An Empirical Approach to Insurance Liability Prediction Error Assessment

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Empirical Risk Margin Analysis

- Directly models variability in outcomes from the central estimate.
- Doesn't rely on assumptions that are often not satisfied (that other common methods do rely on).
- Allows for important dependencies and associations.

Core Assumption

- Past deviation from the central estimate could repeat.

Hindsight Re-estimates

Definition

- Revised opinion of a previous liability estimate based on knowledge of :
  - Payments since last review.
  - A new estimate of the residual liability.

Example

- 31 December 2003 estimate = $5m.
  - Payments in 2004 for 2003 incidents and prior = $1.5m.
  - Residual Liability at 31 December 2004 for 2003 incidents and prior = $3.8m.
- 31 December 2004 hindsight re-estimate = $5.3m.

‘I Value’

$$I_x$$

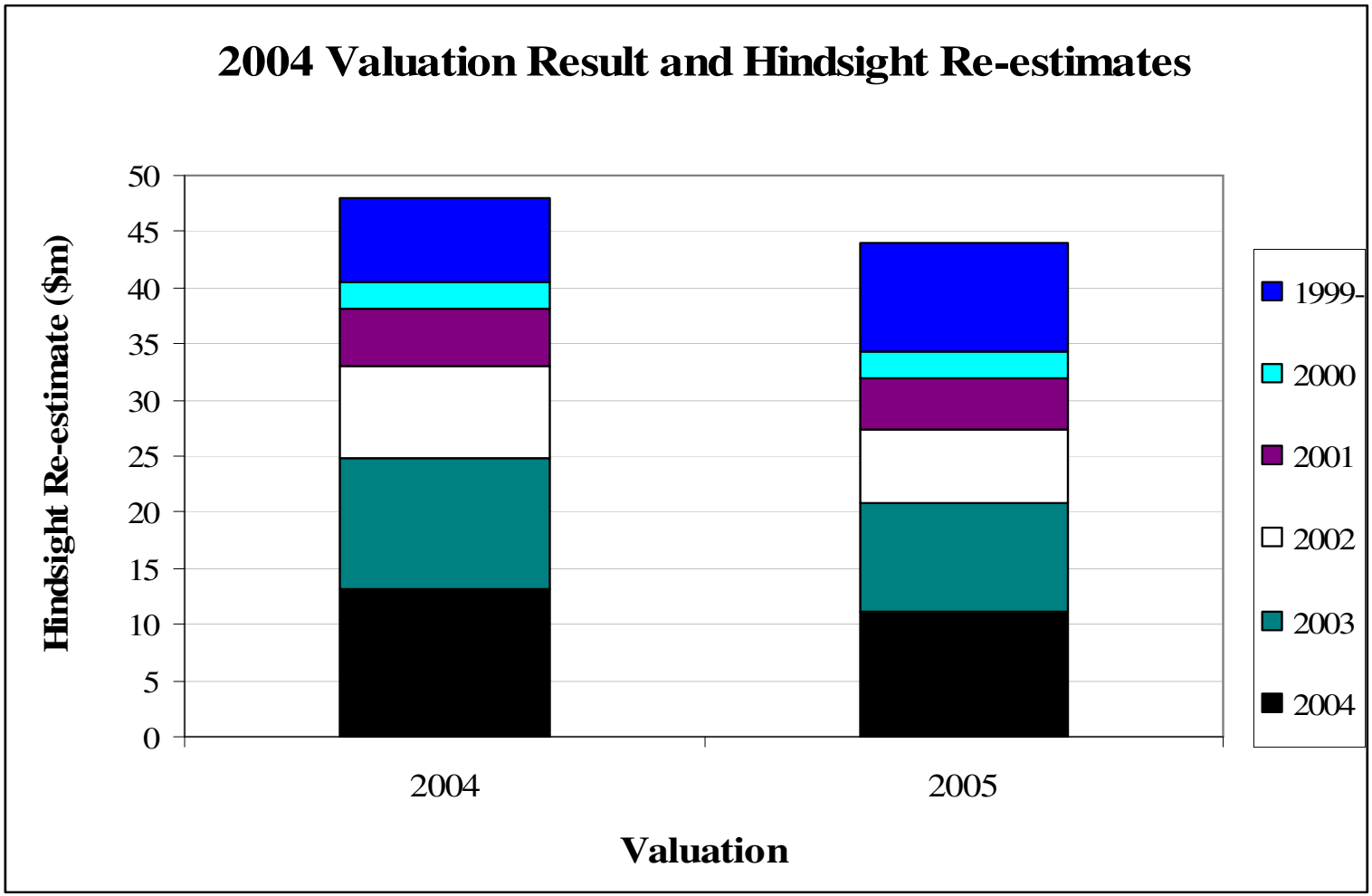
- Accident year ‘X’ outstanding claims liability estimate at the end of transaction year ‘X.’
- Used as an exposure measure.
- Hindsight re-estimate progression for is modelled as a function of each accident year’s ‘*I Value*.’

