The Myth of the Discounted Cash Flow

(How Uncertainty Affects Liabilities)

by

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1. Précis

This paper is about the application of uncertainty to the calculation of discounted liabilities.

Formula non-linearities plus cashflow forecasting uncertainties, combined with non-parametric and non-linear investment return distributions and uncertainty of future inflation rates are combined to create a probability distribution of discounted liabilities.

It is demonstrated that where equity investments are used, the chaotic nature of investment returns renders standard techniques (such as a single pass Markov chain) invalid. It is also demonstrated that many of the approaches normally used in dynamic financial analysis are also incomplete.

The only apparently valid approach utilises simulation techniques to calculate the probability distributions of discounted liabilities, essentially using multiple Monte Carlo Markov chains.

The result of an actual calculation is presented using actual investment and CPI rates plus their uncertainty, combined with combined cashflow forecast uncertainty. Risk levels (probability of failure) are calculated and compared with current standard techniques.

It will be shown that the uncertainties and non-linearities combine to give higher probabilities of failure (defined as insufficient premium to meet liabilities) than is commonly understood, particularly for long tailed insurance classes.

The ‘contamination’ effect of equities is discussed as well as the need to design models to suite the specific circumstances that actually exist. Model design will depend on the investment portfolio profile, the cash inflow/outflow timings and the uncertainties in expected cash payments.

Some ideas of some possible investment and cashflow optimisation techniques are also discussed, indicating how reductions in liability uncertainty can be achieved.

The methodology demonstrated is mathematically straightforward but computationally demanding, but is within the reach of current, high-end workstations.

This paper demonstrates that it is practically possible to calculate solutions to the problem of liability uncertainty, even where there are chaotic investment returns.

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The applicability of this methodology applies to all general and life insurance as well as supperannuation, project finance and investment analysis.

2. Introduction

Discounted liability is the cornerstone of pricing in insurance.

Stated simply, what amount of money do I need to collect today (premium) to meet my payment obligation in the future. The time honoured method is to forecast the expected cashflow (by some method), then calculate the present value, taking into account expected investment income and the effects of inflation (where appropriate). The formula is:

$$\sum_{i-1}^{n} \frac{CF_i\left(\left(1+\inf_i\right)^i\right)}{\left(1+inv_i\right)^i}$$

Where Cf_i is the payment expected to be made at time i, and *inf* and *inv* are the expected inflation and investment earning rates at time *i*.

Typically, though not always, only single investment earning and inflation rates are applied to all years.

This equation (dating back to at least the 1700s) is based on the following assumptions:

(1) The cashflow will actually happen at time *i*.
(2) The investment earning rate will actually be *inv* at time *i*.
(3) The inflation rate will actually be *inf* at time *i*.

However, the value calculated is only valid if the cashflows are accurate, the investment earning rates actually achieved and inflation is what is predicted. The formula was originally developed to calculate the expected returns on loans and calculate annuities. Long used in life insurance, very stable data (life expectancy) was combined with conservative investments (rents and later government bonds).

Currently it is typical for an organisation to calculate the money required using a nominal (safe) earning rate and an assumption that the inflation rate is some (on average) amount lower than the nominal investment rate. Loadings will be added to create a prudential margin. Often organisations will normally invest their money in higher earning investments, often with a large component in equities. The assumption is that the higher investment earning rate achieved will exceed the effects of any uncertainty in cashflow and inflation.

Unfortunately these assumptions are unrealistic. Any forecast of future payments is uncertain, inflation exhibits random uncertainty and any investment has an uncertain return (unless they are Government bonds that are held to maturity).

The problem is how to combine these uncertainties and present a probability distribution of total liabilities, making it possible to determine what risks are actually being faced. If this can be achieved, rational decisions can then be made about what premium levels and investment portfolio mixes are appropriate.

3. Adding Uncertainty to Net Present Value

There are four sources of uncertainty:

(1) Inflation varies. Even in times of low, stable inflation rates there is an inherent uncertainty in inflation. Where there are trend changes (rising or falling) uncertainty is magnified.
(2) Forecast cashflow. Every forecast has an inherent uncertainty e. The distribution of e depends on the forecasting method and the uncertainty in the originating data.
(3) Investment earning rates. All investments are uncertain, interest rates have significant uncertainty, bond yields vary considerably and equities have been proven to be a chaotic, fractal series. Again any trend changes magnify uncertainty.
(4) The forecast of future cashflows is typically non-linear, declining through time. (Little success has been made in fitting single continuous functions to long tailed insurance cashflows, the closest being weibull functions). This non-linear sinking fund creates a 'gearing' effect, small changes in values at the beginning of the sinking fund can have large effects on the total cash outcomes. This amplifies the effects of cashflow and earning uncertainty. In simple terms slightly lower returns in the earlier years, when the cash balance is large, would have to be offset by much higher returns in later years, when the cash balance is small.

The total uncertainty is the product of all the uncertainties in all possible combinations. This uncertainty is multiplied by the non-linearity of the equation (for example; the ratio of a 4% inflation rate and a 7% investment earning rate is 0.97 at year 1 and 0.57 at year 20).

Can all uncertainties be combined and calculated to create a probability distribution of discounted liabilities and hence calculate the risk probabilities of insufficient premium?

3.1 The Chaotic Problem of Equities

It is necessary to digress to examine the issue of equities and why their chaotic, fractal nature renders standard solutions ineffective.

