

Capital Adequacy and Dependence

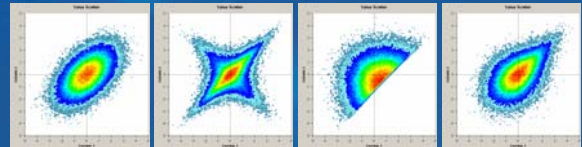
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Different Dependencies

- Linear correlation is not always an appropriate measure of dependence
- Equally correlated bivariate are not necessarily the same



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Introduction

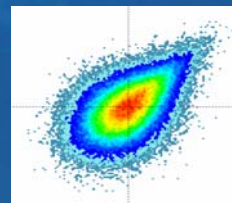
- Impact of dependency structures on capital adequacy
- A practical guide rather than a technical treatise
- Important for:
 - APRA Internal Models
 - Determination of internal risk appetites



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Copulas and DFA

- Copulas allow DFA practitioners to choose the shape and the strength of the relationships
- One useful copula that allows dependence in the right hand tail is the Gumbel



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APRA Internal Models

- APRA Guidance Note GGN 110.2 sets out clear guidelines for the:
 - Evaluation of the interrelationships between risks
 - Stress testing key parameters including the interrelationships



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A Worked Example

- Company in start up
- 1% market share in all 8 classes
- Working Losses modelled using Lognormal distribution
- Very low retention on cats
- \$75m starting capital



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A Worked Example

- Mythical Insurer writing 8 classes
 - Liability
 - NSW CTP
 - Workers' Compensation
 - Professional Indemnity
 - Commercial Property
 - Commercial Motor
 - Domestic Property
 - Domestic Motor



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Fitting a Gumbel

- There are three key steps:
 - Assess pair-wise best fits
 - Overcome issues with multi-dimensionality
 - Determine an appropriate relational structure



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Pair-Wise Best Fits

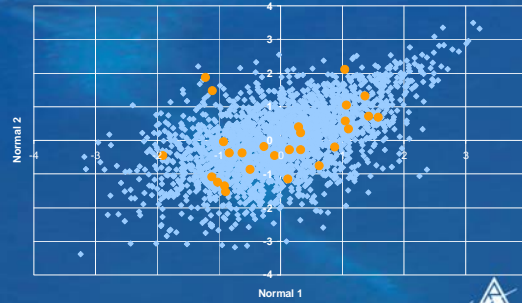
- Different approaches to determining best fits
 - Judgemental
 - Statistical
 - Kendall's Tau
 - Chi-Squared Tests



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Pair-Wise Best Fits

- Determining the chi-squared statistic helps predict actual versus expected



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Pair-Wise Best Fits

- Judgemental approach incorporates prior views

	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property
Liability							
Workers Comp	Med-High						
CTP	Med-High	Med-High					
Prof Indemnity	High	Med-High	Med				
Comm Property	Low-Med	Low	Low	Low			
Comm Motor	Low	Low	Low-Med	Low	Low-Med		
Dom Property	Low	Low	Low	Low	Low-Med	Low-Med	
Dom Motor	Low	Low	Low-Med	Low	Low-Med	Low-Med	Low-Med



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Pair-Wise Best Fits

- These are used to form a pair-wise best view

	Liability	Workers Comp	CTP	Prof Indemnity	Comm Property	Comm Motor	Dom Property
Liability							
Workers Comp	1.50						
CTP	1.30	1.25					
Prof Indemnity	1.50	1.50	1.25				
Comm Property	1.025	1.025	1.025	1.025			
Comm Motor	1.025	1.025	1.15	1.025	1.05		
Dom Property	1.025	1.025	1.025	1.025	1.10	1.05	
Dom Motor	1.025	1.025	1.15	1.025	1.05	1.15	1.10



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Multi-Dimensionality

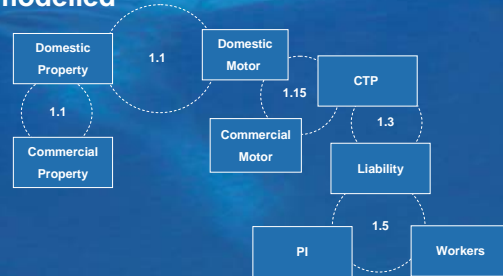
- Gumbel allows only $(n-1)$ parameters to describe $n(n-1)/2$ pair-wise relationships
- Does not handle negative dependence
- Can be overcome by reasonable choice of relational structure



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The Relational Structure

- Represents the explicit dependencies modelled



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The Relational Structure

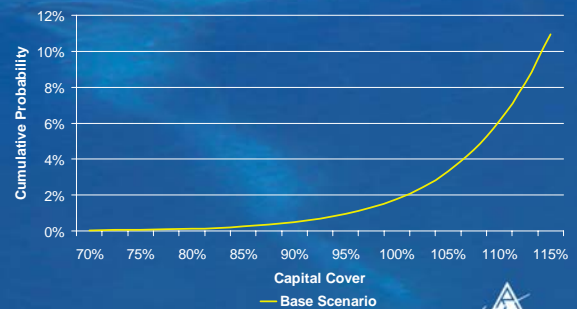
- Can choose which pair-wise relationships are to be modelled explicitly. Based on:
 - Which classes have the strongest pair-wise relationships?
 - What are the largest classes for the insurer?
 - Is there a reasonable justification for “linking” two classes?



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Capital Adequacy for our Insurer

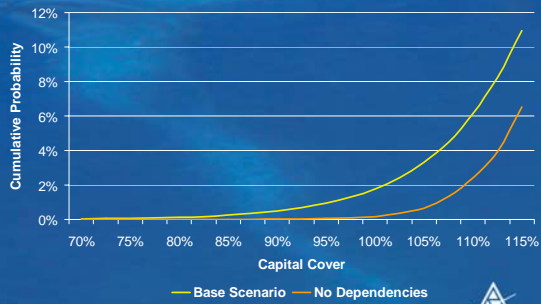
- The estimated probability of failure is 1.8%



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Does Dependence Matter?

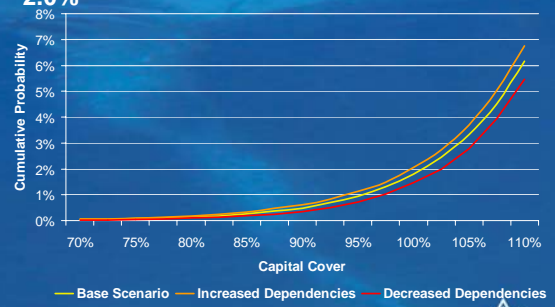
- Removing all dependencies reduces the estimated probability of failure to 0.2%



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Parameterisation Error

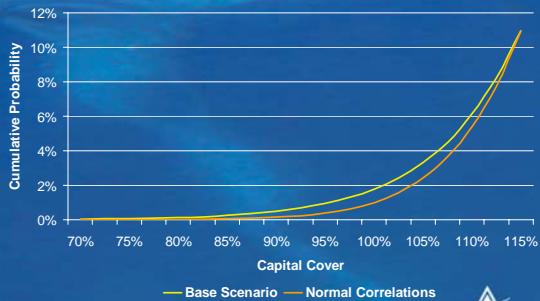
- Range due to parameter selection is 1.5% to 2.0%



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Is the Structure Important?

- Choosing linear correlations reduces the estimated probability of failure to 1.0%



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Conclusions

- Only allowing for linear correlation can seriously underestimate the probability of failure
- Choice of dependency crucial to the conclusions drawn from DFA models
- Parameter error in dependency structures can be significant

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