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Financial Services Forum
Expanding Our Horizons

Weather Derivative Pricing and Risk Management Applications

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Overview

1. Introduction
2. Pricing Principles
3. Temperature Derivatives
4. Rainfall Derivatives

1. Introduction

Introduction

First formal recorded transaction in 1996 – Enron and Energy-Koch .

- HDD swap – Milwaukee, winter 1997
- De-regulation of energy industries – mainly in US and Europe.
- Initially used as a hedge against variability in electricity supply.

US Department of Commerce estimates that weather adversely affects:

- 70% of all US companies;
- 22% of total GDP.

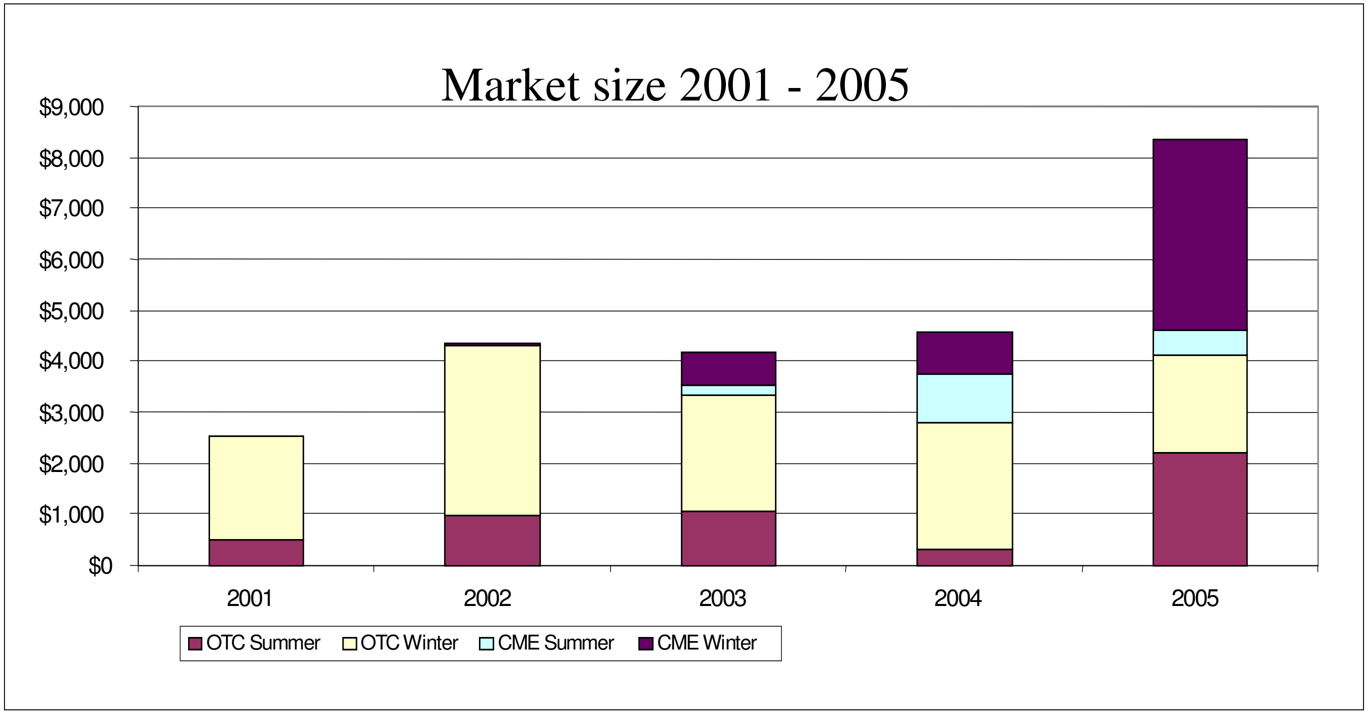
Weather Markets

- Mature Over-the-Counter (OTC) market:
  - Existed since early 1990's.
  - Specifically 'tailored' products.
  - Large European banks and insurance brokers.

- Chicago Mercantile Exchange (CME):
  - operates electronic exchange for weather derivatives.
  - futures and option contracts over US and Canadian cities.
  - 55% of total global turnover in 2005.

- L.I.F.F.E – Closed in 2004
  - series of contracts based on daily average temperatures in London, Paris and Berlin.

Market Size



* Source: PwC 2005 Market Survey

- Stagnant after Enron collapse.
- 2005 shows strong growth may be returning.

