



Valuing Performance-Linked Share Based Payments

A Report from the Coal Face

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Fair Value

The fair value of an option equals the risk neutral expected value $E(X)$ of the present value of the payoff. For an option that is exercised at time t , the present value of its payoff is

$$X = \exp\left(-\int_0^t r(s) ds\right) (S - K)$$

where S is the share price at time t and K is the strike price. If the option is not exercised, its payoff is zero.

Total Shareholder Return

Total shareholder return is the value of the following investment:

1. Invest \$1 in a stock at time 0;
2. When the stock goes ex-dividend, borrow at the risk free rate and invest in the stock;
3. When the dividend is paid, use it to pay off the loan.

We model its natural logarithm which we call the natural return.

Black-Scholes Model

Suppose r is a function of $t > 0$ and C is a symmetric, positive definite $n \times n$ matrix. There is a unique lower triangular matrix L with positive eigenvalues such that $C = LL^t$. Let μ denote the vector valued function of $t > 0$ given by

$$\mu_i(t) = \int_0^t r(s) ds - \frac{t}{2} C_{ii}.$$

Then we simulate the natural return of n shares by $\mathbf{y} = \mu + L\mathbf{z}$ where \mathbf{z} is an n -dimensional Weiner process.

Risk Free Interest Rate

22 Nov 2005	5.30%
15 Feb 2006	5.25%
15 Nov 2006	5.11%
15 Oct 2007	5.24%
15 Aug 2008	5.32%
15 Sep 2009	5.31%
15 Aug 2010	5.28%
15 Jun 2011	5.42%
22 Nov 2012	5.37%

Covariance Matrix

The parameter C_{ij} is the covariance between changes in y_i and y_j .

Since the covariance of natural return is the same with respect to the risk neutral measure and the real world measure, we can estimate C_{ij} by scaling the sample covariance of the daily changes in the historical natural return for shares i and j .

Practical Issues

This estimate runs into one or more of the following problems:

- a small sample implies a higher sampling error;
- a higher sampling frequency is complicated by the fact the intra-day and inter-day steps are different; and
- over a long period, the covariances wander.

Early Exercise

There are several factors that undermine the collection of historical data:

- plan rules change every few years;
- market sentiment varies, e.g. bear, bull;
- company restructures;
- employee mix varies in age or level;
- individual employee preferences change over time; and
- sample sizes can be tiny.

Hull-White

1. Exercise is allowed only after a vesting period.
2. An exercisable option is exercised if the share price is at least m times the strike price.
3. There is a probability λdt that a vested option will be terminated in a given instantaneous interval of length dt . When an option is terminated, it is exercised if it is exercisable; otherwise it lapses.

Voluntary Early Exercise

The multiple m at which an option is exercised varies from employee to employee.

- 2, as this is approximately the median at many companies;
- 1.5, for impatient people;
- infinity (i.e., no voluntary early exercise); and
- γ , defined as follows:

$$\alpha = (r - q - \frac{\sigma^2}{2}) / \sigma; \quad \beta = \sqrt{\alpha^2 + 2r}; \quad \gamma = \frac{1}{1 - \frac{\sigma}{\beta - \alpha}}.$$

Involuntary Early Exercise

We assume that the rate λ of involuntary exercise is 10% p.a.

Share Based Payments

- Current share price \$10.00
- Strike price
 - \$10.00 (options) or
 - \$0.00 (performance shares)
- Vesting at 3 years
- Expiry at 7 years

Hurdles

At the end of the vesting period, if the company has done better than the hurdle, the SBP vests; otherwise it lapses.

- the median TSR of a small group of competitors
- the median TSR of a broad group of established companies
- the *S&P ASX 100 total return index*
- a sector specific return index

Banks

Australia and New Zealand Banking Group
Commonwealth Bank of Australia
Macquarie Bank
National Australia Bank
Saint George Bank
Westpac Banking Corporation

S&P ASX 200 banks total return index

Banks

Commonwealth Bank of Australia

- Dividend yield of 5.42% p.a. (cts)
- Volatility of 17% p.a.

Price

	Share	1.5	1.65	2	∞
None	8.500	1.292	1.272	1.221	1.164
Large	5.815	1.166	1.148	1.100	1.043
Small	4.976	0.990	0.976	0.934	0.883
ASX	4.658	1.018	1.003	0.959	0.906
Sector	4.543	0.913	0.900	0.862	0.814

Commonwealth Bank of Australia

Probability

	Share	1.5	1.65	2	∞
None	1.000	0.428	0.417	0.411	0.410
Large	0.589	0.354	0.343	0.337	0.337
Small	0.515	0.293	0.284	0.279	0.278
ASX	0.456	0.292	0.282	0.277	0.277
Sector	0.469	0.267	0.259	0.254	0.254

Commonwealth Bank of Australia

Materials

Amcor

Alumina

BHP Billiton

CSR

Rio Tinto

Sims Group

S&P ASX 200 materials total return index

Materials

BHP Billiton

- Dividend yield of 2% p.a. (cts)
- Volatility of 27.3% p.a.