- Firstly, it has been well established that stock returns are non-Gaussian[1]. This means that approaches based on Brownian random walk models are invalid.
- Secondly there is evidence that there are no 'short memory' effects and contradictory evidence for 'long memory' effects. This means that a value in the near future is not dependent on an earlier value and a value in the long term future may or may not be dependent on an earlier value [1&2] .
- Thirdly, stock indices exhibit scale invariance, that is, patterns appear the same on whatever time scale they are viewed. A minute by minute series will appear similar to a year by year series.
- Fourthly, large fluctuations can occur in the both short and long terms.
- Fifthly, heteroscedasticity is common for values and returns. This means that variances change through time and these variance changes are in themselves random and unpredictable.

[1] See, for example, Brock, W.A., and de Lima, P.J.F. (1995), "Nonlinear Time Series, Complexity Theory, and Finance", Preprint, Social Systems Research Institute, University of Wisconsin-Madison.

[2] I.N. Lobato and N.E. Savin (1998), "Real and Spurious Long Memory Properties of Stock Market Data", Journal of Business and Economic Statistics.

Current theory suggests that markets are the impact of mass psychological effects of all participants, with non-linear feedback mechanisms that self-reinforce independent events and cause irregular behaviour.[3]

From the point of view of solving current practical problems, these characteristics (and others) render predictability of stock market returns impossible across any time period.

This contrasts with bond and cash yields, which though they exhibit uncertainty, do not have the chaotic effects nature of equities. For example, Bonds are moderately correlated (R^2 of 0.54) to CPI, and exhibit Gaussian properties.

(1) It is common to maintain a mixed portfolio to manage risk, the idea being that a proportion of lower risk investments (e.g. bonds) create a base amount that relatively safe, reducing the overall risk of the total portfolio.

(2) What actually happens is that the more volatile (and chaotic) proportion of the investments that is held in equities 'contaminates' the total portfolio. Their characteristics and probability distribution of returns influencing the whole portfolio to a greater and greater extent as their proportion grows.

(4) This is not the same as the combination of two similar (parametric) distributions, which is well understood and can easily be calculated. A well behaved distribution combined with a chaotic distribution will produce a chaotic distribution.

(5) This can be demonstrated by the following 'thought experiment':

- Take an amount of bonds combined with a smaller amount of equities.
- In a given year the bonds deliver a return, well within expected (and narrow) confidence limits.
- In the same year the equities exhibit a large negative fluctuation in returns, well within the range of such returns made in the past, but well outside the range of returns in the bond portfolio.
- The total portfolio has now a lower return than expected.
- To meet payment demands a sale of investments is required, crystallising the fluctuation.
- The whole portfolio is now in deficit.
- With bond returns being predictable within reasonable confidence limits, the only way the fund can return to balance is if the equities return a correspondingly high return, or a long period of above average returns.

What is happening is that the chaotic uncertainty in returns of the smaller number of equities is driving the total portfolio return probabilities. In other words:

Chaos Contaminates

Of course if the proportion of equities is very small then the effects will be more muted, as the proportion grows then the distribution of returns will then tend to take on the distribution of the equity part. Essentially the variance of the total portfolio will increase, with the distribution becoming more chaotic like as the proportion increases.

[3] " Chinese Stock Market – Is it Chaos?" [Ref]

What is just as important is that other chaotic characteristics will contaminate the whole portfolio, with correlations of returns between years dropping and heteroscedasticity increasing.

Three additional points should be added:

(1) It is not known what proportion of equities is required to have a noticeable effect. Intuitively a very small proportion (say 1%) will be insignificant, but a larger amount (20%, 30%?) will be noticeable. This is a fruitful area for research.

The proportion may be smaller than common sense would expect because of the large fluctuation effect of chaos.

(2) There may be ways to buffer bonds from equity effects, provided the proportion is not too great. A cash 'float' that is sufficient to cover most probabilities (which can be calculated by a stochastic EOQ* formula) could be utilised to prevent forced sales, thus avoiding the crystallisation of losses and hence buying time for the fluctuation to smooth out.

It should be noted that this amount will be related (in a currently unknown way) to the proportion of equities and will probably be much larger than would commonly be thought, though theoretically calculable.

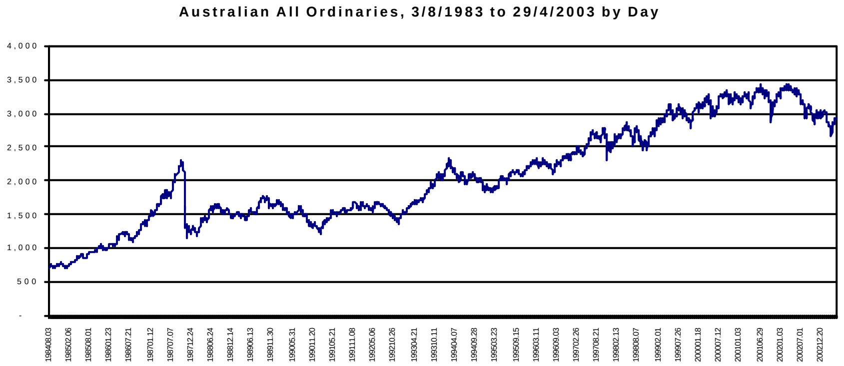
(3) This issue of contamination is of particular relevance to investment in overseas equities, while the jury is still out on whether exchange rates are chaotic and whether they do or do not exhibit 'long memory' effects, their returns will be chaotic in nature, combined with further exchange rate uncertainty.

