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The frequency of valuation for long tail classes

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Abstract

The purpose of this paper is to explore the consequences of frequent valuation of long tailed classes, using various strategies for assumption setting.

A simple simulation of a long tailed portfolio was used to compare the consequences of applying different valuation frequencies and assumption setting strategies, including

- Annual;
- Quarterly with no assumption changes;
- Quarterly, changing assumptions only if there is significant change in experience;
- Quarterly with moving average assumptions;
- Quarterly with an adaptive reserving approach.

It was found that the nature of the portfolio, its size, the valuation models employed, and the specific superimposed inflation environment affecting the portfolio, will all have a large influence on the prediction errors of any quarterly updating approach. The framework and analysis presented in this paper is one way that objective decisions can be made about the appropriate valuation frequency for a particular portfolio.

Keywords: Reserving, frequency, long tailed classes, annual, quarterly.

1 Introduction

Long tail classes have features that make them difficult to value. In particular the low frequency, high severity nature of the claims in long tail classes gives rise to highly variable claims data. Further, the fact that claims in these classes show, on average, long delays to settlement also adds to the difficulty.

A challenge for an actuary valuing a long tail portfolio is to detect any systemic changes affecting the portfolio amongst the high claims volatility. Long tail classes are subject to a number of environmental influences that impact on the value of liabilities, such as business cycles, changes in claimant behaviour and judicial decisions. These environmental influences often result in superimposed inflation in the portfolio.

The difficulty of detecting and modelling any underlying systemic changes in the presence of the claims volatility raises the question of how frequently one should value long-tail classes. In other words, if claims volatility is high, how much additional data is needed before meaningful changes can be made to a valuation model?

This is the question that is explored in this paper.

1.1 Frequency of Valuations

Long tailed classes are typically subject to an annual valuation which involves updating the valuation models and their assumptions. Such a valuation is considered a "full valuation" and involves significant actuarial input.

However estimates of liabilities are often required on a more frequent basis. For example, APRA requires that insurers provide updates of their insurance liabilities on a quarterly basis.

Additionally, reporting time pressures often mean that the full valuation is carried out at a date prior to the Company's reporting date. In these cases, the full valuation will be "rolled-forward" to the reporting date providing that the intervening claims experience does not indicate that this is an unreasonable thing to do.

The question of how frequently one should value long tail classes raises impacts on two important practical issues:

- If one is preparing interim valuations between the annual cycles of "full valuations" how much value is there in performing anything more than a roll-forward of the most recent annual valuation? If one tries to take account of the intervening claims experience are you more likely to be responding to claims volatility rather than picking up systemic changes?

- If an actuary prepares the annual valuation prior to the Company's reporting date, what is the magnitude of the potential prediction errors that can arise?

1.2 Aim and Approach

The aim of this paper is to try and answer the questions raised above.

The approach taken was one of simulation. A simple simulation of a long tailed portfolio was performed with the aim of creating a large number of datasets each containing a "completed" triangle of claims data. Each simulated dataset contained both claims variability and some systematic change.

The claims simulation models were based on data from an Australian Motor Bodily Injury portfolio. The models were parameterised to reflect the claim payment patterns and claims volatilities observed in this portfolio.

The systematic changes were simulated by using models of superimposed inflation. Three different models of superimposed inflation were used each one representing a different scenario about how superimposed inflation may evolve over the course of the year. The scenarios were:

- A **stable environment** – where there was about a two in three chance that superimposed inflation will not change by more than 3% in a year;
- A **variable environment** – where superimposed inflation is modelled so there is about a two in three chance that superimposed inflation will change by more than 6% in a year; and
- A **trend environment** – in which a deterministic 5% p.a. trend in superimposed inflation was added to the variable environment.

Having simulated a large number of datasets under the three different superimposed inflation scenarios the prediction error of different quarterly valuation strategies was measured. The prediction error was taken to be the difference between the true value of the liabilities and the value of the liabilities as predicted by the quarterly valuation model.

The quarterly valuation strategies that were tested were:

- **Basic roll-forward –** in this method, no modifications were made to the future projected cashflows from the previous annual valuation. There were no changes made to the ultimate incurred losses and the outstanding claims were simply the sum of the expected forecast cashflows from the previous annual valuation less the expected claim payments between the previous annual valuation and the current quarterly balance date.
- **Full Roll-forward** – where the projections made at the previous annual valuation were adjusted for any differences between actual and expected for the number of claim reports and claims closures. This means applying the same model parameters as estimated at the last annual valuation to the latest available data.
- **Moving average** – remodelling was carried out each quarter using a moving average of historical experience. Smoothing was carried out on the estimated tail parameters.

- **AvE Threshold** – This involved performing a full roll-forward, but with remodelling and assumption changes if there was a significant change in experience. A significant change in experience involved the deviation of actual v expected claim amounts above some predefined threshold. Any required remodelling was carried out using the moving average method.

- **Adaptive Filtering** – This involved modelling the data with a regression function and updating the parameters using the Kalman Filter (Kalman, 1960; De Jong and Zehnwirth, 1983). This approach is a form of dynamic stochastic modelling. Others have used these types of methods for automatic reserving with considerable success (McGuire, 2007).

The purpose of this exercise was to compare the prediction errors of the roll-forward methods with the remodelling methods at each quarter over the course of a year. Any reduction in prediction error is a measure of the value of the remodelling process. By examining how the prediction error changes over the course of the year, an indication of the value of remodelling at different time intervals is given.

2 Methods

In the following section details are provided about:

- The claim models used to simulate the datasets;
- The superimposed inflation models used to generate different superimposed inflation scenarios;
- Simulation of datasets;
- The different valuation strategies used; and
- How prediction error was measured when the valuation strategies were applied at quarterly intervals over the course of a year.

2.1 Claim simulation models

The claims simulation models were based on data from an Australian Motor Bodily Injury portfolio covering the years 1978 to 2007.

Two claim simulation models were developed, one based on a Payments Per Claims Incurred (PPCI) model and the other on a Payments Per Claim Finalised (PPCF) Model.

2.1.1 PPCI Model

This model has been adapted from the one presented in Taylor (2000). The PPCI in a particular accident quarter, i, and development quarter, j are defined as:

$$PPCI_{ij} = C_{ij}/U_i \qquad \text{2-1}$$

where C_{ij} are the gross claim payments (inflation adjusted) in accident quarter i and development quarter j and U_i is the estimated ultimate number of claims incurred in accident quarter i.

It was assumed that the PPCI were log-normally distributed and that the pattern of PPCI followed a Hoerl curve (De Jong and Zehnwirth, 1983; Wright, 1990). The model had the following form:

$$\log PPCI_{ij} = \beta_0 + \beta_1 \log(j+1) + \beta_2 j + \gamma(i,p) + \varepsilon(j), \qquad j = 0,1,\ldots \qquad \text{2-2}$$

where β_0, β_1 and β_2 are parameters assumed to be fixed over time, $\gamma(i, p)$ is a term to recognise superimposed inflation in accident quarter i and experience quarter $p(= i + j)$ and $\varepsilon(j)$ is a normally distributed error term which is stochastically independent.

A number of different models were used to describe the evolution of $\gamma(i, p)$ and these are discussed in Section 2.2.

Figure 1 shows a plot of E[$PPCI_{ij}$] as a function of development quarter assuming that the superimposed inflation term equals zero. Note that the calculation of E[$PPCI_{ij}$] requires the following bias correction:

$$E[PPCI_{ij}] = \exp[\beta_0 + \beta_1 \log(j+1) + \beta_2 j + \gamma(i,p) + 1/2Var[\varepsilon(j)]] \quad 2\text{-}3$$

The variance of the PPCI data, Var[$\varepsilon(j)$] was described by a function which varied by development quarter. The relative variance (Var[$\varepsilon(j)$] / Var[$\varepsilon(j=0)$]) at each j is shown in Figure 2.

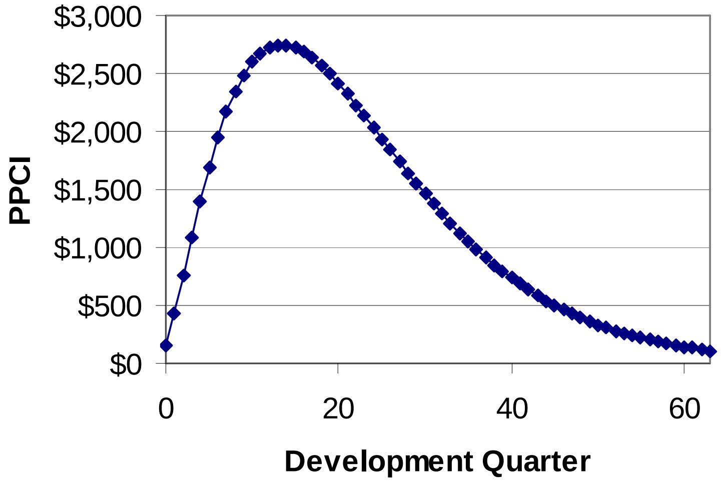

Figure 1 – Plot of E[$PPCI_{ij}$] for the PPCI model with parameters β_0 = 4.998, β_1 = 1.660, β_2 = −0.115 and $\gamma(i, p)$ = 0.

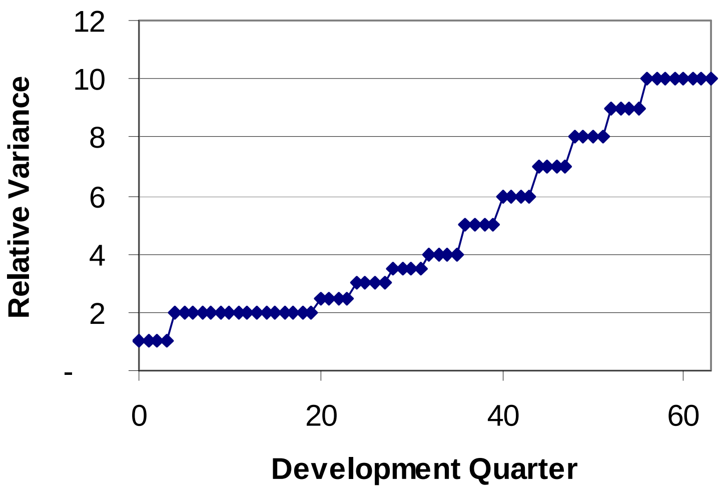

Figure 2 – Plot of the relative variance of log(PPCI)

Var[$\varepsilon(j=0)$] was assumed to depend on the size of the portfolio that was modelled. For the **Large Portfolio PPCI Model** Var[$\varepsilon(j=0)$] = 0.068736. This was based on measurements from a portfolio which had an ultimate number of claims incurred per annum of 1000. This assumption, combined with the relative variance function shown in Figure 2 gave coefficients of variation for PPCI values which varied from 27% in development period 0 up to 100% in development period 60 and later. The coefficient of variation of $PPCI_{ij}$ is given by the following relationship:

$$CV[PPCI_{ij}] = \sqrt{e^{Var[\varepsilon(j)]} - 1} \qquad 2\text{-}4$$

For our **Small Portfolio PPCI Model**, Var[ε($j=0$)] was assumed to be 4 times larger and thus relates to a portfolio which is about one quarter of the size of the Large portfolio.