Contracts

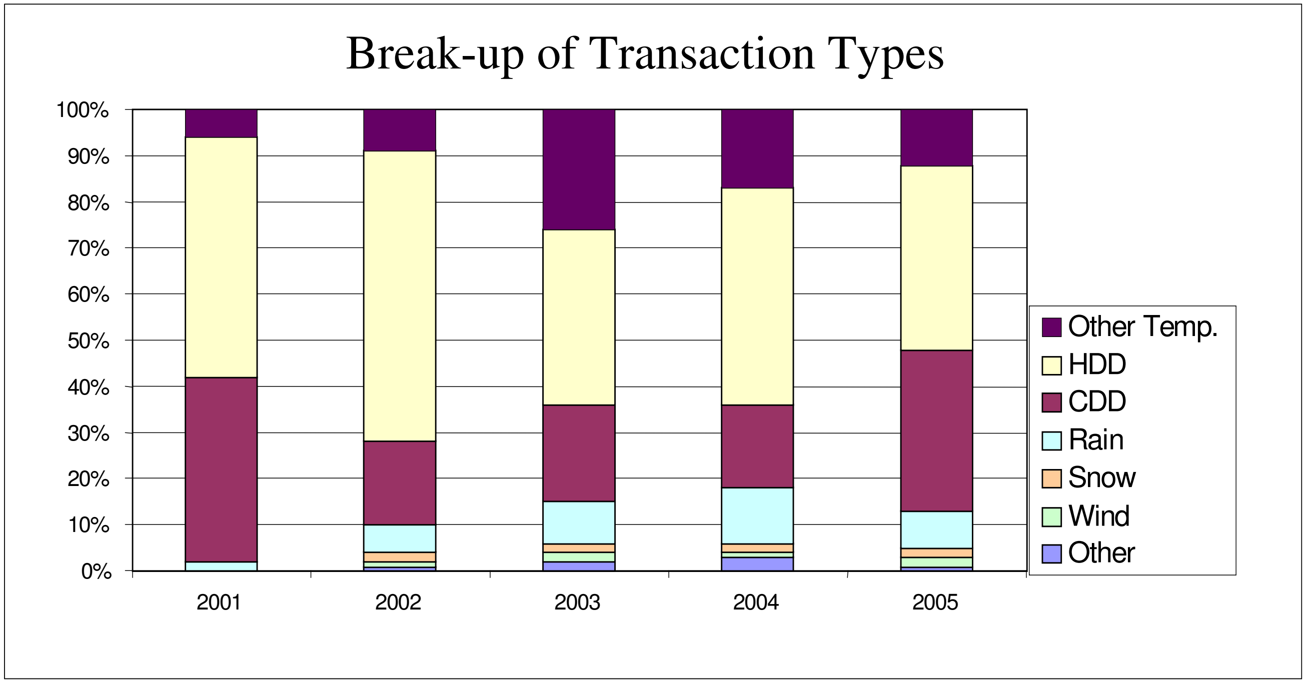
Contract Types:

- Futures - CME, OTC.
- Options – Majority of transactions to date.
- Swaps - increasing in popularity.

Underlying Variables:

- Temperature
- Rainfall
- Wind Speed
- Snow Fall
- Barometric Pressure

Contract Types



* Source: PwC 2005 Market Survey

- ➢Large increases in rainfall contracts.
- ➢CDD now equal with HDD contracts.

Temperature Derivatives

- ➢ Average daily temperature $T_i = \dfrac{T_{\max} + T_{\min}}{2}$

- ➢ The most popular derivative contracts are over Heating Degree Days (HDD) and Cooling Degree Days (CDD).

$$HDD = \sum_{month} \max\{0, (\overline{T} - T_i)\}$$

$$CDD = \sum_{month} \max\{0, (T_i - \overline{T})\}$$

  - Where the reference level, T, is usually 18º.
  - Heating is generally required below the reference temperature and cooling above it.
  - Cumulative number of degrees the average temperature was below the reference level

Rainfall Derivatives

Much less common than temperature-based derivatives.

- Market was born out of temperature exposure.
- ‘Discreetness’ of Rainfall
    - Basis risk - greatest barrier to expansion.
    - Modelling difficulties.
- Lack of counter-parties – only water supply companies as a possible partner.

