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Institute of Actuaries of Australia
Thriving on Change
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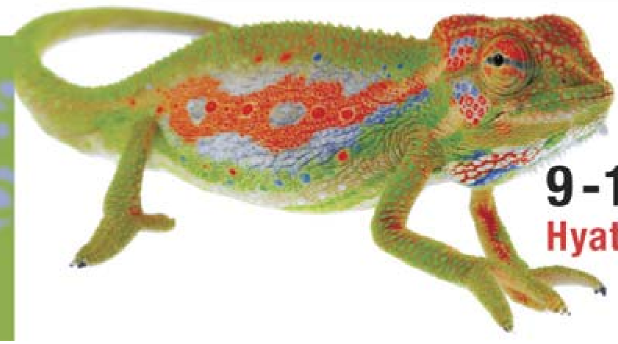
Hyatt Regency Coolum



Stochastic reserving – case study using a Bayesian approach

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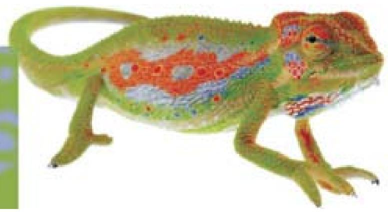
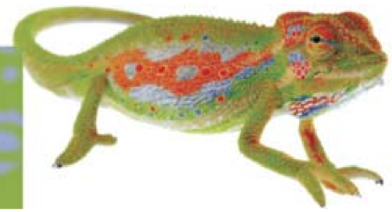


Agenda

- Introduction
- General Bayesian modelling process
- Non-reserving illustration
- Advantages and Disadvantages
- Case study

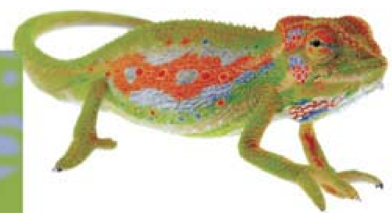
Introduction

- Historical perspective:
  - Stochastic reserving in general insurance
  - Implementation of Bayesian techniques
- Existing papers on Bayesian stochastic reserving
- Key objectives of this paper
  - Introduction to the theory
  - Demonstration in the reserving context



General Bayesian modelling process

- Specify the **data distribution**
- Specify **prior distributions**
- Derive the **likelihood function**
- Derive the **posterior distribution**
- Obtain **parameters** from the **posterior distribution**
- Obtain **forecasts** from the **predictive distribution**



Non-reserving example

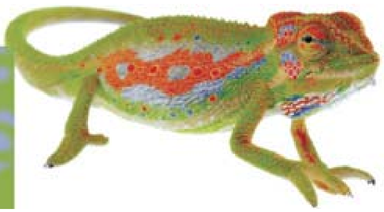
- Steps 1 and 2

$$x_i / \theta \sim Independent\ \ Poisson(\theta)$$

$$\theta / \alpha, \beta \sim Gamma(\alpha, \beta)$$

- Step 3

$$L(\theta / \underline{\mathbf{x}}) = \prod_{i=1}^{n} \frac{\theta^{x_i} e^{-\theta}}{x_i!}$$



Non-reserving example (continued)

- Step 4

$$f(\theta / \underline{\mathbf{x}}, \alpha, \beta) \propto \left(\prod_{i=1}^{n} \frac{\theta^{x_i} e^{-\theta}}{x_i!} \right) \frac{\beta^{\alpha}}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}$$

$$f(\theta / \underline{\mathbf{x}}, \alpha, \beta) \propto \theta^{\alpha + \sum_{i=1}^{n} x_i - 1} e^{-(\beta+n)\theta}$$

- Step 5

$$\theta / \underline{\mathbf{x}}, \alpha, \beta \sim Gamma\left(\alpha + \sum_{i=1}^{n} x_i, \beta + n \right)$$

Non-reserving example (continued)

