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Financial Services Forum

Expanding Our Horizons

Living Life Optimally with a Mean-Reverting Price of Risk

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Objective

- The objective of this paper was to analyse the behaviour of an investor when:
  - the risk premium is mean-reverting and is correlated with the risky asset.
  - In addition, event risks (jumps) exist in the risky asset.

Motivation

- Poterba and Summers (1988) have observed that serial negative correlation seems to exist for returns in the longer horizon in the US and 17 other countries, suggesting mean-reversion.
- It is well known that stock prices are susceptible to sudden changes.

Kim and Omberg

- Risky asset

$$\frac{dS}{S} = \alpha_t dt + \sigma dZ_1$$

$$\rho$$

- Risk premium

$$X_t = \frac{\alpha_t - r}{\sigma} \qquad X_t = -\kappa_x (X_t - \bar{X}) dt + \sigma_x dZ_2$$

Kim and Omberg

- The investor’s utility

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

- Objective of the investor is to maximise expected terminal-utility.

Analytic Solution

- Asset allocation can be broken up into two parts: myopic demand (Merton ratio) and intertemporal demand.

$$\theta(X_t,\tau)=\underbrace{\frac{X_t}{\sigma\gamma}}_{\text{Myopic demand}}+\underbrace{\frac{\rho\sigma_x[C(\tau)X_t+B(\tau)]}{\sigma\gamma}}_{\text{Intertemporal demand}}$$

Myopic and Non-myopic behaviour

- An investor is said to be myopic if he/she only considers this current period when making investment decisions.
- Non-myopic behaviour, on the other hand, occurs when the investor considers the problem as a whole. Intertemporal demand is part of non-myopic behaviour.

Analytic Solution

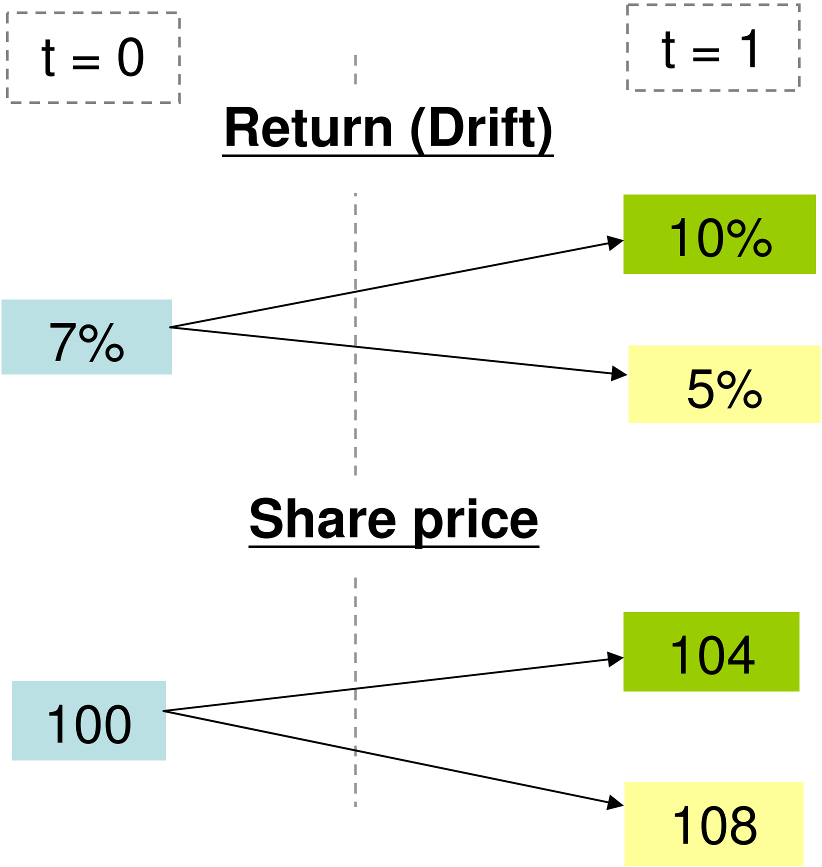
- Asset allocation can be broken up into two parts: myopic demand (Merton ratio) and intertemporal demand.

$$\theta(X_t,\tau)=\underbrace{\boxed{\frac{X_t}{\sigma\gamma}}}_{\text{Myopic demand}}+\underbrace{\boxed{\frac{\rho\sigma_x[C(\tau)X_t+B(\tau)]}{\sigma\gamma}}}_{\text{Intertemporal demand}}$$

Example

- Suppose the correlation is negative and the investor has a risk aversion of 4.