Hindsight Development Factor

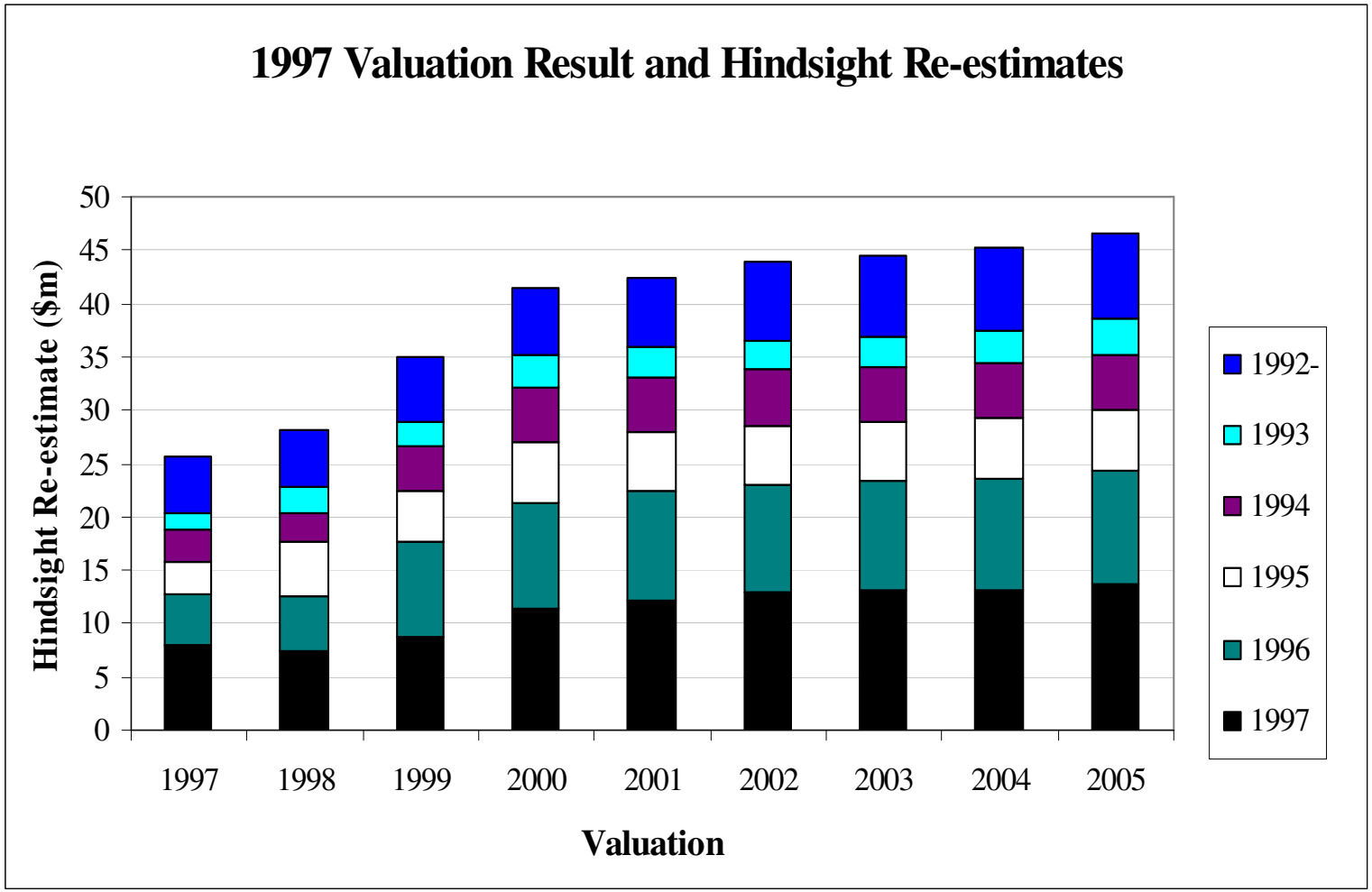
$$Dev(z,d)$$

Hindsight re-estimate development at valuation year Z for

- the accident year transitioning to development year d,
- expressed as a proportion of its '*I Value*.'


2004 Valuation Result and Hindsight Re-estimates
Hindsight Re-estimate ($m)
50
45
40
35
30
25
20
15
10
5
0
2004
2005
Valuation
1999-
2000
2001
2002
2003
2004

Longitudinal Association



Hindsight Re-estimate Projection

AY (x)			${}^{x}H^{2004}_{2004}$	${}^{x}H^{2005}_{2004}$		
1987			205	70		
1988			130	210		
:			:	:		
1999			2,183	2,763		
2000			2,455	2,300		
2001			5,167	4,709		
2002			8,105	6,407		
2003			11,631	9,764		
2004			13,229	11,112		
Total			**47,791**	**43,996**		

Hindsight Re-estimate Projection

AY (x)		I_x	${}^{x}H^{2004}_{2004}$	${}^{x}H^{2005}_{2004}$		
1987		5,092	205	70		
1988		5,503	130	210		
:		:	:	:		
1999		8,568	2,183	2,763		
2000		9,292	2,455	2,300		
2001		11,036	5,167	4,709		
2002		13,561	8,105	6,407		
2003		13,844	11,631	9,764		
2004		13,229	13,229	11,112		
Total			**47,791**	**43,996**		

