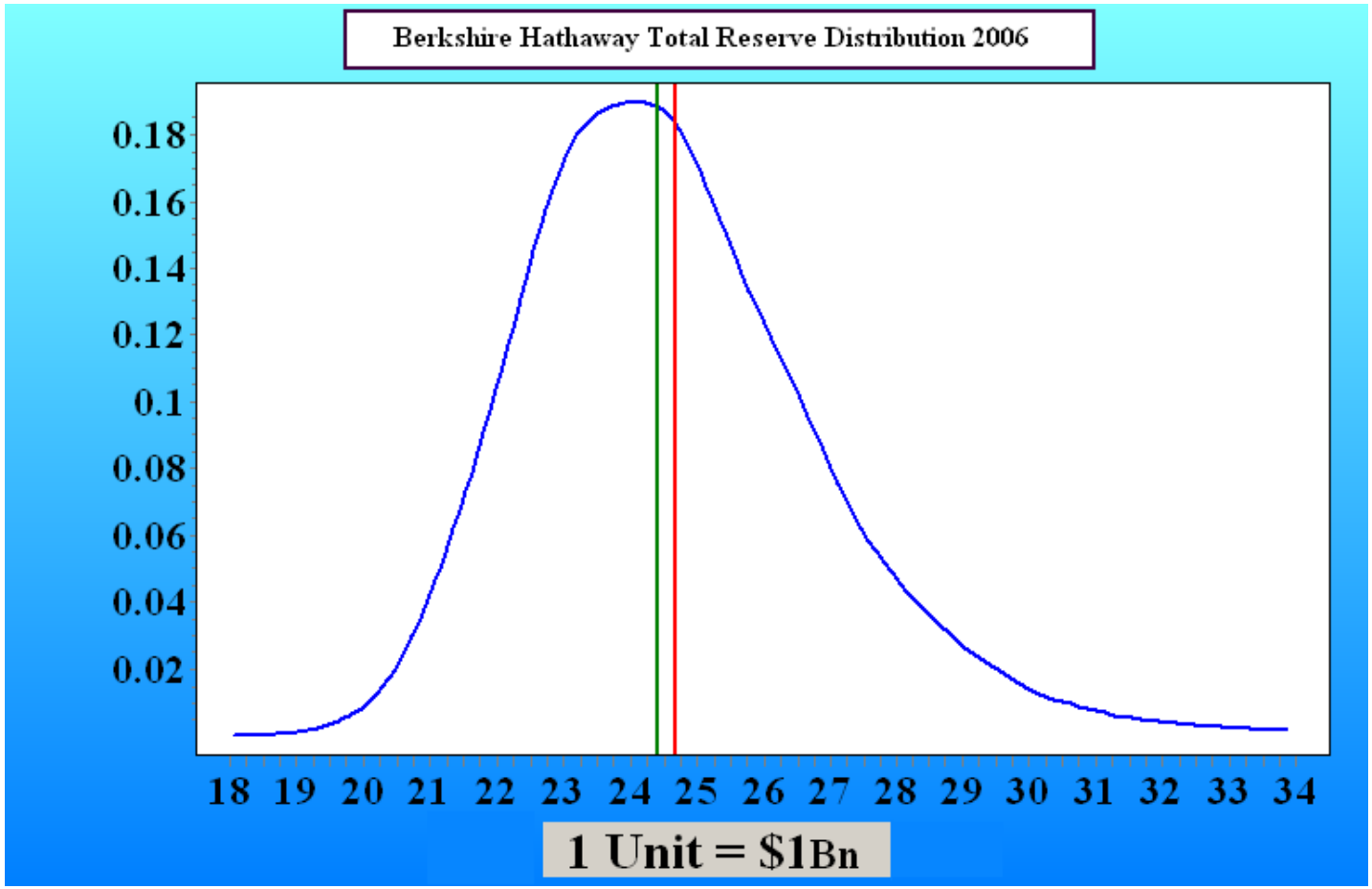
Means, the green areas are additive.

Risk capital component, the orange areas, are sub-additive.

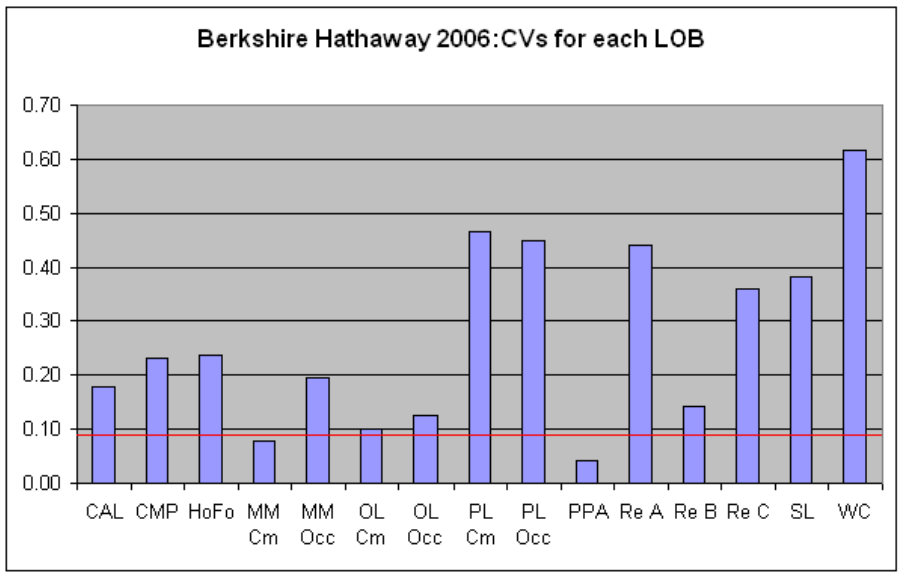
It is more capital efficient to allocate risk capital from top down.