* *Economic Order Quantity, note that this approach can also be used to determine the cash balance necessary to avoid bond tranding (and thus potential capital losses) for a purely cash/bond porfolio.*

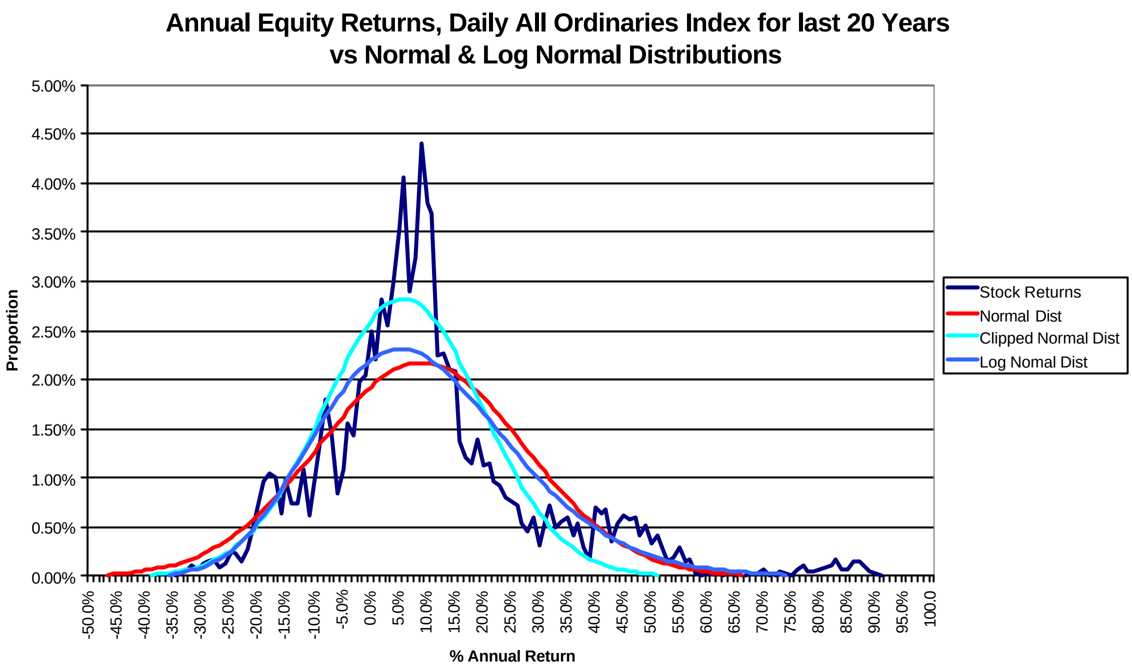
3.2 Example – Australian Equities

It is worthwhile to examine these characteristics to determine whether they apply to Australian equities. The daily indices of the Australian All ordinaries for the last 20 years are:

The annual rates of return by day is:
Chart 1



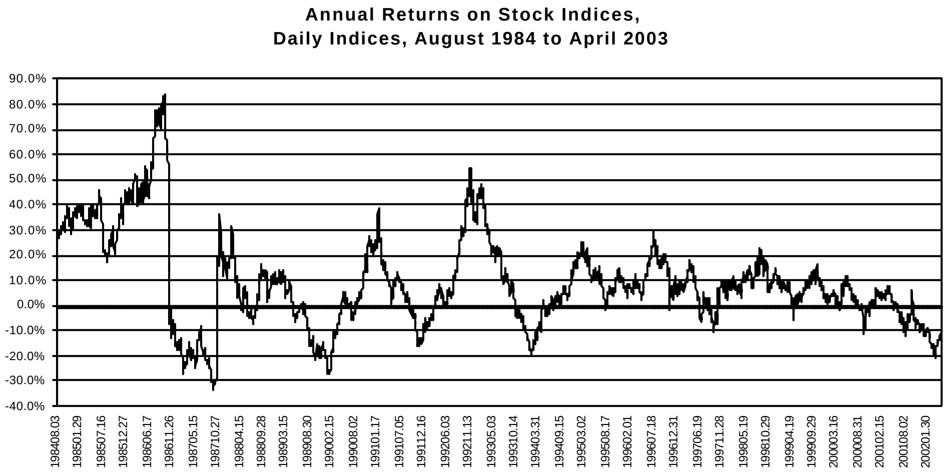
The distribution of annual returns, compared with normal and log-normal distributions are:
Chart 2



As can been seen, any calculation methodology based on using Gaussian distributions will run risks of under or overstating returns depending on the calculation and/or sampling methods (which is why Scholes-Black does not work).

Annual returns by day (note, not annualised returns) demonstrate the issue of heteroscedasticity.

Chart 2b



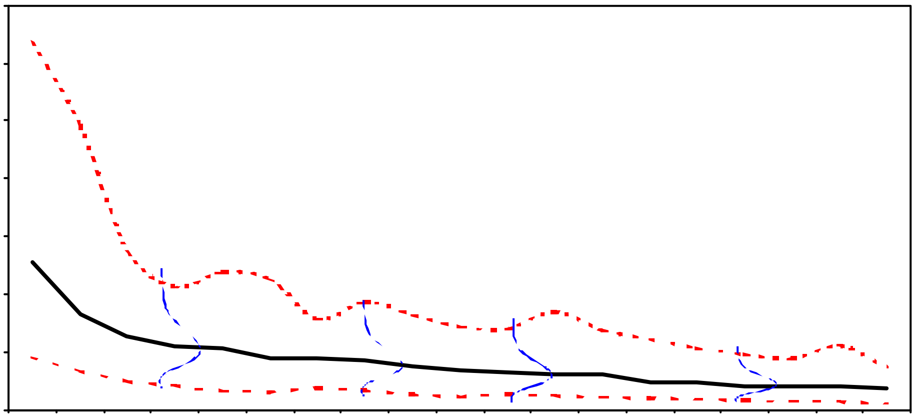
4. Computational Description (largely graphical)

The model can be described as a multi-stage Monte Carlo Markov chain calculation.