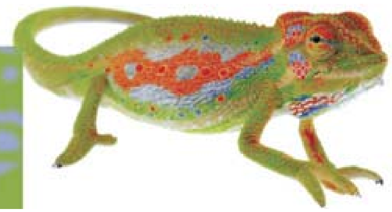
- Step 6

$$f\left(\tilde{x} / \underline{\mathbf{x}}\right) = \frac{\Gamma\left(\alpha_1 + \tilde{x}\right)}{\Gamma\left(\alpha_1\right)\Gamma\left(\tilde{x}+1\right)} \left(\frac{\beta_1}{1+\beta_1}\right)^{\alpha_1} \left(\frac{1}{1+\beta_1}\right)^{\tilde{x}}$$

$$\alpha_1 = \alpha + \sum_{i=1}^{n} x_i \qquad \beta_1 = \beta + n$$

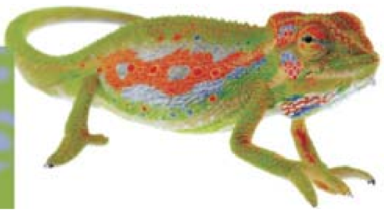
$$\tilde{x} \sim NegativeBinomial\left(y, q\right)$$

$$y = \alpha_1 \qquad q = \frac{1}{1+\beta_1}$$



Advantages and Disadvantages

- Key advantages
  - Robust statistical framework
  - Flexibility to incorporate actuarial judgement and external information
- Key disadvantage
  - Apparent complexity compared to other stochastic reserving methods



Case study – model specifications

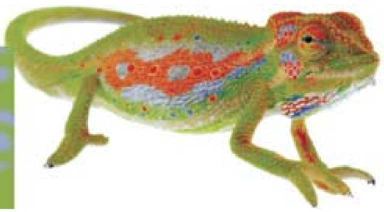
- Over-dispersed Poisson chain ladder model

$C_{ij} / \mathbf{x}, \mathbf{y}, \varphi \sim$ independent ODP, with mean $x_i y_j$, and $\sum_{j=1}^{n} y_j = 1$

$$E\left[C_{ij}\right] = x_i y_j$$

$$V\left[C_{ij}\right] = \varphi E\left[C_{ij}\right]$$

$x_i, \ y_j \sim$ independent non-informative Gamma distributions



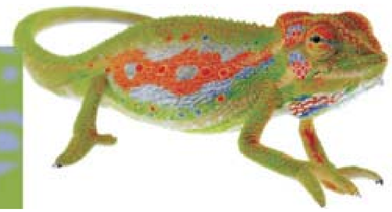
Case study – model specifications

- Bornhuetter-Ferguson model

$$LR_i^{Ini} \sim Independent\ Normal\left(\mu_i, \sigma_{(1)i}\right)$$

$$LR_i^{ODP} / LR_i^{Ini} \sim Independent\ Normal\left(LR_i^{Ini}, \sigma_{(2)i}\right)$$

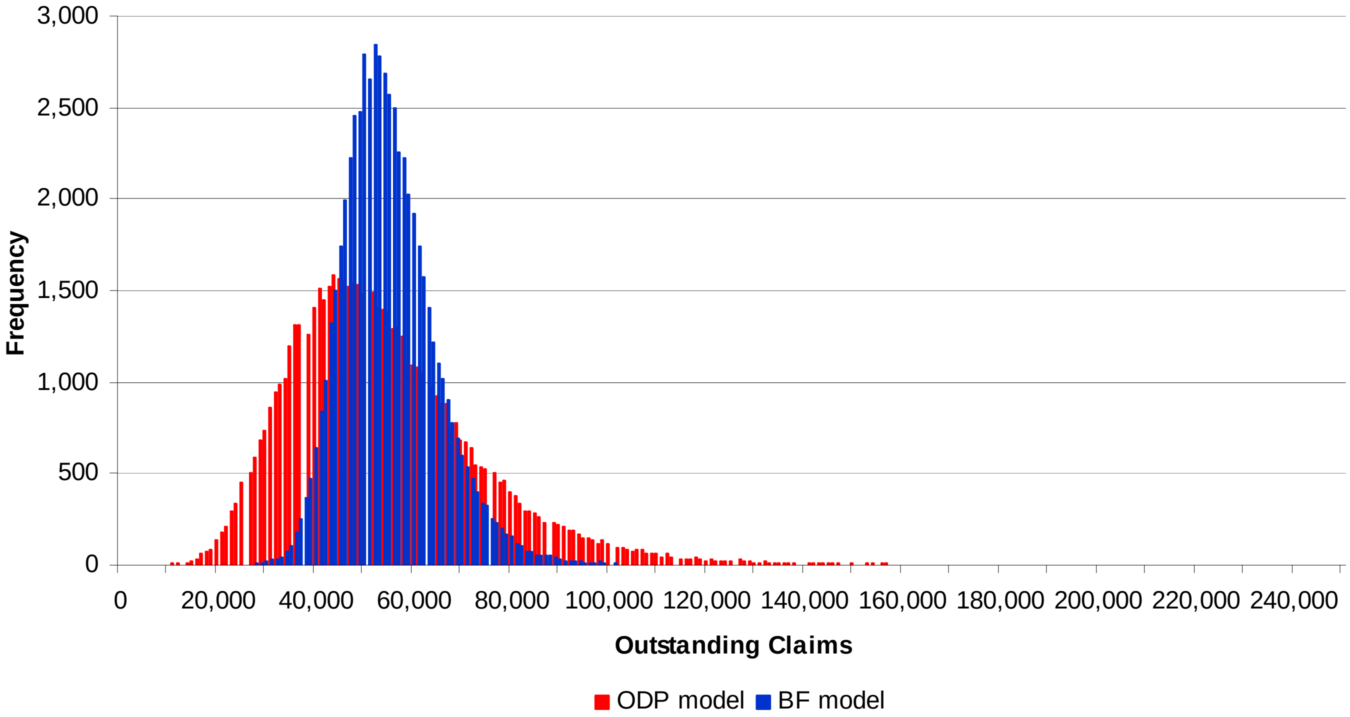
  – Credibility mechanism: relativity of $\sigma_{(1)i}$ to $\sigma_{(2)i}$