A model of ultimate claim numbers was not developed for the PPCI model and hence forecast payments were made assuming a fixed and equal number of claims for each accident quarter. This allowed the focus to remain on the ability to update the PPCI assumptions, rather than the claim numbers assumption.

2.1.2 PPCF Model

This model is based on a number of GLM models that were fitted to individual claim data from a motor bodily injury portfolio. The models were developed as part of a reserving exercise for one of Taylor Fry's clients.

The PPCF model actually consists of three sub-models:

- Average payments per claim finalised;
- The probability a claim finalises in a particular quarter; and
- A model of claim notifications.

2.1.2.1 Average payments per claim finalised model

The PPCF in a particular *i* and *j* are defined as:

$$PPCF_{ij} = C_{ij} / F_{ij} \qquad 2\text{-}5$$

where C_{ij} are the gross claim payments (inflation adjusted) in accident quarter *i* and development quarter *j* and F_{ij} number of claim closures in development period *j* for accident period *i*.

In a similar manner to the PPCI model, the PPCF were assumed to be log-normally distributed. The model had the following form:

$$\log PPCF_{it} = \beta_0 + \beta_1 t + \beta_2 \max(t - 40, 0) + \beta_3 \max(t - 95, 0) + \gamma(i, p) + \varepsilon(t) \qquad 2\text{-}6$$

where *t* is the operational time at finalisation which is defined as the proportion of all claims incurred for the relevant accident quarter that have been closed at the development quarter of finalisation.

Figure 3 shows a plot of E[$PPCF_{it}$] as a function of operational time assuming that the superimposed inflation term equals zero. Again a bias correction of the same form as that used for the PPCI model is needed to calculate E[$PPCF_{it}$].

The coefficient of variation of the PPCF data was described by the following function:

$$CV[PPCF_{it}] = \frac{\sqrt{\frac{0.4}{F_{it}} E[PPCF_{it}]^{2.25}}}{E[PPCF_{it}]} \qquad \text{2-7}$$

Var[$\varepsilon(t)$] was derived from 2-7 using an equation with the same form as 2-4.

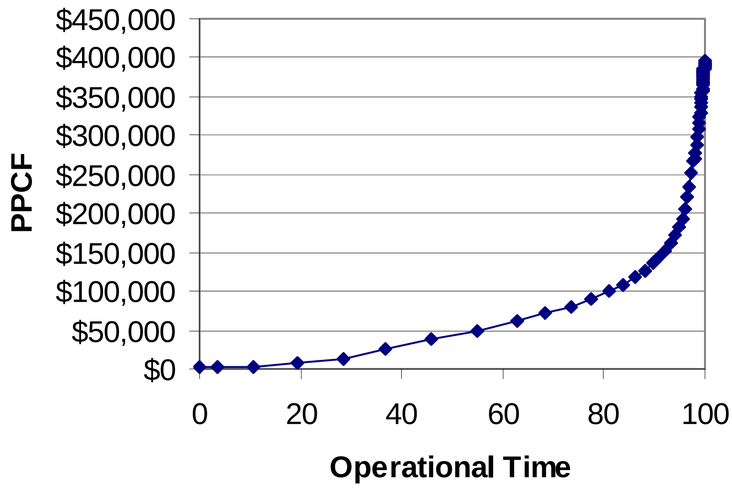

Figure 3 – Plot of E[$PPCF_{it}$] with parameters β_0 = 7.3942, β_1 = 0.07559, β_2 = –0.05042, β_3 = 0.17272 and $\gamma(i, p)$ = 0.

The following figure illustrates how the co-efficient of variation of PPCF vary by operational time when F_{it} = 1.

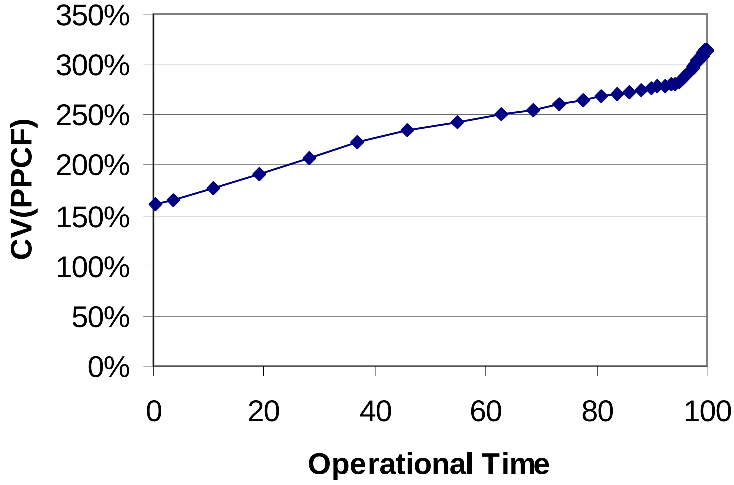

Figure 4 – Plot of the coefficient of variation of for a single finalised claim at different operational times

2.1.2.2 Probability of claim finalisation model

Finalisation rates (PRF_{ij}) were defined as follows:

$$PRF_{ij} = F_{ij} \Big/ \left(O_{ij} + R_{ij} / 3 \right) \qquad \text{2-8}$$

where F_{ij} is the number of claim closures, O_{ij} is the number of claims open, and R_j is the number of claims reported. The term involving the factor of 1/3 allows for the fractional exposure of newly reported claims to finalisation.

Finalisations were simulated from a binomial distribution where the number of claims available to be finalised (the exposure) is given by the denominator in 2-8 and

$$\begin{aligned} \operatorname{logit} E[PRF_{ij}] = \beta_0 & \\ & +\beta_1 I(j=0) + \beta_2 I(j=1) + \beta_3 I(j=2) \\ & +\beta_4 m(j;1,9) + \beta_5 m(j;1,9)^2 + \beta_6 m(j;1,9)^3 \\ & +\beta_7 m(j-8;0,8) + \beta_8 m(j-16;0,14) + \beta_9 m(j-30;0,20) \end{aligned} \qquad \text{2-9}$$

where $m(x;u,v) \quad = \quad \min(v, \max(u,x))$.

A plot of the expected value of the finalisation rates as a function of development quarter is shown in the following figure.

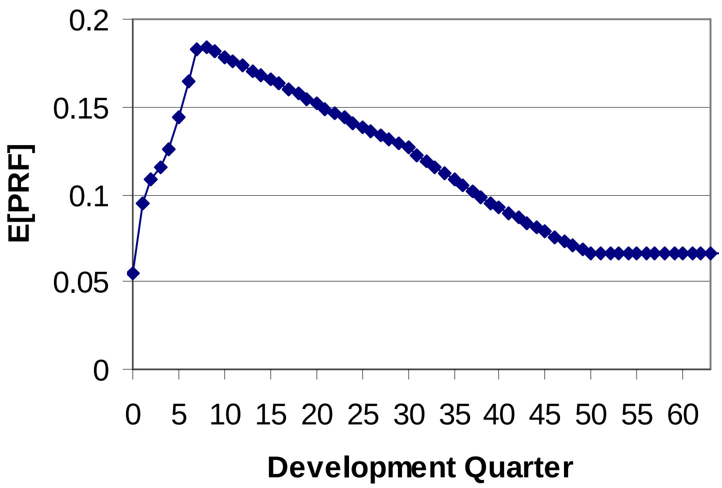

Figure 5 – Finalisation rates with β_0 = −1.618, β_1 = −0.9339, β_2 = −0.3346, β_3 = −0.0601, β_4 = −0.3994, β_5 = 0.1072, β_6 = −0.0068, β_7 = −0.0188, β_8 = −0.0212, β_9 = −0.0354.

2.1.2.3 *Claim notification model*

Claim notifications were modelled as a Gamma-Poisson mixture distribution with:

$$R_{ij} \sim \text{Poisson}\left[E_i \mu_{ij}\right]$$

$$\mu_{ij} \sim \text{Gamma}\left[\frac{\mu_{ij}}{\phi - 1}, \phi - 1\right],$$

where E_i = number of vehicles registered in accident quarter i;

$\phi = 1.3$, and

$$\log \mu_{ij} = \beta_0 I(j=0) + \beta_1 I\left(j>0\right) + \beta_2 \log(j+1) \qquad \text{2-10}$$

A plot of E[R_{ij}] as a function of development quarter is shown below for a portfolio with 2 million motor vehicles registered

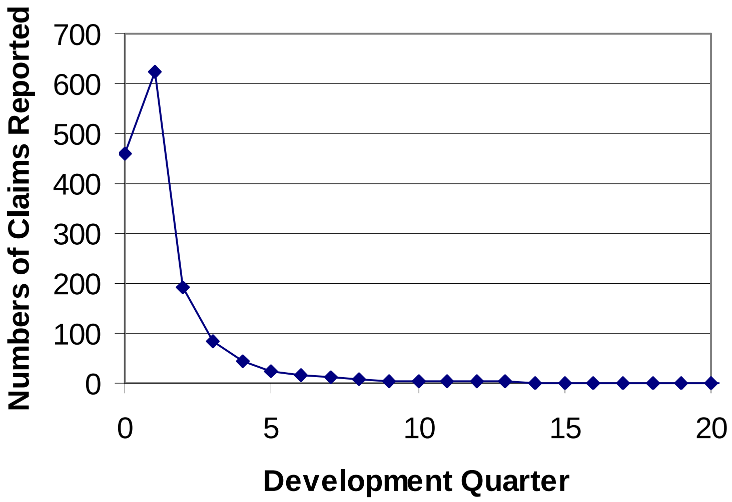

Figure 6 – A plot of E[R_{ij}] with $\beta_0 = -8.3739$, $\beta_1 = -6.0537$, $\beta_2 = -2.9110$ and E_i = 2 million.

Simulations which are referred to as the **Large Portfolio PPCF Model** are from a portfolio with 2 million vehicles. Those referred to as the **Small Portfolio PPCF Model** are from a portfolio with 0.5 million vehicles.

2.2 Models of superimposed inflation

The equations used to simulate PPCI and PPCF included a term, $\gamma(i, p)$, for superimposed inflation in accident quarter i and experience quarter $p(= i + j)$.

This term was assumed to follow a random walk with drift of the form:

$$\gamma_{k+1} = \gamma_k + \mu + \sigma Z \qquad \text{2-11}$$

where $k = i$ or p, μ = drift, σ is the standard deviation of the random walk, and Z is a standard normal random variable. When $k = p$ I am modelling superimposed inflation that varies by payment quarter (**payment quarter SI**) and when k = i I am modelling superimposed inflation that varies by accident quarter (**accident quarter SI**).

The purpose of this model is to simulate how superimposed inflation may change over the course of a year. Without loss of generality, I have chosen a base of zero so that $\gamma(i, p)$ was set equal to 0 for all $p \leq p`$ where $p`$ is the last quarter prior to the start of the year. During the course of the year, the superimposed inflation term is assumed to evolve in accordance with 2-11.