Essentially, to calculate the distribution of probabilities of liabilities requires calculating all the probabilities of all interactions, between the expected cashflow, investment returns and inflation (if added).

A typical forecast cashflow with confidence limits is shown below (output from ICRFS).

Chart 3



This has to be combined with the probabilities of returns and inflation (if applicable). This requires combining every probable cashflow with every probable return with every probable inflation value.

Where the distributions are Gaussian, then mathematical short cuts can be applied. Mathematically it is possible (though tedious) to combine similar distributions into a single probability function (though this can become difficult with non-parametric functions).

As has been shown, where equity investments are involved this cannot be achieved and numerical methods are all that are available.

The problem is that there is an infinite number of calculations. Even if discrete samples are used (say, 100 cashflow points, 1000 investment returns and 1000 inflation returns), then to calculate the 20th year's results would require 101^{60} separate calculations (which on my PC would take approximately 4.10^{143} years).

The calculation problem is compounded by the fact that it cannot be calculated in a single pass. Each payment year has to be calculated separately, with the 20th year requiring 20 years of calculations, the 19th 19 years, etc. Then all the probable results for all years have to be combined to give a single distribution of total liabilities. (This is one reason why a standard single pass Markov chain approach will not work, rather a far more computationally intensive, essentially brute force, approach is required).

There is also a 'gearing' problem. Since the 20th year has 20 separate combinatorial calculations then small changes (or errors) can magnify, which is why approximate distributions can significantly under or overstate total results.

Since this is impossible to do then a Monte Carlo approach is all that is available. This is a simulation technique where random samples of numbers are taken from distributions, then used for calculation. These samples of numbers are then used to do a Markov chain type calculation for every payment year. A further Markov chain calculation is then required to combine every year's liabilities into a single distribution. These then have to be repeated often enough to get a stable result, since every sample will have different values.

Though it is impossible to do all the calculations, it is possible to do enough of them to get a stable and reliable result.

The methodology can be outlined as follows:

(1) First design the model. Care must be taken to model the portfolio's characteristics, such as premium income scheduling, payment timing, the forecasting error, the distribution of applicable investment returns and the distribution of applicable inflation effects.

A portfolio that (say) receives all income at one time, pays only annually and is completely invested in cash will require a very different model structure (trivial actually) to one that receives money continuously, pays continuously and is invested in equities.

A short tail portfolio may be as complex as a long-tailed one if cash inflows/outflows and uncertainties are sufficiently large and complex.

(2) Determine the forecasting period and number of periods. Again, an unstable short-tailed portfolio may require as complex a model as a stable long-tailed one.

(3) Input actual distributions that apply. This is one failing of many Dynamic Financial Models, as they often use assumed distributions.

(4) If Gaussian distributions do apply, then try mathematical short cuts to combine distributions to reduce the calculations required.

(5) If not Gaussian, then determine the level of discreteness in values used for sampling . Essentially the more coarse the values sampled from a distribution, then the more simulations that have to be ran and the less stable will be the final result. Then apply Monte Carlo Markov techniques.

5. An Actual Example Using Real Data

The following model was created with these characteristics, choices being limited by computational capacity and data availability.

(1) A 20 year annual payment cashflow with an log normal e and a CV of 40%. This forecast cashflow was based on actual workers' compensation data and used the ICRFS system to forecast trends and uncertainties.

(2) Inflation is the same as Australian CPI.

(3) The total premium is invested in Australian equities in indexed funds and premiums were paid at random times through the year.

(4) The choice of distribution to chose to select out probable investment earning rates was the daily 20 year All Ordinaries index. The rationale was that:

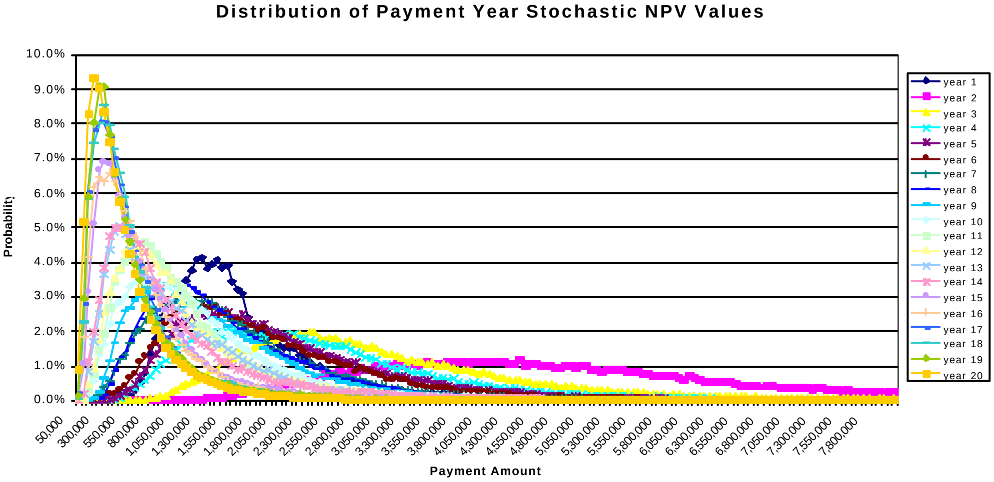
- It is impossible to predict short and long term earning rates.
- Chaotic systems do exhibit regularity over sufficient time.
- Therefore a sufficiently large period will cover all probabilities of future returns (though we can never actually say when they may occur).
- This is probably the most contentious assumption and does require further research. Is a 20 year history sufficient? Analysis of year to year (over even 3 or 5 year) returns show significant differences between periods. Does it require 30 or 40 years to capture all probable outcomes?

