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Asset Allocation in the light of Liability Cash Flows

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Asset / Liability Matching

- One of the profession's claimed areas of expertise
- Need a definition of “closest match” between the stochastic cashflows representing a portfolio of liabilities and its associated assets
- Does the stochastic asset model matter?
- If so, which is the “best” asset model?

Closest Match

Ultimate Surplus

- Project asset and liability cashflows
- When all liabilities extinguished what assets remain are the “ultimate surplus”
- “Closest match” defined by reference to this ultimate surplus
- Most important risk is insolvency
- Insolvency = negative ultimate surplus

“Closest Match”

- With stochastic cashflows the probability of insolvency is always non-zero [unless infinite initial asset amount]
- As initial assets \uparrow
probability of insolvency \downarrow
- Two variables
 - Probability of insolvency
 - Initial asset amount
- Need to fix one of these to produce a unique solution

“Closest Match” – [2]

Closest Match is

The asset portfolio which, for a given probability of negative ultimate surplus, requires the lowest initial asset amount.

Worked Example

Liabilities

- Long tail outstanding claims
- Runs off in 10 years
- Quarterly time intervals

Liability Model

- Stochastic chain ladder (Renshaw & Verrall)
- Modelled as a GLM
- Log link function
- Gamma error distribution
- Linear predictor
 - $\mu + \alpha_i + \beta_i$

Asset Model

- Asset Classes
 - 90 day bank bills
 - 10 year government bonds
 - Australian equities : All Ordinaries Index
- Jon Carter's model
 - An expanded Wilkie type cascade structure
 - Fitted to the Australian markets