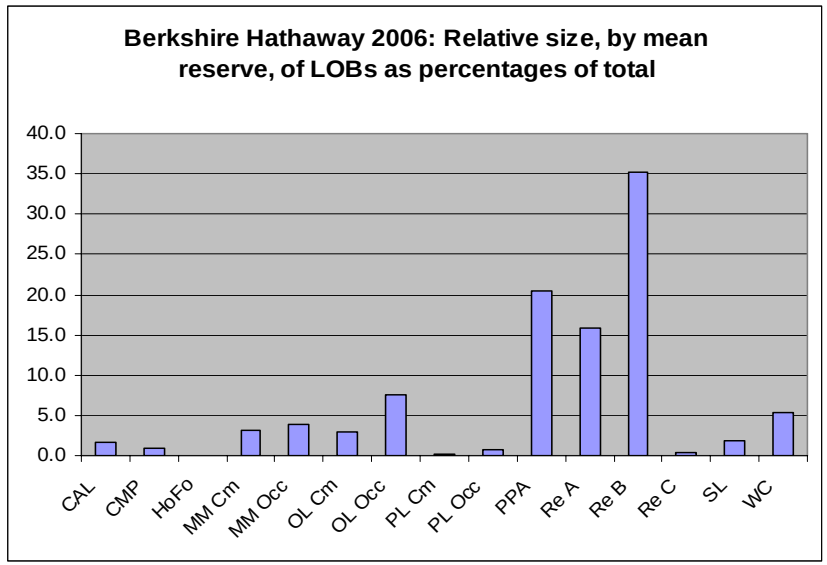


# Capital Allocation Example: Berkshire Hathaway

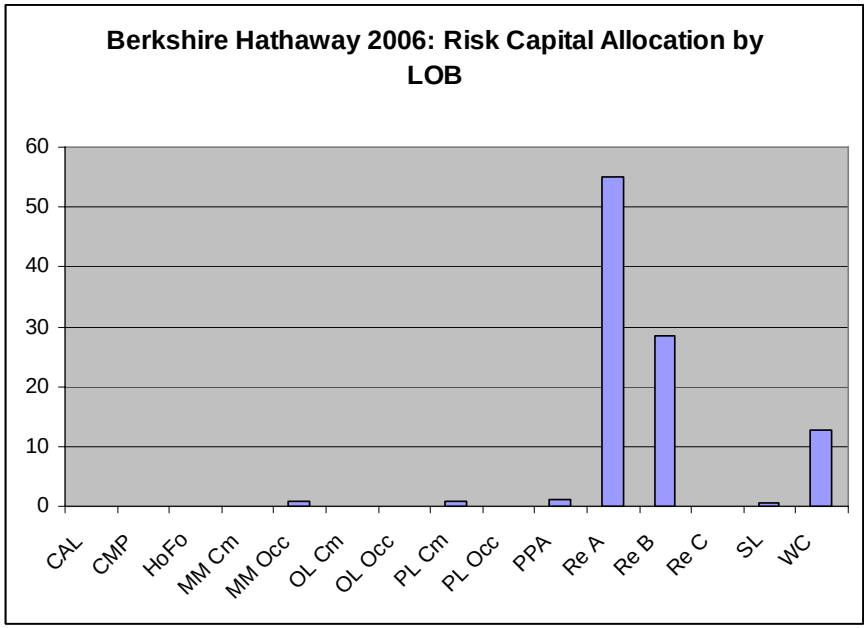


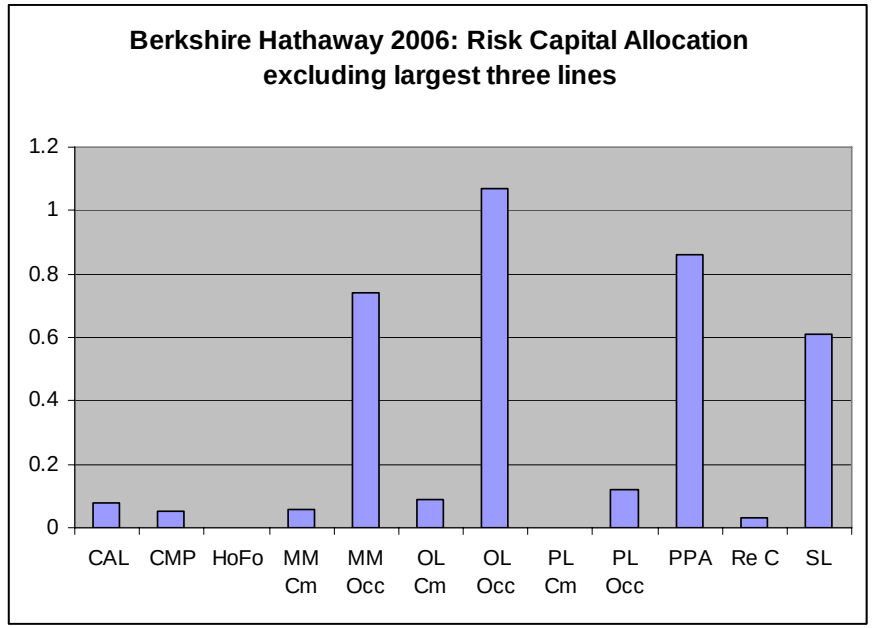

Mean reserve = $24.7B  CV = 9%



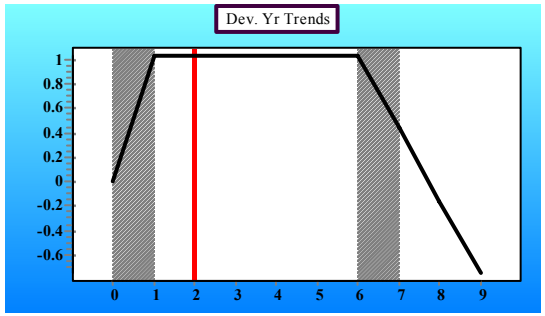

Berkshire Hathaway Lines of Business, CVs (left), relative sizes (right).

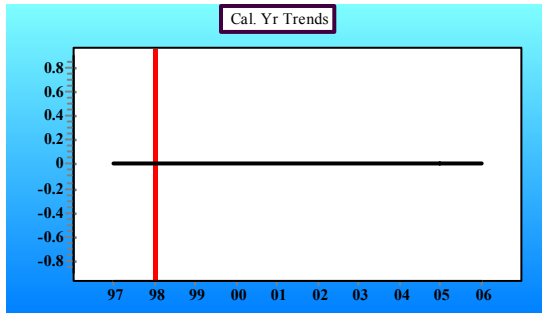


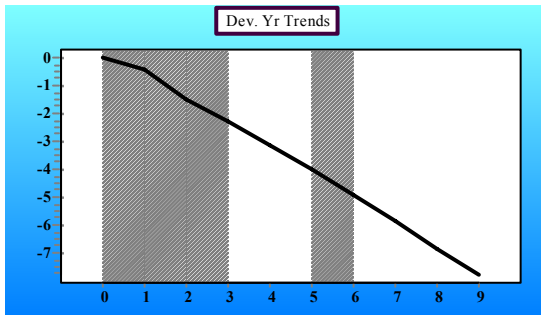



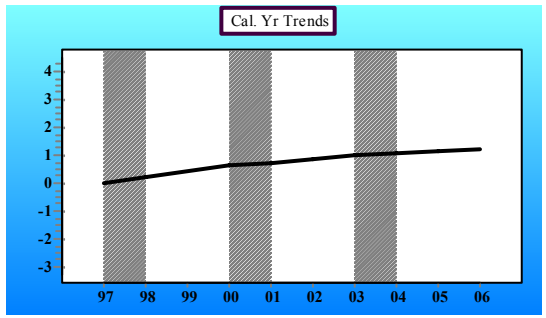

Risk capital allocation by LOB. Graph on right excludes the three largest lines.