Three different sets of parameters are used in 2-11 to represent different scenarios. The scenarios were:

- A **stable environment** – μ = 0, σ = 0.016. In this environment there was about a two in three chance that superimposed inflation will not change by more than 3% in a year. Although this environment is labelled "stable" it does not mean that superimposed inflation is static – just that it has lower variability compared to the other scenarios.

- A **variable environment** – μ = 0, σ = 0.032. There is about a two in three chance that superimposed inflation will change by more than 6% in a year; and

- A **trend environment** – μ = 0.0125, σ = 0.032. Here a 5% p.a. trend has been added to the variable environment.

An alternative superimposed inflation model for the stable environment was used in some simulations. This model called the random **jump model** assumed that at each quarter there was a 25% chance of a +2.5% change in $\gamma(i, p)$, a 25% chance of a -2.5% change and a 50% chance of no change. In this model, there is about a 75% chance that superimposed inflation will not have changed by more than 2.5% over the course of the year.

2.3 Simulation of datasets

By combining the claim simulation models (Section 2.1) and superimposed inflation models (Section 2.2) a large number of datasets were simulated. In these datasets, data prior to $p`$, which represents the last quarter prior to the start of the year of interest, were simulated without any claims volatility (process error) and assuming that superimposed inflation was zero. Data subsequent to $p`$ did include process error. While superimposed inflation was assumed to evolve according to the models described in Section 2.2 for a period of 1 year i.e. from $p`$ to $p`+ 4$.

The decision to exclude process error and variation in superimposed inflation from the data prior to *p*` was so that all valuation modelling methods (discussed below) yielded the same estimated value for the outstanding liabilities when applied to the data up to *p*`. In other words, the data have been simulated so that no matter what valuation method is used, we are able to get a true picture of the stable and unchanging claim generating process that has been in existence up until *p*`. Thus there is a single outstanding claims estimate at *p*`. This estimate is often referred to as the previous annual valuation result. The purpose of this was to shift the debate away from the accuracy of method used for the base valuation.

Then after *p*`, process error and systematic changes (superimposed inflation) are introduced. The aim then, is to see how well the different valuation strategies are able to pick up the systematic changes amongst the claims volatility as they evolve over the course of the year.

I have not simulated any further change in superimposed inflation beyond experience quarter *p*`+ 4 as the last valuation date of interest is *p*`+ 4 and obviously no valuation technique is able to pick up systemic changes that occur after the valuation. This means that the numerical results compare the relative reliability of different approaches, not the absolute reliability or error.

2.4 Valuation methods

There are 5 basic valuation strategies that are applied to the dataset at quarterly intervals from *p*`+1 to *p*`+ 4. In each case the valuation method was applied to the size components of the PPCI and PPCF models (that is $PPCI_{ij}$ and $PPCF_{ij}$) only. That is, claim number and finalisation models were not updated – for each quarterly valuation, model assumptions for these models were retained from the previous annual valuation. Obviously systemic changes could have an impact on numbers and finalisation rates. However in the interest of keeping things as simple as necessary, I have focussed only on superimposed inflation in the size components.

2.4.1 Basic roll-forward

In this method, no modifications are made to the future cashflows or ultimate incurred losses from the previous annual valuation result.

2.4.2 Full Roll-forward

Here no modifications are made to the model parameters that were estimated at the previous annual valuation, however new cashflow forecasts are made which take account of the actual number of claim reports and open claims at the valuation date.

2.4.3 Moving average

In this method E[$PPCI_j$] or E[$PPCF_j$] values at each *j* are estimated by averaging actual $PPCI_j$ or $PPCF_j$ values from the historical data. Two different averaging periods were used; the past 16 experience quarters and the past 8 experience quarters.

For both models a seven development period moving average technique was used to smooth estimates of E[$PPCI_j$] and E[$PPCF_j$] in the tail.

2.4.4 AvE Threshold

This involved performing a full roll-forward, but with remodelling and assumption changes in each individual cell only if the magnitude of (actual experience/ expected experience – 1) exceeded some threshold in the cell. Two different thresholds were used: a **high threshold** of 50% and a **low threshold** of 30%. A moving average smoothing technique was applied to the claims experience in the tail before the method was applied.

Any required remodelling was carried out using the 16 quarter moving average method.

An alternative method would have been to vary the threshold by development quarter so that a larger threshold was applied to the more volatile tail. This would be a more appropriate method in practice. But for current purposes, the use of the simpler method is not expected to change the qualitative conclusions of this study.

2.4.5 Adaptive Filtering

The moving average techniques applied above pool measurements from a number of experience quarters in an attempt to "see through" the claims volatility to detect the underlying systemic movements. A moving average involving many experience quarters will be less sensitive to the claims volatility but will be slower at picking up any systemic changes. Conversely, an average involving less experience quarters will be more sensitive to claims volatility but will be quicker at picking up any systemic changes that occur.

When using moving average valuation techniques the actuary is required to use judgement and knowledge of the portfolio at hand to decide how many periods of data should be used in the average.

An alternative to moving average techniques are adaptive filtering techniques. In essence, these techniques also produce weighted averages of parameter estimates from the current and past data quarters. However the weight that is assigned to current and historical data quarters depends on both the expected claims volatility and expectations about how the parameter estimates may move. Adaptive filters are formal statistical methods of trying the balance the need to identify systemic changes with the need to pool the results from many time periods.

The adaptive filter that has been used in this paper is known as the Kalman Filter (Kalman, 1960; De Jong and Zehnwirth, 1983). Details of the method can be found in Taylor (2000) and McGuire (2007).

To apply the Kalman Filter, PPCI and PPCF have been modelled using the regression functions 2-2 and 2-6. The Kalman Filter was then used to update the regression parameters at each quarterly valuation.

Two different versions of the Adaptive Filter were used: a **low variance filter** which was specifically calibrated for the **stable environment** and a **high variance filter** which was specifically calibrated for the **variable environment**. The low variance filter was applied in the case of the Stable environment and the low variance filter was applied in the case of the Variable and Trend environments.

2.5 Prediction Error

2.5.1 Mean Squared Error of Prediction

Having applied various valuation strategies at quarterly intervals over the course of a year, the prediction errors of each of the strategies was then compared at each quarter. Prediction error was measured using the following procedure:

- For each model and superimposed inflation scenario simulate 100 datasets as described in Section 2.3.
- At each quarter p = p`+ 1, $p$`+ 2, p`+ 3, and $p$`+ 4 apply one of the valuation methods of Section 2.4 using only the data available at the time of valuation.
- At each quarter estimate the outstanding claims liability that is predicted by each valuation model in relation only to payments that occur after p`+ 4 and in relation to accident quarters $i \leq p$`. Each estimate is denoted $E_{p,m,s}$ where the subscripts m differentiates the estimate by model, and the subscript s differentiates the estimate by simulated dataset.
- For each dataset determine the true outstanding claims liability in relation to payments that occur after p`+ 4 and in relation to accident quarters $i \leq p$`. Denote these L_s.
- The prediction error for each simulated dataset is then calculated as L_s - $E_{p,m,s}$. The set of prediction errors created from all simulated datasets gives an estimate of the distribution of the prediction error.
- One summary statistic that is used to quantify the prediction error is the Root Mean Squared Error of Prediction (RMSEP) which is defined as:

$$RMSEP_{p,m} = \sqrt{\frac{\sum_{s=1}^{100}(L_s - E_{p,m,s})^2}{100}} \qquad \text{2-12}$$

  In this paper, the RMSEP is usually expressed as a percentage of mean value of L_s.

There are a couple of points to make about this process. First, the outstanding liability is always measured in relation to payments made after p`+ 4. The intention of this choice is to illustrate how the ability to estimate this quantity changes as more data becomes available over the year. To do this it is preferable to always measure prediction error in relation to the same quantity.

Second, in Section 2.2 I mentioned two different types of superimposed inflation: payment quarter SI and accident quarter SI. When we are considering payment quarter SI, the RMSEP is measured in relation to all past accident years. However when we are considering accident quarter SI, the RMSEP is measured in relation to the accident quarters that are affected. In all simulations the accident quarters that were affected are: i = p`- 3, $p$`- 2, p`- 1 and $p$`. In other words, when accident year SI emerges it is assumed to do so 3 quarters prior to the last full valuation (p`).

And finally, the estimate of prediction error focuses only on the ability to make central estimates of undiscounted claim costs that have been adjusted for normal economic inflation. The impact of discounting, future economic inflation, changes in superimposed inflation occurring after p `+ 4, risk margins and expenses have been ignored.

2.5.2 Annual valuation update error

Another quantity of interest is the annual valuation **update error.** If an actuary was to adopt a roll-forward valuation strategy for quarterly valuations, but with a full annual valuation at quarter 4, there is the possibility of a large movement in liabilities when moving from the roll-forward valuation at quarter 3 to the annual valuation at quarter 4 if systemic changes have emerged during the year.

We have defined the update error for a particular quarterly valuation method, m, to be:

$$E_{p`+4,m=a,s} - E_{p`+3,m,s} \qquad \text{2-13}$$

where the first term denotes the liability estimate made at quarter 4 using the annual valuation methodology.

Determining an appropriate value for $E_{p`+4,m=a,s}$ is problematic. Most annual valuations are a mixture of judgement and modelling and it is not possible to replicate this over the each of the 1000s of datasets created for this study. Additionally, it is not suitable to use the actual liability L_s as there is some prediction error for all valuation methods.

We have used $E_{p`+4,m,s}$ as predicted by the Adaptive Filter as a proxy for $E_{p`+4,m=a,s}$. The reason for this was that the adaptive filter consistently outperformed all the other modelling methods used in this study.

I note however that if one does use L_s in place of $E_{p`+4,m=a,s}$ there is no change to the conclusions that have been drawn by this study.

3 Results

The results of this study are presented in the following three sub-sections:

- Prediction errors throughout the year
- Annual valuation update errors from different quarterly valuation strategies
- Prediction error resulting from early valuation.

The first of these subsections compares the prediction errors of the roll-forward methods with the remodelling methods at each quarter over the course of a year under a number of different superimposed inflation scenarios.

The second of the subsections then looks at the likely distribution of liability adjustments (update errors) when moving from a 3rd quarter roll-forward valuation to a 4th quarter full valuation.

While the final section looks at the distribution of prediction errors that can occur under different superimposed inflation scenarios when the annual valuation is carried out one quarter prior to the balance date.

3.1 Prediction errors throughout the year

3.1.1 Stable Superimposed Inflation scenario

Two different models were used to represent the stable superimposed inflation scenario. One was a random walk model and the other a random jump model (Section 2.2). No qualitative differences were found between the two models and I have only presented the results of the random walk model here.

3.1.1.1 Payment quarter SI

The following two figures show the prediction errors throughout the year for each of the quarterly valuation methods in an environment with stable payment quarter SI.

