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Assessing & Monitoring Insurance Liability Uncertainty

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Abstract

Financial reporting of general insurance liabilities to the Australian Prudential Regulatory Authority (APRA) has required, for a number of years now, such liabilities to be determined:

- On the basis of a present value based central estimate liability, plus
- A risk margin to give a present value liability with at least a 75% probability of adequacy (PoA).

Beyond reporting and prudential management, the proper risk management, capital allocation and product pricing processes of modern general insurance management should be based on sound risk-based capital assessment, and liability risk and uncertainty measurement.

In this paper, we review current Australian actuarial practice in estimating liability uncertainty and variability, as well as liability variability correlation and diversification benefits. We identify a number of practical issues arising and consider how current practice relates to the actuarial control cycle.

Issues explored included:

- Comparison of differences in uncertainty results depending on what type of claims data is examined;
- Consideration of the measurement of uncertainty in past data utilising different time units, and whether inconsistent results could be obtained;
- Examination of fitting different probability distributions for insurance claims, including Log-Normal, and what they imply;
- Assessment of diversification between classes, and whether there are practical alternatives to the current approaches;
- Examination of different ways to allocate diversification benefits by class and whether some methods have unexpected shortcomings; and
- Consideration of possible approaches to monitoring risk margin experience.

Keywords: general insurance, risk, risk margin, monitoring, volatility, correlation, diversification, probability, adequacy, sufficiency

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The views expressed herein are entirely those of the authors, Elaine Collins (eccollins@kpmg.com.au), Laurel Kong (laurelkong@kpmg.com.au), Stephen Robertson-Dunn (srobertsond@kpmg.com.au) and Felix Tang (felixtang@kpmg.com.au), and do not necessarily reflect those of the Institute of Actuaries of Australia or their employer, KPMG Actuaries.

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1 Overview

1.1 Introduction

Financial reporting of general insurance liabilities to the Australian Prudential Regulatory Authority (APRA) has required, for a number of years now, such liabilities to be determined:

- On the basis of a present value based central estimate liability, plus
- A risk margin to give a present value liability with at least a 75% probability of adequacy (PoA).

Under the Australian equivalent of International Financial Reporting Standards (IFRS) that commenced this year in Australia, general purpose reporting of general insurance liabilities are also required to include a margin for liability uncertainty (albeit not specified at a particular level of adequacy).

In addition, APRA's regulatory capital requirements are in principle risk based capital requirements that are built on the 75% PoA liability plus additional margins, and in other cases such as insurers in run-off, APRA requires capital reserves on the liability risks to be assessed and maintained at a 99.5% PoA before allowing release of reserves to shareholders.

Internationally, the International Accounting Standards Board (IASB) has recently concluded that a future IFRS for insurance accounting will require non-life insurance liability to be based on a discount reserving approach which will also include margins for risk and uncertainty. Furthermore, the prudential supervision and solvency principles currently being promoted by the International Associations of Insurance Supervisors (IAIS) are clearly moving in the direction of risk based capital management and other develops such as Solvency II in Europe are based on such principles.

Beyond reporting and prudential management, the proper risk management, capital allocation and product pricing processes of modern general insurance management should be based on sound risk-based capital assessment, and liability risk and uncertainty measurement.

Measurement of liability variability is therefore fundamental to all these needs of reporting, regulation and management.

In this paper, we review current Australian actuarial practice in estimating liability uncertainty and variability, as well as liability variability correlation and diversification benefits. We identify a number of practical issues to arising, and consider how current practice relates to the actuarial control cycle.

In exploring the issues in this paper, we have focussed on assessing risk margins at the lower probabilities of adequacy, since probabilities in the proximity of 90% or above often require approaches and considerations different from the standard approaches, which we are investigating. In these circumstances, additional issues in these other types of analyses including copulas and tail dependencies need to be considered.

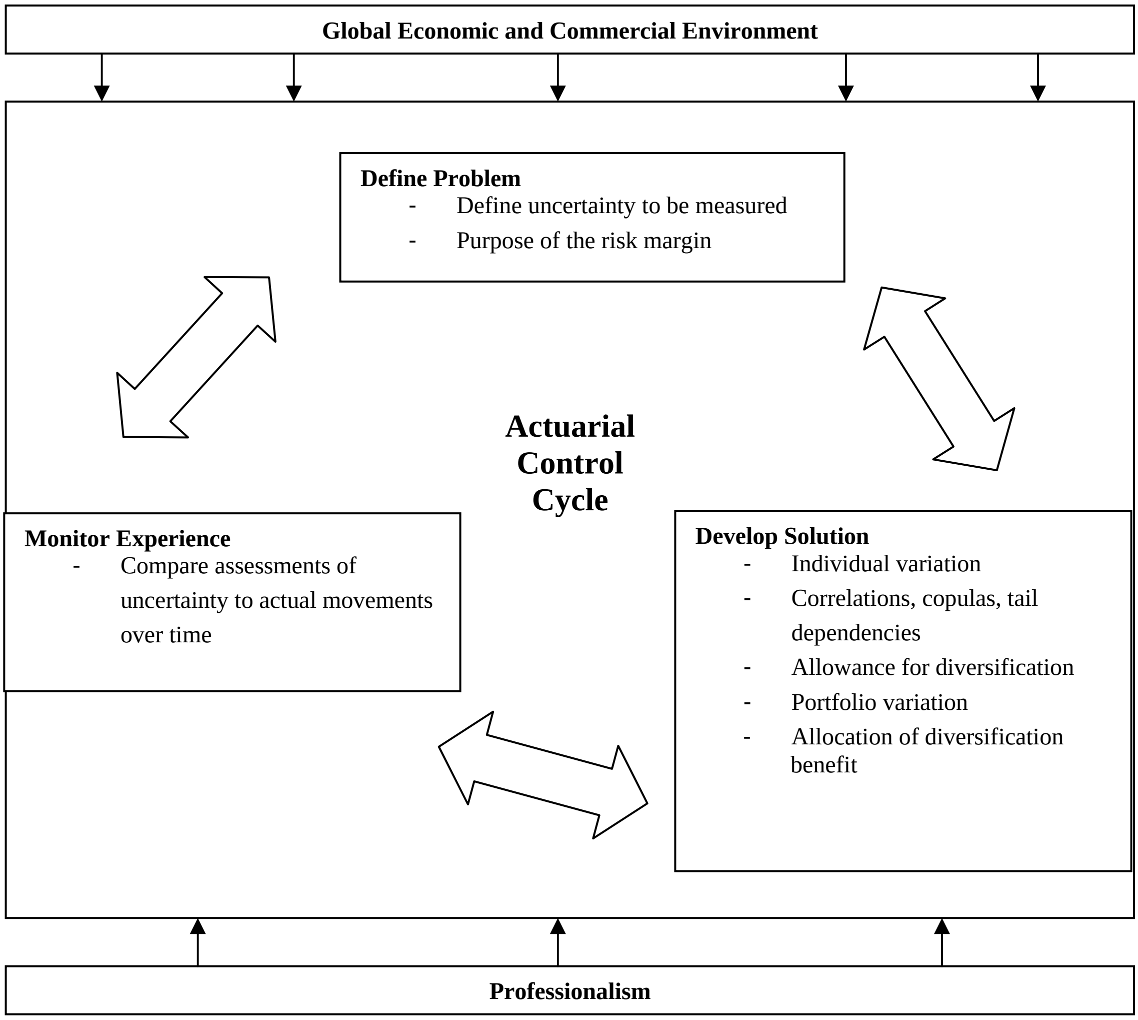
A simplified representation of the current Australian risk margin framework is presented in Appendix A for reference.

1.2 Actuarial control cycle & structure of paper

We have structured this paper around the actuarial control cycle.

The actuarial control cycle approach to problem solving might suggest that the issue of determining suitable risk margins could be expressed as per the following figure.

Figure 1: The actuarial control cycle



The application of the control cycle requires tools for providing a measure of uncertainty as well as monitoring the performance of these measures. The model used to assess uncertainty depends on the characteristics of the data, including how well the data is organised, and whether there is any missing data. Once uncertainty has been assessed, monitoring tools are required to allow one to compare actual variation experienced with the uncertainty expected.

Reflecting the actuarial control cycle, the sections of the paper are as follows.

In section 2, we discuss “defining” the problem of variability assessment, summarising the uncertainty risk margins are currently perceived to incorporate, and looking at the challenges of translating analysis to the required definition of uncertainty.

For measuring insurance liability uncertainty, some tools have been developed in the industry, such as the Mack technique, Bootstrapping and various others. To the extent one may have available some useful variability or correlation information on payments or other liability building blocks, these may need to be converted to a usable measure on liability results via one of these tools.

In section 3, we examine current "solutions" commonly used in Australia to produce estimates of risk margins, highlighting potential shortcomings. We utilise historical data in summarised claim runoff triangles, the Mack technique to assess past variability and Bootstrapping to build empirical probability distributions. We have investigated some of these in more detail, to assess their validity and whether or not an alternative exists.

In section 4, we examine the subject of risk margin "monitoring", in order to put forward potential ways of enhancing the current monitoring processes.

In section 5, we discuss our main conclusions.

This paper focuses on practical issues surrounding assessing and monitoring measures of uncertainty for insurance risk derived from underwritten general insurance contracts, but the principles espoused might be applicable to assessing and monitoring other risks faced by insurers.

2 Interpretation of uncertainty

In this section, we discuss the definitions that underlie the existing Australian insurance liability model, and explore some areas where the translation of analysis to these definitions may be confusing. The latter includes the varying results that can be obtained for a single class, using a single tool, depending on how the data is presented (i.e. underlying type of data and time unit).

2.1 Definitions

The approach used in Australia with regards to assessing and monitoring insurance liability uncertainty, is based on the current regulatory framework of the Australian Prudential Regulation Authority (APRA) and Professional Standards of the Institute of Actuaries of Australia (IAAust). Under the current regulatory framework, expected values are termed central estimates, and the regulation requires insurance liabilities to be discounted with a separate risk margin applied.

Central estimate liability = Expected present value of the liabilities, such that if all the possible values of the liabilities are expressed as a statistical distribution, the central estimate is the mean of that distribution.

Risk margin = the amount by which a liability provision is greater than the central estimate liability to achieve the required probability of adequacy (PoA).