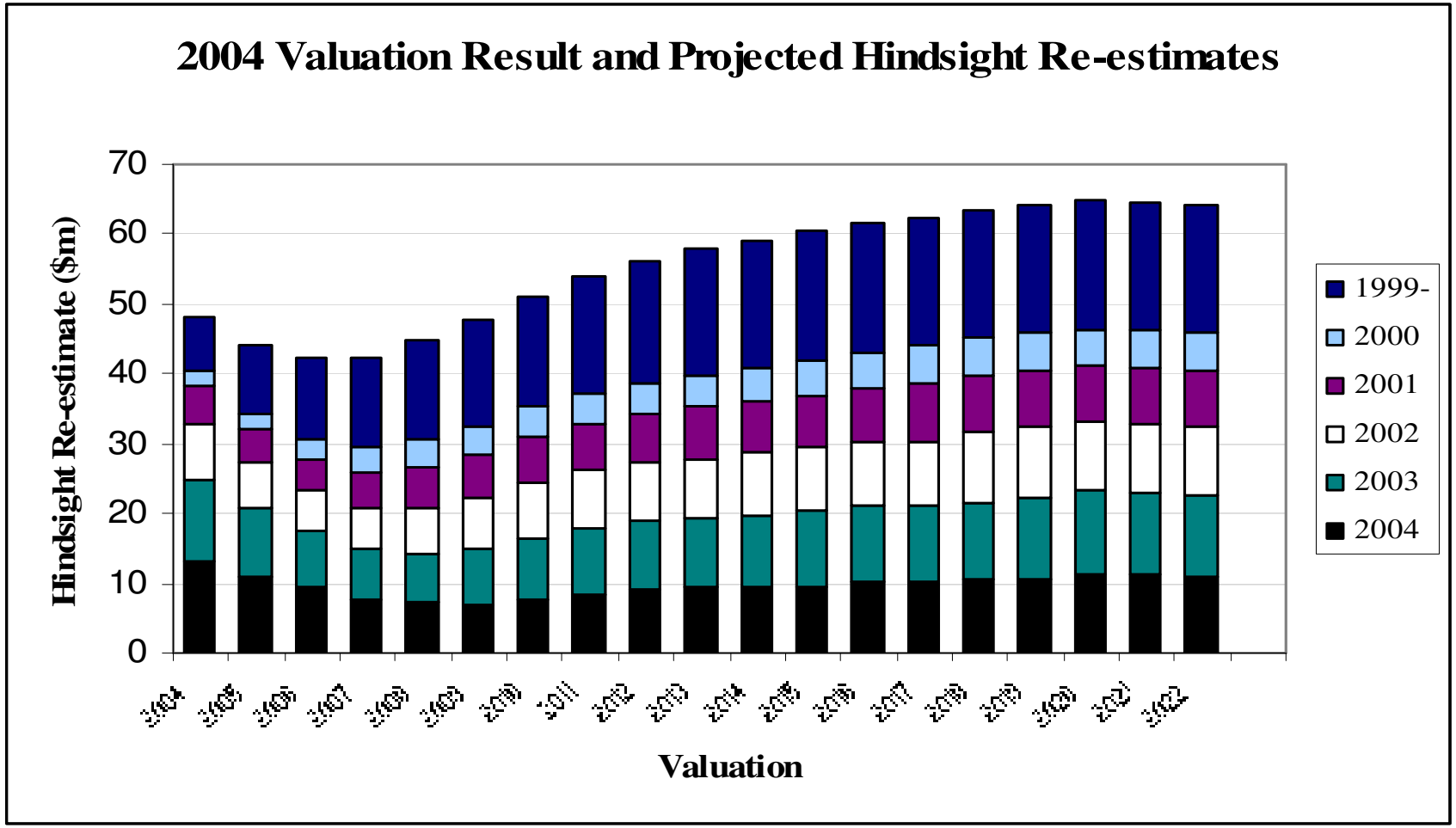
Hindsight Re-estimate Projection

AY (x)	d	I_x	${}^{x}H^{2004}_{2004}$	${}^{x}H^{2005}_{2004}$	DY Trans'n	
1987	18	5,092	205	70	17->18	
1988	17	5,503	130	210	16->17	
:	:	:	:	:	:	
1999	6	8,568	2,183	2,763	5 -> 6	
2000	5	9,292	2,455	2,300	4 -> 5	
2001	4	11,036	5,167	4,709	3 -> 4	
2002	3	13,561	8,105	6,407	2 -> 3	
2003	2	13,844	11,631	9,764	1 -> 2	
2004	1	13,229	13,229	11,112	0 -> 1	
Total			**47,791**	**43,996**		