## Allocation of reserve by calendar year depends on the trend decay structure



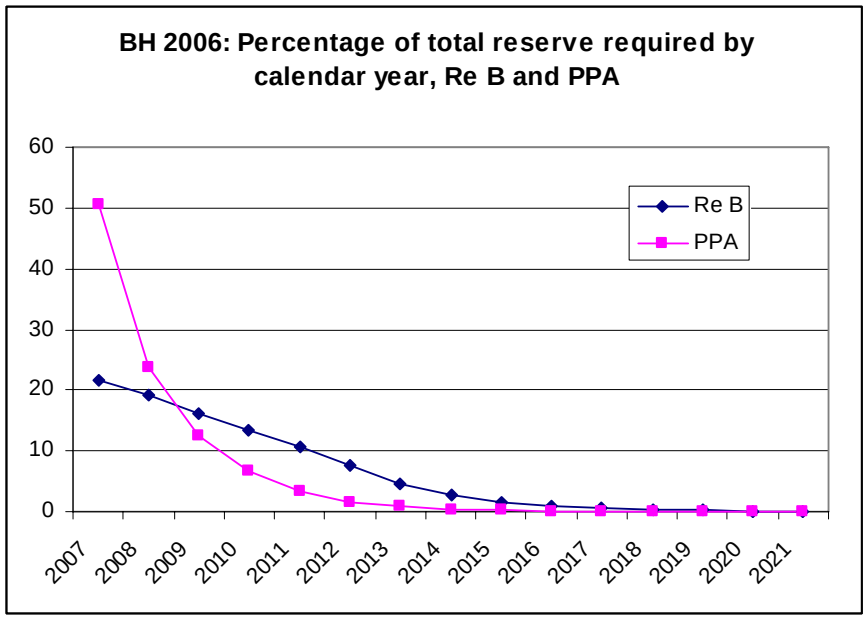



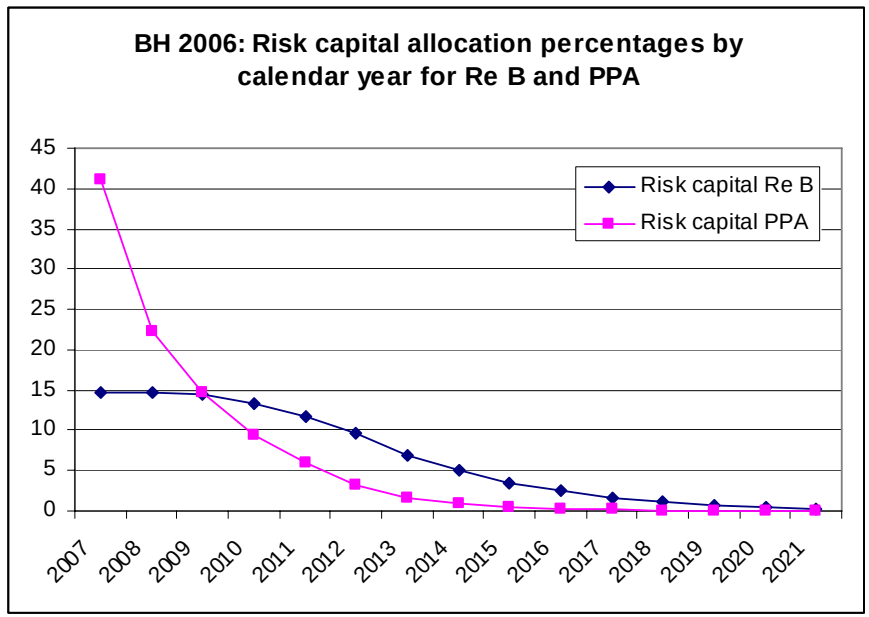



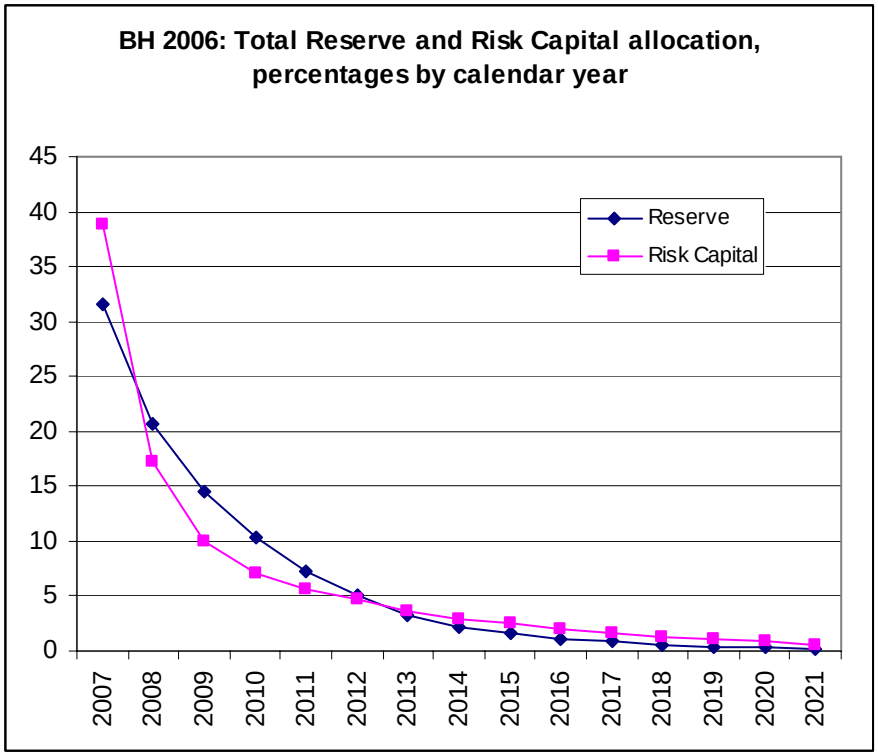

Re B (above) and PPA (below), the two largest lines at Berkshire Hathaway, development and calendar year trends.