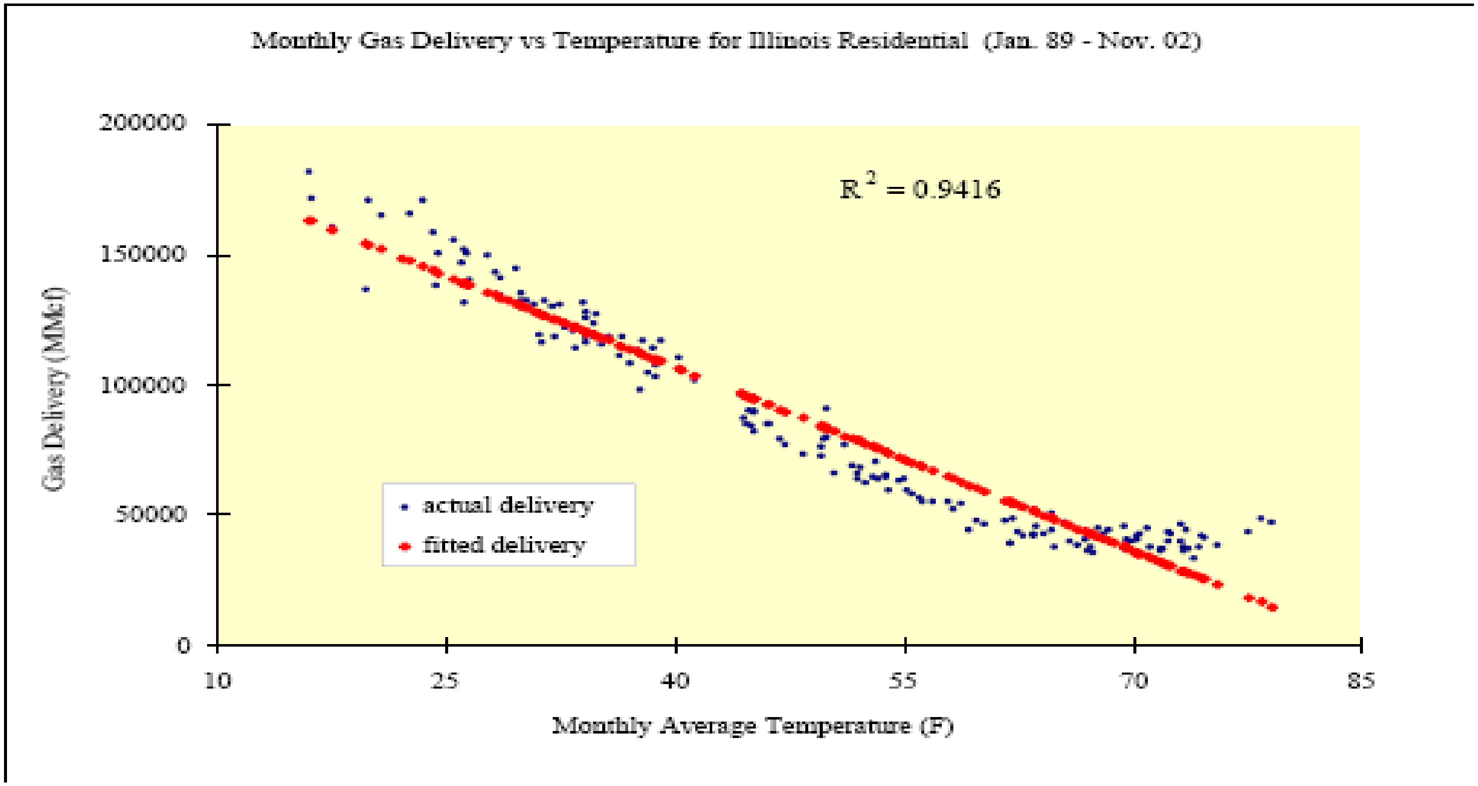
Risk Management Applications

Energy / Utility Companies

- Sales are highly correlated with temperatures.
- Definition of HDD and CDD contracts is based on an energy companies exposure.
    - Heating/cooling reference level (18º)
    - Cumulative based underlying variable.

- Enron
    - Oil and gas pipeline manager.
    - Used weather derivatives to reduce exposure to weather.
    - Soon became a ‘market-maker’ on CME and others.

Hedging Temperature



Risk Management Applications

Construction

- Temperature:
    - Concrete curing (setting) is temperature dependent.
    - Productivity reduces at unusually high and low temperatures.
    - ‘Stop work’ laws.
- Rainfall:
    - Precipitation delays can often represent 10% of contract.
    - Subsidence.
- Other exposures:
    - Snow fall.
    - Wind speed – cranes and other heavy equipment.

Weather Derivatives vs. Insurance

Some key differences:

- Identifiable Loss: There is no need to prove that a loss has occurred. Reduces costs – claims assessors, lawyers etc.
- Moral Risk: Nearly entirely removed – referenced to a transparent index
- Minimal Underwriting: Only counter-party risk requires investigation.
- Immediate Payout – known magnitude.
- Basis risk

2. Pricing Principles

Pricing

Traditional Black-Scholes assumptions:

- A traded underlying asset that can be used to create a hedge, i.e. sold short.
- Log-normal distribution.

Other methods must be found for the pricing of these contracts:

- Alternative BS framework.
- Martingale approach.
- Numerical simulation.

Mean Reversion

- Weather variables do not rise or fall without bound
- Mean reversion strength depends on several factors – most significantly latitude.

Mean-reversion component:

$$\frac{dX_t}{dt} = -\gamma.(X_t - \overline{X})$$

Ornstein-Uhlenbeck (OU) process:

$$dX_t = \gamma(\overline{X} - X_t).dt + \sigma.dW_t$$

Modified OU process:

$$dX_t = \left[\gamma(\overline{X} - X) + \frac{d\overline{X}_t}{dt}\right]dt + \sigma.dW_t$$

Alternative Black-Scholes

- Futures Price:

$$Y_t = X_t . e^{r(T-t)}$$

- Process s.d.e:

$$dY_t = y[(\mu - r)dt + \sigma dW_t]$$

- Modified Black-Scholes p.d.e:

$$\frac{dV_t}{dt} = rV - \frac{1}{2}\sigma^2 y^2 \frac{d^2V}{dy^2}$$

- Solution:

$$\begin{aligned} V(y,t) &= BS(ye^{-r\tau}, t, r, \sigma) \\ &= e^{-r\tau}.BS(y, t, 0, \sigma) \end{aligned}$$

Numerical Methods

‘Burn’ Analysis:

- No assumptions needed re: the process dynamics;
- No parameters to be estimated;
- Agreement on price.

Monte Carlo Simulations:

$$\mathbf{E}[f(X_t)] = \frac{1}{N}.\sum_{i=1}^{N} f(\overline{X}(t,\psi_i))$$

- Model dependant;
- Data intensive.

3. Temperature Modelling and Derivative Pricing

Data

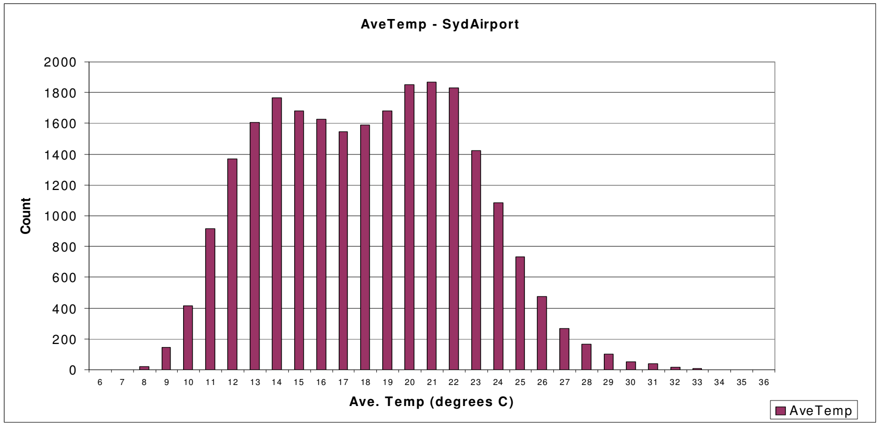
Australian Bureau of Meteorology (BOM)

- Sydney Airport. (Jan 1940 – Dec 2005)
- Observatory Hill. (Jan 1940 – Dec 2005)
- Prospect Dam. (Jan 1965 – Dec 2005)