Insurance liability = Central estimate liability plus risk margin

Outstanding claims liability = Liability for payments in relation to claims that occurred prior to the valuation date

Future claims liability (also known as premium liability) = Liability for payments in relation to claims that are expected to occur after the valuation date during the unexpired risk period related to “unearned premiums”

Diversification benefit = The amount (or proportion) by which the overall portfolio risk margin for an insurer is less than the sum of the individually assessed risk margins, owing to the risks not being fully correlated.

Coefficient of variation (CoV) = Standard deviation divided by mean

GPS310 (to replace GPS210) = General Insurance Prudential Standard on Audit and Actuarial Reporting and Valuation, expected to be issued by APRA in January 2006 with applicability from 30 June 2006

PS300 = Professional Standard 300 on general insurance liability valuations issued by IAAust

2.2 Sources of uncertainty

The categories of uncertainty regarding general insurance claim outcomes are described in the literature as follows:

- **Model selection error**, deriving from the difference between the actual process generating the claim experience and the closest member of the family of claim experience models selected;
- **Parameter error**, deriving from the sampling error in model parameter estimates;
- **Parameter evolution error,** deriving from the inclusion in a model as constants any parameters which are in fact subject to change over time;
- **Process error,** deriving from the random departure of future claim experience from model expectations.

Houltram [2003] adds two further sources of uncertainty:

- **Input data error**, any erroneous data will similarly have introduced uncertainties into the estimate of the parameters;
- **Judgement error,** deriving from the risk that actuary judgement that calls on expected future non-random changes might be wrong.

An alternate way to examine variability is to consider that variability is comprised of two elements – independent and systemic variation, discussed in Bateup & Reed [2001].

Independent risk, as its name implies, is considered to be that portion of variability that is subject to the law of large numbers or pooling of risk. That is, as a class of business grows, its independent variation can be expected to reduce proportionally as this reflects the variability of many smaller risks being combined.

Systemic risk is considered to be variability introduced by environmental, legal, or other factors such as process changes that affect the underlying risks in a correlated way. It is generally considered that systemic variability is not diversifiable. However, we note that systemic influences may not affect different classes in the same way. Therefore, we further clarify that systemic variability is not diversifiable within a class, but might be diversifiable to some extent across classes.

2.3 Cautions & practical issues

There are a number of practical issues and cautionary notes to be considered related to defining the problem of assessing uncertainty including:

- **Timeframe:** APRA’s existing GPS 210 and proposed GPS 310 are consistent in stating that the risk margin is determined on a basis that is intended to value the insurance liability of the insurer at a 75% level of adequacy, with no explicit statement about the timeframe over which this level of adequacy is intended to be applicable. On the other hand, APRA states that the current general insurance capital adequacy framework has a goal to reduce the probability of the insurer’s capital base being eroding to 0.5% over a one-year time horizon. APRA have clarified that an infinite time horizon interpretation of a risk margin on insurance liabilities is the appropriate definition.

- **Translating assessed past volatility to ultimate liabilities:** Uncertainty in ultimate insurance liabilities is being estimated, but past actual data is in the form of, inter alia claim numbers, payments and reported incurred. We note the difficulty of translating variability measures on such historical data to the uncertainty of ultimate insurance liabilities and the use of tools, such as Mack and Bootstrapping techniques for this purpose. Measures of past variability do not allow for future process volatility. Additionally, Houltram [2003] points out that measures that are based only on past data do not take account of actuarial judgement error incorporated in assessing the ultimate liabilities.
- **Skewness:** Although the central estimate is defined as the expected value of a distribution, or the mean, where a distribution is skewed (often applicable and assumed for general insurance), the available sample of data might lead to a biased estimate. As Houltram [2003] points out, it is important to consider any under-representation of larger claims or events in the available data in order not to understate results.
- **Gross or net data:** We have observed that data used in the industry for variability assessment can be either gross or net of reinsurance/non-reinsurance recoveries. Some published papers particularly rely on net data. While the net liability requires the application of a risk margin, if recoveries are not consistently and directly proportional to gross, net data can involve the juxtaposition of two or more distributions, (a gross distribution and a recoveries distribution). This significantly impacts the observed distribution shape, introduce obscure serial correlation in the data and potentially leading to a multi-modal net claims distribution. Variability measures may also be a mixture of true claim variability and changes in recoveries (e.g. changing reinsurance structures over time). Actuaries should carefully consider the level of gross and recovery timing mismatch in the data (and/or recovery or reinsurance structure changes and, if significant, consider basing variability measurement on ground-up gross data. In this case, net results including risk margins may be obtained by applying expected recoveries under the associated PoA scenario to the gross central estimate including implied gross risk margins.
- **Assumptions relevant to liability uncertainty:** It seems clear under GPS310 that variability in all claim and recovery assumptions, as well as future claim inflation assumptions are relevant to APRA's definition of uncertainty. However, it appears that variability from discount rates and management expenses is excluded from the APRA measurement of uncertainty. Under the Australian regulatory regime, a "risk free" rate (or government bond rate) is prescribed for determining discounted insurance liabilities. In practical terms this assumption is effectively fixed, with no consideration given within the insurance liability to either individual insurer investment strategy or potential variation around the implied future investment earnings. On the other hand, management expenses are a source of variation in the central estimate, but their uncertainty is not generally explicitly considered as part of the risk margin. Whether or not this is appropriate is not clear from legislation or standards, but we have assumed that risk margins disregard variability in management expenses and relate only to the size of the underlying pure claims variability.
- **Past / future variability:** Assessment of past variability may require some adjustment prior to application as a measure of future variability. This can be for various reasons, for example, changes in portfolio size or where a full cycle of economic or environmental conditions may not be present in the data. Where a portfolio experiences growth, variability should reduce, but the observed variability will include periods of greater variation, leading to potential overstatement of current risk margins. Likewise, a shrinking portfolio may lead to understatement of margins. If recent economic or environmental conditions have been fairly homogeneous (e.g. the last decade of economic growth and weather conditions), the measured insurance liability variability may have been understated relative to more average conditions.

- **Claims in the unexpired risk period:** The relationship between outstanding claims and future claims liability (or premium liability) variability has not been extensively researched, but it has been highlighted that there should prima facie be more uncertainty associated with claims that have not yet occurred compared with those that have already occurred. The problem is typically approached by applying a multiple to the variability measure for the outstanding claims liability to estimate the future claims liability variability. Bateup & Reed [2001] used factors of 1.25 for long tail and 1.75 for short tail, the higher factor for short tail being attributed to the potential for catastrophes or other adverse events to happen in the unexpired risk period. There is no clear justification for the size of factors being applied in the industry and one can argue they potentially ignore additional available information. For example, it might be reasonable to consider assessments of the future by experts in various areas, such as weather, economic outlook, legal positioning, etc.

2.4 Type of claim data

Past payment and reported incurred data is often used by current methods for measuring volatility, using a chain ladder framework. Given that we are interested in the ultimate liability, which is simply the sum of payments after an infinite timeframe, perhaps payments are the most relevant item around which to measure variability. However, this might disregard variation added or subtracted by the choice of valuation method (relative to a simple chain ladder approach).

It is also worth noting that while choice of valuation method may itself introduce variability, ideally the method chosen should reflect, amongst other things, the drivers of cost for the portfolio. Therefore, the chosen method may in fact already be guided by where stability in experience is expected.

The most common tools for measuring variability pre-suppose that a chain ladder or link ratio method is used to value the central estimate. Therefore, where the underlying central estimate valuation method differs, the measured variability may not be directly applicable to the liability central estimate. We also observe that the results might vary depending on whether payments or reported incurred formed the basis of analysis, which in turn might differ from the data relied upon for the central estimate.

If we were to relate measurement of variability to the method chosen, items of interest for measuring variability might include:

Table 1: Items of interest for measuring variability

Method	Measure variability of…
Chain ladder on payments	Payments
Chain ladder on incurred	Reported incurred
PPCI	Numbers reported Payments PPCIs
PPCF	Numbers reported Numbers finalised or finalisation rates Payments PPCFs
PCE	Payments or payment to outstanding factors Case estimates or case estimate development factors
BF (on reported incurred)	Reported incurred Loss ratios

Now we provide some example variability results utilising different claim data types.

In order to understand the different picture of variability that might be presented by some of these approaches, we have taken a dataset of actual data and applied the Mack technique. We have used the following claim data types for both a short tail class and long tail class:

- Payments
- Reported incurred
- Claim numbers reported

The results are shown in Table 2 below:

Table 2: Variability results for claim payments, reported incurred and reported numbers

	Mack technique measure of variability based on:		
Portfolio	Payments	Reported Incurred	Reported numbers
Short tail	30.4%	-41.3%	17.8%
Long tail	13.6%	32.4%	17.7%
	Implied mean as % of that for Short Tail (Payments):		
	Payments	Reported Incurred	
Short tail	100%	-67%	
Long tail	421%	222%	

Immediate observations include:

- The measures of variability for the three types of data give quite different results. However, we note that they are not expressed on the same basis. We believe this highlights an important point regarding interpretation of the Mack results, that the implied variability is expressed as a proportion of the excess above the base variable. Hence, for payments, it is a proportion of outstanding claims (i.e. ultimate paid less paid to date), while for reported incurred, it is a proportion of IBNR (i.e. ultimate incurred less reported incurred).

- The Mack technique on reported incurred produced negative results for the short tail class. The reason for this outcome was the portfolio experiencing historical downwards movement in the reported incurred cost over the periods of development. This led to negative IBNR, which means that the positive variation measure based on reported incurred analysis, when expressed relative to that negative IBNR, becomes negative.
- It would therefore appear necessary to re-express the reported incurred measures in terms of the outstanding liability, rather than IBNR. Once the reported incurred percentage is re-expressed relative to the liability in each case, a positive variation should result.
- It does not appear possible to re-express claim number variability in terms of the liability. Put simply, we would expect this measure to be incomplete because it does not include variability derived from claim size. Intuitively, given that claim frequency tends to stabilise more quickly than average claim size, for most if not all classes, it might be expected the variability measure from claim numbers alone could be biased towards understatement. However, the actual relationship is complicated by many factors such as existence of large claims or nil claims and structural changes in the data. Also, re-expressing it in terms of the liability would mean we were comparing a numbers count to a dollar figure. This appears intuitively misleading.

The next table includes the re-expressed results. In this table, the percentage variability can be interpreted as an estimate of the coefficient of variation (CoV).

We will focus on the payments and reported incurred measures further in this illustration and dropping the reported numbers measure for simplicity. However, this could still provide useful insight to understanding the components of variability in the chosen valuation methods. For example, claim numbers reported form an input to the PPCI method, where the ultimate number of claims incurred is usually projected using the chain ladder method. However, it should be noted that a claim numbers measure could not be re-expressed in a way that would allow comparison with measures for payments and reported incurred.