For the large portfolio models, the difference in RMSEP between the roll-forward methods and the remodelling methods is, in general, not appreciably different at all quarterly valuation dates. This is true for both the PPCI and PPCF models. In all figures, the prediction error is the same at each quarter for the basic roll-forward because with this method, the projected liability remains unchanged at each valuation quarter.

The exception is when a modelling method overly sensitive to claims volatility is used. For example the 8 period moving average shows increasing prediction error as the time since the last full valuation (valuation quarter = 0) increases.

Note that in the figures, all methods give the same prediction error at valuation quarter 0 (the last full valuation date). The reasons for this were discussed in Section 2.3. Additionally, only a basic roll-forward is shown for the PPCI model. This is because for this model no attempt was made to forecast claim report volatility. Such volatility is expected to have very little influence on the results of the PPCI analysis.

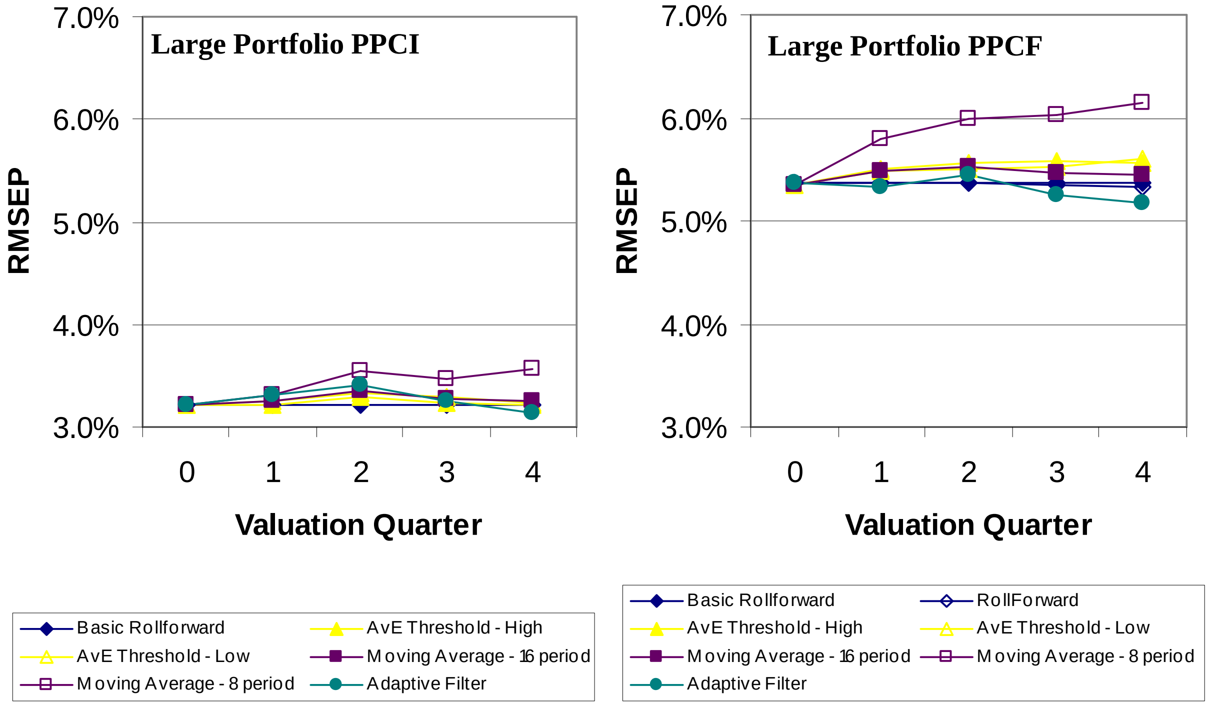

Figure 7 – Large portfolio: RMSEP for different valuation methods in a stable environment (payment quarter SI)

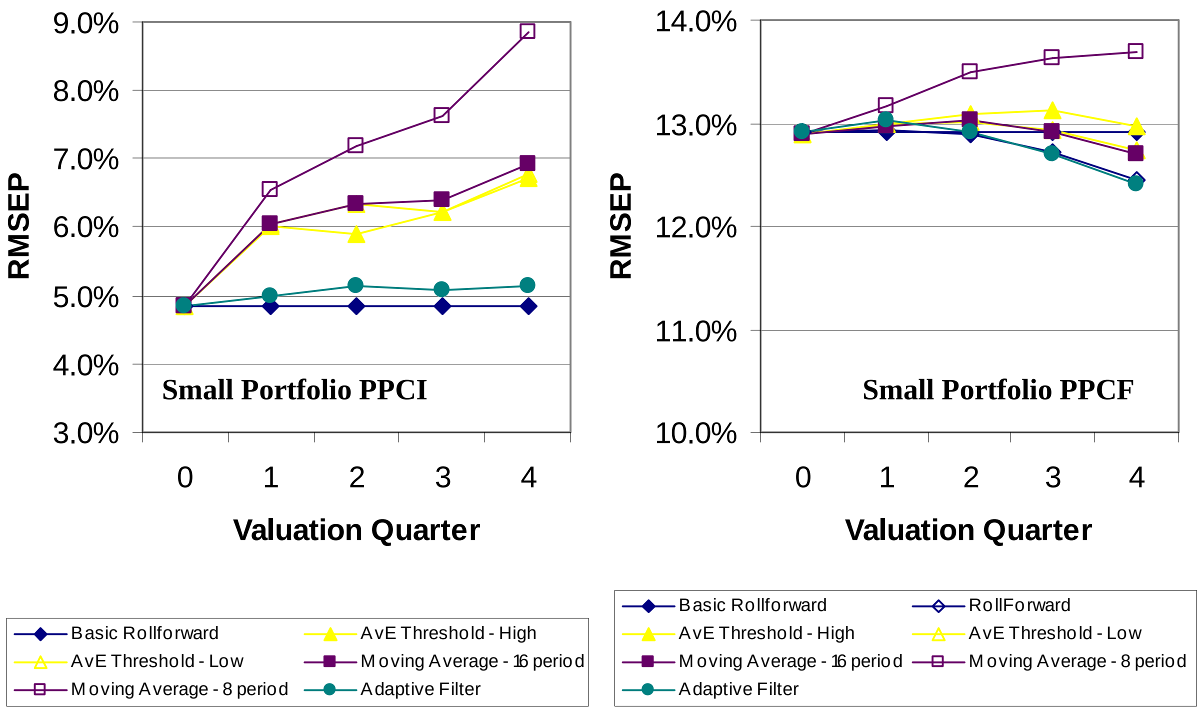

Figure 8 – Small portfolio: RMSEP for different valuation methods in stable environment (payment quarter SI)

However, for the smaller portfolio models, the act of remodelling often led to greater prediction errors than would be obtained by a roll-forward. This was particularly apparent for the PPCI model. The reason for this is that the volatility of the claims data increases as the portfolio gets smaller and it appears that most of the quarterly revaluation methods are responding inappropriately to the increased noise.

Of particular interest for the small portfolio PPCF model is that the results of the basic roll-forward are clearly inferior to the results of the “full” roll-forward. This is particularly visible at valuation quarters 3 and 4. In the figure it is difficult to see the results of both roll-forwards. The basic roll-forward results are a horizontal line just below the 13% line, while the full roll-forward results lie underneath the low variance adaptive filter.

Out of the remodelling techniques, the adaptive filtering approach was seen to have the lowest prediction error.

3.1.1.2 Accident quarter SI

The results for accident quarter SI tend to be qualitatively similar to those for payment quarter SI (Figure 9 and Figure 10). Of interest in the small PPCF model is that again the full roll-forward clearly outperforms the basic roll-forward. Again the results of the full roll-forward are hidden behind the adaptive filter results. The outperformance is slightly more pronounced in the accident quarter SI results as these results relate only to the latest accident year in the previous annual valuation while those for payment quarter SI relate to all past accident years. Changes in the number of open claims have a greater effect on accident years which are less developed.

3.1.1.3 Summary

Overall, in a stable environment an appropriate roll-forward strategy gave MSEP that were not appreciably worse than any remodelling method. In the case of a small portfolio PPCF model, an appropriate roll-forward strategy meant a full roll-forward making allowance for the actual number of open claims.

In cases where the remodelling method was too responsive to claim data volatility, the remodelling method gave worse prediction errors than the roll-forward strategy. This was most apparent in the small portfolio PPCI model where all remodelling strategies apart from the Adaptive Filter gave worse RMSEP over the course of the year.

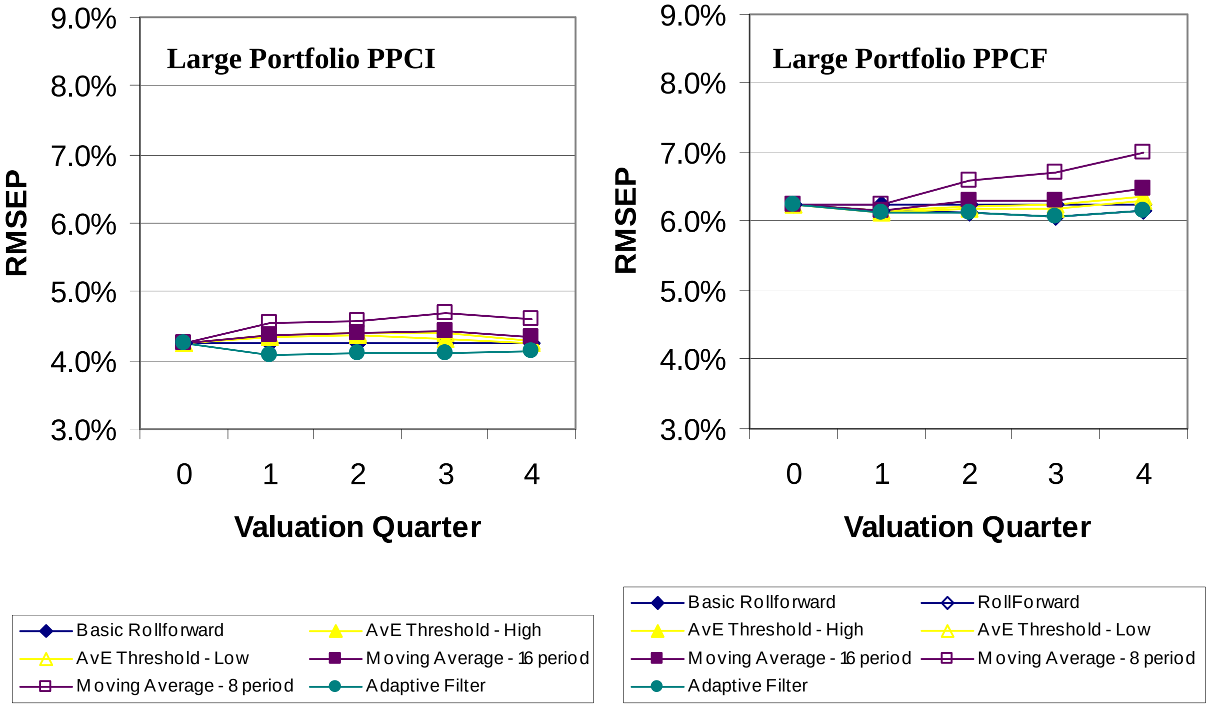

Figure 9 – Large portfolio: RMSEP for different valuation methods in stable environment (accident quarter SI)