Hindsight Re-estimate Projection

AY (x)	d	I_x	${}^{x}H^{2004}_{2004}$	${}^{x}H^{2005}_{2004}$	DY Trans'n	Dev(2005,d)
1987	18	5,092	205	70	17->18	2.7%
1988	17	5,503	130	210	16->17	1.4%
:	:	:	:	:	:	:
1999	6	8,568	2,183	2,763	5 -> 6	6.8%
2000	5	9,292	2,455	2,300	4 -> 5	-1.7%
2001	4	11,036	5,167	4,709	3 -> 4	-4.2%
2002	3	13,561	8,105	6,407	2 -> 3	-12.5%
2003	2	13,844	11,631	9,764	1 -> 2	-13.5%
2004	1	13,229	13,229	11,112	0 -> 1	-16.0%
Total			**47,791**	**43,996**		

Hindsight Re-estimate Projection



Stochastic Empirical Approach

- Requires results from more than 2 valuations.
- Current outstanding claims valuation is the starting point.
- For each projected development year transition, there will be several hdfs to choose from.
- Sample hdf's randomly to project 'final hindsight re-estimate.'
- Repeat random re-sampling many times.
- Use results to construct a 'final hindsight re-estimate' pdf.

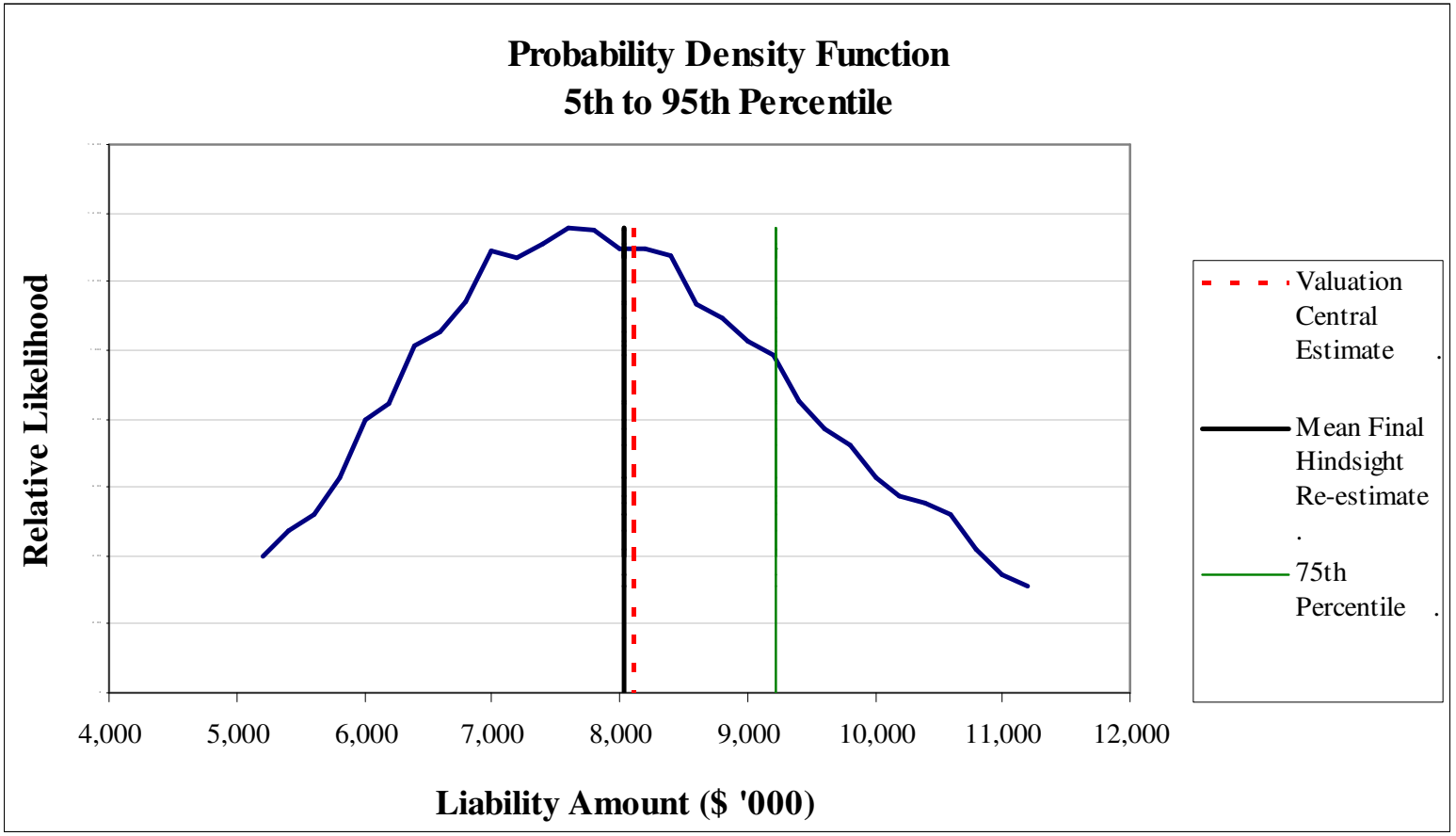
Stochastic Empirical Approach - Example

Dev(z,d) Values **End – Valuation Year**

DY Transition	d	1998	1999	2000	2001	2002	2003	2004	2005
8->9	9	4.2%	-5.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
7->8	8	2.7%	-2.1%	-4.4%	0.0%	0.0%	0.0%	0.0%	0.0%
6->7	7	0.5%	4.4%	0.0%	0.2%	4.1%	0.0%	0.0%	-0.6%
5->6	6	5.0%	2.8%	-2.1%	0.7%	0.8%	0.0%	-2.9%	-1.0%
4->5	5	0.6%	1.5%	-0.4%	0.0%	-0.1%	3.4%	-8.7%	-9.5%
3->4	4	2.6%	0.7%	-0.7%	-1.4%	-3.2%	0.3%	4.6%	-1.3%
2->3	3	4.6%	-1.4%	-3.9%	1.1%	2.0%	20.2%	-2.3%	1.1%
1->2	2	9.9%	-18.5%	-3.3%	-1.7%	2.5%	3.1%	-0.5%	1.0%