Table 3: Reported incurred re-expressed

	Adjusted Mack technique measure of variability based on:	
Portfolio	Payments	Reported Incurred
Short tail	30.4%	46.0%
Long tail	13.6%	13.2%
	Implied mean as % of that for Short Tail (Payments):	
Portfolio	Payments	Reported Incurred
Short tail	100%	60%
Long tail	421%	545%

The results are now shown in a comparable way in Table 3, we can consider further how the variability of payments and reported incurred might be expected to inter-relate, observing the empirical evidence above.

The re-expressed table above shows that:

- For the short tail class of business, the reported incurred measure shows much higher variability than the payments measure.
- For the long tail class of business, the two measures are actually quite similar, although the reported incurred measure is slightly lower.

- The long tail class of business has apparently lower proportionate variability than the short tail class of business. We observe that the former is an overall larger business class, and therefore this may be a size-related factor rather than something inherent to the class.
- Within the classes, we note that the liability implied by the payments and reported incurred data differ, and this appears to influence the relative "percentage" variability observed.

We explore this last point a little more. The Mack technique calculates not only a measure of variability of the underlying data, but also an implied mean insurance liability in its interim steps. If the underlying data type is changed, say from paid to using the incurred triangle, even though they originate from the same underlying claims experience, the estimate of the implied mean is likely to be different, as is the variability percentage.

In the case of the short tail class, the implied mean is substantially different, with the reported incurred data producing much higher percentage variability results. Our investigation indicates similar dollar variability, but a very different implied mean presented by the Mack technique. This might indicate that it would be inappropriate to apply percentage variability derived from the Mack technique to a central estimate derived from alternate valuation methods. Instead, the estimated dollar variability might be a better measure of variability.

In the case of the long tail class, the percentage variability is similar. However, we note that the implied dollar variability for reported incurred is in fact higher than the implied dollar variability for payments. This casts doubt on assuming dollar variability is a reliable constant even where similar percentage variability is obtained from different analyses.

In light of the above issue where the implied central estimate was simply not comparable across different types of data, we have re-performed the analysis on an alternate set of data for short tail and long tail classes. The results are presented below:

Table 4: Variability results in alternate short tail and long tail classes

	Adjusted Mack technique measure of variability based on:	
Portfolio	Payments	Reported Incurred
Alternate short tail	9.5%	8.8%
Alternate long tail	10.2%	9.9%
	Implied mean as % of that for Short Tail (Payments):	
Portfolio	Payments	Reported Incurred
Alternate short tail	100%	97%
Alternate long tail	391%	531%

Results from these alternate data are quite similar for both measures on both the long tail and short tail classes. The alternate long tail class has differing implied means underlying each measure (similar to the original class analysed), although the alternate short tail class has relatively similar implied means. The less divergent sources of variability may be due to the smaller differential, however, for a large portfolio that dominates an insurer's balance sheet, even relatively small differences can result in large dollar movements in risk margins.

We conclude from the above analysis that:

- Resulting variability is expressed as a proportion of the excess above the base variable when utilising the Mack technique. Hence, for payments, it is a proportion of outstanding claims (i.e. ultimate paid less paid to date), while for reported incurred, it is a proportion of IBNR (i.e. ultimate incurred less reported incurred). Therefore, in any such analysis, variability percentages need to be re-expressed to ensure like is compared with like.
- The percentage variability is clearly influenced by the implied mean produced in the variability analysis. Direct application of these percentages (a widespread use of the Mack technique) to central estimates may be inappropriate, particularly where the change in dollar variability is not directly proportional to the change in implied mean. At the very least, the difference in implied mean (either through the Mack technique or by the actuary in setting a central estimate) needs to be considered before applying results of the Mack technique. We observe that Houltram [2003] alluded to this issue, where he noted that the analytical tool used to derive a measure of uncertainty should rely on an underlying method that gives a well-fitted mean close to the adopted central estimate.
- Even where the estimated liability is close, different data sets yield differing (if less so, relatively) views on variability, which might be simply a characteristic of the data or might suggest that direct application of the variability results to a liability estimated from a different method is unsuitable.

In conclusion, we believe it is important to consider different types of data when determining Mack measures of variability, but to apply caution in the application of these variability measures. Additional considerations might include which of the above measures more closely reflects the data used in setting the adopted central estimate.

It might also be considered that where neither a chain ladder on payments or a chain ladder on incurred is adopted for the actual central estimate liability, that the adopted model should provide a better fit and therefore smaller variance than indicated by a chain ladder based Mack method. This might lead to a conclusion that the smaller variance (in dollar terms) is more relevant when results conflict, and in fact might still be an overestimate. However, one should not take this conclusion too far, to the point of discarding evidence that variability is high.

2.5 Length of unit data period

Depending on the volume of history available, claims data is usually tabulated into run-off triangles with yearly, half-yearly, quarterly, or even monthly intervals for actuarial analysis. Although each of these alternatives may merely summarise identical underlying transactional data in different ways, we investigated whether projections for each could produce a different liability estimate and assessed level of risk. We observe that past variations can be offset or amplified as a result of different ways of summarising the data.

A simplistic view might be taken that by summarising data into broader periods, the offsetting effects within a data cell become more prevalent and volatility is dampened. However, this may not be generally true. To illustrate this, let us consider the following two scenarios of triangular general insurance data.

Figure 2: Triangular data with different time units - scenario one

Scenario 1 - incremental payments in time unit 1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	100	100	100	100	100	100	50	50	50	50	50	10	10	10	10	10
B	100	100	100	100	100	100	50	50	50	50	50	10	10	10	10	
C	100	100	100	100	100	100	50	50	50	50	50	10	10	10		
D	100	100	100	100	100	100	50	50	50	50	50	10	10			
E	100	100	100	100	100	100	50	50	50	50	50	10				
F	100	100	100	100	100	100	50	50	50	50	50					
G	50	150	50	150	50	150	25	75	25	75						
H	100	100	100	100	100	100	50	50	50							
I	100	100	100	100	100	100	50	50								
J	100	100	100	100	100	100	50									
K	100	100	100	100	100	100										
L	100	100	100	100	100											
M	100	100	100	100												
N	100	100	100													
O	100	100														
P	100															

Scenario 1 - incremental payments in time unit 2

	0'	1'	2'	3'	4'	5'	6'	7'
A'	300	400	400	250	200	160	40	40
B'	300	400	400	250	200	160	40	
C'	300	400	400	250	200	160		
D'	300	400	400	250	200			
E'	300	400	400	250				
F'	300	400	400					
G'	300	400						
H'	300							

In the above hypothetical scenario the pattern of payments is the same for each incident period except for incident period G. However, this variance is totally offset in incident period D when the triangle is summarised further into two time units per period.

Figure 3: Triangular data with different time units - scenario two

Scenario 2 - incremental payments in time unit 1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	100	100	100	100	100	100	50	50	50	50	50	10	10	10	10	10
B	100	100	100	100	100	100	50	50	50	50	50	10	10	10	10	
C	100	100	100	100	100	100	50	50	50	50	50	10	10	10		
D	100	100	100	100	100	100	50	50	50	50	50	10	10			
E	100	100	100	100	100	100	50	50	50	50	50	10				
F	100	100	100	100	100	100	50	50	50	50	50					
G	50	50	150	150	50	50	75	75	25	25						
H	100	100	100	100	100	100	50	50	50							
I	100	100	100	100	100	100	50	50								
J	100	100	100	100	100	100	50									
K	100	100	100	100	100	100										
L	100	100	100	100	100											
M	100	100	100	100												
N	100	100	100													
O	100	100														
P	100															

Scenario 2 - incremental payments in time unit 2

	0'	1'	2'	3'	4'	5'	6'	7'
A'	300	400	400	250	200	160	40	40
B'	300	400	400	250	200	160	40	
C'	300	400	400	250	200	160		
D'	200	500	300	300	150			
E'	300	400	400	250				
F'	300	400	400					
G'	300	400						
H'	300							

In this second scenario, the payments for incident period G is changed slightly such that effectively the "period" of the oscillation is extended. This time the trend is not offset but rather amplified in period D' when the triangle is summarised.

The above simple scenarios illustrates that the same data summarised in broader time units does not necessarily equate to dampened volatility, as the true relationship is, among other effects, complicated by actual variation in the data, which may be further impacted by cyclical impacts, random variation and potential "anchoring" effects such as incomplete data updates (e.g. data that is grossed up or rolled forward).

With the above scenarios in mind, we used the Mack technique on payment triangles for a long tail and a short tail class with quarterly, half-yearly and yearly time units.

Table 5: Variability results with quarterly, half yearly and yearly time units

Portfolio	Mack technique measure of payment variability: Quarterly	Half-yearly	Yearly
Short tail	32.1%	40.1%	30.4%
Long tail	12.4%	12.9%	13.6%
Portfolio	Implied mean as % of that for Short Tail (Payments): Quarterly	Half-yearly	Yearly
Short tail	100%	104%	100%
Long tail	426%	419%	419%

Immediate observations include:

- For short tail data, Mack's measure of variability increases when the data is summarised from quarterly into half-yearly, but reduces to close to the quarterly level when summarised further to yearly. This likely highlights that yearly data may represent an over summarisation for the short tail data. In the case of the quarterly results, it may be possible that the quarterly result includes some roll forward estimates and/or non-updated data that are artificially understating the variability. This is a point of which to be cautious.
- For long tail data, the three measures of variability seem consistent across time units, although slightly increasing as the time unit broadens. One possible effect contributing to the increase could be that the degrees of freedom (as indicated by the number of data cells in a triangle) reduce as the data triangle is summarised further. The Mack technique does respond to degrees of freedom available in the data. Therefore, a slight increase in the Mack measure of variability could be expected, although the data may be stable regardless of the summarising.

An important actuarial implication from the above discussion is that when an assumption is made for the future uncertainty by assessing past experience, the selected time unit under which the analysis was performed may not present a complete picture of underlying volatility.

3 Assessment of risk margins

In this section, we consider the distribution of insurance liabilities, the assessment of risk margins at the 75th percentile, whether there is any alternative to the current correlation and diversification framework, and possible approaches to allocate diversification benefit to business classes.