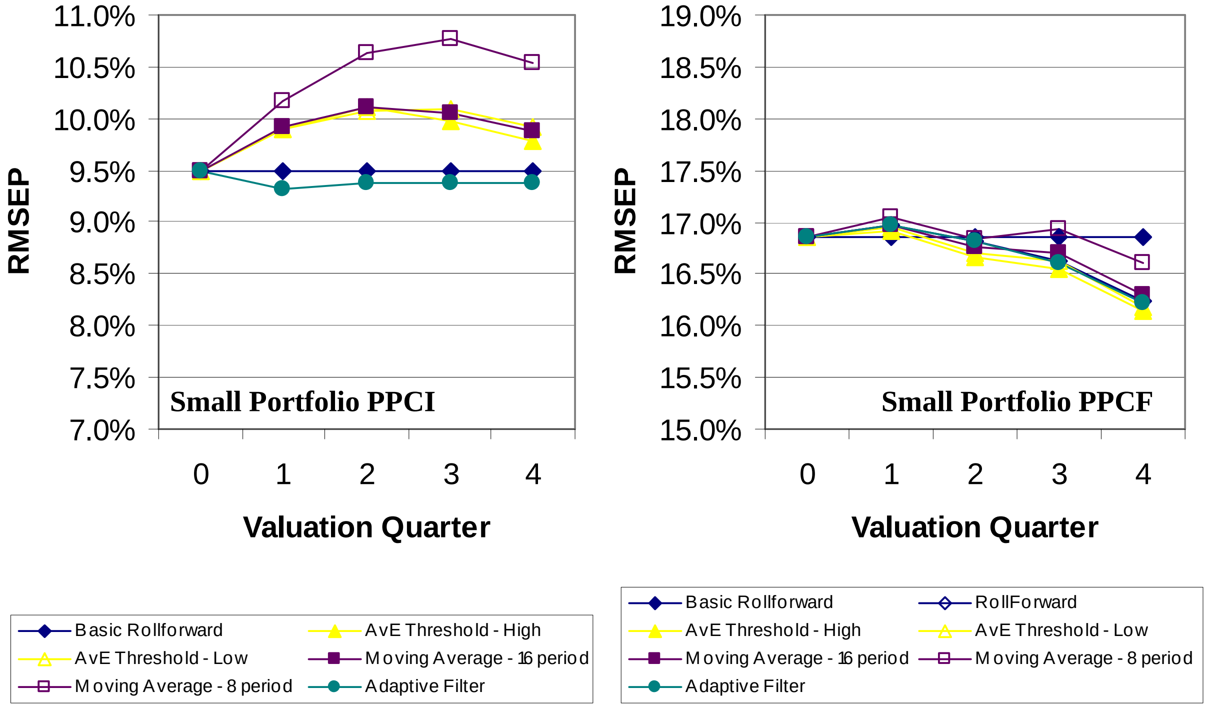

Figure 10 – Small portfolio: RMSEP for different valuation methods in stable environment (accident quarter SI)

3.1.2 Variable and Trend Superimposed inflation scenarios

For the large portfolio models, as the magnitude of the underlying systemic changes increases, the performance of the roll-forward strategy starts to decline relative to the remodelling strategies. This is illustrated in Figure 11 on the following page for the Large PPCI model and in

Figure 13 for the Large PPCF model. For both models, under both the variable and trend SI scenarios, the MSEP of the remodelling techniques tend to decreases as the time since the last annual valuation increases. This is particular noticeable in quarters 3 and 4. In comparison the prediction error of the roll-forward technique remains constant.

Note however that this statement is not true for the remodelling strategies which rely on the moving average when faced with accident quarter SI. Because this SI is emerging only in the most recent accident year (as at the previous valuation), and because the moving average techniques fit a separate parameter for each development quarter, this emerging SI effects PPCI parameters in the moving average model up to development quarter 7 only. In contrast, the regression function used in the adaptive filter responds to the emerging accident year SI by increasing PPCI estimates at all future development quarters.

However for the small portfolio models, the roll-forward method is equal or superior to all remodelling strategies apart from the Adaptive Filter (Figure 12 and

Figure 13).