Price

	Share	1.5	2	4.95	∞
None	9.418	2.776	2.973	3.040	3.030
Large	6.181	2.366	2.525	2.586	2.576
Small	5.898	2.216	2.355	2.412	2.402
ASX	5.441	2.185	2.322	2.379	2.369
Sector	5.146	2.063	2.185	2.238	2.228

BHP Billiton

Probability

	Share	1.5	2	4.95	∞
None	1.000	0.547	0.497	0.484	0.484
Large	0.481	0.402	0.364	0.351	0.351
Small	0.479	0.372	0.338	0.327	0.327
ASX	0.405	0.351	0.318	0.307	0.307
Sector	0.388	0.326	0.297	0.286	0.286

BHP Billiton

Consumer Staples

Coca-Cola Amatil

Coles Myer

Futuris

Fosters Group

Lion Nathan

Woolworths

S&P ASX 200 consumer staples total return index

Consumer Staples

Woolworths

- dividend yield of 3.25% p.a. (cts)
- volatility of 19% p.a.

Price

	Share	1.5	2	2.55	∞
None	9.071	1.944	1.999	1.986	1.964
Large	6.006	1.676	1.725	1.714	1.692
Small	5.704	1.572	1.617	1.606	1.584
ASX	5.097	1.500	1.543	1.532	1.511
Sector	4.973	1.475	1.518	1.507	1.485

Woolworths

Probability

	Share	1.5	2	2.55	∞
None	1.000	0.537	0.514	0.512	0.512
Large	0.549	0.412	0.391	0.390	0.390
Small	0.531	0.383	0.364	0.363	0.363
ASX	0.452	0.351	0.333	0.331	0.331
Sector	0.439	0.343	0.325	0.324	0.324

Woolworths

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- If the company does well, the hurdle is passed and the option is in the money. If it does badly, neither occurs.

Narrow or Broad

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- A hurdle against close peers has more impact on value than one against a broad group.
- Suppose the underlying share does well enough for a significant payoff but not so well that the test is passed. This requires the comparator group to do well as a whole.
- Companies in the same line of business are exposed to common shocks.

Aside

The distribution of TSR is lognormal. If Y is normal with mean μ and standard deviation σ , the expected value of $X = \exp(Y)$ is

$$\begin{aligned} E(X) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(\mu + \sigma z) \exp\left(-\frac{z^2}{2}\right) dz \\ &= \exp\left(\mu + \frac{\sigma^2}{2}\right) \times \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{(z - \sigma)^2}{2}\right) dz \\ &= \exp\left(\mu + \frac{\sigma^2}{2}\right). \end{aligned}$$

Volatility

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- As we assume that all assets have the same mean return, low volatility stocks have a better median return.
- However, the effect of volatility on fair value is subtle and depends on the hurdle.

Group or Index

- It is harder to beat the hurdle against the ASX 100 index than against the large comparator group.
- It is harder to beat the hurdle against the sector specific index than against the small comparator group.

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- It is harder to beat the hurdle against the ASX 100 index than against the large comparator group.
- It is harder to beat the hurdle against the sector specific index than against the small comparator group.
- Mean return is higher than the median return.

High Share Price Paths

- Half of the current value of a share comes from the return for $z > \sigma$, that is, future prices above $\exp(\mu + \sigma^2)$.

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- Half of the current value of a share comes from the return for $z > \sigma$, that is, future prices above $\exp(\mu + \sigma^2)$.
- Most of the value of an option comes from price paths well above the mean.
- Such paths result in early exercise regardless of your model.

Price

Price

	One Test	Retests
None	1.221	1.221
Large	1.100	1.218
Small	0.934	1.170
ASX	0.959	1.180
Sector	0.862	1.112

CBA options—monthly retests

Price

Price

	One Test	Retests
None	8.500	8.500
Large	5.815	7.385
Small	4.976	6.943
ASX	4.659	6.427
Sector	4.543	6.314

CBA performance shares—monthly retests

Retesting

- A hurdle with retesting has a negligible impact on the fair value of options.
- Retesting drastically reduces the impact of a hurdle on fair value.

Questions

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and
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