3.1 Shortcomings of current approach

There are a number of limitations of the current framework for assessing individual class and portfolio level risk margins. This is to be expected, as any simplification of a real life process is imperfect, and in a statistical sense, estimation of higher order moments of a distribution is more difficult than lower order moments. We note that the mean involves the first moment and the standard deviation the second moment.

Some of the limitations include:

- **Inappropriate distributions:** The assumed underlying distribution is often assumed to be Log-Normal, even though it may not be the case.
- **Non-additive nature:** The distribution of the portfolio is assumed to be the same shape as the underlying classes, even though these distributions may not be additive. In particular, Log-Normals are not addictive.
- **Negative risk margins:** For skewed distributions, the mean is not equal to the 50th percentile, and if it is above the 75th percentile, a negative risk margin can result for Australian reporting, which might be considered undesirable, notwithstanding that on a purely statistical basis, a given probability of sufficiency does not change in its degree of conservatism. APRA has addressed this limitation by requiring margins to be no less than one half the coefficient of variation. One might consider whether the potential bias in historical data introduced through a skewed distribution might result in the best estimate liability being determined below the "true mean" (and closer to the median).
- **Correlations:** The matrix of correlations between each pair of classes is frequently determined with a large degree of judgement, as analysis and verification of historical correlations is difficult.
- **Tail dependencies at higher percentiles:** The application of correlations assumes a linear relationship between distributions (i.e. assumes a "Pearson R" correlation coefficient). In situations where a linear relationship is not a good approximation, tail dependencies on extreme events or other non-linear correlations structured by means of copula analysis should be considered.
- **Allocation of diversification:** Methods of allocating diversification benefits back to individual classes may not reflect the actual drivers of the diversification benefits.

3.2 Probability distributions

A typical assumption among actuaries for general insurance outstanding claims liabilities is that of a Log-Normal distribution. The preference for this distribution appears to consider such features as:

- The skewed nature of the distribution – it is generally accepted that liability outcomes are skewed.

- The inability of the distribution to fall below zero, as the liability is not generally expected to fall below zero.

In order to examine whether a Log-Normal distribution is appropriate, we have used the Bootstrapping technique on cumulative payments for two short tail and two long tail classes, and then attempted to fit different distributions to the outcomes of these classes. We have selected Bootstrapping, rather than Mack, as it allows an empirical distribution to be produced based on residuals.

After this procedure has fitted a probability distribution to each class, the fitted distribution's mean is not necessarily the same as the mean derived from our valuation method adopted to evaluate central estimates. However, for each individual class, we have checked that they are similar, as shown in the graphs. If the means derived from the valuation method and variability technique were very different, we would have concerns regarding either or both our implied mean liability and/or the interpretation of the variability results obtained.

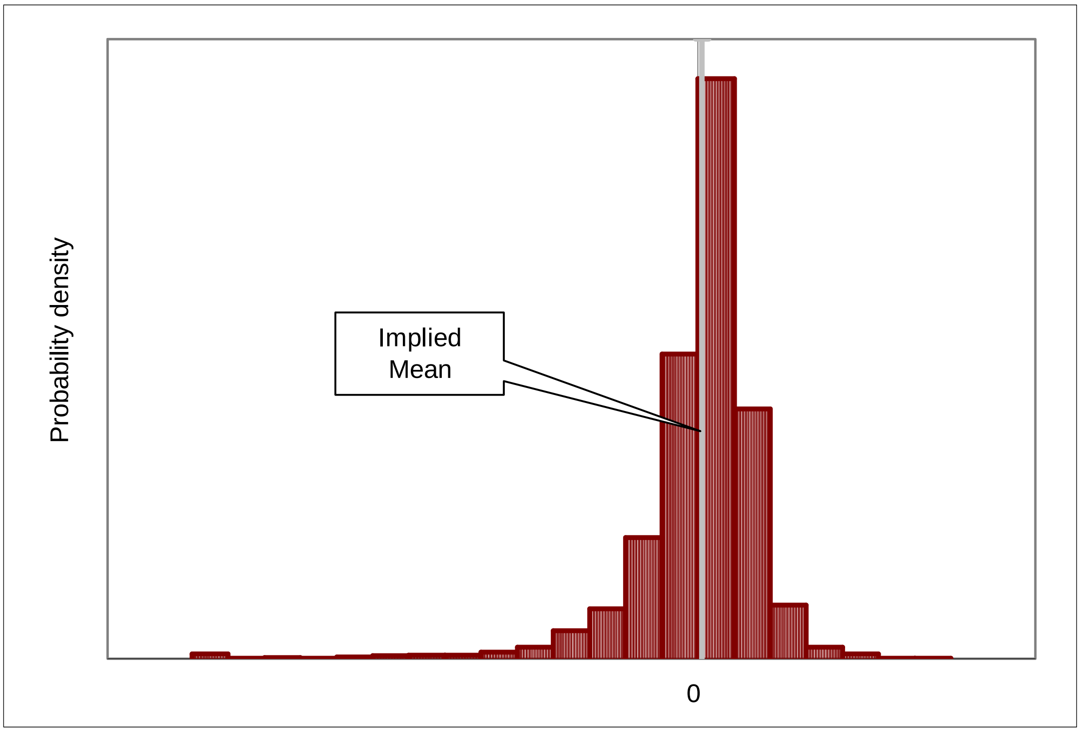
In order to derive a distribution of possible values of the liability, procedures similar to those detailed in England [2002] were followed:

1. The chain ladder factors were mechanically selected and used to back-project historical points of cumulative data, which are then converted to historical back-projected incremental payments.
2. Pearson residuals have been calculated using incremental payments, comparing historical actuals to these historical back-projected payments, adjusted for the degrees of freedom (that is, the volume of data). Pearson residuals were used as they do not superimpose an assumed statistical distribution.
3. The Pearson residuals have been randomised and multiplied to the back-projected historical numbers, to produce 5,000 theoretical variations on the fitted numbers. Our simulations produced 5,000 such variations.
4. Each of the variations has had a cumulative chain ladder approach re-applied to produce a revised projection each time.
5. The 5,000 revised projections can be graphed to produce an empirical distribution.
6. Different statistical distributions are then fitted to the empirical distribution to see which provides the best fit. We have used the package @Risk for this purpose.

3.2.1 Short tail classes

A histogram of liability outcomes has been presented below for the selected short tail classes.

Figure 4: Histogram for short tail class A

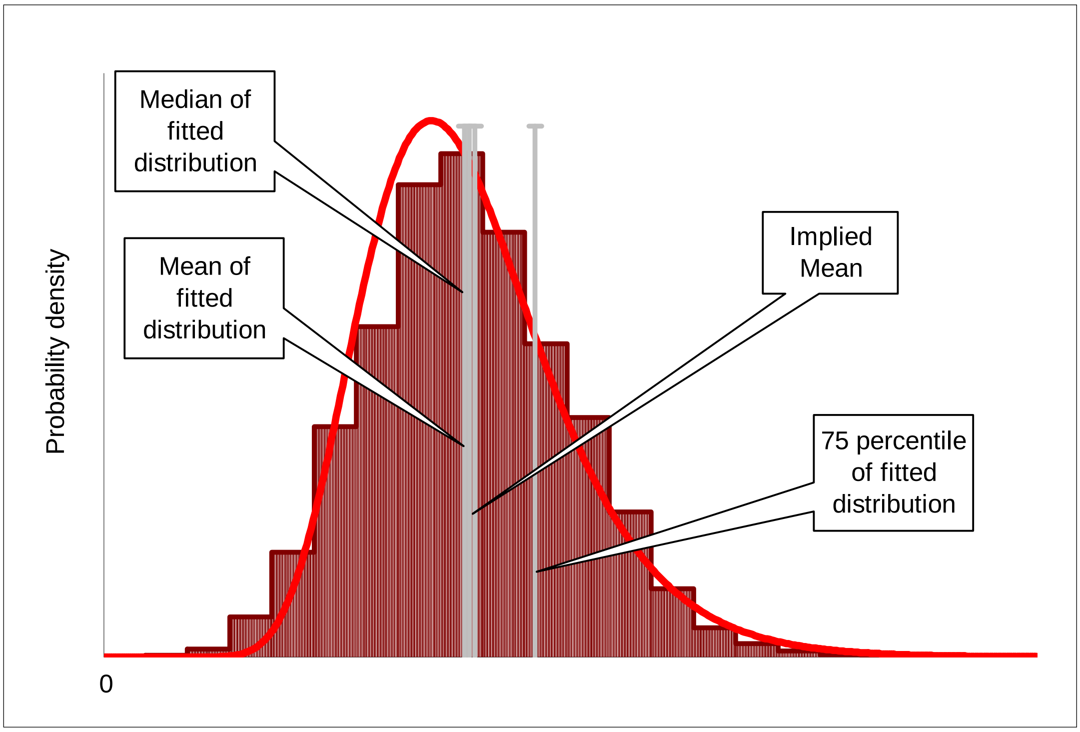


Immediate observations on short tail class A include:

- That some outcomes are indeed below zero. Whether or not this could occur depends on the data available for analysis. As our data was provided net of recoveries, where time lags to recovery can be significant, it is possible for a negative net liability to be calculated. However, while it might be no surprise that some claims are fully paid and effectively have negative estimates until recoveries are received, it would seem unusual that the entire liability for an ongoing class could be negative. Therefore, although the second "desirable" feature of the Log-Normal above has perhaps not proven true using this tool, this may suggest ill-behaved data rather than violation of what would intuitively appear a sensible assumption. Nonetheless, this data and result highlights the points made earlier about net versus gross variability analyses.
- The variation is large due to some extreme outcomes. This large variation may not be readily apparent due to the infrequent nature of the more extreme outcomes. There appears to be some skewness present, although it appears to be the reverse of usual expectation (skewness to the left instead of to the right).

Subsequent to the above observations, we attempted to fit a range of distributions to the above outcomes. Unfortunately, no distribution could be found which would suitably fit this class.

Figure 5: Histogram for short tail class B



Immediate observations on short tail class B include:

- There were no negative outcomes.
- There is evidence of skewness.

Although other distributions, such as Inverse Gauss and Gamma, provided better statistical fits, the Log-Normal distribution was also found to provide a reasonable fit for this data, as illustrated on Figure 5 above.

3.2.2 Long tail classes

A histogram of outcomes for each of the long tail liability class has been presented below. We also attempted to fit a range of probability distributions.