0->1	1	-5.8%	4.3%	13.5%	32.6%	-14.8%	-9.8%	-6.0%	-7.9%

Re-sampling Matrix

	1	2	3	4	5	6	7	8	9
0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	0.0%
0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.1%	0.0%	0.0%
34	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	0.2%	0.0%	-5.0%
-1	0.0%	0.0%	0.0%	0.0%	0.6%	-2.1%	0.0%	-2.1%	0.0%
2	0.0%	0.0%	0.0%	2.6%	5.0%	-2.1%	0.0%	-4.4%	0.0%
51	0.0%	0.0%	2.0%	2.6%	-0.1%	0.8%	0.2%	0.0%	-4.2%
524	0.0%	9.9%	-3.2%	3.4%	5.0%	4.4%	-0.6%	2.7%	0.0%
7,492	-14.8%	-3.3%	1.1%	-0.7%	3.4%	-2.1%	0.5%	0.0%	0.0%
8,102									



- **Valuation Central Estimate = $8.1m**
- **Mean Final Hindsight Re-estimate = $8.0m**
- **75th Percentile of Final Hindsight Re-estimate Distribution = $9.2m**

Association and Dependency Effects

Some sources

- Same / similar valuation basis is often applied across AYs.
- Valuation basis shifts often implemented across many AYs.
- Longitudinal association effects.

Important because:

- Random re-sampling destroys dependency structures.
- Important dependency structures will generally be positive.
- If ignored, result would understate probabilistic spread.

Association and Dependency Effects

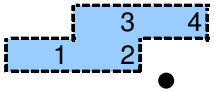
Suggestion:

- Replace random re-sampling with ‘block re-sampling.’
- Preserves dependency effects within blocks.
- Still destroys dependency effects between blocks.
- Bigger blocks will preserve a higher proportion of the underlying structure.
- Choose nxn blocks big enough to reasonably assume elements ‘n’ units apart are nearly independent.