(5) 30 Simulation runs in total were undertaken, with a sampling of 2,000 points for every calculation with constant resampling. This meant that the number of calculations increased linearly rather than geometrically (approximately 840.10^6 calculations). With overheads for frequency counts and other statistics being calculated at every step meant a single simulation run took approximately 15mins (1Ghz AMD processor), with the full 30 runs taking about 7.5 hours.

Deterministic NPV calculation of mean payments,	liability = $24.251 million
With a 'safe' investment earning rate of 2% above CPI,	liability = $29.009 million
With a 'safe' investment earning rate of 2.5% above CPI,	liability = $28.046 million
With a 'safe' investment earning rate of 3% above CPI,	liability = $27.135 million
Stochastic Liability with Actual CPI and earning rates,	liability = $29.116 million

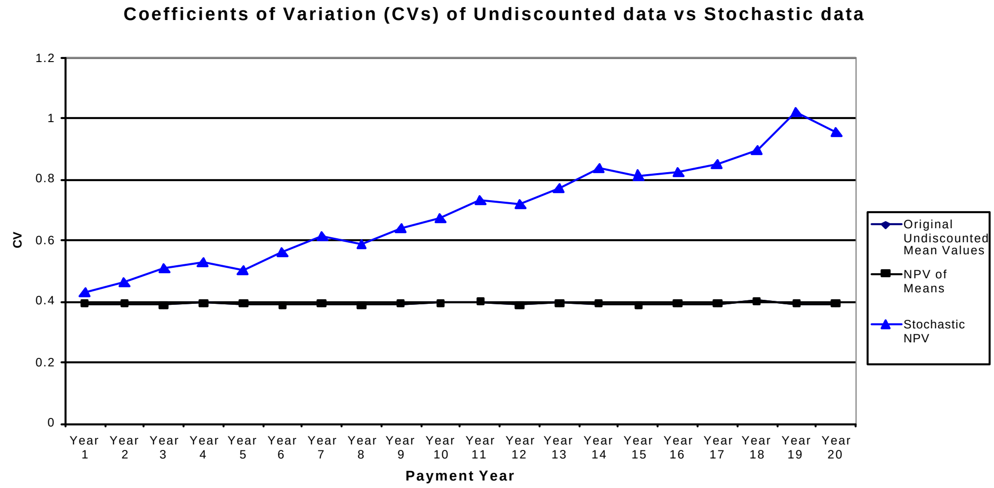
When each payment year's stochastic NPV calculations are completed, they have distributions as shown in the following chart:

Chart 4



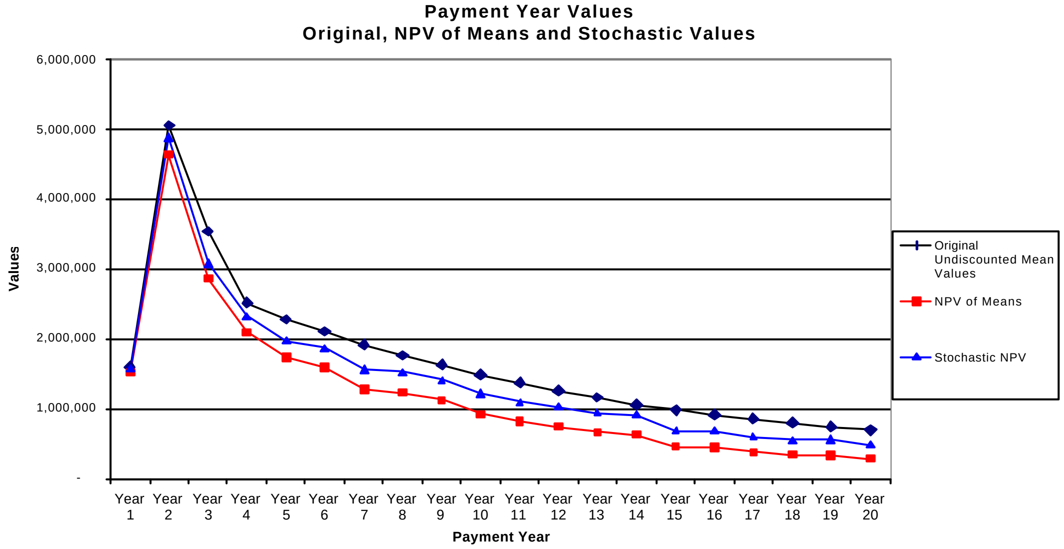
Note that they maintain the original log-normal distribution, though the CV of each payment year was markedly higher than the original data. This should be no surprise, as we are essentially adding additional distributions to each data point. As multiple years are calculated then the cumulative effect is to spread the core distribution out, while maintaining the core (log-normal in this case) distribution of points.