Figure 6: Histogram for long tail class A

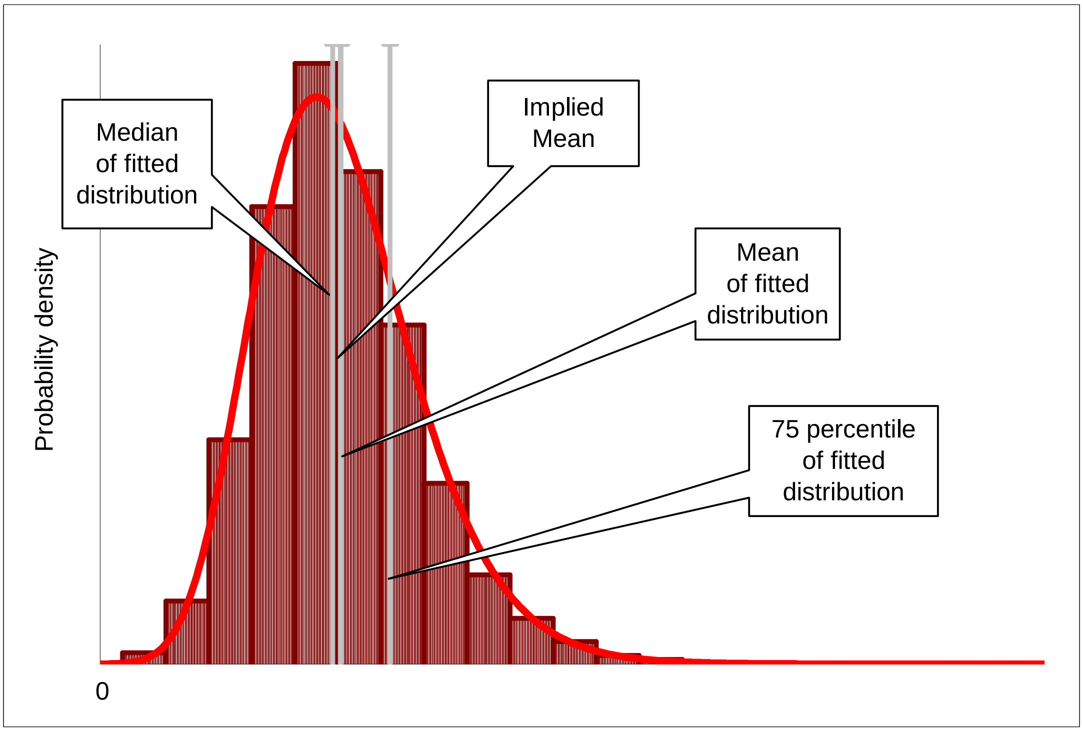
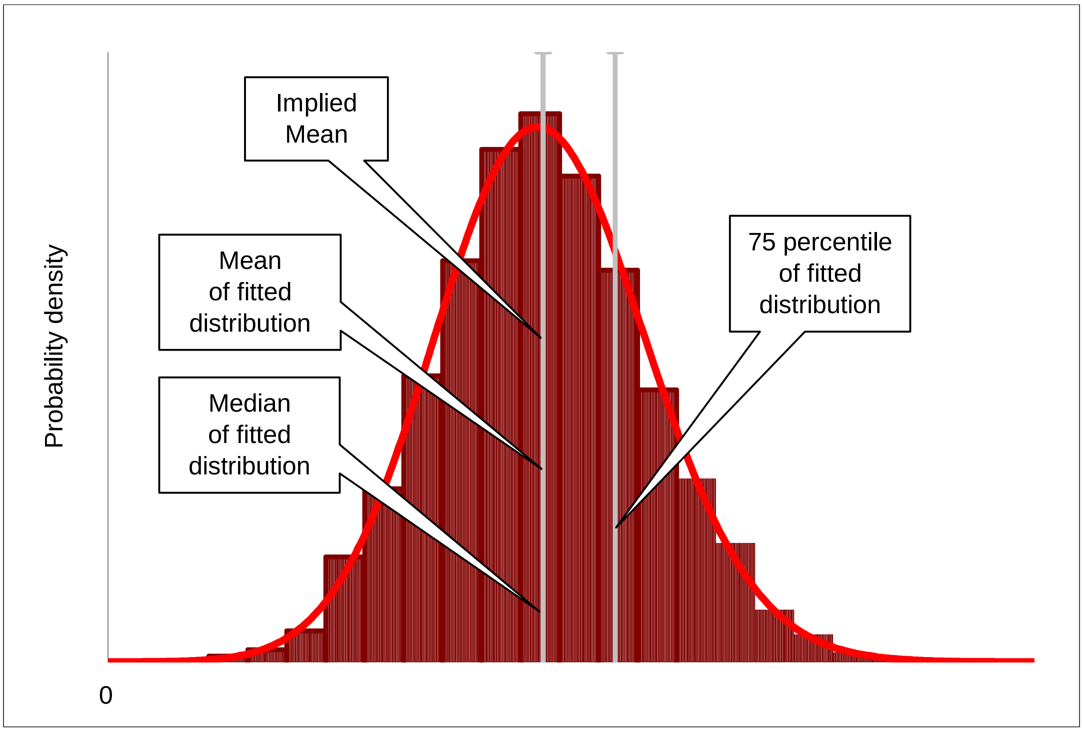


Figure 7: Histogram for long tail class B



Immediate observations on both graphs include:

- There have been no negative outcomes
- There is evidence of skewness.

Subsequent to the above observations, we have explicitly attempted to fit a range of distributions to the liability outcomes for both classes. Again, the Inverse Gauss was found to provide good statistical fit, so did Gamma and Normal respectively for long tail class A and B. Similarly to short tail class B, the Log-Normal provided a reasonable fit.

3.3 Diversification benefit and correlations

It is commonly anticipated that when an insurer underwrites more than one class of business, that the overall portfolio variability will be less than the sum of the individual class variability. This concept is similar to the law of large numbers used in statistics (which also has influences within a class, as it grows), whereby summing independent risks should lead to a proportionally lower variability within the probability distribution. However, as opposed to simple statistical examples, the example of insurance includes both independent variability and systemic variability.

As previously mentioned, the issue of reduction in variability by diversifying across different classes is generally referred as the "diversification benefit" (DB). The measured impact of this reduction is generally evaluated using standard statistical formulae.

First, a quick recap on correlations:

$$Var(X+Y) = Var(X) + Var(Y) + 2 \times \sqrt{Var(X) \times Var(Y)} \times \rho_{X,Y}$$

where,

X, Y = the central estimate liability for classes X and Y
$Var(X+Y)$ = combined variance of central estimate liability for the sum of class X and Y
$Var(X), Var(Y)$ = variance of central estimate liability for class X and Y respectively
$\rho_{X,Y}$ = correlation coefficient between the central estimate liability for classes X and Y

The correlation coefficient between classes X and Y is calculated as follows:

$$\rho_{X,Y} = \frac{CoVar(X,Y)}{\sqrt{Var(X) \times Var(Y)}}$$

where,

$$CoVar(X,Y) = E\{[X - E(X)] \cdot [Y - E(Y)]\}$$

$\rho_{X,Y}$ is the Pearson R correlation coefficient. It lies between -100% and 100%, and if X and Y are not independent, it measures how strongly they are linearly related to one another.

It is of note that the same sign linear transformations of the variables would not distort the level of correlation, i.e. if *a, b, c* and *d* are constants, and *a* and *c* are of the same signage, then:

$$\rho_{a \cdot X+b, c \cdot Y+d} = \rho_{X,Y}$$

It is also of note that perfect correlation, whether +100% or –100%, occurs if and only if one variable is a linear combination of the other. Priest [2003] observes that any Normal distribution can be expressed as a linear function of any other Normal distribution, so the correlation coefficient is a full descriptor of the relationship between two Normal distributions. However, a correlation coefficient cannot fully describe a non-linear relationship, the effects of which can become important between variables from non-Normal distributions. When dealing with skewed distributions, one must consider where the correlation measure (or what part of the density curve) is most relevant to its application.

We have earlier noted that we believe systemic variability can to some degree be diversifiable across classes of business. This is consistent with Bateup & Reed [2001], where correlation between classes of business has been expressed relative to systemic risk. As these correlations are less than 100%, it appears that Bateup & Reed [2001] also considers that systemic risk can be diversified to some extent.

Correlations could be said to be one of the most uncertain assumptions and possibly with the least empirical support in the task of estimating overall risk margins. We have observed the following:

- **Correlation between liabilities:** As previously noted, the variability in which we are interested is that of the central estimated liability, not payments or some other element. Similarly, the correlation is the correlation of the variability between the liabilities rather than between claim payments. Consequently, to the extent one may have available some useful correlation information on payments or other liability building blocks, these may need to be converted to a usable measure via some tool (such as Bootstrapping or Monte Carlo simulation) to produce indicative liability variability correlations (similar to the way some credit rating migration models that convert company profitability models to credit rating migration correlations). These would still then be subject to underlying assumptions, dependent on the tool and application of the result.
- **Correlation matrix based on judgement**: Given the difficulty in measuring liability correlations, it is not uncommon for the correlation matrix to be populated based on judgement or other intuitive arguments related to the qualitative characteristics of each of the insurance class pairs. As an approximate example, for Public Liability and Professional Indemnity one may expect that a higher correlation (say 80%) is more convincing than a lower correlation (say 25%), but a lower correlation may not be unreasonable for Motor and Aviation . The judgement behind these assumptions might be based on the perception that Public Liability and Professional Indemnity are alike – both related to claims relating to litigations from injured third parties, but such linkage by general reasoning is difficult to establish between Motor and Aviation.
- **Use of diversification rule of thumb:** Published rule of thumb calculations of diversification benefits give results as high as 40% or 50%. Derivation of diversification benefits utilising correlation coefficients from Bateup & Reed [2001] might give quite high diversification benefits, We observed, in around 2002, shortly after the publication of various research papers on industry benchmark risk margins, diversification benefits adopted by the industry were not as high as the rules of thumb implied, but were usually around 20% to 30%. In some cases, the insurers and actuaries used their own coefficient of variation and correlation models, and these seemed to support lower DBs. This highlights that changing assumptions can significantly alter the outcome of such models.

- **Recent decrease in diversification benefits:** Recently, we have observed insurers reducing DBs further, sometimes to around 10% to 15%. In some instances, the actuary has judgementally overridden the outcome of statistical models in favour of the lower DB. However, in other cases, the lower DBs have been derived assuming greater correlations within the existing models.

We note that issues around the difficulty of analysing and observing diversification and correlations, and the additional complications of actuarial judgements have already been covered by other literature. We have not explored further observation of correlations. However, we have briefly summarised other alternatives to the current approaches to calculating diversification benefits.