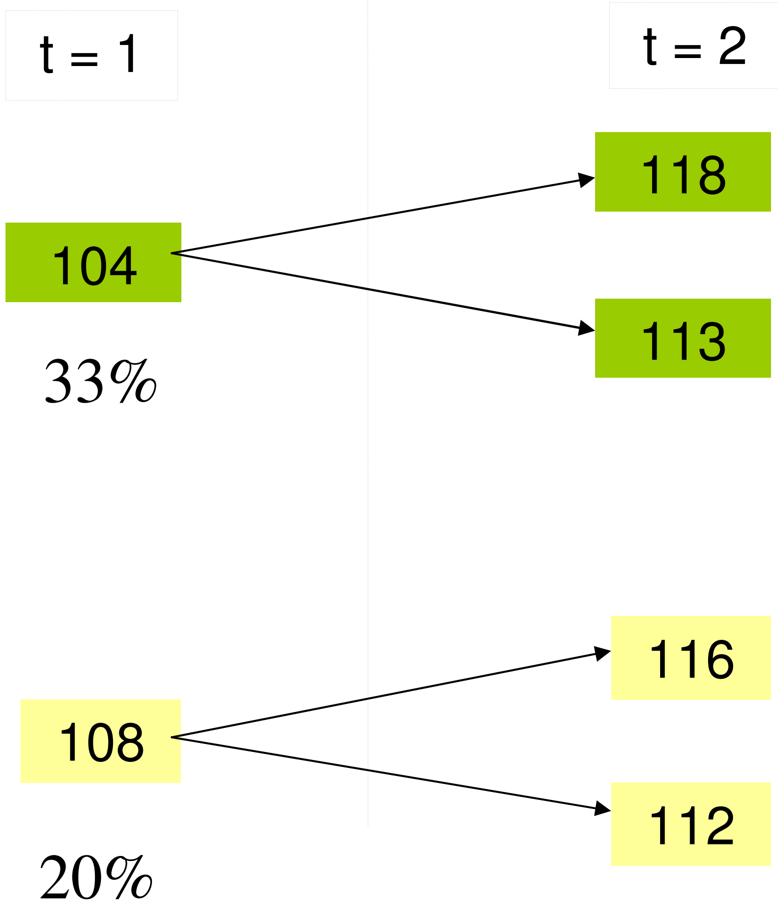
Example



Risk-free = 6%

Myopic investor :25%	Non-Myopic investor :30%
105.6	105.5
106.5	106.6

Example



Myopic	Non-myopic
114.42	114.31
112.75	112.65
113.19	113.30
112.40	112.51

Higher utility

Wu (2003)

- Wu considers the case when jump occurs.

$$\frac{dS}{S} = (\alpha_t - \lambda g)dt + \sigma dZ_1 + (e^q - 1)dQ$$

Jump Demand

- The asset allocation is:

$$\theta(X_t,\tau)=\frac{X_t}{\sigma\gamma}+\frac{\rho\sigma_x[C(\tau)X_t+B(\tau)]}{\sigma\gamma}+\frac{\lambda\hat{g}}{\sigma\gamma}$$

- Negative jumps reduce the asset allocation.