# Berkshire Hathaway ReB and PPA reserve by calendar year and risk capital allocation by calendar year.

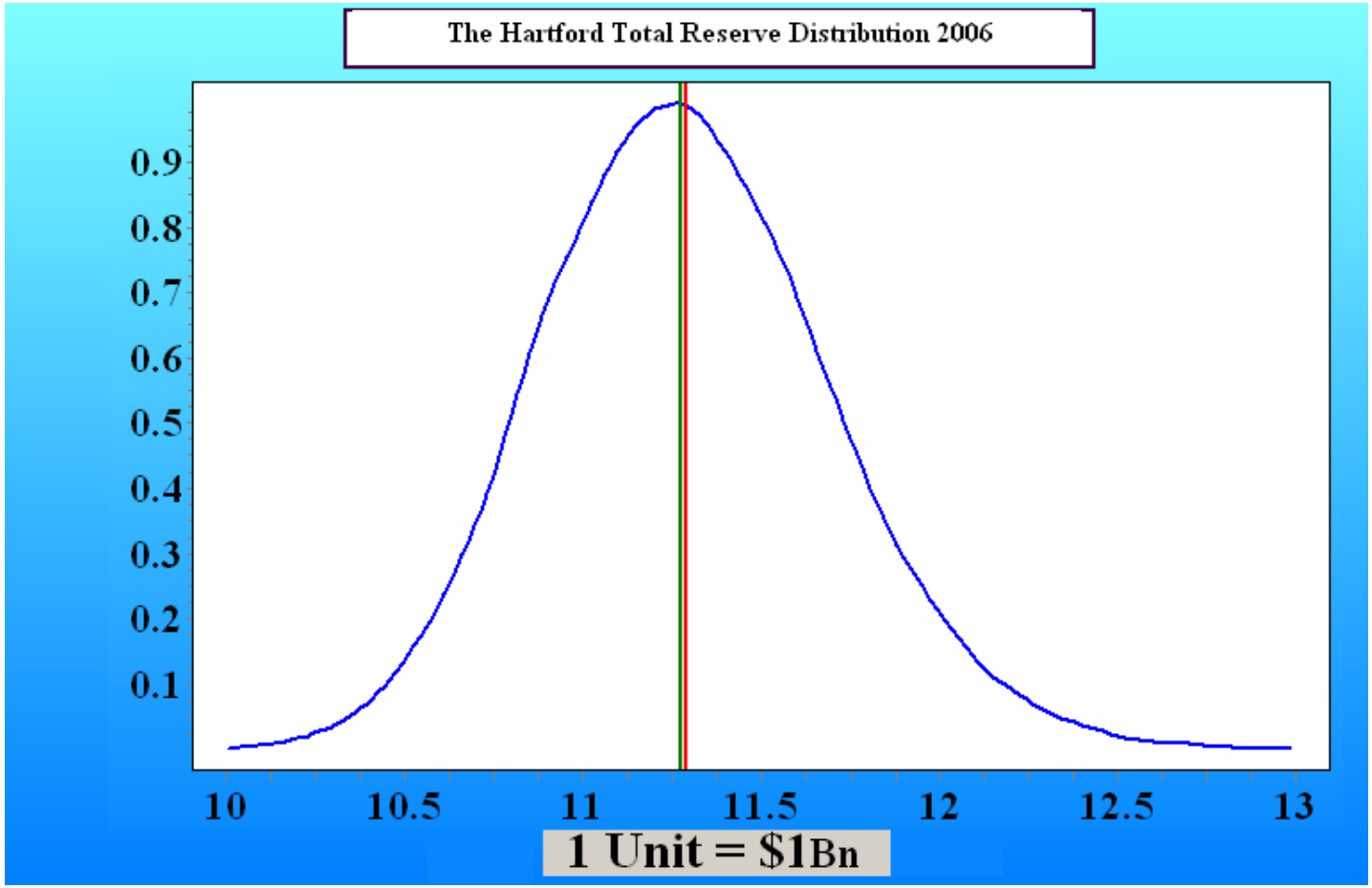


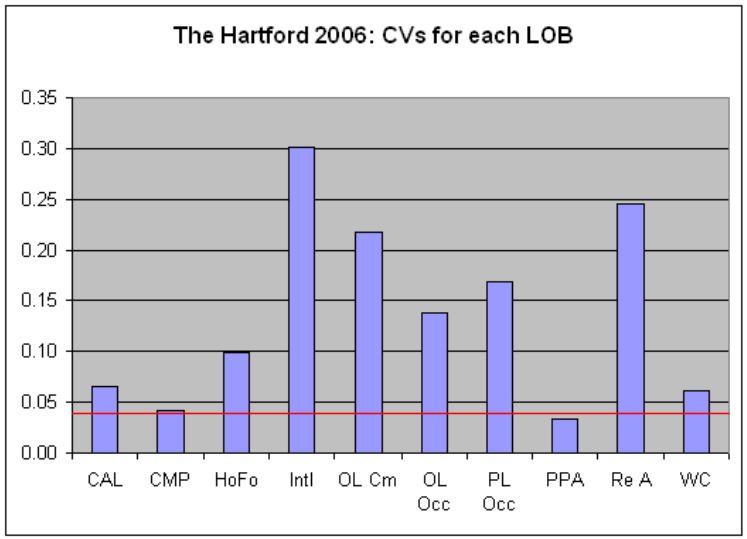
# Berkshire Hathaway Total Reserve and Total Risk Capital allocation by calendar year.

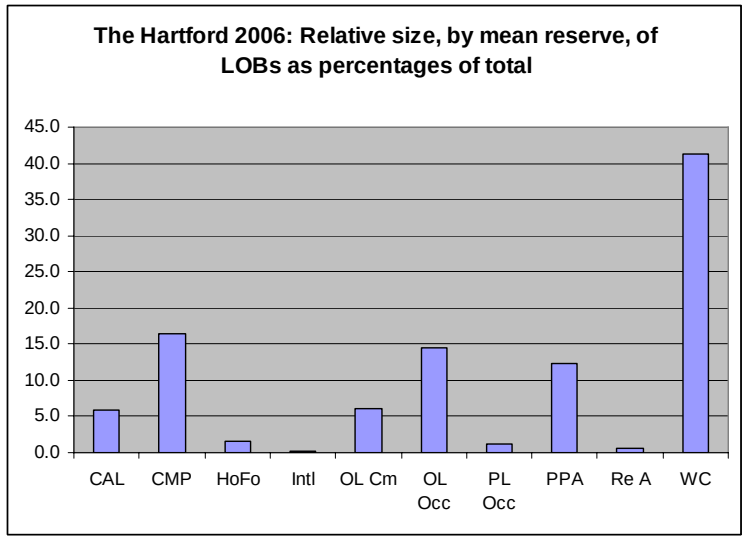


# Capital Allocation Example: The Hartford

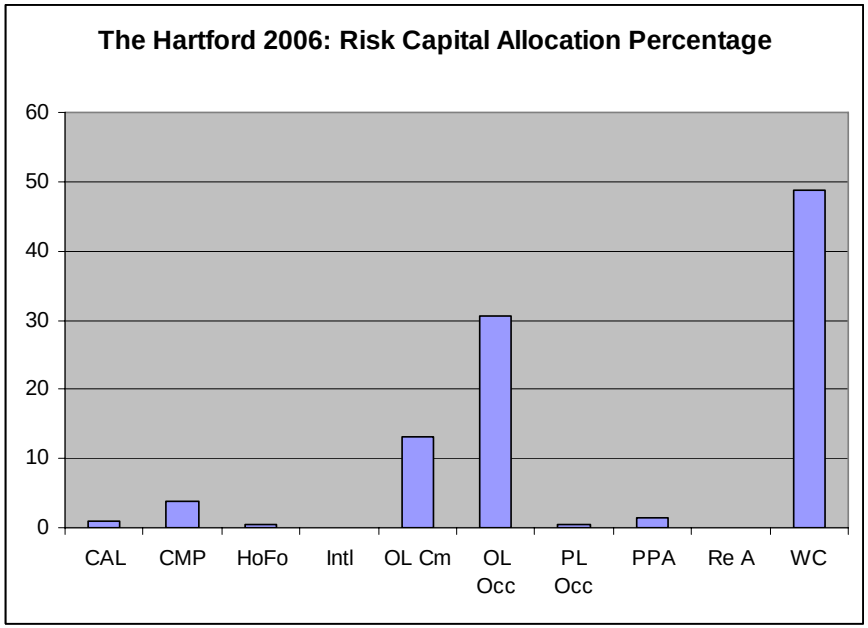


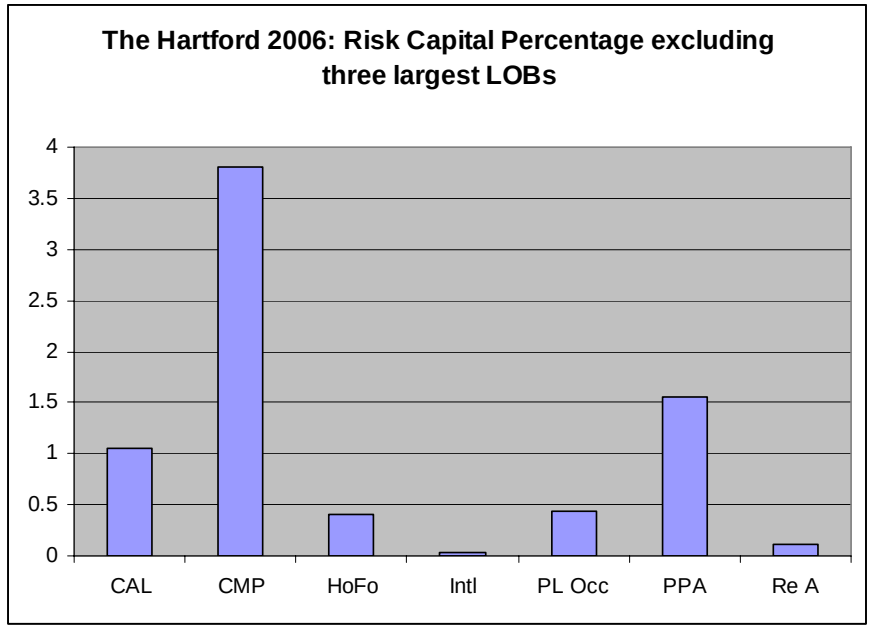

Mean reserve = $11.3B  CV = 4%



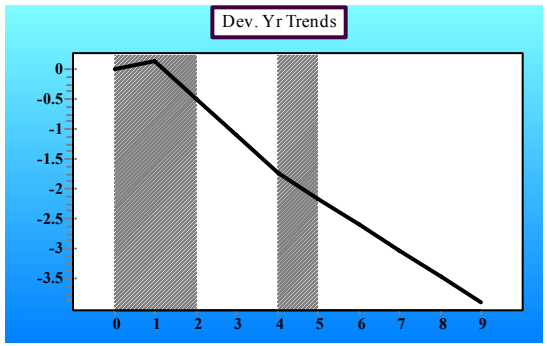

The Hartford Lines of Business, CVs (left), relative sizes (right).