3.3.1 Examining portfolio data in aggregate

For an insurance group with subsidiaries within its group, it is common practice to use a full correlation matrix listing all classes of business, and to make an assumption for each pair-wise correlation. The assumed correlations are usually based on judgement supported by considerations at the class level, such as the similarity of the insurance class and geographical overlaps. The end result of this analysis is a portfolio level coefficient of variation, which is applied to the portfolio central estimate to obtain the diversified risk margin.

However, we investigated whether an alternative method for examining a portfolio coefficient of variation might simply be to combine the original class data, which might implicitly incorporate historical correlation between the sets of data being combined. We used a two class example, again one short tail and one long tail, deriving measures of variability for each class in isolation, and for the two classes in combination. The relationship of variability results from the two classes in combination compared to the individual classes might give an indication of the historic diversification benefit. From this, if required, one might solve for the implied correlation using the formulae described above. While not the end goal, such a solution may have been of interest for comparison against commonly assumed correlations.

The implied liability from the Mack technique on the combined portfolios was substantially different to the sum of the implied liability from two individual Mack analyses. This is perhaps not surprising, for were this not the case, cautions against combining unlike classes for central estimate analysis would not be so prevalent.

This casts doubt on whether solving for the correlation would have meaning when the implied projections were so different. Therefore, this confirms that the caution regarding combining different classes for central estimate analysis appears to also be applicable to variability analysis. It is possible this approach could work for other combinations of classes, but at a minimum a check should be completed on the implied liability, and even were this to prove consistent, some caution might still be applied to the results.

It is noted that, in any event, such a total portfolio approach would at best provide a correlation indication for the range of historic liability results reflected in the data. Correlation in the tails and tail dependence relevant for high PoA determinations will typically be understated in such analysis (because of low frequency of relevant events in the data and non-Normality impacting the technical calculation of the correlations).

3.4 Allocation of diversification benefit

In the same way that risk margins might be derived for a variety of purposes or reporting channels, so too the allocation of diversification benefits can be used for different purposes. APRA reporting in Australia requires that the central estimate liability and risk margin after allowing for diversification benefit are reported. Therefore, when there is a diversification benefit, the amount needs to be allocated back to the classes. However, the uses of allocation of diversification benefits extend beyond APRA reporting, and can form the basis of capital allocation and risk management, in which case a meaningful attribution of the diversification benefit to the contributing risk factors is required.

There are several possible approaches to the allocation of diversification benefits:

- Size of risk margin, central estimate liability or total liability (the most widespread approach)
- Uniformly reducing the probability of adequacy for each class
- Considering the impact of exclusion of classes

We will examine each one of these approaches in turn.

3.4.1 Allocation by pro rata approach

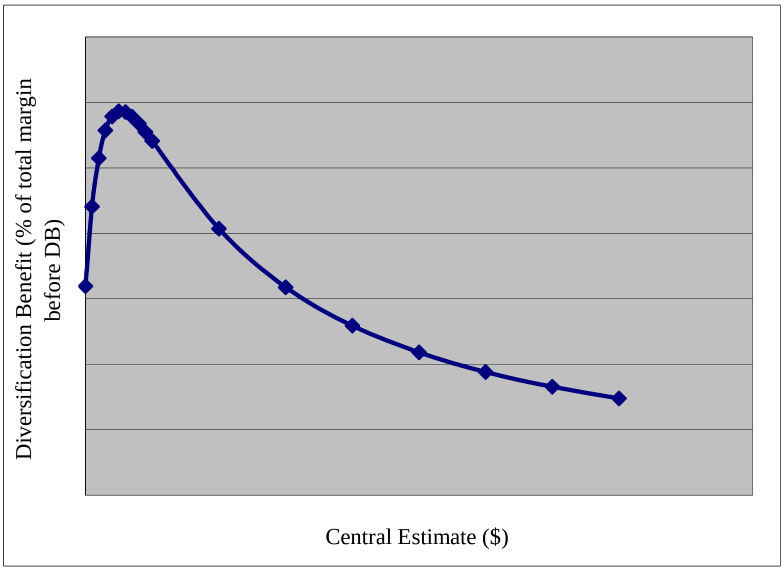
The simplest approach to allocating diversification benefits is a pro-rata allocation. This approach is by far the most widespread due to its simplicity and ease of use. Examples of figures that could be used as weights for the allocation include:

- Dollar amount of central estimate liability
- Dollar amount of total liability
- Coefficient of variation
- Dollar amount of undiversified risk margin
- Risk margin as percentage of central estimate

The first two weights are analogous to allocation by size of liability, and the last three weights are analogous to allocation by level of risk. However, the pro-rata approach can often lead to counter-intuitive results. We show that using these weights as per the pro-rata approach implies a higher level of diversification benefits being attributed to the most concentrated part of the portfolio or the most uncertain class. To illustrate, we first consider two simple numeric examples.

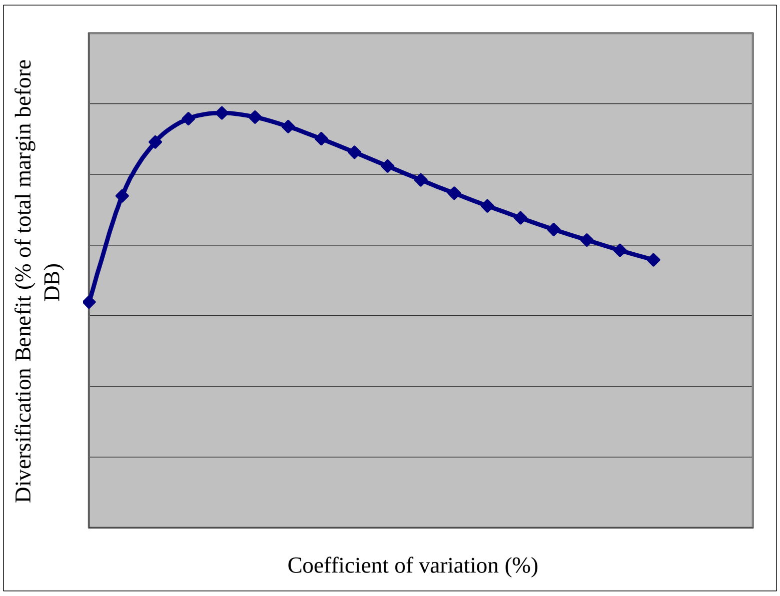
If we were to allocate diversification benefit pro-rata by size of liability and assume less than perfect correlation between classes, as the central estimate of one class grows in dominance over other classes, the diversification benefit % for the whole portfolio (expressed as a percentage of total undiversified risk margin) reduces. By general reasoning, this means diversification is achieved when one class grows from nil, reaching a maximum level of diversification benefits when the portfolio is balanced, and if the class grows further, the higher the concentration of the portfolio in one class, the lower the overall percentage of diversification available. Therefore pro-rata allocation by size of liability is counter-intuitive. This is illustrated in Figure 8.

Figure 8: Diversification benefit % as size of class A increases



If we were to allocate diversification benefit pro-rata by level of risk and continue to assume less than perfect correlation between classes, as the uncertainty (CoV as a proxy) of one of the classes increases relative to all other classes, the diversification benefit % for the whole portfolio (expressed as a percentage of total undiversified risk margin) reduces. By general reasoning again, this means as the uncertainty or CoV of one class grows, its variability becomes more dominant, and the lower the overall percentage of diversification benefit available. Therefore the level of diversification reduces as the uncertainty of one class increases, and allocating the benefits pro-rata to the level of risk is also problematic. This is illustrated in Figure 9.

Figure 9: Diversification benefit % as uncertainty of class A increases



3.4.2 Allocation by reducing probability of adequacy

The second approach of applying a uniform reduction in the probability of adequacy until the resulting risk margins sum to the diversified risk margin produces very similar results to pro-rata allocation to CoV. These two approaches are identical when the assumed distribution is Normal, because the risk margin is simply a multiple of the standard deviation parameter, or equally, the CoV, given the central estimate. A similar allocation result is also expected under a Log-Normal distribution assumption, with the more skewed classes (characterised by higher CoVs) being allocated higher diversification benefits.

3.4.3 Allocation by exclusion approach

To deal with shortcomings in the previous approaches, one may seek to identify the source of the diversification benefit and allocate it accordingly. One possible approach considers the relative impact on the overall risk margin and diversification benefit if each class were excluded one by one. This is essentially a scenario analysis based approach.

Under each exclusion scenario, two effects are observable when a class of business is excluded from the portfolio:

- Dollar amount of total risk margin, before diversification, reduces
- Dollar amount of total diversification benefit reduces

The ratio of reduction in diversification benefit and reduction in undiversified risk margin gives an indication of the level of contribution to the overall diversification benefit by the excluded class. It is effectively a rate of change measure, indicating the change in diversification benefit given changes in the total risk margin owing to that class. We determine one ratio for each class of business in this way and multiply it by the respective individual undiversified risk margin to obtain a measure of the relative amount contributed to diversification by each class of business.

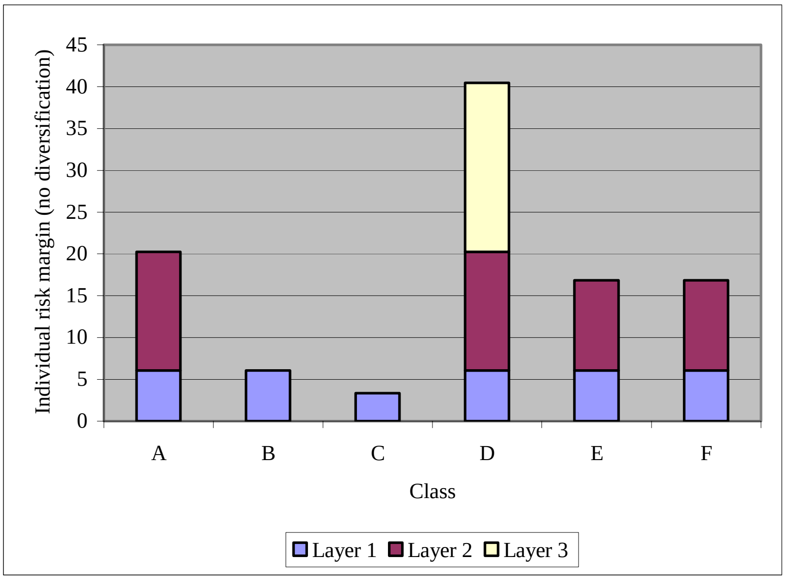
Now we utilise these relative amounts for diversification contribution by class of business as weights to allocate the original amount of diversification benefit for the portfolio to each class of business. Using this method has ensured that the impact of contribution to diversification by any class of business has been taken into account. One remaining shortcoming is the changes in diversified risk margin that occur for classes that have remained very stable from year to year caused solely by the behaviour of other classes.