Chart 5



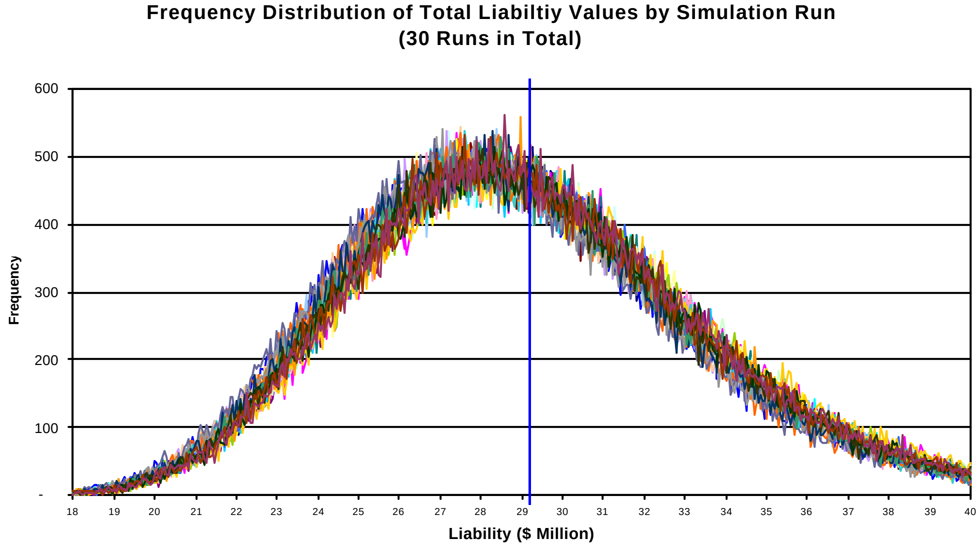
This increase in uncertainty raises the mean value of each payment year's NPV. The following chart compares the mean stochastic payment year values against the original (undiscounted) mean values and the discounted mean values (standard method).

Chart 6



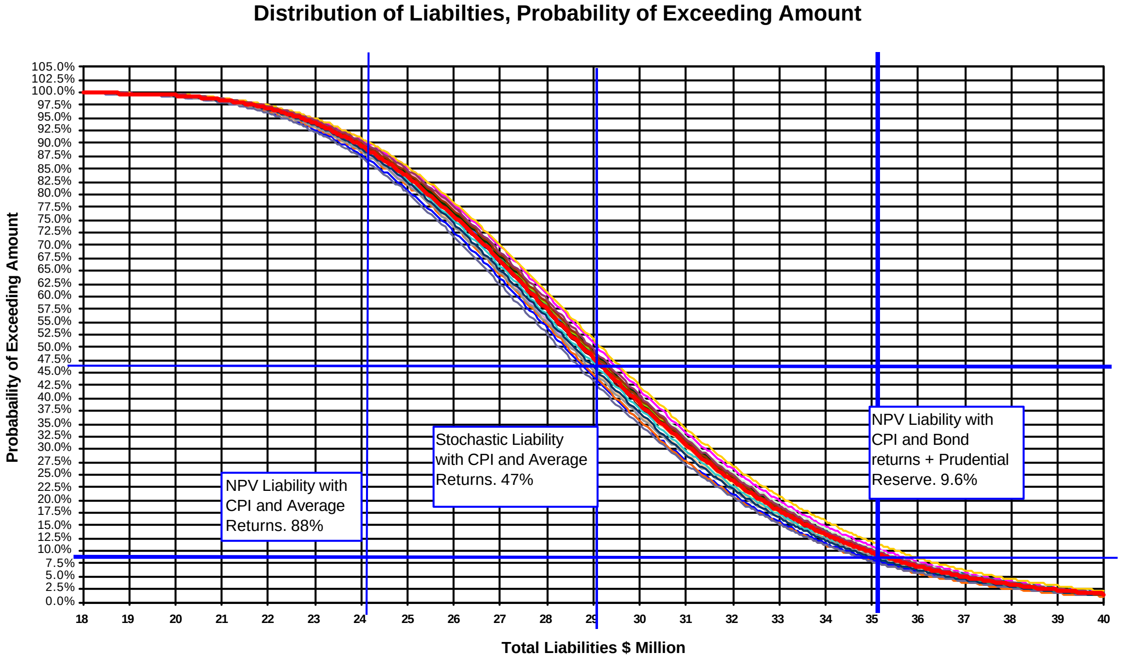
Each payment year has a distribution of values, when these are combined into a single distribution of all values, for all simulation runs (30 runs in all) the following distribution of total liabilities is:

Chart 7



After this can be turned into a failure chart, with the probability of the total liability exceeding a given value being shown. This, along with the average value of the runs is given in the following chart (with the mean values, NPV of mean values and NPV of mean values with a no risk return rate of CPI +2.5% plus 25% prudential reserve, shown for comparison).

Chart 8


Distribution of Liabilties, Probability of Exceeding Amount
Probabaility of Exceeding Amount
Total Liabilities $ Million
NPV Liability with CPI and Average Returns. 88%
Stochastic Liability with CPI and Average Returns. 47%
NPV Liability with CPI and Bond returns + Prudential Reserve. 9.6%

6. Comparison of the Probability Distribution of Liabilities to Standard Practises

The stochastic liability values are compared with standard calculations in the following table:

Table 1

Liability Value	Liability Value as % of Mean value	Probability of This or lesser Amount Occurring	Probability of This Amount Being Exceeded	Comment
24,251,379	88.1%	11.8%	**88.2%**	NPV of Mean Values, average CPI & Equity returns
29,379,952	100.0%	53.2%	**46.8%**	Mean Stochastic Liability Value
30,000,000	103.0%	61.1%	**38.9%**	
31,000,000	106.5%	69.2%	**30.8%**	
32,000,000	109.9%	76.8%	**23.2%**	
33,000,000	113.3%	82.0%	**18.0%**	
34,000,000	116.8%	86.7%	**13.3%**	
35,057,790	120.4%	90.4%	**9.6%**	NPV of Mean Values, Average CPI rates. Safe investment earning rate of CPI + 2.5%. 25% Prudential Margin added.
36,000,000	123.6%	93.0%	**7.0%**	
37,000,000	127.1%	95.3%	**4.9%**	
38,000,000	130.5%	96.6%	**3.4%**	
39,000,000	133.9%	97.6%	**2.4%**	
40,000,000	137.4%	98.4%	**1.6%**	

As can be seen there is nearly a 1 in 10 chance of the standard mean value (with a ‘safe’ investment rate, plus a prudential margin), being exceeded. There is nearly 1 in 20 chance that the mean value will be exceeded by more that 27%.