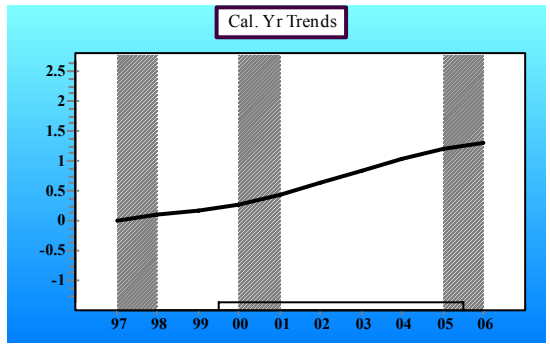


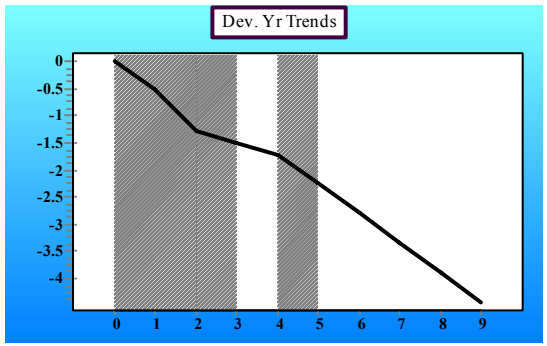



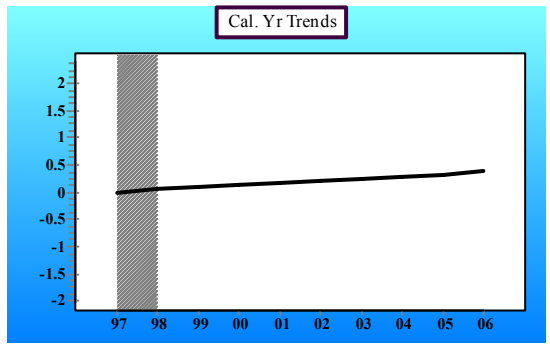

Risk capital allocation by LOB. Graph on right excludes the three largest lines.