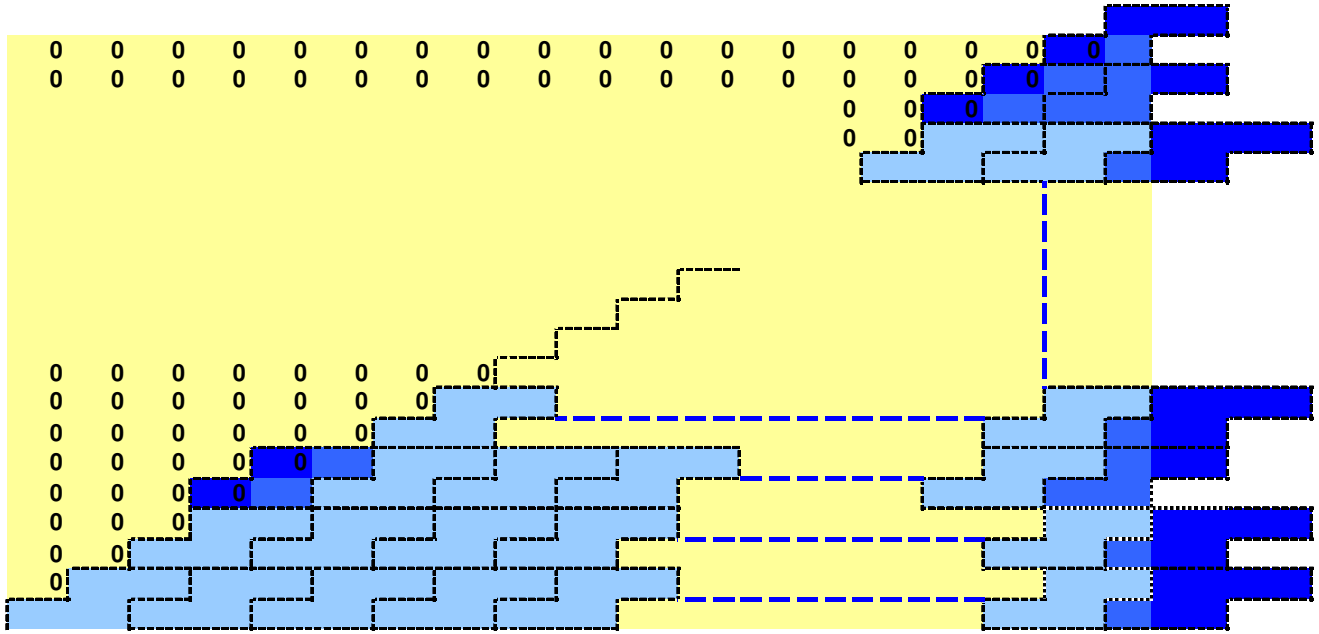
Block Re-sampling

DY Transition	d	1998	1999	2000	2001	2002	2003	2004	2005
8->9	9	4.2%	-5.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
7->8	8	2.7%	-2.1%	-4.4%	0.0%	0.0%	0.0%	0.0%	0.0%
6->7	7	0.5%	4.4%	0.0%	0.2%	4.1%	0.0%	0.0%	-0.6%
5->6	6	5.0%	2.8%	-2.1%	0.7%	0.8%	0.0%	-2.9%	-1.0%
4->5	5	0.6%	1.5%	-0.4%	0.0%	-0.1%	3.4%	-8.7%	-9.5%
3->4	4	2.6%	0.7%	-0.7%	-1.4%	-3.2%	0.3%	4.6%	-1.3%
2->3	3	4.6%	-1.4%	-3.9%	1.1%	2.0%	20.2%	-2.3%	1.1%
1->2	2	9.9%	-18.5%	-3.3%	-1.7%	2.5%	3.1%	-0.5%	1.0%
0->1	1	-5.8%	4.3%	13.5%	32.6%	-14.8%	-9.8%	-6.0%	-7.9%

Example 2 x 2 Block Re-sampling

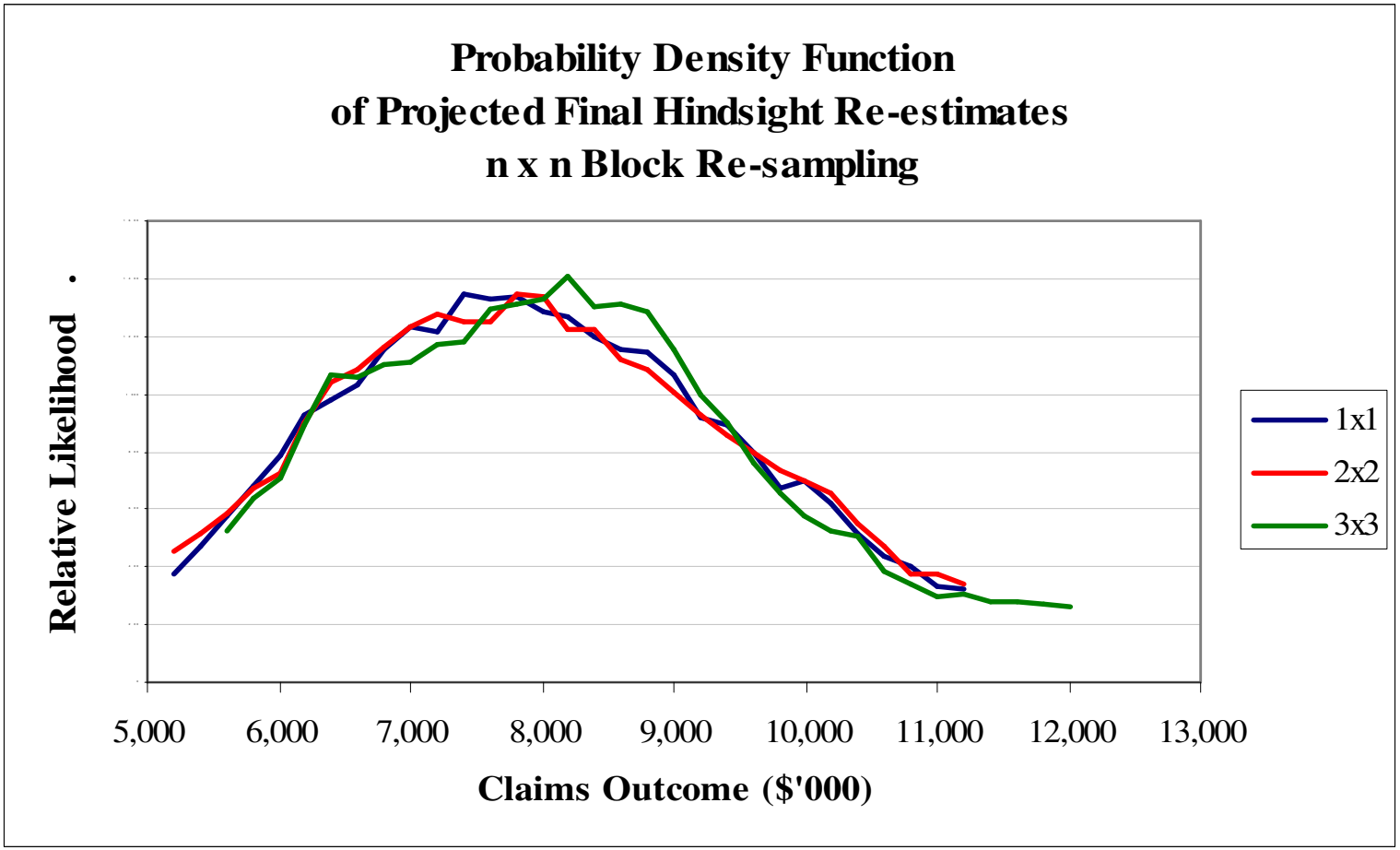


- Use tiles with the above shape to cover the bottom right triangle of the re-sampling matrix.