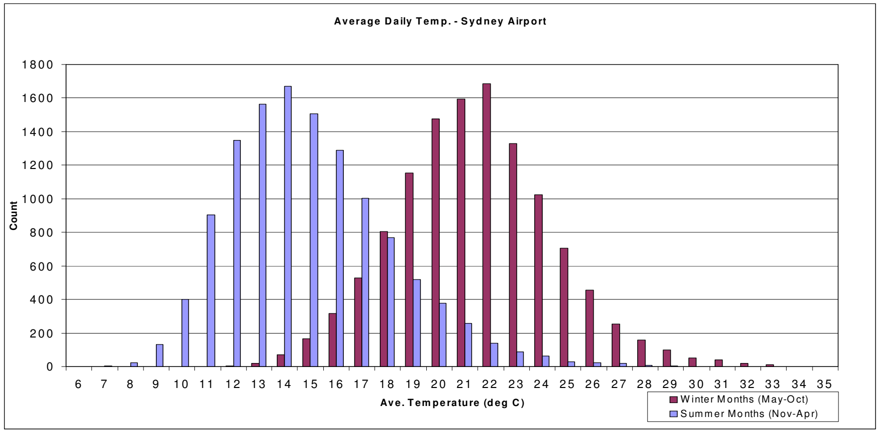
Missing data:

- Temperature – Backup stations.
- Rainfall – much more difficult. No accurate measure due to discreetness of rainfall.

Temperature Distributions



- Bi-modal Distribution



Modelling Temperature

Steps:

- De-trend data;
- Choose functional form for seasonal fluctuations;
- Estimate the parameters, including mean-reversion;
- Simulate the process;
- Analyse residuals.

Long-term Trends

All temperature data sets revealed a significant positive slope

$$T_{Long} = a + b.t$$

$$T_{long} = a + b.t + c.t^2$$

- Time series over 70 years should de-trend with a quadratic function.
- Natural geological based heating + human induced global warming

Seasonal Trends

Fourier series to model seasonal component:

$$T_{Seasonal} = \varepsilon\alpha_0 + \sum_i \alpha_i.Sin(\gamma t + \phi) + \sum_i \beta_i Cos(\lambda t + \theta)$$

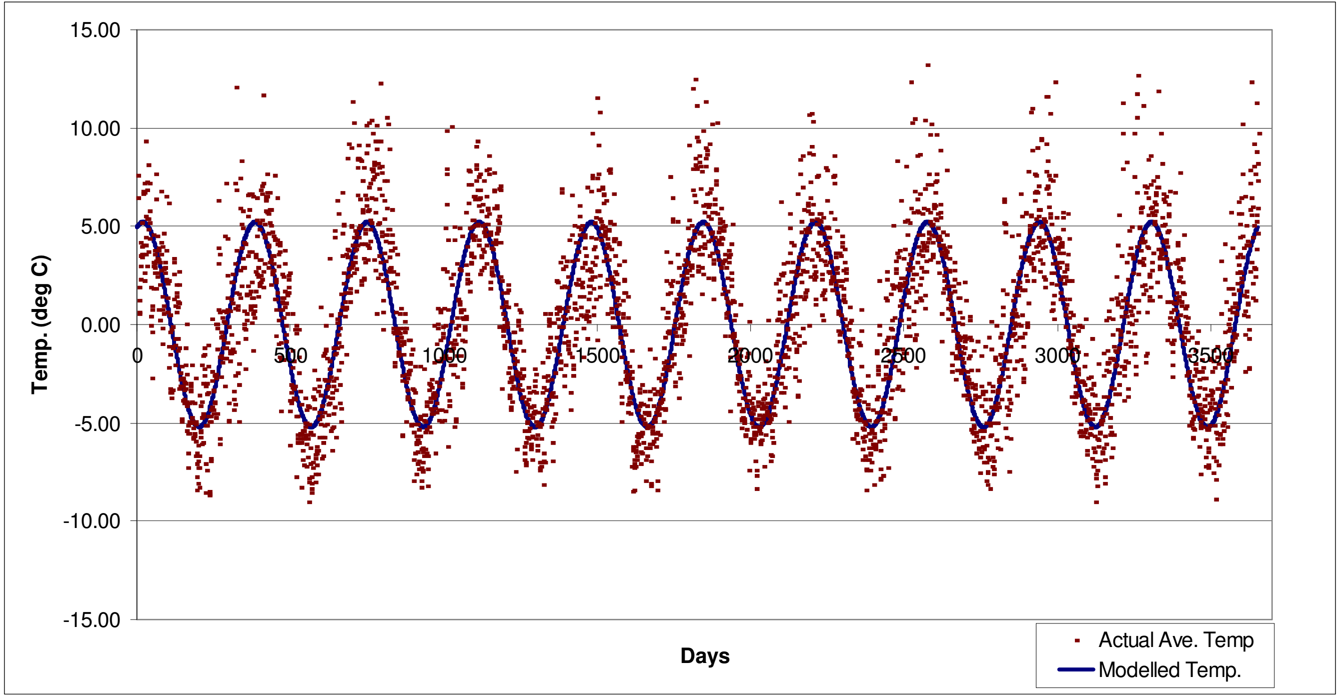
- A first order series is sufficient to capture seasonal pattern.