## Allocation of reserve by calendar year depends on the trend decay structure



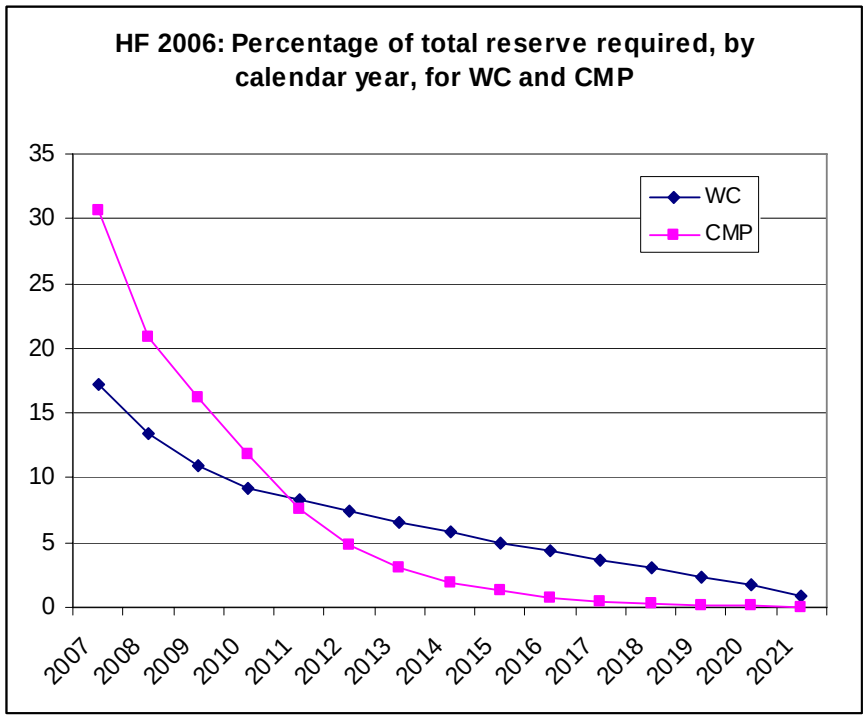



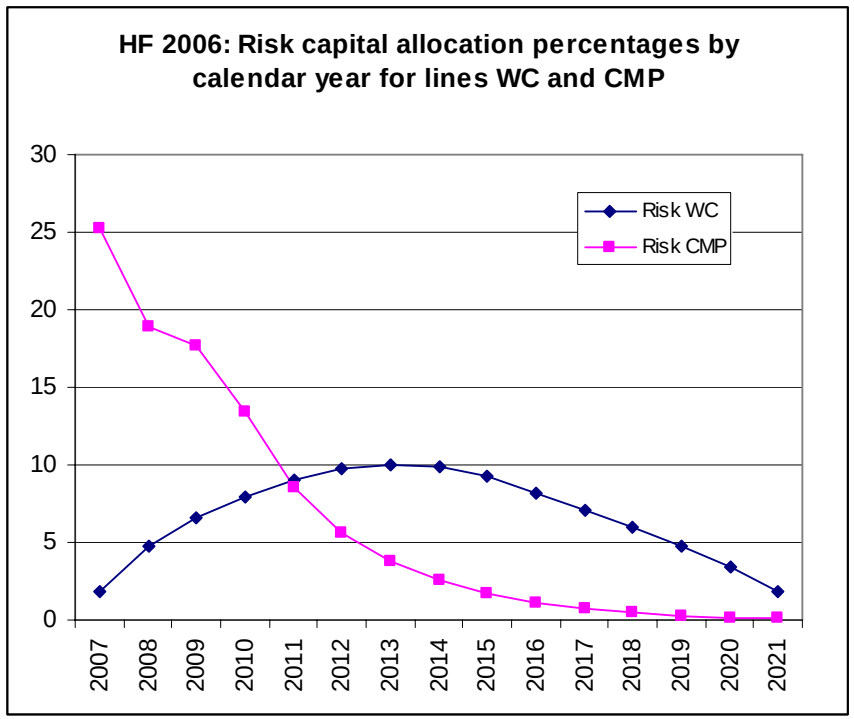



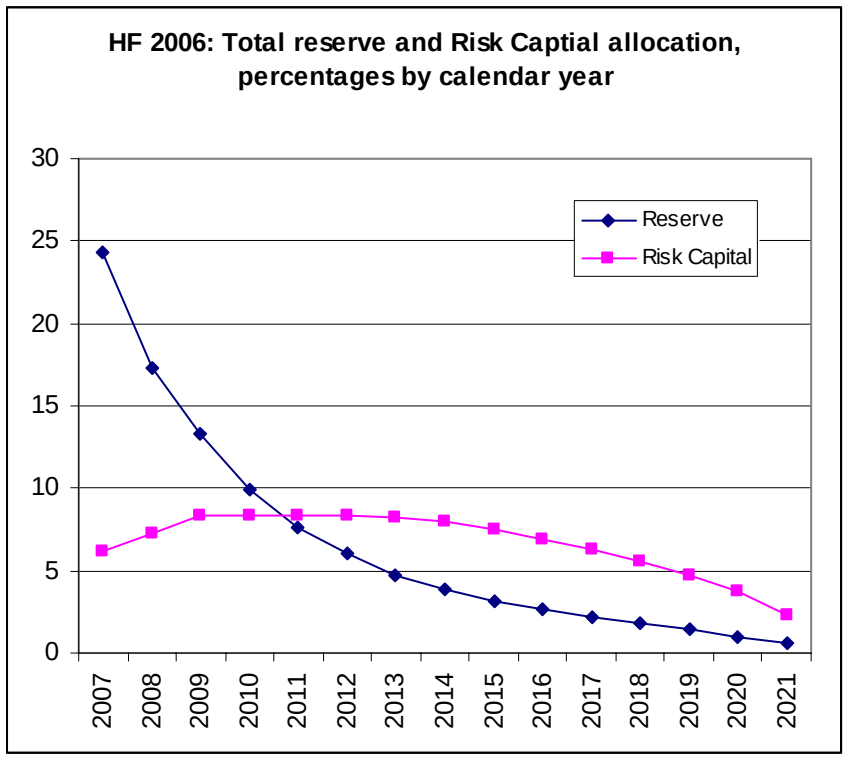

WC (above) and CMP (below), the two largest lines at The Hartford, development and calendar year trends.

# The Hartford WC and CMP reserve by calendar year and risk capital allocation by calendar year.



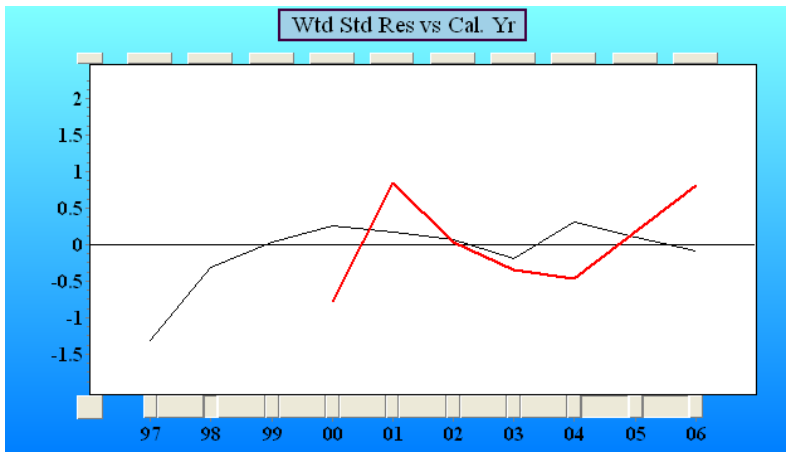



# The Hartford Total Reserve and Total Risk Capital allocation by calendar year.

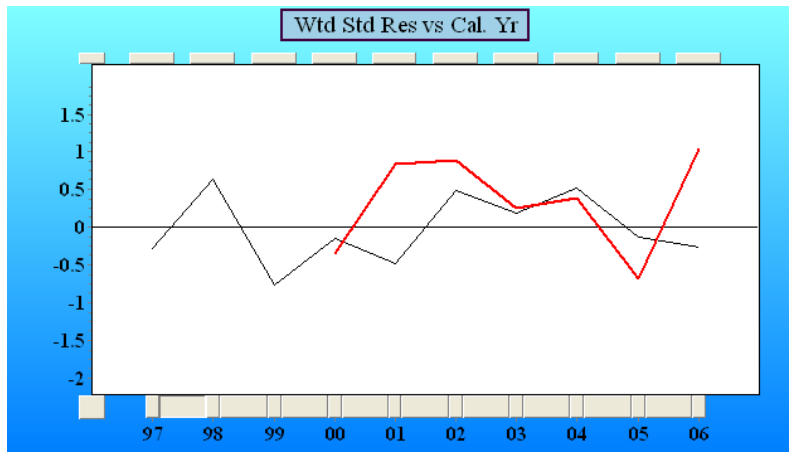


# No two companies are exactly alike and no company is the same as the industry

WC and CMP in Total US Industry show a high process correlation of 0.46



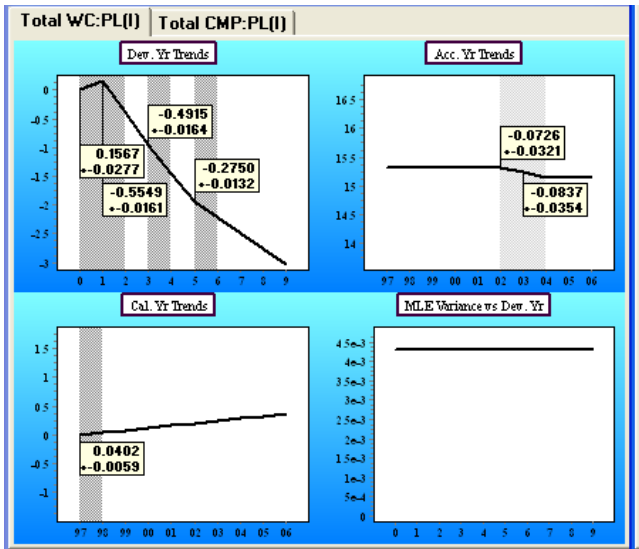

CMP Calendar trends

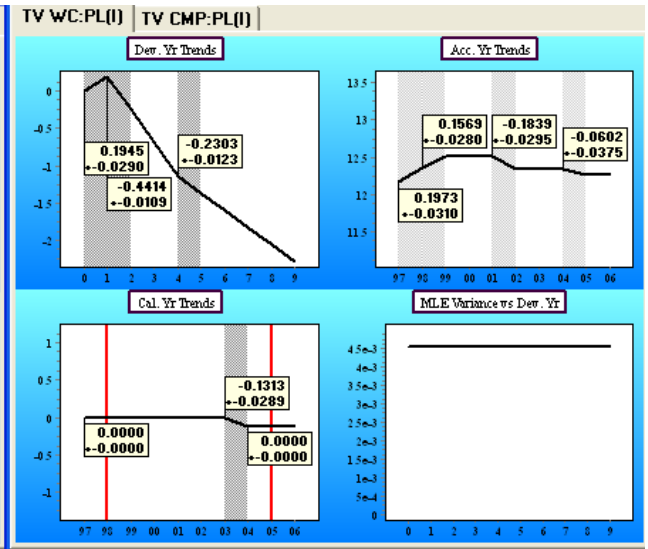



WC Calendar trends

Year 2000 is highlighted in red.