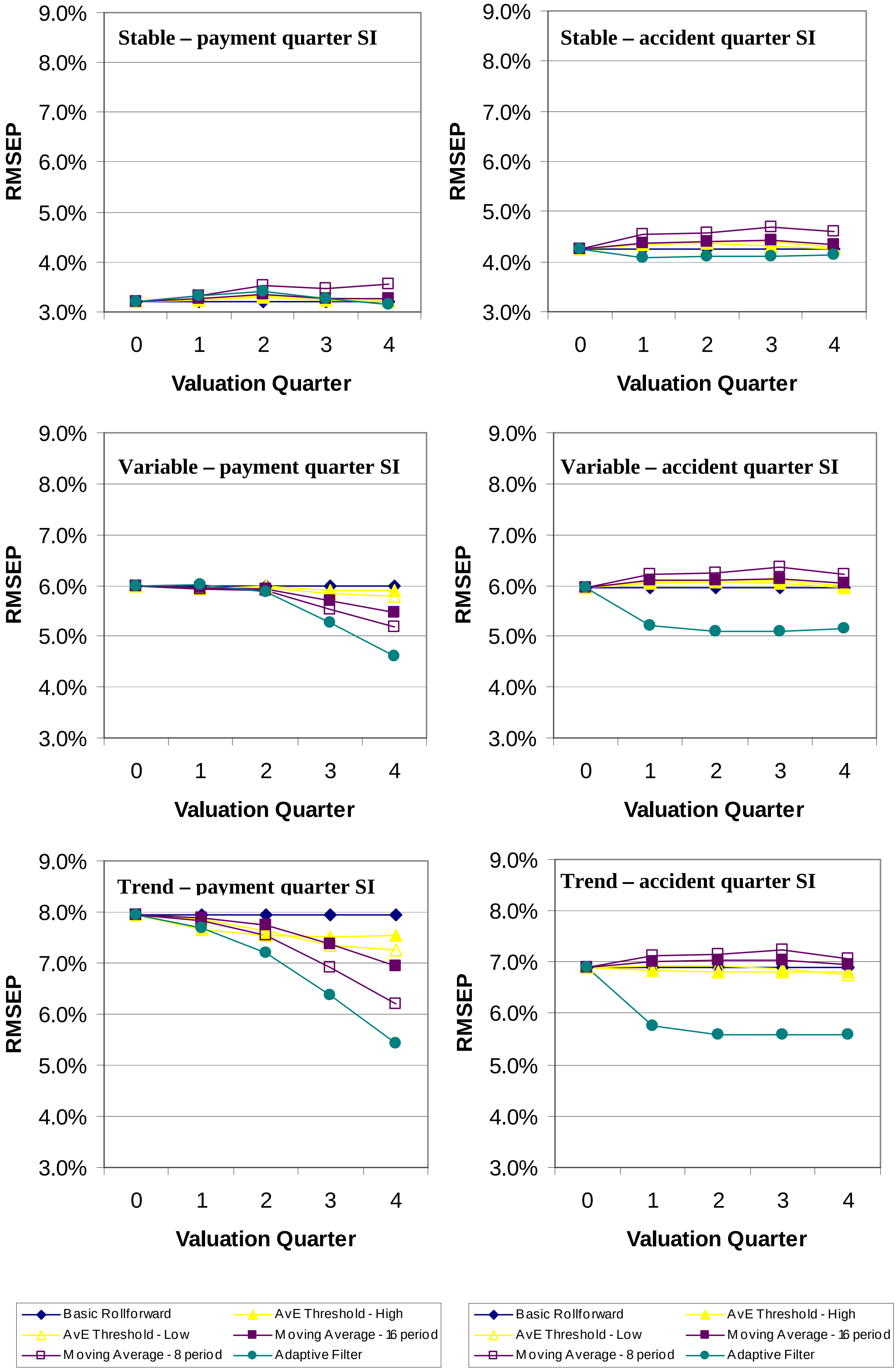

Figure 11 – Large portfolio PPCI: affect of SI environment on RMSEP

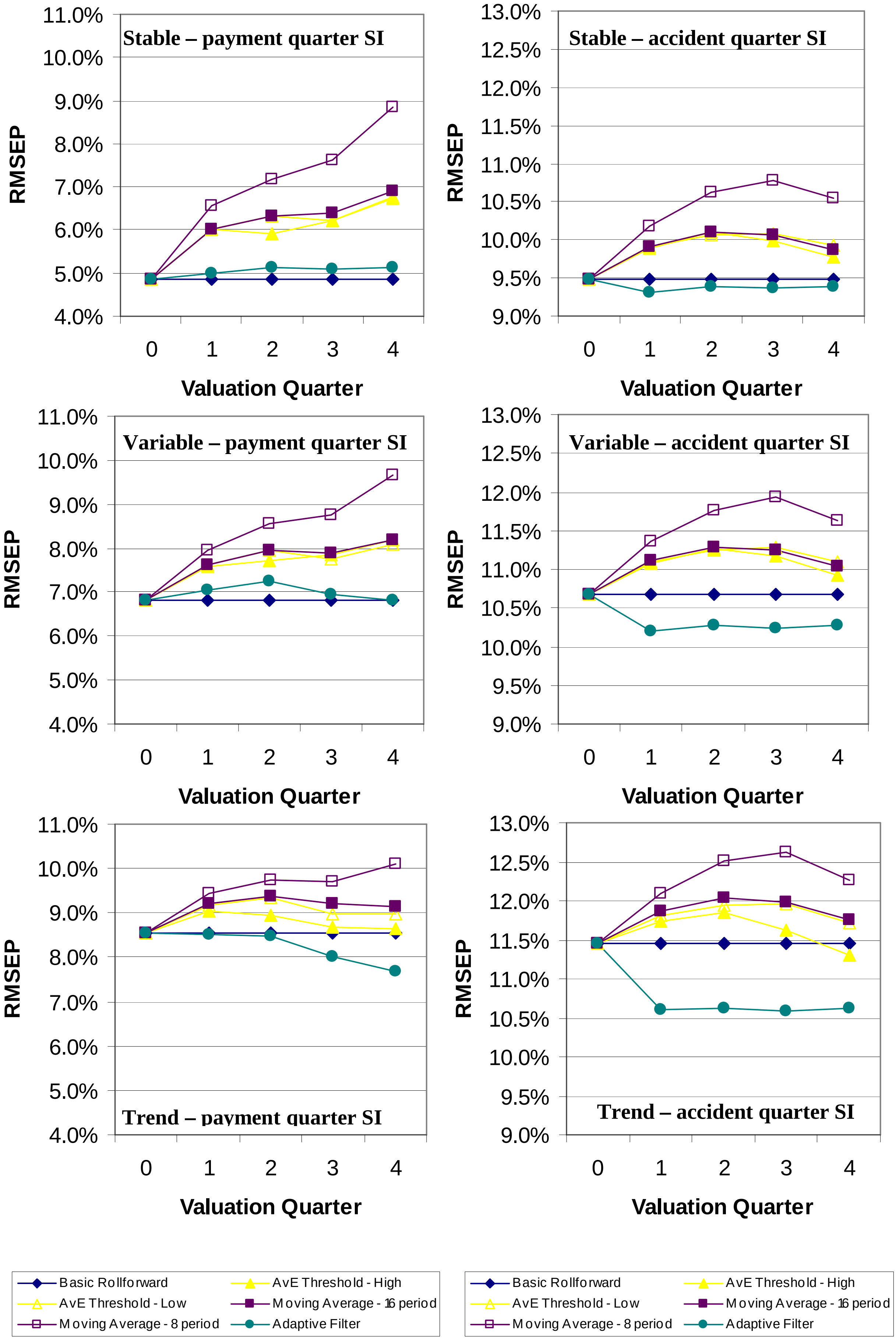

Figure 12 – PPCI on small portfolio: affect of SI environment on RMSEP

Valuation frequency for long tail classes

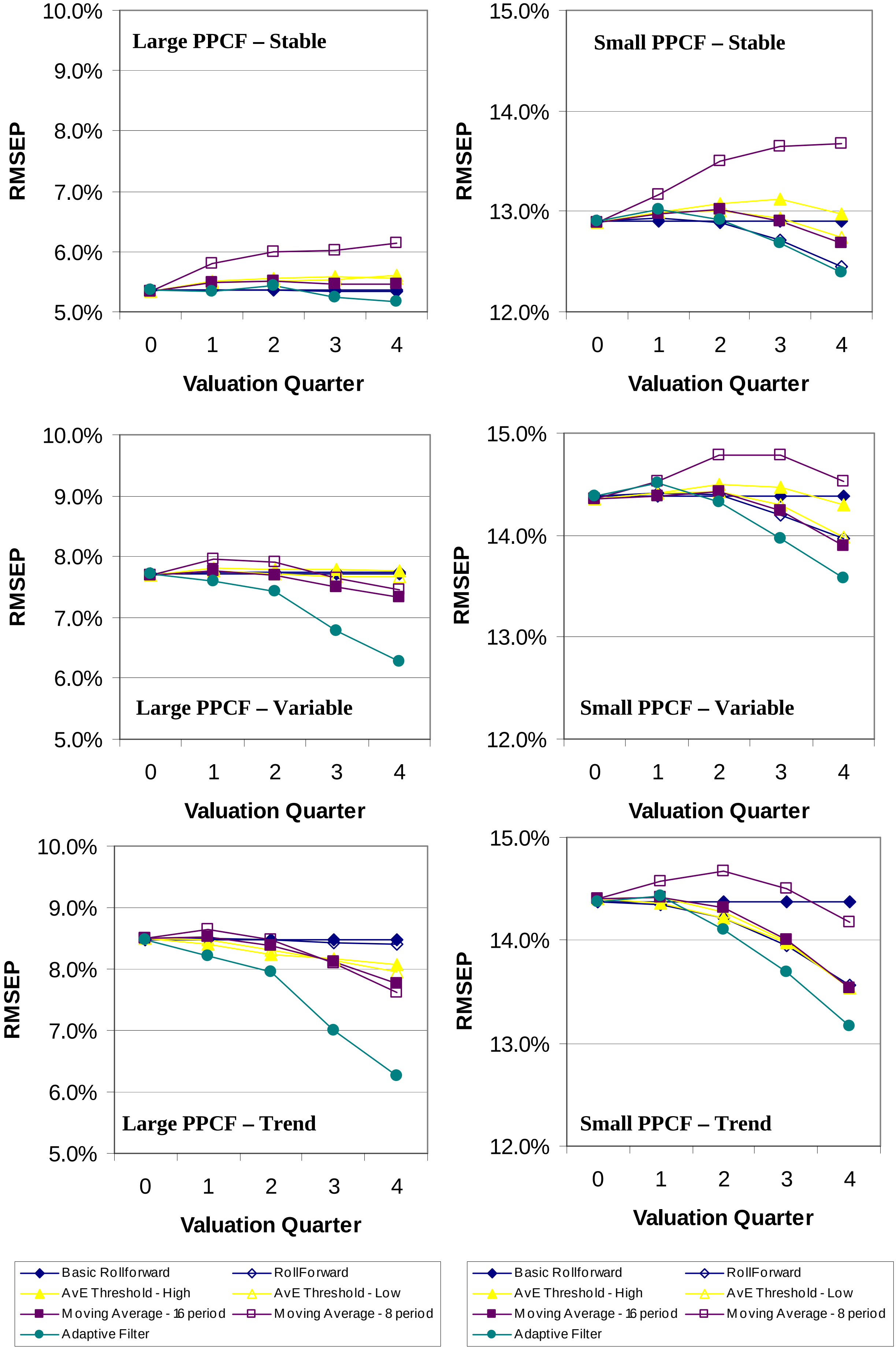

Figure 13 – PPCF: affect of payment quarter SI environment on RMSEP

3.1.3 Example of model fitting process

In the final part of this subsection, an example of the model fitting process is given. The figures below show the model updating process in 2 quarterly valuations following the previous annual valuation. The dataset used has been generated in the Trend – payment quarter SI environment.

The first figure shows how in the face of quarter 1 claims experience, the 16 period moving average and adaptive filter models are updated.

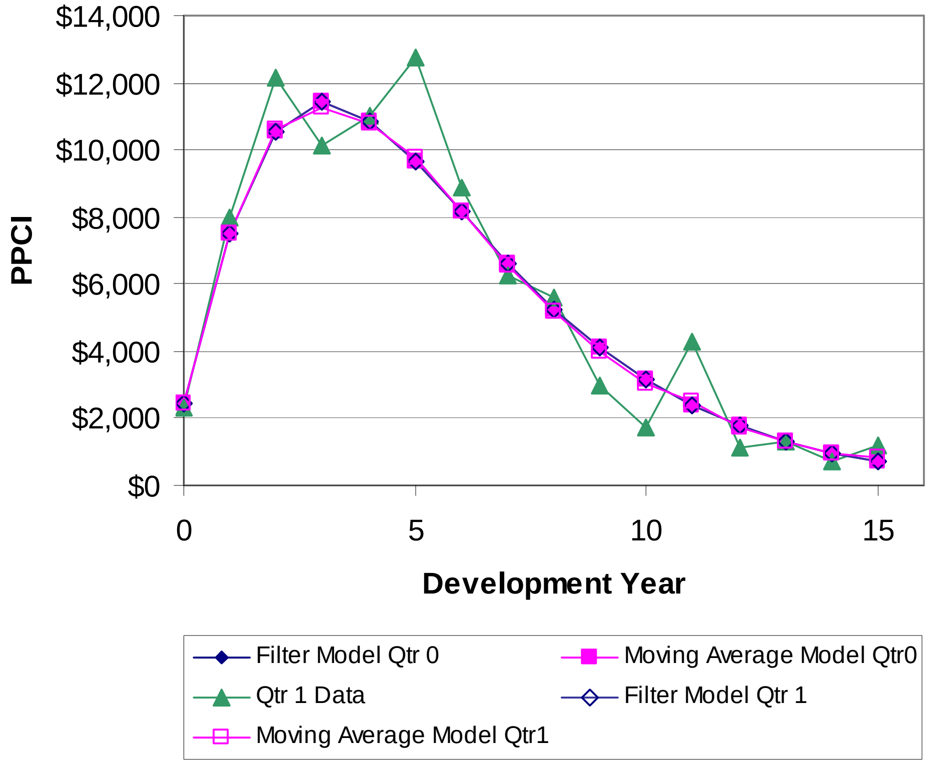

Figure 14 – Actual and fitted data in the first payment quarter after the previous annual valuation

In the figure the quarter 0 model predictions are identical. These are the predictions of the annual valuation. After 1 quarter of data there has been little movement in the model predictions.

Figure 15 shows the responses of the different modelling methods to the 2nd quarter of data.

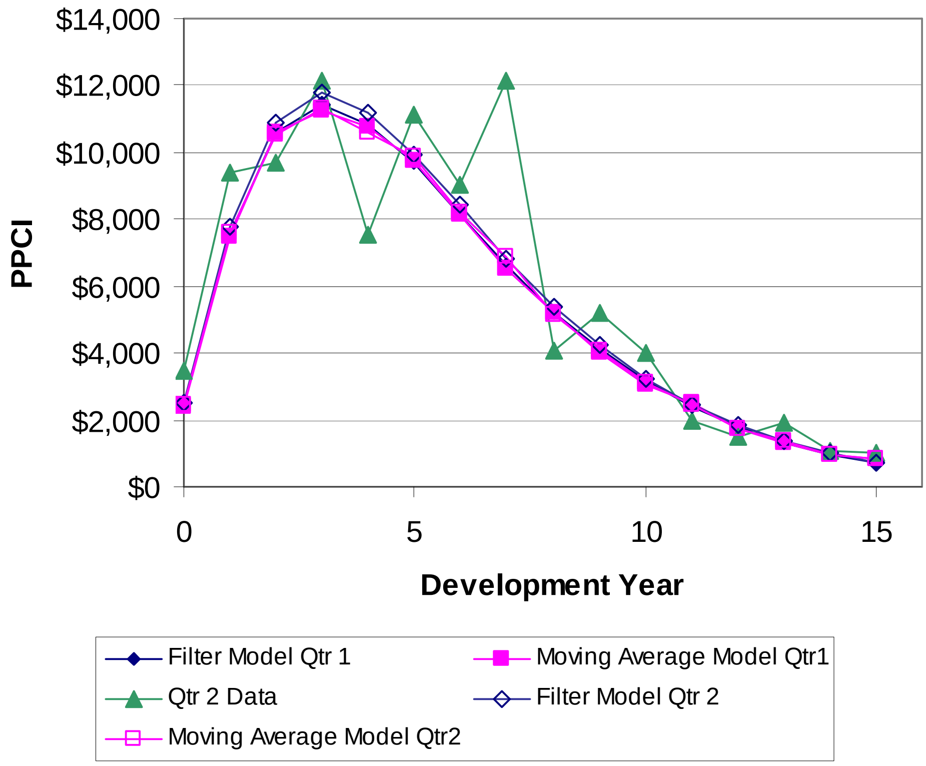

Figure 15 – Actual and fitted data in the second payment quarter after the previous annual valuation

After the second quarter of data we are starting to see a response of the filter to the trend environment. However the moving average model remains largely unchanged.

3.2 Annual valuation update errors from different quarterly valuation strategies

One way to get a feel for the practical implications of the measurements made in the previous section is to use those measurements to estimate the likely distribution of liability adjustments when moving from a 3rd quarter roll-forward valuation to a 4th quarter full valuation. These liability adjustments are defined as update errors in Section 2.5. As discussed in that Section, we have used the Adaptive Filter estimates of liability at quarter 4 as a proxy for a 4th quarter full valuation. This was justified on the grounds that out of all the valuation methods used in this study, it was the one that consistently gave the lowest prediction error, particularly at quarter 4.