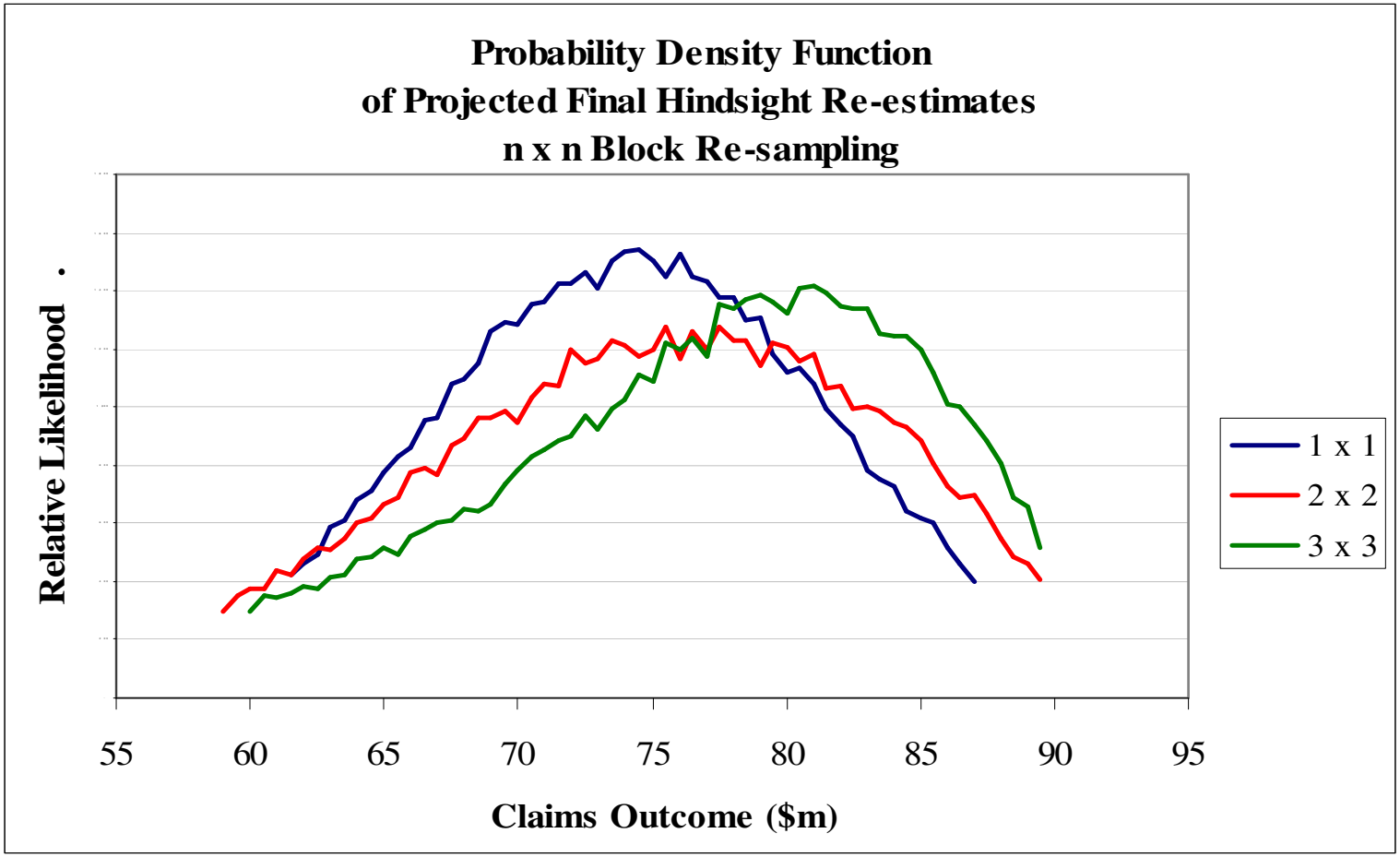


- Populate the matrix one block at a time, rather than one point at a time.