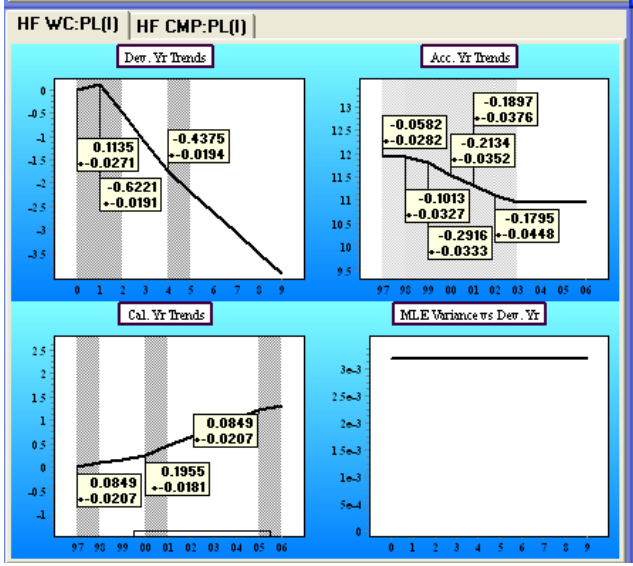
Total US Industry

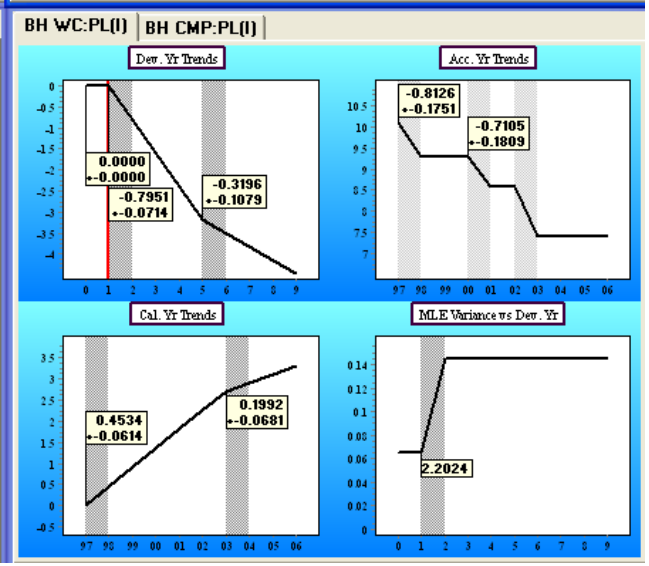



Travellers

The Hartford

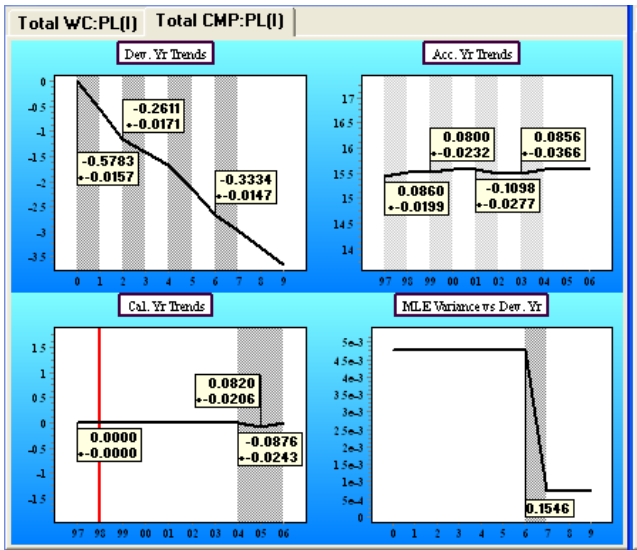


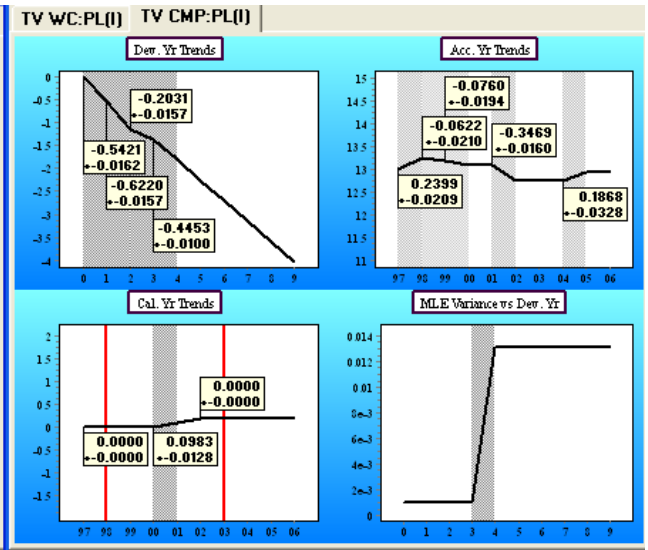

Berkshire Hathaway

Models for Workers Comp. (WC)