6.1 Impact of Increasing Payment Forecast Uncertainty

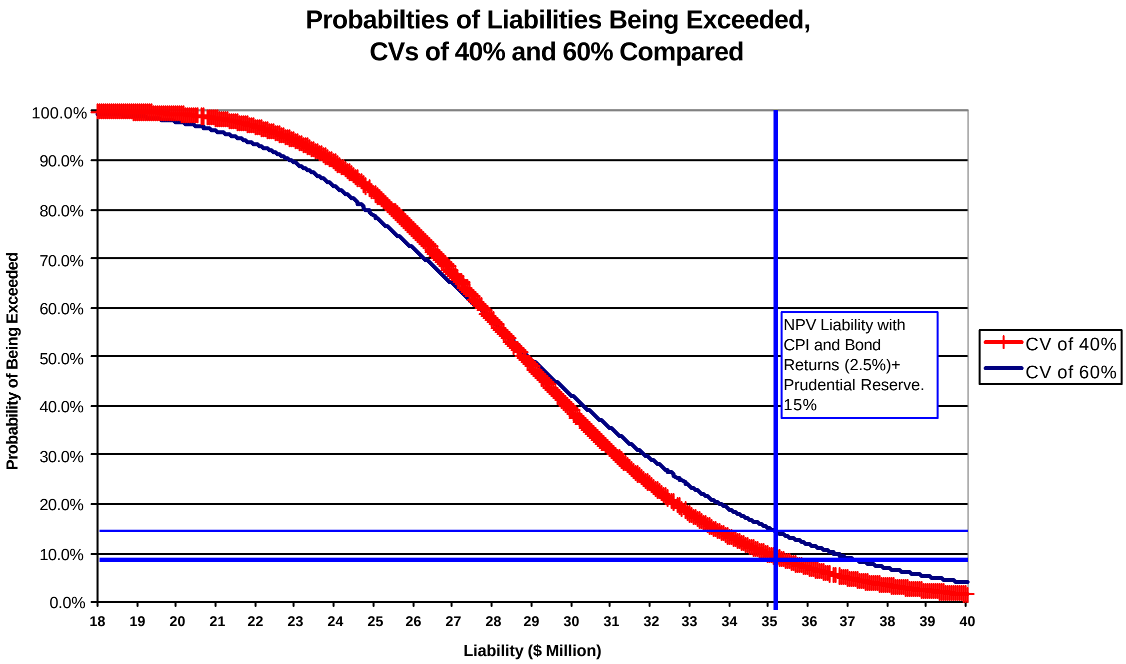
As might be expected the results are sensitive to increases in the uncertainty in the payment forecast, with *failure risks increasing dramatically at higher values*. Comparative runs with a forecasting uncertainty increased to 60% (not uncommon) were undertaken

If the CV of the original payment distribution increase to 60% then the mean value only increases by 1%, however the risk of the ‘risk free’ liability being exceeded increases by over 50%. The chance that the mean value will be exceeded by more that 27% increases by 80%.

Table 2

	Probability of This Amount Being Exceeded		
Liability Value	**CV of 40%**	**CV of 60%**	**Ratio**
24,251,379	88.2%	83.2%	0.94
29,000,000	47.9%	49.4%	1.03
30,000,000	38.9%	42.1%	1.08
31,000,000	30.8%	35.3%	1.15
32,000,000	23.2%	28.5%	1.23
33,000,000	18.0%	23.7%	1.32
34,000,000	13.3%	19.0%	1.42
35,057,790	9.6%	14.8%	1.55
36,000,000	7.0%	11.7%	1.68
37,000,000	4.9%	8.8%	1.79
38,000,000	3.4%	6.9%	2.02
39,000,000	2.4%	5.3%	2.22
40,000,000	1.6%	3.9%	2.44
Mean value	29,115,566	29,379,952	1.01