Block Re-sampling



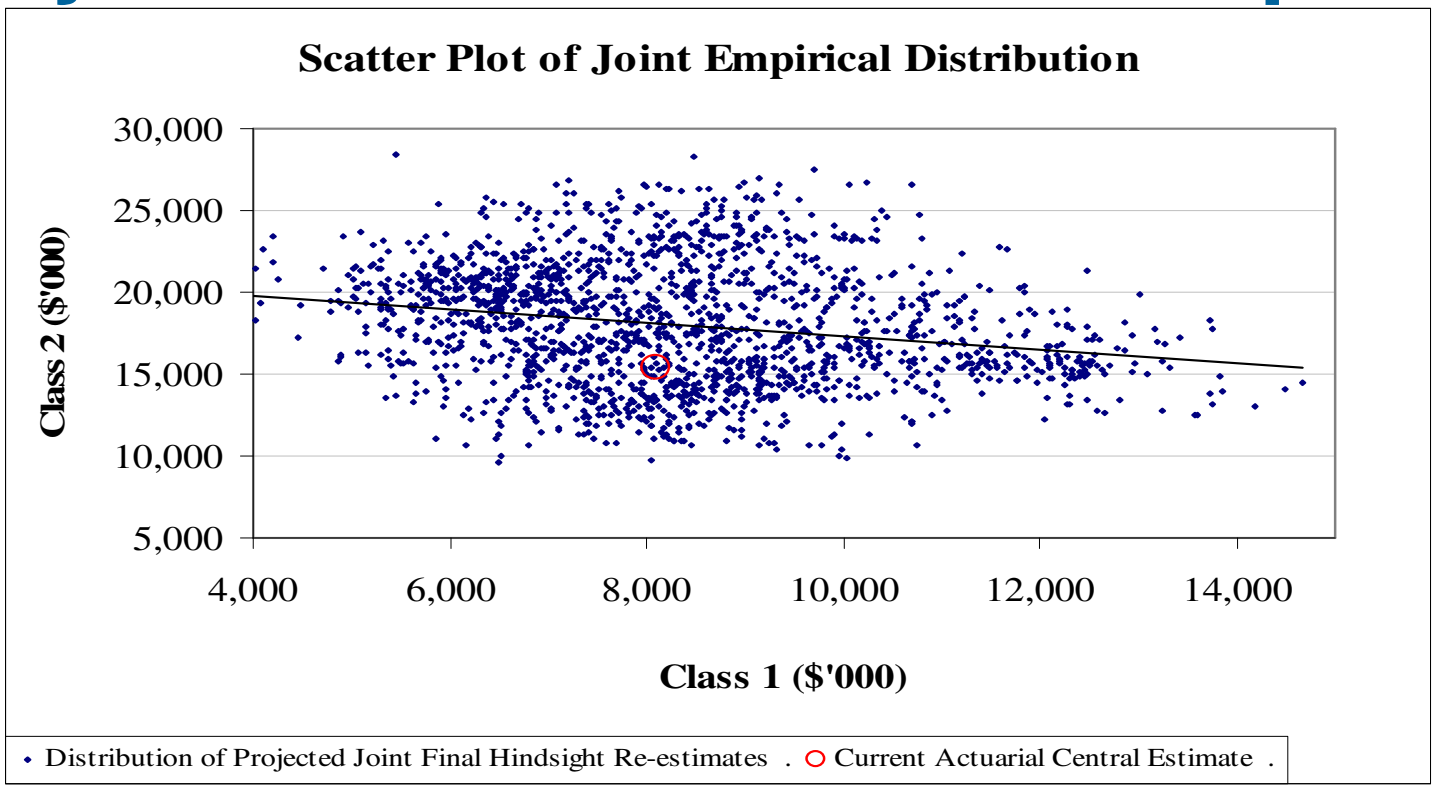
Block Re-sampling



Between-class Associations
Synchronous Block Re-sampling

- For each re-sampling of development factors, select the same values of Z & d across all classes, for re-sampling matrix elements that correspond.
- In this way, the re-sampling is synchronised.
- Captures past associations in estimate adequacy between classes.
- Gives an empirical diversification benefit assessment.

Synchronous Block Re-sampling



Class	Actuarial Central Est	Final Hindsight Re-estimate Distribution Mean	75th Percentile	Empirical Risk Margin for 75% sufficiency $	% of Actuarial Central Est
1	8,102	8,260	9,315	1,213	15%
2	15,408	17,897	20,570	5,162	34%
Joint	23,510	26,157	28,533	5,023	21%

Sum of empirical risk margin for 75% sufficiency for each class	6,375
Empirical risk margin for 75% sufficiency - joint distribution	5,023
Empirical diversification benefit 1 - (5,023) / (1,213 +5,162)	21%

Empirical Diversification Benefit Assessment

Takes into account:

- Process variability and associations between process variability across business lines.
- Past '*estimation error*' and associations between past '*estimation error*' across business lines.

Premium Liability Empirical Risk Margins

- Requires residual liability estimates for past '*premium liability years.*'
- Could then determine hindsight development factors.
- Follow a similar process to outstanding claims to project final hindsight re-estimates.

Insurance Liability Empirical Risk Margins

- Synchronised population of re-sampling matrices for outstanding claims and premium liabilities across classes.
- Gives rise to a projected final hindsight re-estimate pdf that captures association effects:
  - Within classes over time.
  - Between classes.
  - Between outstanding claims and premium liabilities.

An Empirical Approach to Insurance Liability Prediction Error Assessment

The end