The results of this analysis are shown in 4 tables which follow on the next few pages. Each table shows summary statistics from the estimated annual update error distributions. In each table the update error value is expressed as a % change from the 3rd quarter result. Results are shown for payment quarter SI models only. Qualitatively similar results are found for the accident quarter SI models.

Table 3.1 shows a summary of the update errors from the large portfolio PPCI model. The first column of the table shows that under a stable environment the update error distributions are similar for all valuation methods. This is consistent with the results of Figure 7 where prediction errors were similar across all methods at quarters 3 and 4.

However as we move to the rightmost columns of the table which show the update errors under the Variable and Trend superimposed inflation scenarios, the update error distributions start to change. Under the Variable scenario and using a roll-forward quarterly valuation strategy there is a 25% chance that the annual valuation will result in an increase in reserves of 3% or more and a 10% chance that the increase will be 5% or more. These increases correspond to the 75th and 90th percentiles of the update error distribution (shown at P75 and P90 in the table). Had a full valuation been carried out at quarter 3 (by using the adaptive filter) the size of the increases corresponding to the 75th and 90th percentiles are estimated to be 1% and 2%, respectively.

Under the trend scenario, there is a 25% chance that the annual valuation update error will be more than 5%, and a 10% chance the revision will be more than 8%. The corresponding errors using the adaptive filter for the quarter 3 valuation would have been 2% and 3%, respectively.

Table 3.1 – Large Portfolio PPCI: update error summary statistics

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	0%	0%	0%	3%
AvE Threshold - High	0%	0%	0%	2%
AvE Threshold - Low	0%	0%	0%	2%
Moving Average - 16 period	0%	0%	0%	2%
Moving Average - 8 period	0%	0%	0%	1%
Adaptive Filter	0%	0%	0%	1%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	1%	3%	3%	3%
AvE Threshold - High	1%	2%	3%	3%
AvE Threshold - Low	1%	2%	3%	3%
Moving Average - 16 period	1%	2%	2%	2%
Moving Average - 8 period	1%	2%	2%	2%
Adaptive Filter	1%	2%	2%	2%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	1%	2%	3%	5%
AvE Threshold - High	1%	1%	2%	4%
AvE Threshold - Low	1%	1%	2%	4%
Moving Average - 16 period	1%	1%	2%	4%
Moving Average - 8 period	1%	1%	2%	3%
Adaptive Filter	1%	1%	1%	2%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	2%	3%	5%	8%
AvE Threshold - High	1%	2%	4%	6%
AvE Threshold - Low	1%	2%	4%	6%
Moving Average - 16 period	1%	2%	4%	5%
Moving Average - 8 period	2%	2%	3%	4%
Adaptive Filter	1%	2%	2%	3%

Table 3.2 shows a summary of the update errors from the small portfolio PPCI model. In a stable superimposed scenario, the update errors for the roll-forward and the adaptive filter are similar. However, in the variable and trend environments, the update error is reduced if the adaptive filter is used at quarter 3. The reduction in error that results from performing a full valuation at quarter 3 is less in the small portfolio compared to the large. This is a result of it being more difficult for a full valuation to pick up systemic changes in a small portfolio. Evidence for this statement is the greater bias at quarter 4 for all remodelling valuation methods in the small portfolio compared to the large portfolio (results not shown).

Also of interest in Table 3.2 is the observation that the other valuation methods gave worse update errors compared to the roll-forward. This is consistent with the results shown in Figure 8.

Table 3.2 – Small Portfolio PPCI: update error summary statistics

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	0%	0%	0%	2%
AvE Threshold - High	-1%	-1%	-1%	0%
AvE Threshold - Low	0%	0%	0%	1%
Moving Average - 16 period	0%	0%	1%	1%
Moving Average - 8 period	0%	0%	0%	1%
Adaptive Filter	0%	0%	0%	0%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	1%	3%	3%	3%
AvE Threshold - High	3%	3%	4%	4%
AvE Threshold - Low	3%	4%	4%	4%
Moving Average - 16 period	3%	4%	4%	4%
Moving Average - 8 period	5%	5%	5%	5%
Adaptive Filter	1%	2%	1%	1%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	1%	2%	2%	4%
AvE Threshold - High	1%	2%	2%	3%
AvE Threshold - Low	2%	3%	3%	4%
Moving Average - 16 period	3%	3%	3%	4%
Moving Average - 8 period	4%	4%	4%	4%
Adaptive Filter	0%	1%	1%	2%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	2%	4%	4%	5%
AvE Threshold - High	3%	4%	4%	5%
AvE Threshold - Low	4%	4%	5%	5%
Moving Average - 16 period	4%	4%	5%	6%
Moving Average - 8 period	6%	6%	7%	7%
Adaptive Filter	1%	2%	2%	2%

Tables 3.3 and 3.4 show summary statistics of the update errors for the Large portfolio PPCF model and Small Portfolio PPCF model, respectively. Both PPCF models behave similarly to the their equivalent PPCI model.

Table 3.3 – Large Portfolio PPCF: update error summary statistics

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	0%	0%	0%	2%
RollForward	0%	0%	0%	2%
AvE Threshold - High	0%	0%	0%	2%
AvE Threshold - Low	0%	0%	0%	2%
Moving Average - 16 period	0%	0%	0%	2%
Moving Average - 8 period	0%	1%	0%	2%
Adaptive Filter	0%	0%	0%	1%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	2%	3%	4%	4%
RollForward	2%	2%	4%	4%
AvE Threshold - High	2%	3%	4%	4%
AvE Threshold - Low	2%	3%	4%	4%
Moving Average - 16 period	2%	3%	4%	4%
Moving Average - 8 period	3%	4%	4%	4%
Adaptive Filter	1%	2%	2%	2%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	1%	2%	3%	5%
RollForward	1%	2%	3%	5%
AvE Threshold - High	1%	2%	2%	5%
AvE Threshold - Low	2%	2%	3%	5%
Moving Average - 16 period	2%	2%	3%	5%
Moving Average - 8 period	3%	3%	4%	5%
Adaptive Filter	1%	1%	1%	2%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	3%	4%	5%	7%
RollForward	2%	4%	5%	7%
AvE Threshold - High	2%	4%	4%	6%
AvE Threshold - Low	3%	4%	4%	6%
Moving Average - 16 period	3%	4%	4%	6%
Moving Average - 8 period	5%	6%	6%	7%
Adaptive Filter	1%	3%	3%	3%

Table 3.4 – Small Portfolio PPCF: update error summary statistics

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	1%	1%	1%	2%
RollForward	0%	0%	0%	1%
AvE Threshold - High	0%	0%	0%	0%
AvE Threshold - Low	0%	0%	0%	1%
Moving Average - 16 period	1%	1%	0%	1%
Moving Average - 8 period	1%	1%	0%	1%
Adaptive Filter	0%	0%	0%	1%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	2%	3%	3%	3%
RollForward	1%	2%	2%	2%
AvE Threshold - High	3%	3%	3%	3%
AvE Threshold - Low	3%	3%	3%	3%
Moving Average - 16 period	3%	3%	3%	3%
Moving Average - 8 period	5%	5%	5%	5%
Adaptive Filter	1%	2%	1%	1%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	3%	3%	3%	4%
RollForward	1%	2%	2%	3%
AvE Threshold - High	1%	2%	1%	2%
AvE Threshold - Low	2%	2%	2%	3%
Moving Average - 16 period	3%	3%	2%	3%
Moving Average - 8 period	3%	4%	4%	4%
Adaptive Filter	1%	2%	1%	2%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	5%	5%	5%	6%
RollForward	2%	3%	3%	4%
AvE Threshold - High	2%	3%	3%	4%
AvE Threshold - Low	3%	4%	4%	5%
Moving Average - 16 period	3%	4%	4%	5%
Moving Average - 8 period	6%	6%	6%	6%
Adaptive Filter	2%	3%	2%	3%

3.3 Prediction error resulting from early valuation

The results of Section 3.1 tend to show that the prediction error of the roll-forward one quarter out from the previous full valuation is similar to that of the best remodelling method. This indicates that the prediction error that results from performing a valuation one quarter prior to the required balance date would in many cases not be too different to that from a full balance date valuation.

The four tables shown on the following pages summarise the prediction error distributions one quarter out from the previous annual valuation in the case of payment quarter SI. They show that the 75th and 90th percentile prediction error values for the roll-forward (basic and full) are never more than 1% from the best remodelling methods.

Conversely, if one attempts to update a full valuation after one quarter, use of a method overly sensitive to noise in the claim data can increase the prediction error. This is illustrated for the Small Portfolio PPCI model in Table 3.6 where the 75th and 90th percentiles of the 1 quarter prediction error are lowest for the roll-forward method, while those of the 8 quarter moving average method are measurably larger.

Table 3.5 – Large Portfolio PPCI: distribution of prediction error 1 quarter after full valuation

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	0%	0%	0%	6%
AvE Threshold - High	0%	0%	0%	5%
AvE Threshold - Low	0%	0%	0%	5%
Moving Average - 16 period	0%	0%	0%	6%
Moving Average - 8 period	0%	0%	0%	5%
Adaptive Filter	0%	0%	0%	5%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	3%	4%	6%	6%
AvE Threshold - High	3%	4%	6%	6%
AvE Threshold - Low	3%	4%	6%	6%
Moving Average - 16 period	3%	4%	6%	6%
Moving Average - 8 period	3%	4%	6%	6%
Adaptive Filter	3%	4%	6%	6%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	2%	2%	4%	10%
AvE Threshold - High	2%	2%	4%	9%
AvE Threshold - Low	2%	2%	4%	9%
Moving Average - 16 period	3%	2%	4%	9%
Moving Average - 8 period	3%	2%	4%	9%
Adaptive Filter	2%	2%	4%	10%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	4%	6%	8%	13%
AvE Threshold - High	4%	6%	7%	13%
AvE Threshold - Low	4%	6%	8%	14%
Moving Average - 16 period	5%	7%	8%	14%
Moving Average - 8 period	5%	6%	8%	14%
Adaptive Filter	4%	7%	8%	14%

Table 3.6 – Small Portfolio PPCI: distribution of prediction error 1 quarter after full valuation

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	0%	0%	0%	5%
AvE Threshold - High	0%	0%	0%	5%
AvE Threshold - Low	0%	0%	0%	6%
Moving Average - 16 period	0%	0%	1%	6%
Moving Average - 8 period	0%	0%	1%	6%
Adaptive Filter	0%	0%	0%	5%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	5%	6%	6%	6%
AvE Threshold - High	6%	6%	7%	7%
AvE Threshold - Low	6%	6%	7%	7%
Moving Average - 16 period	6%	6%	7%	7%
Moving Average - 8 period	6%	7%	7%	7%
Adaptive Filter	5%	6%	7%	7%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	4%	3%	4%	10%
AvE Threshold - High	4%	4%	6%	11%
AvE Threshold - Low	4%	5%	6%	12%
Moving Average - 16 period	5%	5%	6%	12%
Moving Average - 8 period	5%	5%	6%	12%
Adaptive Filter	4%	4%	6%	11%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	7%	9%	9%	15%
AvE Threshold - High	8%	9%	10%	16%
AvE Threshold - Low	8%	9%	10%	17%
Moving Average - 16 period	8%	9%	11%	17%
Moving Average - 8 period	9%	10%	12%	18%
Adaptive Filter	7%	9%	10%	16%