Combining this with the linear trend we obtain:

$$\overline{T} = a + b.t \ + \alpha.Sin(\gamma t + \phi) + \beta Cos(\lambda t + \theta)$$

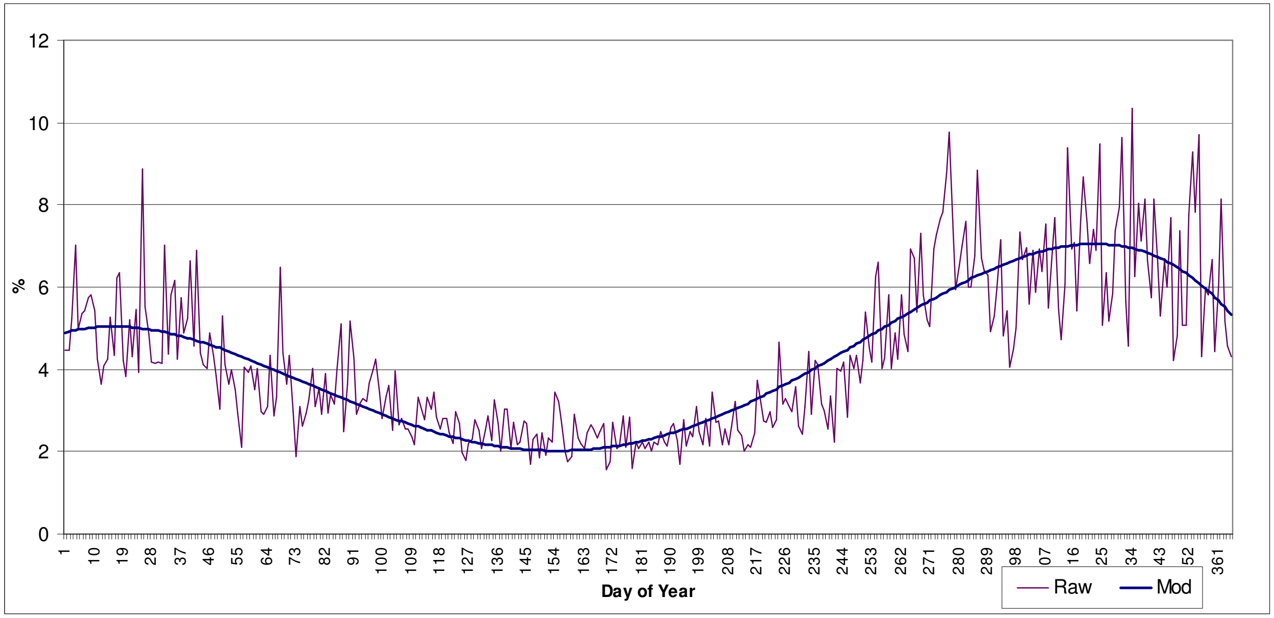
Model Fit

Actual vs Expected – Sydney Airport (10 years)



Temperature Volatility

Daily Temperature Volatility – Sydney Airport (10 years)



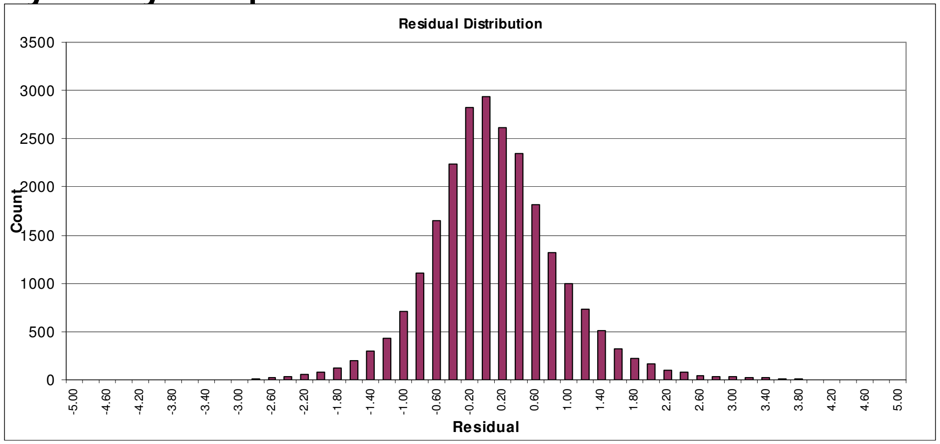
- ➢ Degree-4 polynomial fitted to volatility distribution.

Parameter estimation

- Parameter estimation – least squares

	Syd. Airport	Observatory	Prospect
a	16.925	17.434	17.26
b	6.30×10^{-5}	5.16×10^{-5}	4.91×10^{-5}
α	5.14	4.91	5.194
β	0.69	-0.20	0.986
φ	1.097	1.25	1.100
θ	0.97	1.10	0.675

- Residuals – Sydney Airport.



Pricing Example

- CDD option - January

Period:	January
Measure:	Cumulative CDD
Exercise Prices:	170 / 180 / 190 / 200 CDD's
Tick::	$100,000 /CDD
Location:	Sydney Airport (Kingsford Smith)

- Pricing via:
  1. Normal approximation.
  2. 'Burn' analysis – 66 years of data.
  3. Monte Carlo simulations

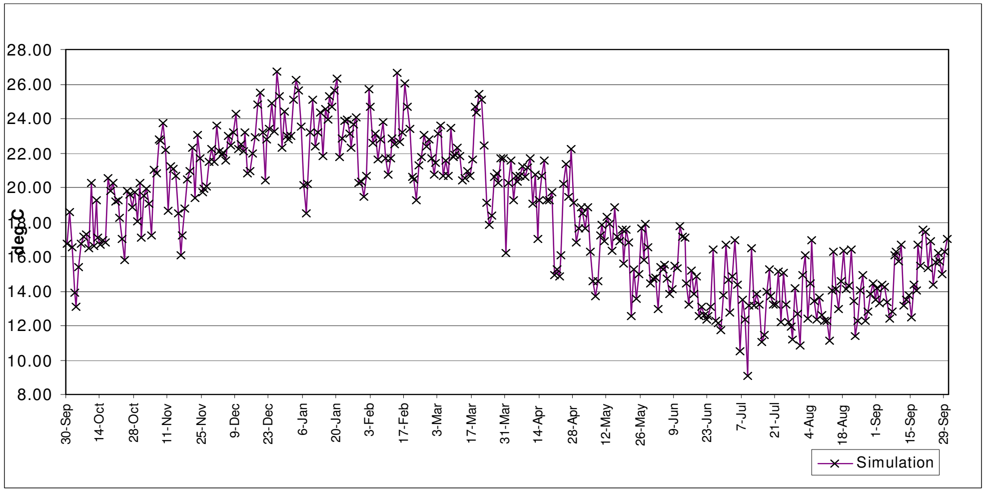
Monte Carlo

- Stochastic form:

$$T_t = \overline{T}_t + (T_0 - \overline{T}_o).e^{-\gamma\Delta t} + \int_s^t e^{-\gamma\Delta t}.\sigma_\tau dW_\tau$$

- Euler approximation - discrete

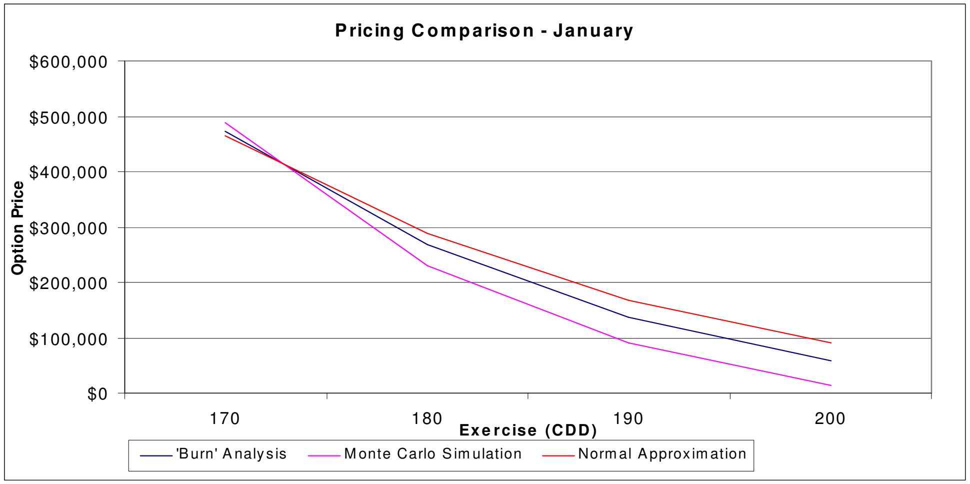
$$T_{t+1} - T_t = \gamma(\overline{T} - T_y). + \frac{d\overline{T}_t}{dt} + \sigma.Z$$



Pricing Example

Sydney Airport
January

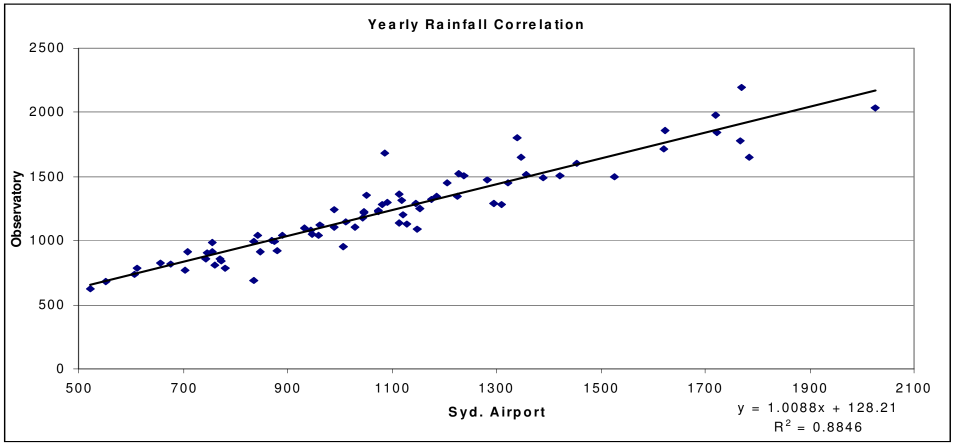
	Exercise (CDD)			
Method	**170**	**180**	**190**	**200**
'Burn' Analysis	$473,306	$268,763	$137,865	$59,582
Monte Carlo Simulation	$489,044	$230,479	$90,848	$13,990
Normal Approximation	$463,670	$288,627	$167,993	$91,012