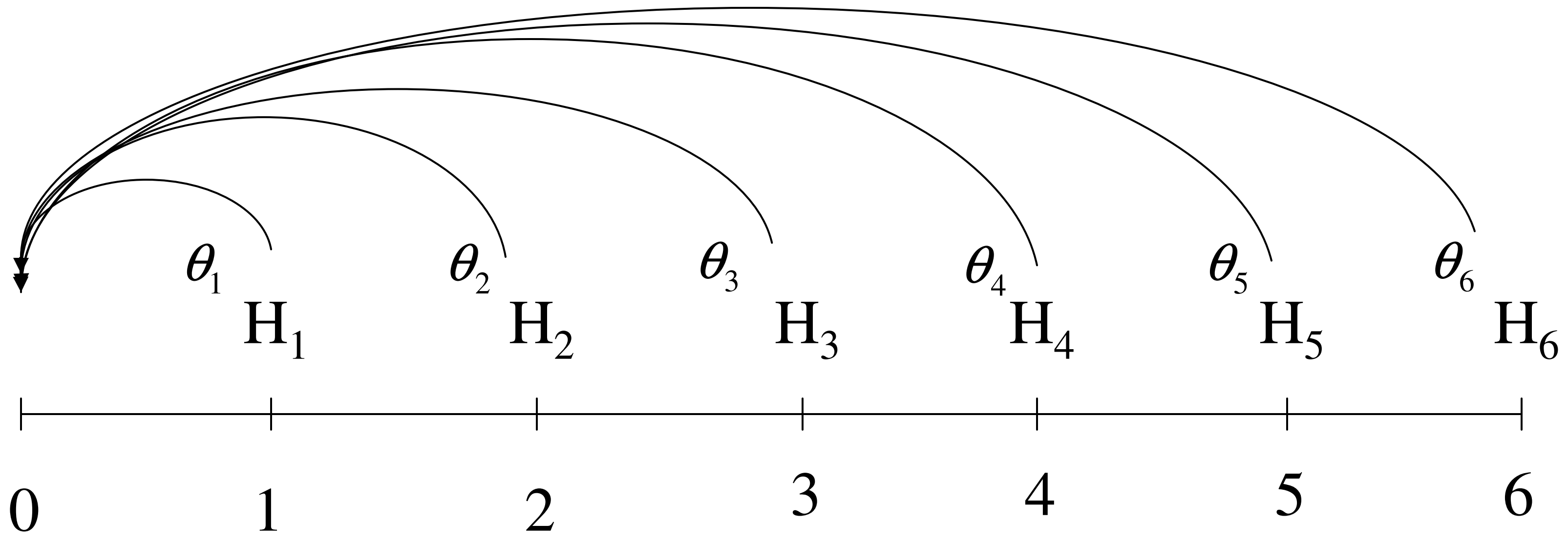
Wachter (2002)

- Considers consumption without jumps.

- Consumption is important because one of the main reasons we invest is to consume the wealth.

Weighted Average Formula

- Wachter shows that the asset allocation is actually the weighted average of future consumption value



Impact of consumption

- It follows that consumption actually impacts the asset allocation.

Model

- Risky asset

$$\frac{dS}{S} = (\alpha_t - \lambda g)dt + \sigma dZ_1 + (e^q - 1)dQ$$

- Risk premium

$$X_t = \frac{\alpha_t - \lambda g - r}{\sigma} \qquad X_t = -\kappa_x (X_t - \bar{X})dt + \sigma_x dZ_2$$