3.4.4 Allocation by stratification approach

An alternative way to allocation could be a stratification approach. This approach has a similar aim to the exclusion approach, that is, to allocate diversification benefits to their source. It might also have an aim of keeping margins or capital allocated to any class relatively constant, where that class has not been subject to change. That is, explicit changes made by other classes might be largely insulated from the unchanged classes.

The stratification approach separates individual undiversified risk margins into layers (or strata), where upper and lower limits depend on the number and characteristics of the classes of business. We illustrate this in Figure 10.

Figure 10: Allocation of diversification by stratified approach



The first step to this approach involves allocating the undiversified risk margin of each business class to the chosen strata (we have used three strata here), where classes with similar sized risk margins are in the same group. Lower and upper bounds of risk layers would be set, where the upper bound of the first layer might be based on the highest risk margin of the subgroup of classes with the lowest margins. This upper bound becomes where the next risk layer starts, which would have an upper bound determined by the risk margins of the group of classes with the "middle sized" risk margins. The risk margins could then be segmented into these layers, as illustrated above.

The diversification benefits are then calculated for each layer of risk margins, with the result being a diversification amount for each layer (i.e. stratified diversification benefits, rather than a constant diversification benefit). For simplicity, the diversification benefits for each layer could be allocated using a simpler pro-rata approach. This does not fully recognise the differences in correlations, but avoids excessive quantity of calculations. For our example portfolio, any change in overall diversified risk margin due to a change in the non-diversified risk margin for class D in layer 3 would be fully allocated to that class.

3.4.5 Comparison

The following example compares the resulting allocated diversification benefits under different allocation approaches, excluding the stratification approach. For the sake of simplicity, this example assumes there are six classes of business and liability is Normally distributed.

Table 6: Comparison of diversification allocation methods

Summary of claims liabilities

Class	Central estimate	Coeff of variation	Standard deviation	Prob of sufficiency	Margin	Margin % of CenEst	Divers'n Benefit	DB % of Margin	Divers'd margin	Divers'd margin % of CenEst
A	200	15%	30	75%	20.235	10.12%				
B	60	15%	9	75%	6.070	10.12%				
C	100	5%	5	75%	3.372	3.37%				
D	200	30%	60	75%	40.469	20.23%				
E	250	10%	25	75%	16.862	6.74%				
F	250	10%	25	75%	16.862	6.74%				
Total	1060	11%	114.361	81.8%	103.871	9.80%	26.736	25.74%	77.136	7.28%
DB					-26.736	-2.52%				
Total-DB	1060		114.361	75%	77.136	7.28%				

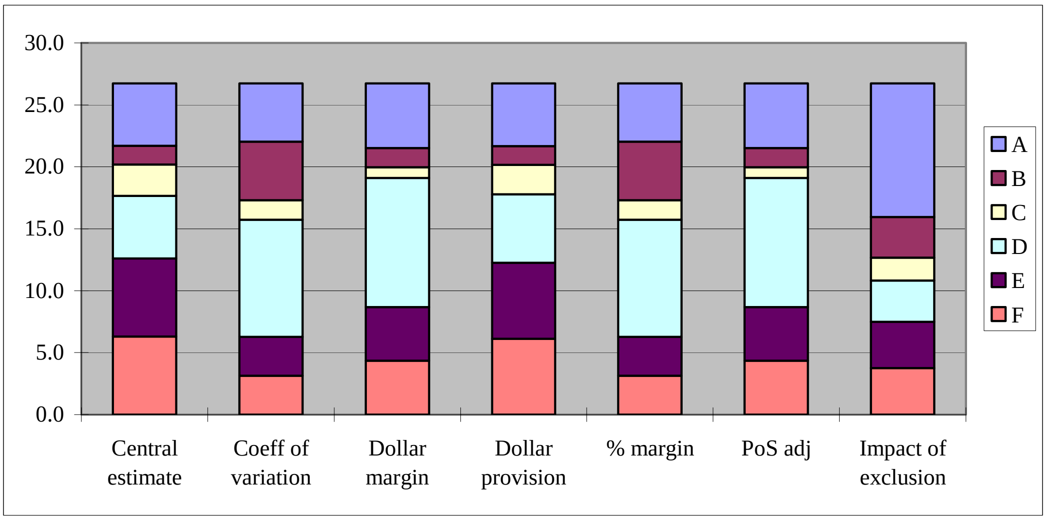
Assumed correlation matrix

	A	B	C	D	E	F
A	100%	50%	50%	25%	0%	0%
B	50%	100%	50%	25%	0%	0%
C	50%	50%	100%	25%	0%	0%
D	25%	25%	25%	100%	75%	75%
E	0%	0%	0%	75%	100%	75%
F	0%	0%	0%	75%	75%	100%

In the above example, we can see that class E and F are the relatively larger classes by size of liability, and class D is the most variable by assumed CoV.

Results from different allocation approaches are shown in the graph below.

Figure 11: Graph of diversification allocation results



In simple terms, the allocation methods produce three broad groupings of outcomes illustrated in the graph:

1 Diversification benefit allocated primarily to the largest classes. The central estimate and dollar provision allocation methods allocate most benefit to classes E and F.

2 Diversification benefit allocated primarily to the most variable class. Methods other than exclusion impact allocated most benefit to class D.

3 Diversification benefit allocated primarily based on “contribution to diversification” and low correlation. The impact of exclusion approach allocated most benefit to class A.

We believe the third group, allocating the most benefit to class A and utilising allocation by exclusion, is the most appropriate allocation, since it is most closely related to the assumed drivers of diversification benefit, namely lack of concentration of portfolio in a single large class and low correlations between classes. We note that class A takes away some of the concentration of the portfolio in classes E and F, giving a greater spread of liabilities. Additionally, Class A has relatively low correlation with other major classes. It is intuitive that two classes that are highly correlated would produce a lower diversification benefit.

We therefore conclude that the primary advantage of the exclusion approach rests on the recognition of the key drivers of diversification benefit. However, the calculations required become more time-consuming and can be potentially more difficult to explain. Where capital is allocated to classes based on diversified risk margins, it becomes very important to recognise the true diversified contribution of classes to overall risk.

The stratification approach will produce similar allocation results to the exclusion approach but will have the advantage that any change in overall diversification benefits will be allocated to the individual business classes that changed in size / characteristics, with the allocation tending to be more stable per business class (size / characteristics). This characteristic can be helpful when capital allocation is used for management reporting of business unit performance - business unit capital is subject to change because of changes in the risk profile of another unrelated business unit.

4 Monitoring risk margins

This section aims to explore possible approaches to monitoring, risk margin estimation albeit noting that such monitoring will become more meaningful when the monitoring has been performed over accumulated periods of time.

4.1 Tracking uncertainty

Currently within Australian actuarial practice we observe that risk margins monitoring techniques are still relatively underdeveloped, with monitoring the central estimate given far greater attention. Given the sensitivity of portfolio risk margins to underlying assumptions around variability and correlations, which have significant reliance on judgement, monitoring variability and correlations is worthy of specific consideration.. This becomes a greater interest given that assumptions can tend to become "anchored" across successive valuations.

As discussed earlier in this paper, the risk margin is intended to be assessed such that the resulting provision is at a specified probability of "ultimately" being adequate. However, achieving adequacy ultimately with a particular probability does not mean the same level of probability of adequacy applies to every development year (individually) leading up to the ultimate year. The monitoring dilemma can therefore become further confused when information for monitoring is only available over shorter timeframes.

Tracking margin variables (i.e. measured coefficients of variation, assumed correlation matrices, theoretical diversification benefits and others) over successive actuarial valuations may provide useful information regarding the suitability of previously assessed risk margins. Over time, a history of assessed level of uncertainty could be monitored.

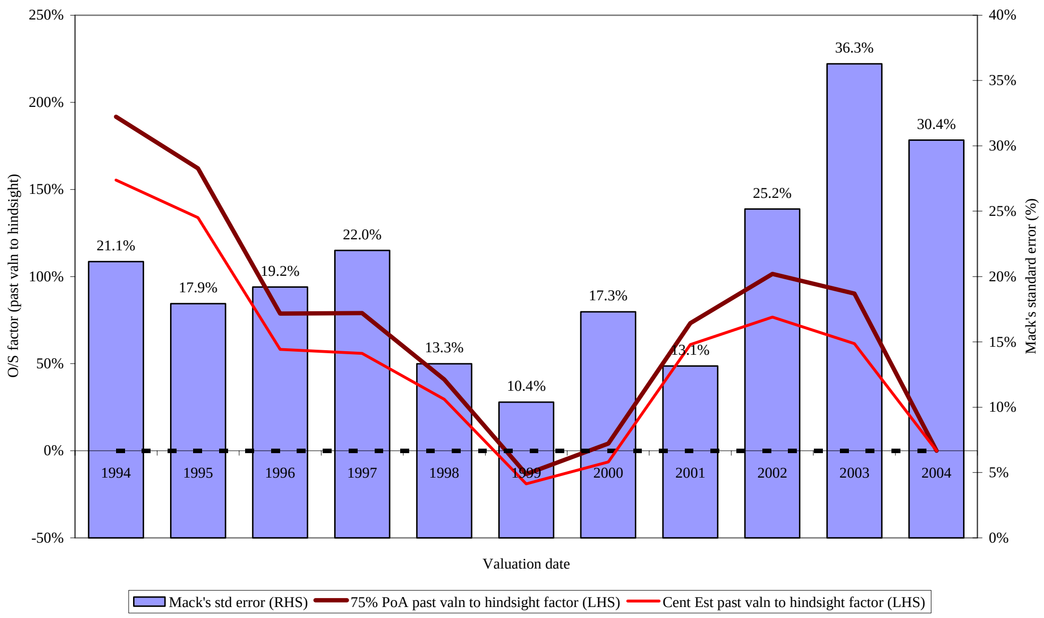
4.2 Hindsight estimates

A technique to monitor the adequacy of risk margins might include assessing over time central estimates, with and without risk margins, with the benefit of hindsight. We have illustrated a technique to monitor risk margins in Figure 12. The X-axis represents the valuation date, with the Y-axis demonstrating the percentage variation of the initial estimate of liabilities at the time (with and without risk margin) against the current "hindsight" liability (i.e. current outstanding plus actual payments between the relevant valuation date and now) for those incident periods that were relevant at that valuation date.