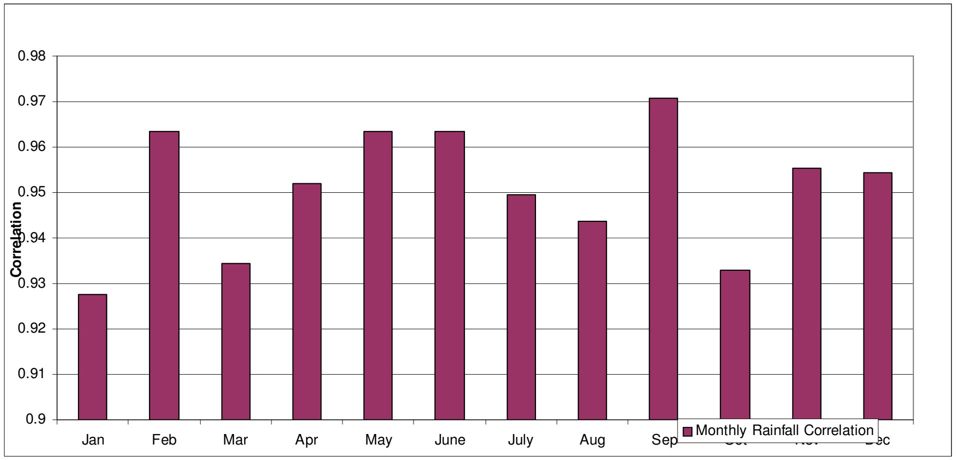
4. Rainfall Modelling

Rainfall Correlations

- Annual



- Monthly



Modelling Rainfall

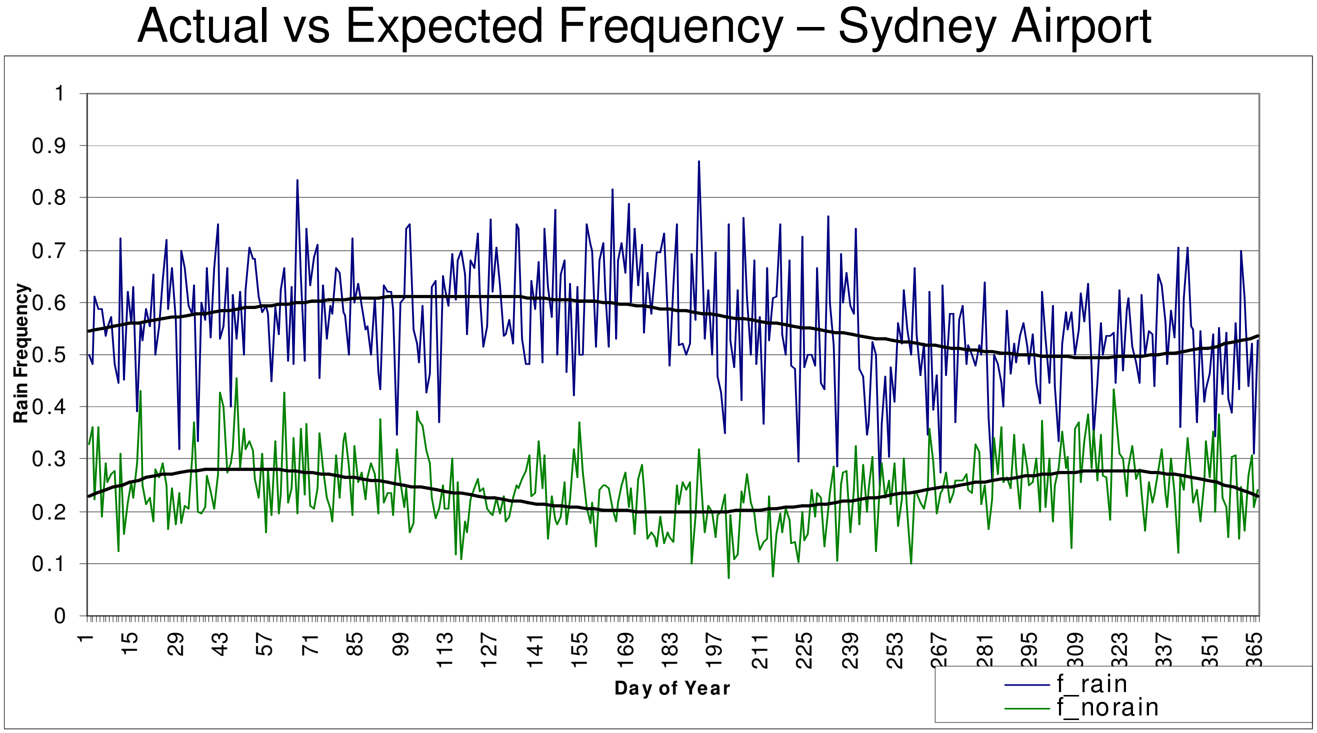
Compound Model – size & frequency.

- Frequency: Markov Chain.
- Size: Gamma distribution (4 segments).

Transition probabilities:

	Rain	No rain
Rain	0.55	0.45
No rain	0.28	0.72

Frequency Simulation



- Errors are greatest in winter - i.e. errors not proportional.
- Clearly defined seasonal patterns.

Magnitude

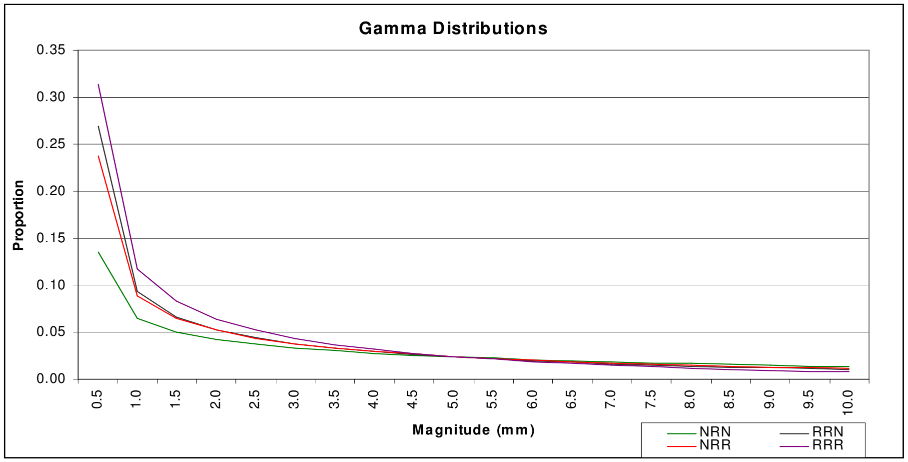
- 4 segments conditional on t-1 and t+1.