Model

- The investor's utility

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

- Insurance premium

$$P_t = \mu_t(Z_t - W_t)$$

Model

- Budget constraint

$$dW = rWdt + (\frac{dS}{S} - r)\theta Wdt - c_t dt - P_t dt$$

Objective function

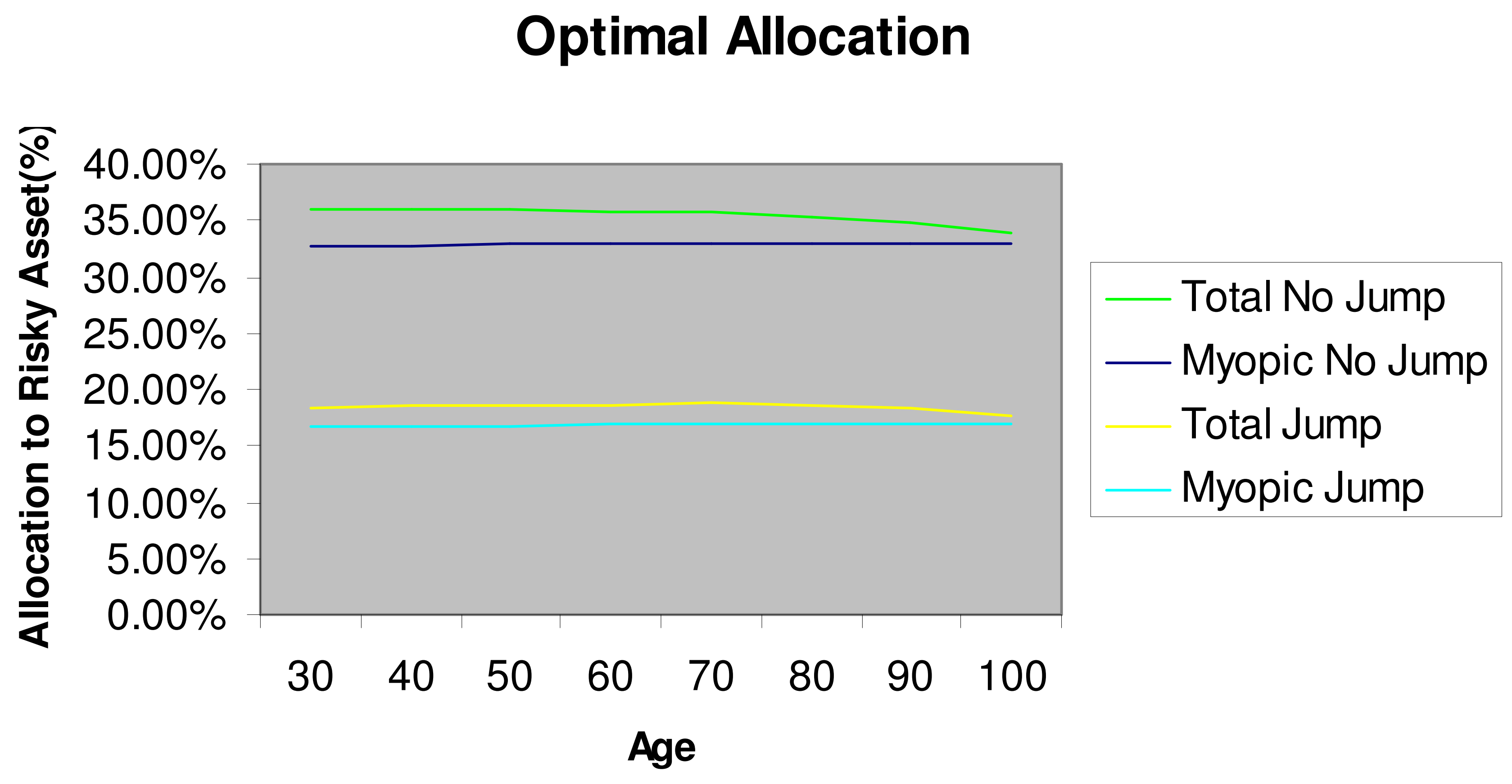
- The objective of the investor is to maximise utility. This is mathematically given by:

$$J(W, X_t, t) = \max_{\theta, C} E_t \left\{ \int_t^{\omega} {}_T p_t \mu_{t+T} \left[\int_t^T U(C_s) ds + U(Z_T) \right] dT \right\}$$

Numerical method

- A numeric approach was used to solve this problem and the Japanese economy was used as parameters.

Results


Optimal Allocation
Allocation to Risky Asset(%)
40.00%
35.00%
30.00%
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
30
40
50
60
70
80
90
100
Age
Total No Jump
Myopic No Jump
Total Jump
Myopic Jump

Results: Consumption

Age	Myopic		Overall	
	c_{ms}	c_m	c_s	c
30	0.1781	0.1746	0.1789	0.1749
40	0.2022	0.1989	0.2030	0.1992
50	0.2357	0.2326	0.2363	0.2329
60	0.2841	0.2813	0.2846	0.2815
70	0.3583	0.3558	0.3586	0.3561
80	0.4836	0.4817	0.4840	0.4819
90	0.7253	0.7245	0.7262	0.7246

Conclusion

- Investors will exploit the correlation between the risky asset and the risk premium to maximise their consumption utility.