Table 3.7 – Large Portfolio PPCF: distribution of prediction error 1 quarter after full valuation

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	-1%	0%	-1%	4%
RollForward	-1%	0%	-1%	4%
AvE Threshold - High	-1%	-1%	-1%	4%
AvE Threshold - Low	-1%	-1%	-1%	4%
Moving Average - 16 period	-1%	0%	-1%	4%
Moving Average - 8 period	-1%	0%	-1%	4%
Adaptive Filter	-1%	0%	-1%	4%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	5%	4%	7%	7%
RollForward	5%	4%	7%	7%
AvE Threshold - High	5%	4%	7%	7%
AvE Threshold - Low	5%	4%	7%	7%
Moving Average - 16 period	5%	4%	7%	7%
Moving Average - 8 period	5%	5%	7%	7%
Adaptive Filter	5%	4%	7%	7%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	3%	2%	4%	10%
RollForward	2%	1%	4%	10%
AvE Threshold - High	2%	1%	3%	9%
AvE Threshold - Low	3%	2%	3%	9%
Moving Average - 16 period	2%	2%	4%	9%
Moving Average - 8 period	2%	2%	4%	9%
Adaptive Filter	3%	1%	4%	9%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	6%	5%	9%	15%
RollForward	6%	5%	9%	15%
AvE Threshold - High	6%	5%	9%	15%
AvE Threshold - Low	6%	5%	10%	16%
Moving Average - 16 period	6%	5%	10%	16%
Moving Average - 8 period	7%	5%	10%	16%
Adaptive Filter	7%	4%	9%	15%

Table 3.8 – Small Portfolio PPCF: distribution of prediction error 1 quarter after full valuation

Valuation Method	Mean			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	-2%	-2%	-2%	3%
RollForward	-2%	-2%	-3%	2%
AvE Threshold - High	-2%	-2%	-3%	2%
AvE Threshold - Low	-2%	-2%	-2%	3%
Moving Average - 16 period	-2%	-2%	-2%	3%
Moving Average - 8 period	-2%	-2%	-2%	3%
Adaptive Filter	-2%	-2%	-3%	2%
	Standard Deviation			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	11%	11%	12%	12%
RollForward	11%	11%	12%	12%
AvE Threshold - High	11%	11%	12%	12%
AvE Threshold - Low	11%	11%	12%	12%
Moving Average - 16 period	11%	11%	12%	12%
Moving Average - 8 period	12%	11%	13%	13%
Adaptive Filter	11%	11%	13%	13%
	P75			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	6%	4%	7%	13%
RollForward	6%	5%	6%	12%
AvE Threshold - High	7%	5%	7%	13%
AvE Threshold - Low	7%	6%	7%	13%
Moving Average - 16 period	7%	6%	7%	13%
Moving Average - 8 period	7%	7%	8%	14%
Adaptive Filter	6%	5%	7%	12%
	P90			
	Stable - rw	Stable - jump	Variable	Trend
Basic Rollforward	17%	18%	19%	26%
RollForward	17%	17%	18%	25%
AvE Threshold - High	17%	18%	18%	25%
AvE Threshold - Low	17%	18%	18%	25%
Moving Average - 16 period	17%	18%	18%	26%
Moving Average - 8 period	17%	19%	18%	26%
Adaptive Filter	17%	18%	18%	25%

4 Discussion

4.1 The effect of claims environment and portfolio size on the prediction error of different quarterly valuation methods

This study attempts to quantify a point that is well understood by actuaries. That is in long tail classes it is difficult to measure underlying systemic changes in claims experience amongst the volatility of the claims data. Because of this difficulty the value of re-estimating liabilities at too frequent an interval is uncertain.

It was found that in a relatively stable superimposed inflation environment - an environment where there was a two in three chance that annual superimposed inflation would change by less than 3% in the year - that a quarterly valuation strategy that involved re-modelling on a quarterly basis did not appreciably improve the prediction error.

This was true for a number of quarterly re-modelling methods including:

- a 16 quarter moving average;
- a method in where remodelling occurred only once new claims experience exceeded a predefined threshold; and
- A regression method where the parameters were updated by the Adaptive (Kalman) Filter.

In fact, if the portfolio was small enough and/or the valuation method was particularly sensitive to claims data volatility, then remodelling could lead to worse prediction errors. For example, we found that an 8 quarter moving average would generally lead to the worst prediction errors in a stable environment.

Also, in our small portfolio PPCI model - a model for a portfolio which experienced about 250 motor bodily injury claims per annum - all quarterly remodelling techniques that we tested, apart from Adaptive Filter method, gave worse prediction errors. This indicated that these remodelling methods were inappropriate for this particular portfolio. in that they were overly sensitive to claims data volatility.

However, if the claims environment was less stable then re-modelling tended to decrease prediction error. This improvement was usually marginal in the first quarter since the last full revaluation. However by the third quarter the improvement could be significant. Again, for portfolios and models where the claims data volatility was high enough to drown out the underlying systemic changes, then inappropriate modelling techniques could lead to worse prediction errors. This was again seen in our small portfolio PPCI model.

4.2 Alternative roll-forward methods

The most basic quarterly valuation strategy simply involves no adjustment to the projected cash-flows from the previous annual valuation.

An alternative method is to adjust any projections made at the annual valuation for differences in the numbers of claim reports, claims closures, active claims etc compared to what was projected.

We found that the latter approach gave superior results particularly when the portfolio was small and/or the liability of interest relates to a relatively underdeveloped accident period.

4.3 Superiority of the Adaptive Filter to other remodelling methods

An examination of the results in Section 3.1 shows that the Adaptive Filter tended to outperform the other modelling methods.

This is to be expected for a number of reasons. Adaptive filtering methods are specifically designed to detect a moving signal in noisy data. In essence they produce weighted average of parameter estimates from the current and past data quarters. However, the weight that is assigned to current and historical data quarters depends on both the expected claims volatility and expectations about how the parameter estimates will move. If the parameters are expected to move significantly from quarter to quarter, then more weight will be given to newer data quarters. Alternatively, if claims volatility is high then the weights are spread more evenly amongst old and new data quarters. In all cases however newer data periods get more weight than older periods.

This contrasts to the moving average techniques where the weights tend to be all or nothing – a past data quarter is either included in the average or it is not.

Also, adaptive filters use statistical reasoning to balance the need to identify systemic changes with the need to pool over many data periods in order to deal with claims data volatility. The other methods tend to do this in a more ad hoc way – the number of data periods used in an average or the levels of the thresholds adopted tend to be determined using judgement alone.

The adaptive filter that has been used in this study is the Kalman Filter. A limitation of this tool is that it requires the assumption of normally distributed data. For this study the limitation was not an issue because the payment data that we simulated was log-normally distributed and by taking logs of the data it could be dealt with by the Filter.

However, in practice there tends to be problems in assuming that the data is log-normally distributed. The main difficulty is the requirement for a bias correction which results from modelling the log transformation of the data. The bias correction requires accurate determination of the variance in different cells of the data triangle. The variance of some cells can be difficult to determine (particularly in the tail of the dataset) and this can lead to the use of inappropriate bias corrections.

Another difficulty is that the log-normal assumption may not always be appropriate for the data at hand. For example, claim reports or finalisations can be difficult to model with a normal or log normal assumption.

For these reasons in general it would be preferable to use alternative filters such as the GLM filter (Taylor, 2008). This is an extension of the Kalman filter to certain members of the Exponential Dispersion Family of distributions.

4.4 Annual valuation update errors

A quarterly valuation method that does not pick up systemic changes as well as the methods adopted for an annual valuation may lead to large step changes in the value of liabilities when one moves from the 3rd quarter valuation to the 4th quarter full valuation.

We have termed such changes annual valuation update errors (Section 2.5.2) and these were quantified in Section 3.1.3. We found that in a relatively stable superimposed inflation environment, that an appropriate roll-forward procedure did not give appreciably worse update errors at the 75th and 90th percentiles of the update error distribution.

However, if an inappropriate quarterly remodelling strategy was chosen, the inappropriate strategy could lead to worse update errors than if no remodelling was done at all. However the magnitude of the update errors caused by remodelling tends to be small: excluding the 8 period moving average method, under the stable scenario there is a 90% chance that remodelling will cause an update error that is at most 2% more than that obtained using an appropriate roll-forward.

When superimposed inflation becomes more variable or exhibits a clear emerging trend, the roll-forward strategy did start to give larger update errors. The worst update errors were seen in the trend environment (as expected). In a trend environment the largest update error for a roll-forward at the 75th percentile was 5%. However had an annual valuation remodelling methods been used at quarter 3 the equivalent update error would have been 2%. So the difference in update errors at the 75th percentile was 3%. At the 90th percentile this difference increased to 5%.

There would be typically many influences on the decision about what if any quarterly valuation strategy is adopted. However the above analysis seems to indicate that:

- Although remodelling methods can lead to increased update errors in some environments, the magnitude of these errors for any reasonable remodelling method appear to be small.

- The magnitude of update errors will increase as the claims environment becomes more variable. Under the scenarios tested the difference in update errors between a roll-forward strategy and a full remodelling strategy were at most 5% at the 90th percentile. The magnitude of this error needs to be considered in the face of other uncertainties that are faced at any valuation. For example, for liabilities that are long tail in nature, how the systemic influences continue to evolve after the last balance date will have a large impact on actual prediction errors. The impact of these changes has not been considered in this study.

4.5 Errors associated with preparing an annual valuation one quarter prior to balance date.

The results of this study indicate that remodelling with only one extra quarter of data at best leads to a marginal improvement in the prediction error.

The improvement in prediction error was at most estimated to be around 1 to 2% at the 75th percentile. This suggests that for the models and scenarios tested in this paper, the prediction error is not significantly increased by performing the valuation one quarter early.

4.6 Valuation approaches in practice

The current study does not explore potential behavioural bias effects (conscious or sub-conscious) when actuaries update valuations. The analyses presented in this study assume that the chosen updating method is applied in full and without any subjective/judgemental adjustments. All such adjustments may have the potential to make the results better or worse.

There may be circumstances in which the update approach that is adopted is probably best tailored to the particular circumstances that have arisen. For example:

- a partial “freeze” on claim settlements in latest quarter due to operational constraints, and/or

- known deficiencies in the data available for the latest quarter.

However, exploring these ideas was beyond the scope of the paper.

4.7 Measurement Framework

The nature of the portfolio, its size, the valuation models employed, and the specific superimposed inflation environment affecting the portfolio, will all have a large influence on the prediction errors of any quarterly updating approach. The framework and analysis presented in this paper is one way that objective decisions can be made about the appropriate valuation frequency for a particular portfolio.

5 References

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