For this analysis, we have apportioned the risk margins in the "hindsight" estimate to each incident year, adopting the same percentage to each incident year for simplicity. For illustrative purposes, the graph has been constructed by taking a triangulation containing fifteen years of data, and performing successive valuations at each year end. Therefore, the 1994 valuation had only five years of data, the 1995 valuation six years of data and so forth. For simplicity, no discounting is applied.

We have used a cumulative paid chain ladder method, and assumptions have been selected manually (primarily to allow for a lack of tail data in the early valuations) but have tended to remain close to the observed averages. That is, we have tried to avoid subjective adjustments away from the averages in order to remove variability related to actuarial judgement. Risk margins at the 75th percentile have been based on the standard error as derived from the Mack technique at each valuation date.

Figure 12: Monitoring uncertainty measures



Ideally (i.e. if experience unfolds as expected), the percentage variation of the hindsight central estimate to the estimate made at the time should equal 0%, since then the estimate made at the time was exactly enough to cover the future payments. One anticipates the ratio of the hindsight 75% adequacy liability estimate to the 75% adequacy estimate made at the time being greater than 0% in most years.

The graph illustrates that there was an overestimate in the early years, up to 1998, followed by a short period of underestimating, followed by overestimating again until 2003. The differentials for the last year will always equal 0%, since the hindsight estimate is equal to the estimate made at the time.

For the 1998 year valuation, the central estimate has proven to be close to 30% higher than the amount actually required for that liability as assessed now. On a 75% adequacy basis, the estimated liability was around 40% higher than the amount currently assessed as required to meet liabilities.

While the intention of this graph was to illustrate a potential method for monitoring risk margin (and central estimate) adequacy, rather than an investigation of this data in particular, we feel it is worth making some specific observations about these results:

- Most valuations appear to be proven overestimates, at first glance, which is suggestive of bias in the central estimate valuation process.
- The very large overestimation evident for the early years may be due to the lack of data, which may have led to an element of error in parameter selection.
- The class of business is a property class, and therefore the underestimation suggested for the 1999 valuation in particular may be as a result of catastrophes in that year (the 1999 Sydney April Hailstorm). This would not have been allowed for within the parameters derived from earlier years. This highlights the importance of considering items that may not be represented in the data.

- The 2000 valuation, although underestimated on a central estimate basis, was sufficiently reserved on a 75% adequacy basis compared to that assessed at the most recent valuation, although the 1999 valuation was comparatively not sufficient even on a 75% adequacy basis.
- Valuations subsequent to the 2000 valuation appear to again be generally overestimated. We note that the valuation factors (which were largely based on averages, with little judgement incorporated) may have included undue weight given to the high development arising from the 1999 incident year. However, in this particular instance, we would expect an actuary estimating outstanding claims would have knowledge of the major catastrophe that had occurred, and might adjust the adopted factors by incident year to accommodate this.
- The gap between the central estimate and 75% adequacy lines does not appear consistent. However, this gap is correlated to the degree of overestimation. As the hindsight liability is the denominator of the Y-axis, as the differential between the hindsight and initial estimate increases, initial risk margin calculated as a percentage of the initially calculated liability experiences a gearing effect against the hindsight estimate. This effect might support a replacement of the Y-axis with a measure of the dollar over/under estimation, but this is inappropriate in a growing portfolio.
- Our adopted risk margins have changed over time, with the 2003 and 2004 valuation dates experiencing a shift in the observed standard error.
- The estimated standard error was at its lowest immediately prior to and during the 1999 valuation process. This highlights the danger of reliance on historical data alone, as this data clearly did not incorporate allowance for variations caused by catastrophes.

5 Findings

In this paper, we have reviewed current Australian actuarial practice in terms of the actuarial control cycle and identified a number of practical issues arising.

Our main findings are as follows:

- **Estimating liability uncertainty and variability:** Uncertainty results depend on the type of claim data and unit time periods into which it is summarised. We found some surprisingly inconsistent results, which we subsequently examined and re-expressed to find that it seemed appropriate to make a number of assessments of variability of the past data in order to obtain a better a picture.
- **Liability variability correlation:** We reviewed the practice of estimating correlations and their impact, noting that adopted diversification benefits in the industry have been decreasing over recent years. We found that it was important to consider all the outputs of variability and correlation assessments, including the implied liability.
- **Diversification benefits:** Diversification benefits assessed are allocated to the underlying business classes giving rise to them. We identified significant shortcomings in some allocation procedures and suggested some alternative approaches, which consider and credit the key drivers of diversification benefits.
- **Monitoring:** We observed that risk margin monitoring techniques are still relatively underdeveloped in the industry, with monitoring the central estimate given far greater attention. We concluded that unless such monitoring was undertaken, the suitability of previous assessments was unknown. Therefore, we illustrated a technique to monitor the adequacy of risk margins, including assessing over time the insurance liability with the benefit of hindsight.

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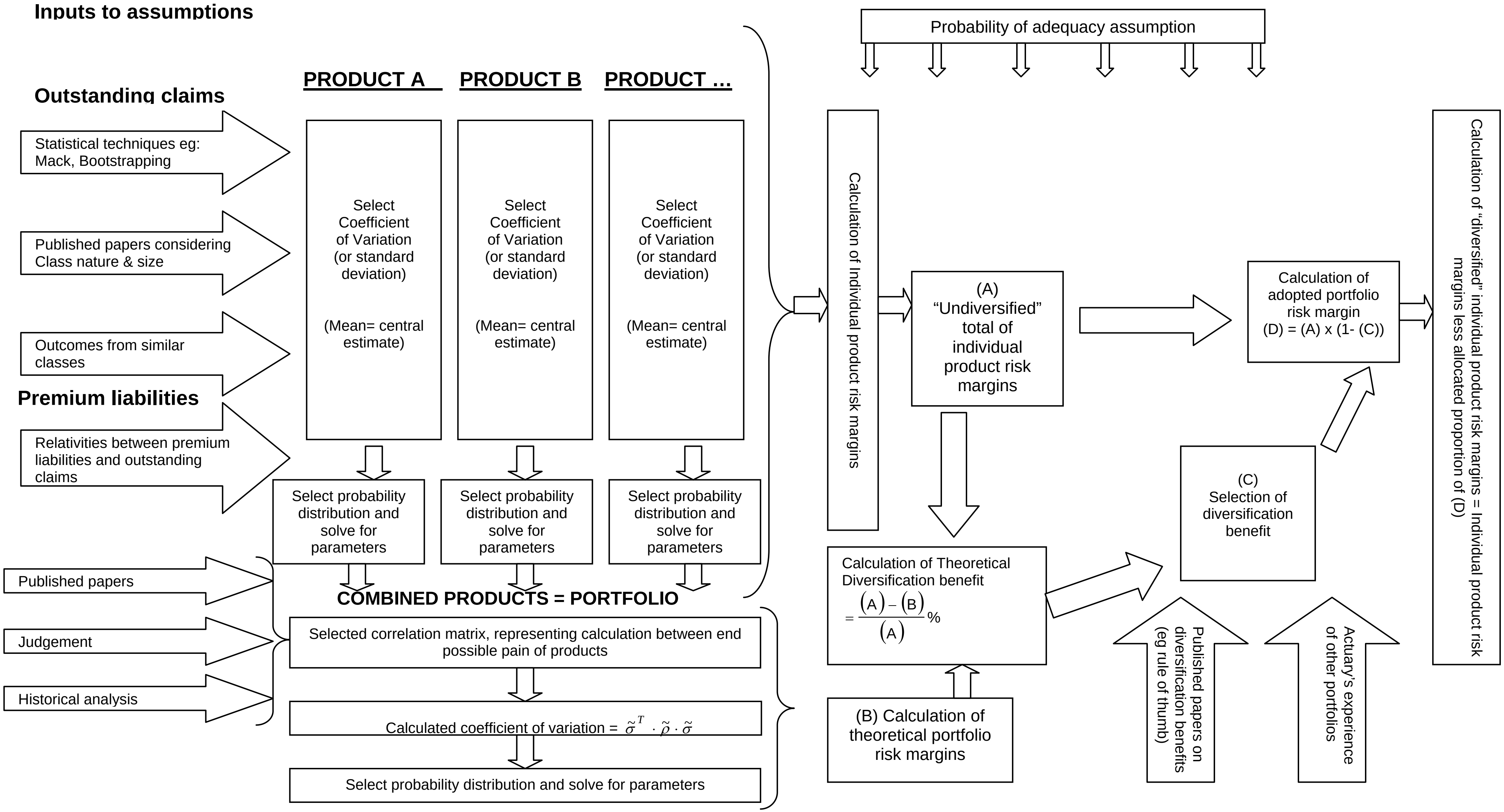
7 Appendix


Inputs to assumptions
Outstanding claims
Statistical techniques eg: Mack, Bootstrapping
Published papers considering Class nature & size
Outcomes from similar classes
Premium liabilities
Relativities between premium liabilities and outstanding claims
PRODUCT A
PRODUCT B
PRODUCT …
Select Coefficient of Variation (or standard deviation)
(Mean= central estimate)
Select Coefficient of Variation (or standard deviation)
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(Mean= central estimate)
Select probability distribution and solve for parameters
Select probability distribution and solve for parameters
Select probability distribution and solve for parameters
COMBINED PRODUCTS = PORTFOLIO
Published papers
Judgement
Historical analysis
Selected correlation matrix, representing calculation between end possible pain of products
Calculated coefficient of variation = $\tilde{\sigma}^T \cdot \tilde{\rho} \cdot \tilde{\sigma}$
Select probability distribution and solve for parameters
Probability of adequacy assumption
Calculation of Individual product risk margins
(A) "Undiversified" total of individual product risk margins
Calculation of adopted portfolio risk margin (D) = (A) x (1- (C))
Calculation of "diversified" individual product risk margins = Individual product risk margins less allocated proportion of (D)
Calculation of Theoretical Diversification benefit
$= \frac{(A)-(B)}{(A)}\%$
(B) Calculation of theoretical portfolio risk margins
(C) Selection of diversification benefit
Published papers on diversification benefits (eg rule of thumb)
Actuary's experience of other portfolios