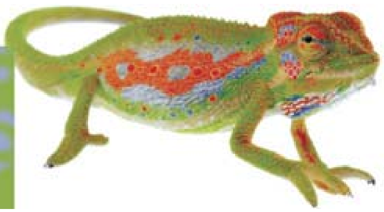


Case study - results (1)

Accident Year	ODP model Mean	ODP model Standard Deviation	ODP model Coefficient of Variation	ODP model 75th Percentile	BF model Mean	BF model Standard Deviation	BF model Coefficient of Variation	BF model 75th Percentile	Mean Ultimate Loss Ratio Initial	Mean Ultimate Loss Ratio ODP model	Mean Ultimate Loss Ratio BF model
1	0	NA	NA	NA	0	NA	NA	NA	71%	65%	65%
2	164	619	378%	0	144	356	247%	107	71%	82%	82%
3	641	1,201	187%	1,087	585	713	122%	799	71%	83%	83%
4	1,688	1,892	112%	2,174	1,609	1,104	69%	2,124	71%	75%	75%
5	2,815	2,343	83%	4,347	2,984	1,375	46%	3,711	71%	61%	62%
6	3,707	2,553	69%	5,434	3,447	962	28%	4,009	71%	80%	79%
7	5,521	3,233	59%	7,607	5,258	1,110	21%	5,916	71%	77%	76%
8	11,070	5,266	48%	14,130	10,420	1,891	18%	11,540	71%	79%	77%
9	10,800	6,293	58%	14,130	12,370	2,577	21%	13,780	71%	55%	60%
10	17,200	14,320	83%	23,910	17,720	6,413	36%	20,850	71%	66%	67%
Total	53,606	19,660	37%	64,120	54,538	9,626	18%	59,980	71%	71%	72%

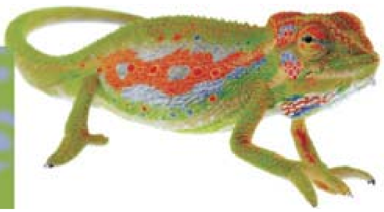
Case study - results (2)





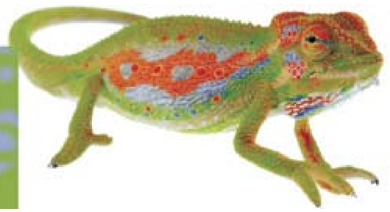
Case study – some model extensions

- Informative prior distributions for parameters
- Changes to standard deviations $\sigma_{(1)i}$ and $\sigma_{(2)i}$
- Prior distribution for the over-dispersion parameter φ



Conclusions

- Bayesian stochastic reserving is worth considering
- WinBUGS and other programs make implementation fairly straightforward
- Important benefits:
  – Sound statistical framework
  – Incorporation of actuarial judgement



Questions ???