		Rain t+1	
Rain t-1		No	Yes
No	Average	3.74	7.20
	StdDev	6.21	12.23
	Max	79.0	182.1
	Min	0.1	0.2
Yes	Average	6.43	13.89
	StdDev	11.22	21.46
	Max	132.6	216.2
	Min	0.2	0.2

- Fit 4 Gamma distributions

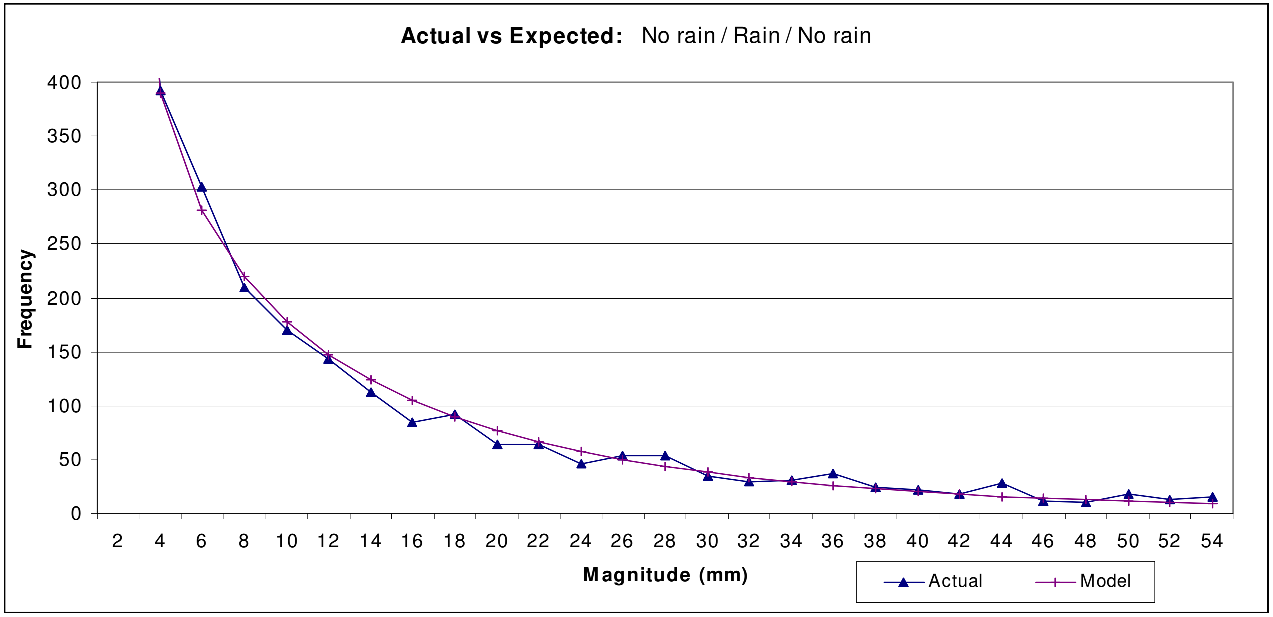
Gamma distribution parameter values

	NRN	RRN	NRR	RRR
alpha	0.58	0.45	0.47	0.49
beta	19.32	12.09	13.07	6.39



Magnitude

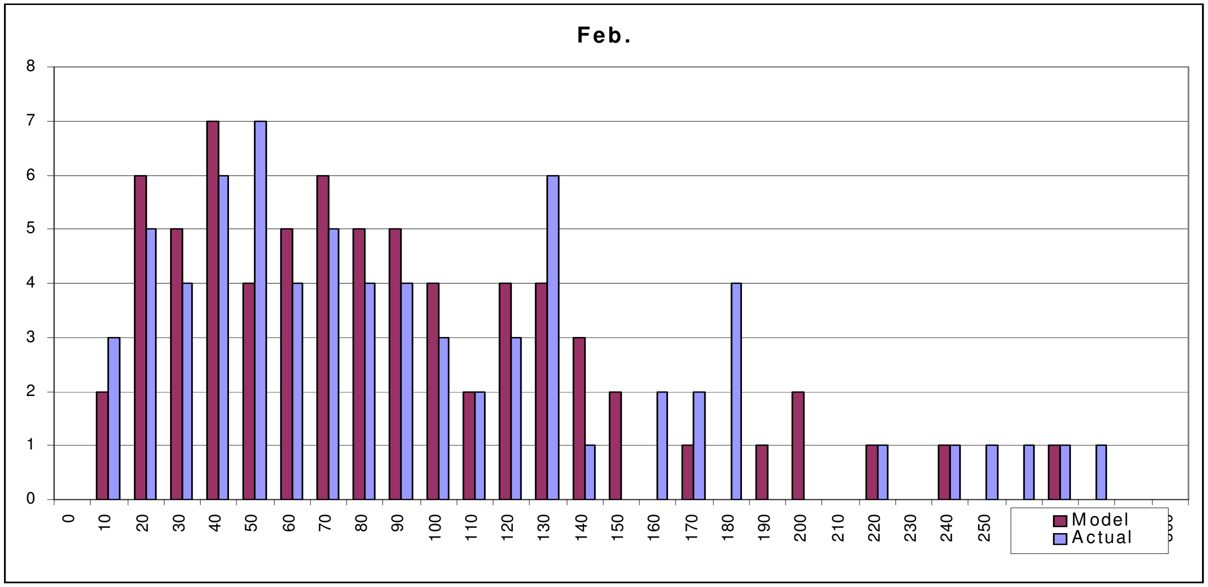
Actual vs Expected Magnitude – Sydney Airport



- Close Gaussian fit.
- Clearly defined seasonal patterns.

Simulation

Actual vs Expected Magnitude – Sydney Airport



Where to from here?

- New Markets:
  - Australian market practically non-existent – agricultural based economy.
  - Must promote to seek out suitable counter-parties.
  - Improve product design – reduce basis risk.
  - Centralised data recording methodologies – Europe in particular.

- New Interest:
  - Hedge funds – attracted to immature market.
  - Diversification tool – minimal correlation to debt and equity markets.
  - Weather-based indexed insurance contracts.