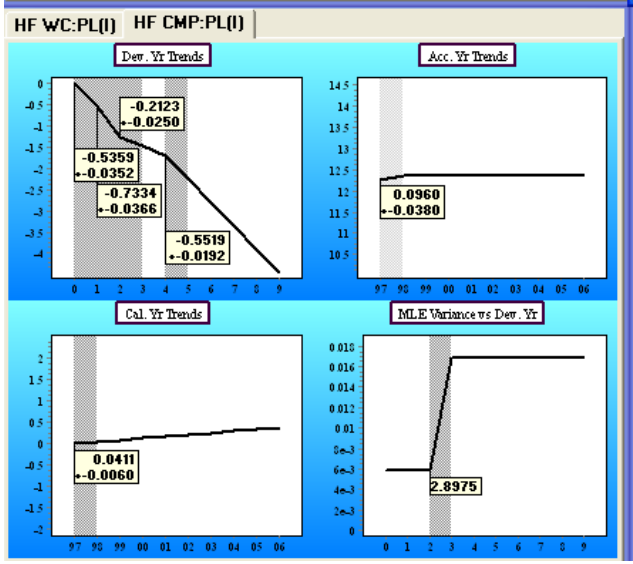
Total US Industry



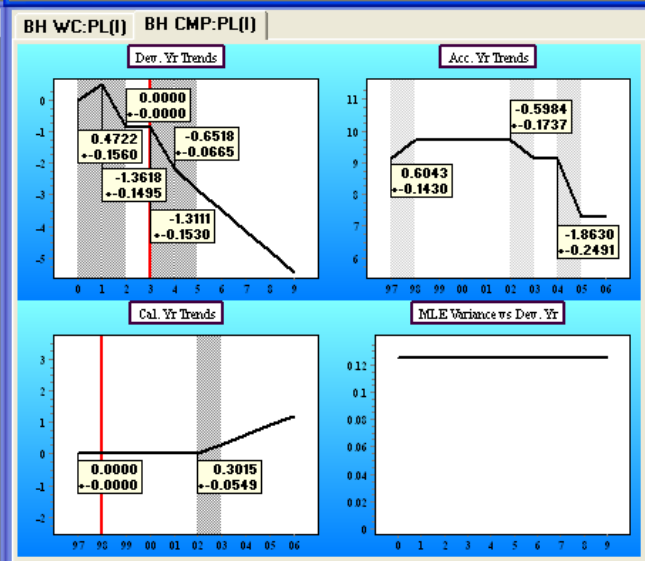

Travellers



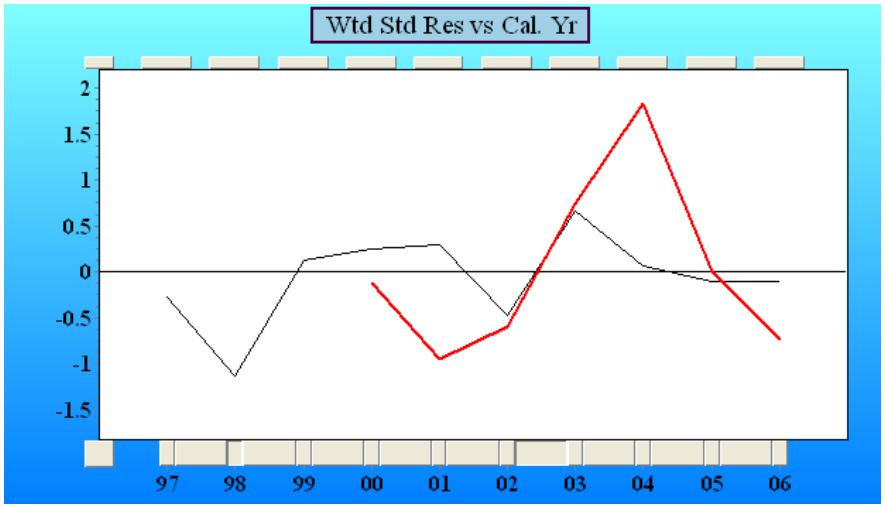

The Hartford

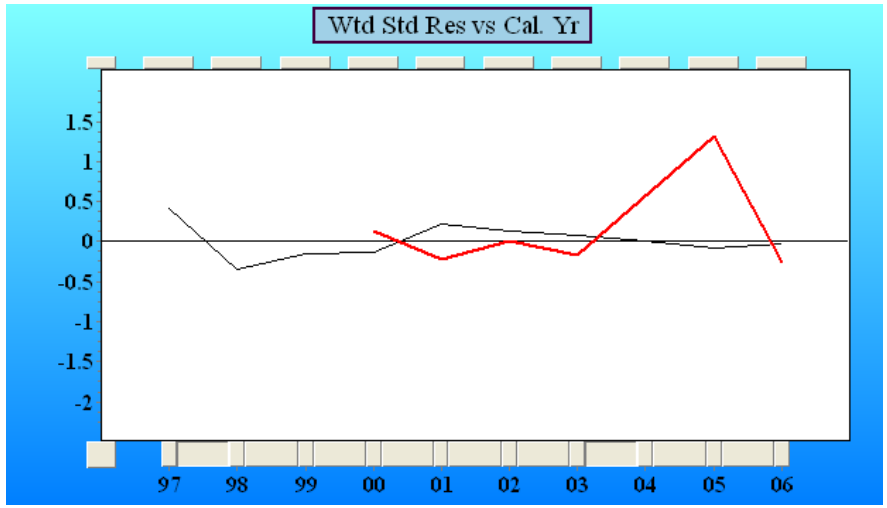



Berkshire Hathaway



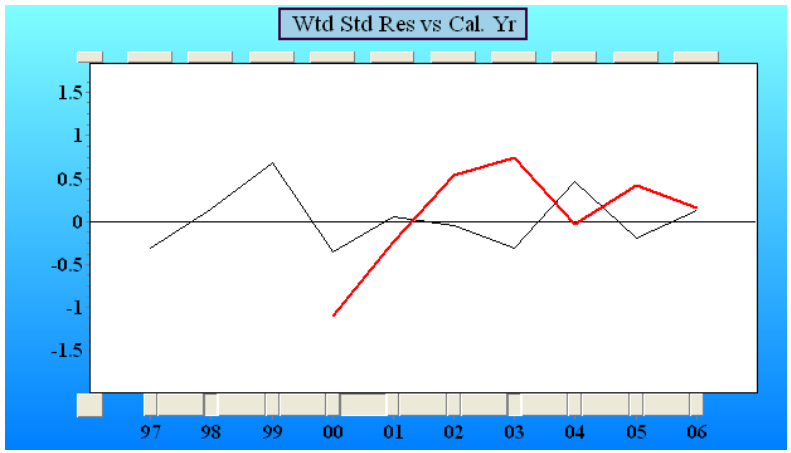

Models for Commercial Multi Peril (CMP)

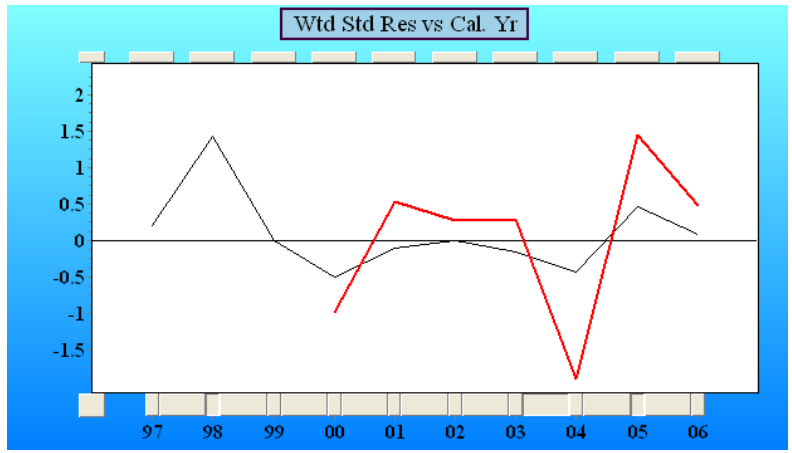


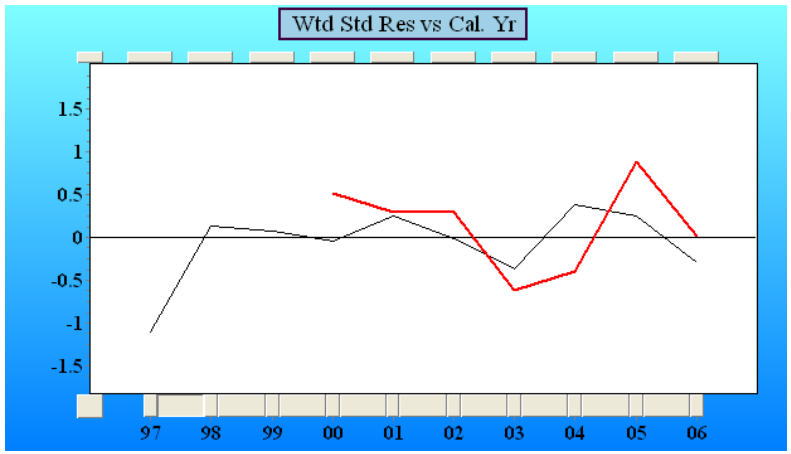

Berkshire Hathaway CMP (left) WC (right) Process correlation = 0.36

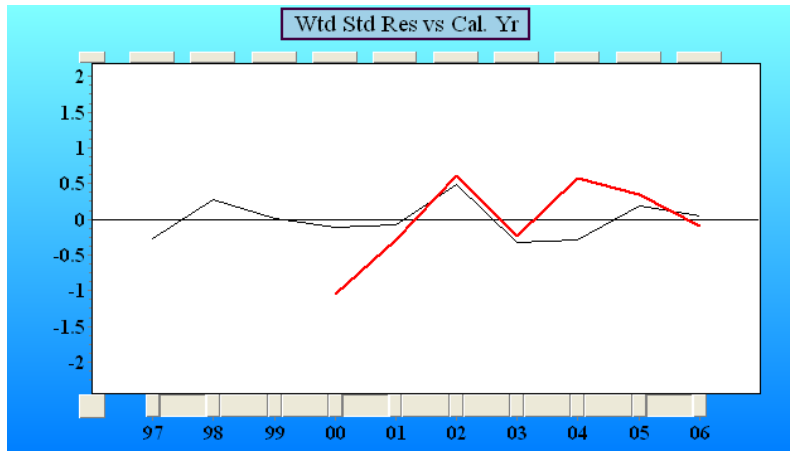
Market share (by Ultimates) = 0.5% CMP, 1% WC




Travellers: CMP (left), WC (right) Corr. = 0.34 Market Share = 10% and 5% resp.




The Hartford: CMP (left), WC (right) Corr. = 0  Market Share = 5% and 4% resp.

# Some Conclusions

- Use of ad hoc correlations and industry-wide development factors is inadvisable if an alternative exists.
- Estimating correlations from model residuals is feasible provided all trends are accounted for in the model.
- Risk modelling is an exchange of information among a number of interested parties and should involve a common language that facilitates this.
- Modelling frameworks are available that go a good way towards meeting the desiderata for “half-decent predictive capability.”