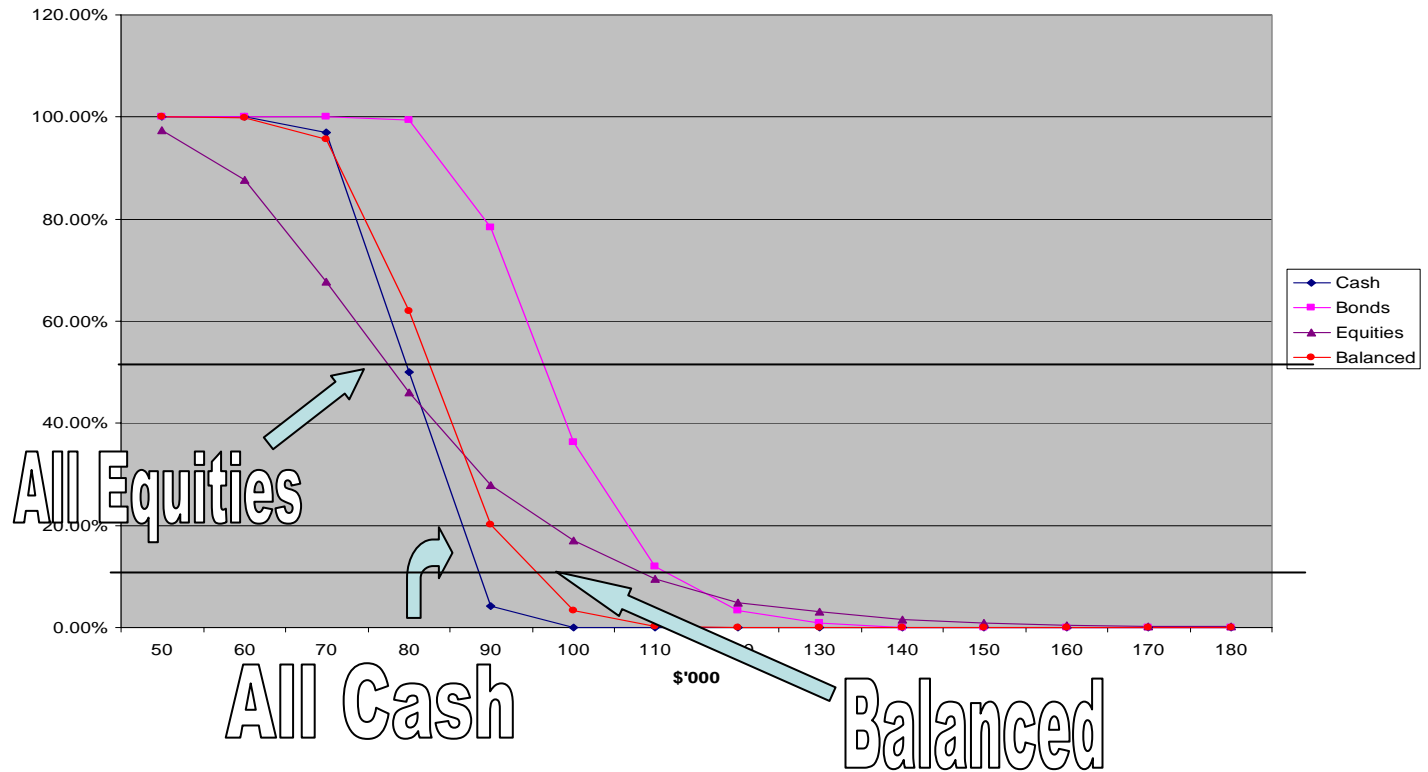
Asset Portfolios

- 100% cash
- 100% bonds
- 100% equities
- Balanced – 30% cash, 30% bonds, 40% equities

Probabilities of Insolvency

Initial Asset Amount	All Cash	All Bond	All Equity	Balanced
50000	100.00%	100.00%	97.33%	100.00%
60000	99.99%	100.00%	87.57%	99.96%
70000	96.90%	100.00%	67.78%	95.68%
80000	50.08%	99.37%	46.15%	62.09%
90000	4.27%	78.44%	27.98%	20.16%
100000	0.06%	36.22%	16.99%	3.42%
110000	0.02%	12.06%	9.48%	0.33%
120000	0.03%	3.42%	4.88%	0.02%
130000	0.01%	0.83%	3.02%	0.05%
140000	0.01%	0.10%	1.50%	0.01%
150000	0.04%	0.03%	0.93%	0.03%
160000	0.05%	0.02%	0.43%	0.04%
170000	0.05%	0.04%	0.21%	0.03%
180000	0.01%	0.00%	0.16%	0.01%

Probabilities of Insolvency



Does the Asset Model Matter?

Asset Models

- Random Walk Model
- Jon Carter
- Vector Autoregressive (VAR)
- Regime Switching VAR (RSVAR)

Asset Models Comparison

Asset Models	Closet Asset Match	Approximate Initial Asset Required for less than 5% probability of insolvency
RW	All Cash	82000
JC	All Cash	87500
VAR	Balanced	94500
RSVAR	All Cash	90500

Comparison of Alternative Definitions of “Closest Match”

Comparison of Different Definitions

- Assume we have initial asset amount of 95000 and we use VAR(1) as our asset model

Comparison of Different Definitions

Lowest Mean Squared Ultimate Surplus

Lowest

Portfolios	Probability of Insolvency	Mean Ultimate Surplus	Median Ultimate Surplus	Mean Squared Ultimate Surplus (closest billion)	Mean Shortfall
All Cash	6.03%	7030	10306	47	-3903
All Bond	6.25%	10369	14821	51	-4156
All Equity	14.41%	87358	65568	190	-10135
Balanced	4.20%	27586	30466	79	-5036

Mean

Shortfall

Lowest Probability of Insolvency

Which is the “Best” Asset Model?

Comparison of Asset Models – Mean forecasts

- Sophisticated models do not beat Random Walk model in terms of mean forecast
- RSVAR outperforms with in-sample data—but largest number of parameters

Evaluation of Interval Forecasts

Christoffersen(1998)

- $P(X \text{ falls in its } p\% \text{ confidence interval})=p$
- So we should expect approximately $p\%$ of the actual data falls in the $p\%$ confidence interval
- Likelihood ratio test can be used to test whether the observed % is significantly different from what is supposed to be.
- If likelihood ratio is >3.82 , statistically significant at 5%

Evaluation of Interval Forecasts

- Quarterly sample data is split into in-sample data(1981Jan-1995Oct) and out-of-sample data (1996Jan-2004April)
- In-sample data is used for parameter estimation
- Construct one-step-ahead $p\%$ confidence intervals by using the four models. ($p=50,55,60,65,70\dots95$)

Interval Forecasts Comparison

Models	Width of 95% Confidence Interval
Random Walk	0.46
VAR(1)	0.46
RSVAR(1)	0.38
Regime Switching Random walk (3 regimes)	0.52

Evaluation of Interval Forecasts -Share Price Return

- All Models provide wide intervals for Share Price Return
- Regime Switching model provide narrowest intervals with reasonable coverage.
- For long term projection, structural changes are likely to occur, regime switching model might be more robust

Further Research

Further Research

- Continue interval forecast comparisons
- Develop a new asset model?
- Does the liability cashflow model matter?