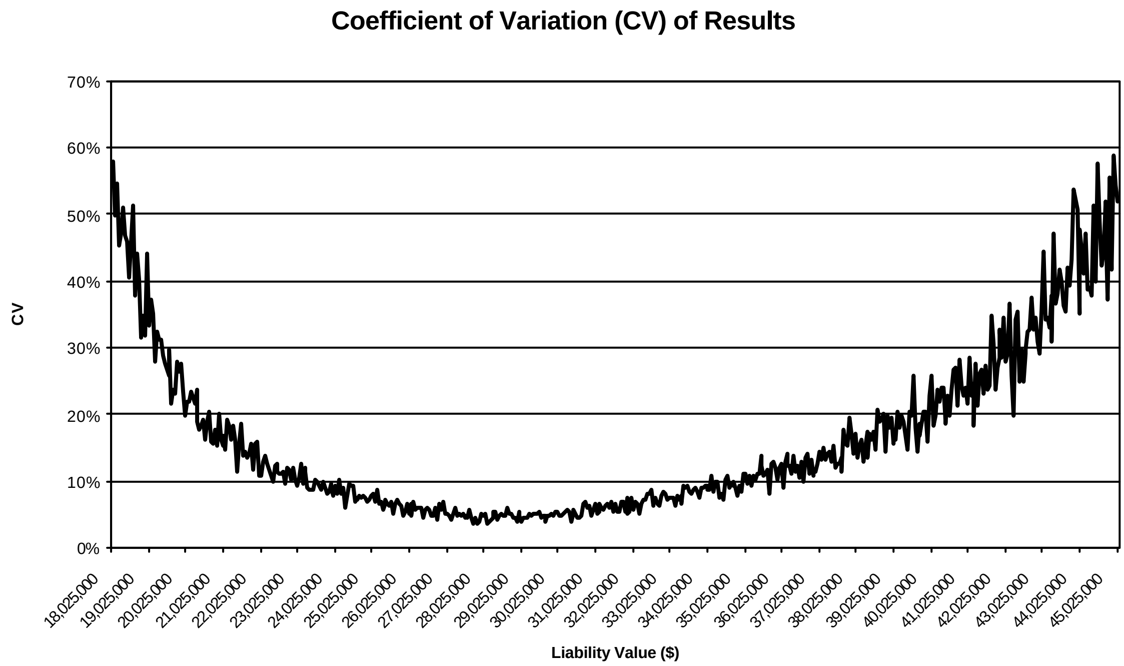
Chart 9



6.2 Increasing Uncertainty at Extreme Values

The uncertainty of the liabilities increase at extreme value. The following chart shows the CV by liability value.

Chart 10



This result should be no surprise, firstly low probabilities at extreme values will necessarily have fewer calculations and therefore be more uncertain. Secondly , the probability distribution of equity returns has reasonably flat extreme tails, with a number of small, but still significant, probabilities of extreme values. For example, the probability of a positive return of 36% is virtually the same as the probability of a return of 51%.

This means that, from a risk of failure point of view, probabilities of extreme failure (that is, the actual liability is far larger than the mean value) will always be inherently more uncertain.

It is possible to improve the sampling rate at extreme probabilities by using segmented sampling, with lower sampling rates at close to mean values and higher rates at the more extreme values. This would reduce the CV at extreme values, reducing the problem but *not* eliminating it. There will always be higher uncertainties at extreme values, reflecting the fact that there are small but finite probabilities of extreme positive or negative returns and any sampling methodology will combine these with more 'normal' return probabilities.

From a practical point of view this means that the probability of extremely poor results could be much higher than the mean value. For example, in the example calculations (40% CV) the mean probability of the liabilities being greater than $40 million (37% greater than the mean value) is 1.6%. However, in reality the value could quite easily be 3% or 0.5%.

This can be included into the calculation, giving confidence limits of each liability probability.

6.3 Combining Multiple Injury Years and/or Product Lines

The example shown is for a single injury year, but in principle combining multiple injury years results, which can then be used for a stochastic NPV calculation, is straightforward.

- Where multiple injury years exhibit the same type of error(e) distribution (e.g. log normal) then combining payments years (at the same point of development) distributions (albeit with different means and standard deviations) is trivial.

- Where error distributions differ in type, they can be combined using numerical methods.

- The same principles apply to combining different product lines, though caution should be applied as it t may be advantageous to model widely differing products separately to enable better investment portfolio matching.

- *The decision to combine/split stochastic NPV calculations of different products may be another fruitful area of research to determine the optimum splits and combinations that should be made (e.g. all short tail combined, all long tail combined).*

7. Conclusions and Future Research to be Completed

7.1 Conclusions

- The methodology outlined of applying stochastic principles to NPV calculations is valid.
- It is capable of handling chaotic equity returns, which few other methodologies attempt without resorting to unfounded assumptions.
- It can be expanded to mixed investment portfolios as well as other investment types (such as overseas equities).
- The True Risk of investment portfolios not meeting liabilities can be calculated, along with the uncertainty of the calculation.
- *This enables rational decisions to be made about the risk/returns that organisations wish to undertake.*
- The methodology is computationally demanding but well within the reach of most organisations (additionally it is ideal for concurrent processing, with multiple processors or PCs calculating separate sections then results being combined). It does vastly exceed the capacity of any spreadsheet and would be difficult to undertake in a macro based language such as SAS. Any 3rd generation programming language (such as C) would be suitable (albeit after ensuring that high precision is available) though scientific based languages such as Fortran or APL are more suitable, due to the need to manipulate very large matrices. *It is essential that any language (or version/compiler) used has a good random number generator.*

7.2 Further Research to be Undertaken

- Further research needs to be undertaken on determining the optimum number of years of equity returns to cover all probabilities that are appropriate for a given calculation.
- Further research to determine the optimum splits and combinations of product lines that should be made (e.g. all short tail combined, all long tail combined).
- What proportion of equities is required to have a noticeable chaotic effect on a mixed investment portfolio?
- Other investment types (such as overseas equities raising the additional issue of exchange rate uncertainty) need to be researched.
- Low cashflow uncertainty product lines (e.g. life insurance, where the cashflow uncertainty comes from sampling uncertainty from a population mortality table) need to be investigated.
- Further work needs to be undertaken to investigate segmented sampling to reduce (but wont eliminate) high CVs for extreme values.
- The balance between sampling coarseness, number of simulations and result stability needs to be researched to determine the best selections for optimum run time and result stability.

7.3 Caveat

A stochastic cashflow forecasting methodology (such as Insureware's ICRFS forecasting system) is essential.

If forecast cashflow uncertainty cannot be calculated then this methodology cannot be applied, therefore standard industry techniques, such as chain-ladder/ratio forecasting models, are inapplicable.