Institute of Actuaries of Australia

XVth GENERAL INSURANCE SEMINAR

Evolution of the Industry

Linear Correlation as a Measure of Dependency

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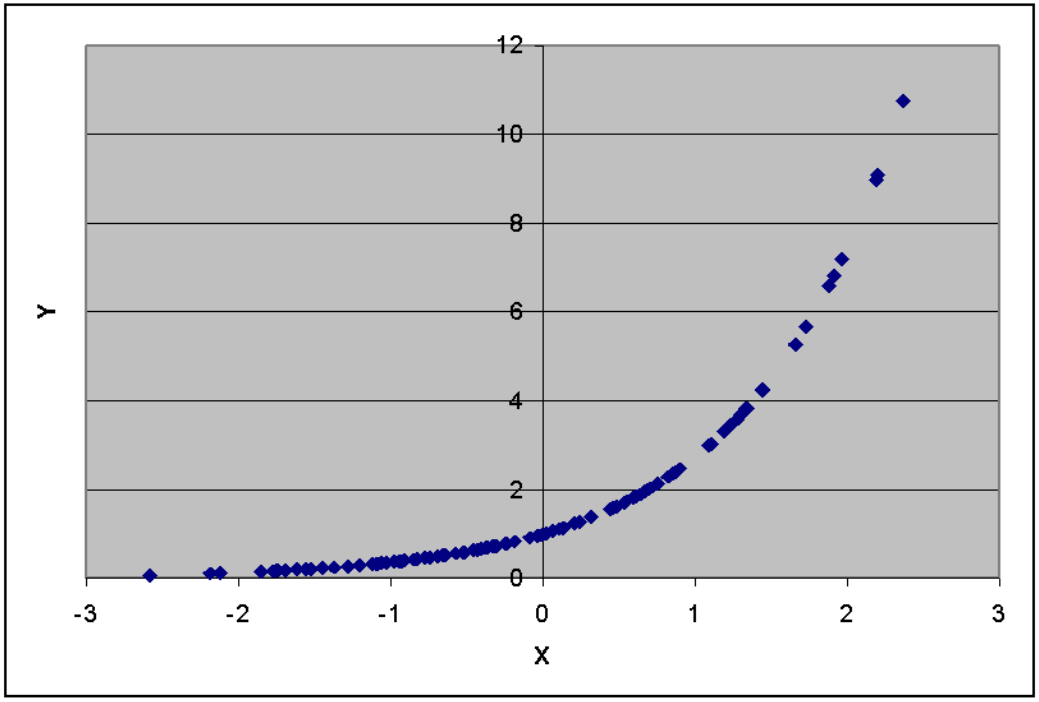
Applications of Pearson Correlation to General Insurance

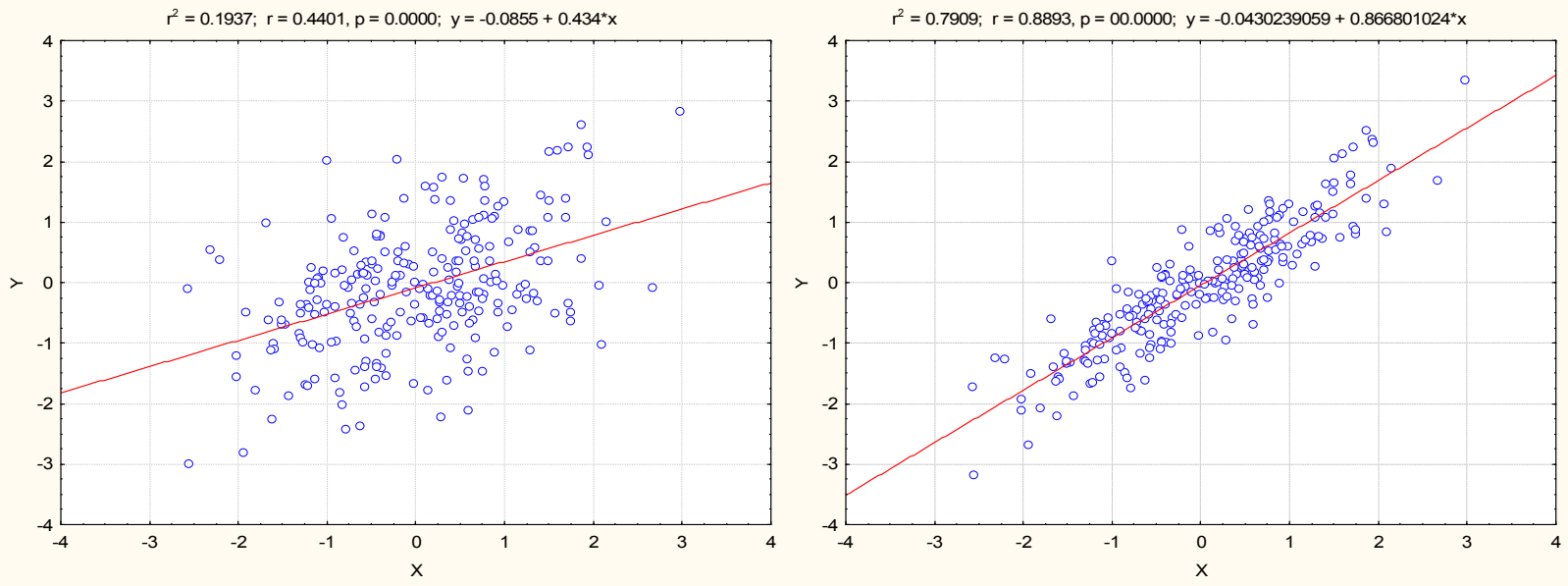
Correlation is not what it seems

N(0,1) X	Exp(N(0,1)) Y
-0.300232	0.7406463
-1.277683	0.2786822
0.2442573	1.2766728
1.2764735	3.5839787
1.1983502	3.314644
1.7331331	5.6583544
-2.183588	0.1126367
-0.234181	0.7912184
1.0950225	2.98925
-1.086701	0.3373276
-0.690204	0.5014737
-1.690432	0.1844398
-1.846911	0.1577236
-0.977629	0.3762018
-0.773507	0.4613921
-2.117931	0.1202802

Correlation 84% =CORREL(B7:B106,C7:C106)

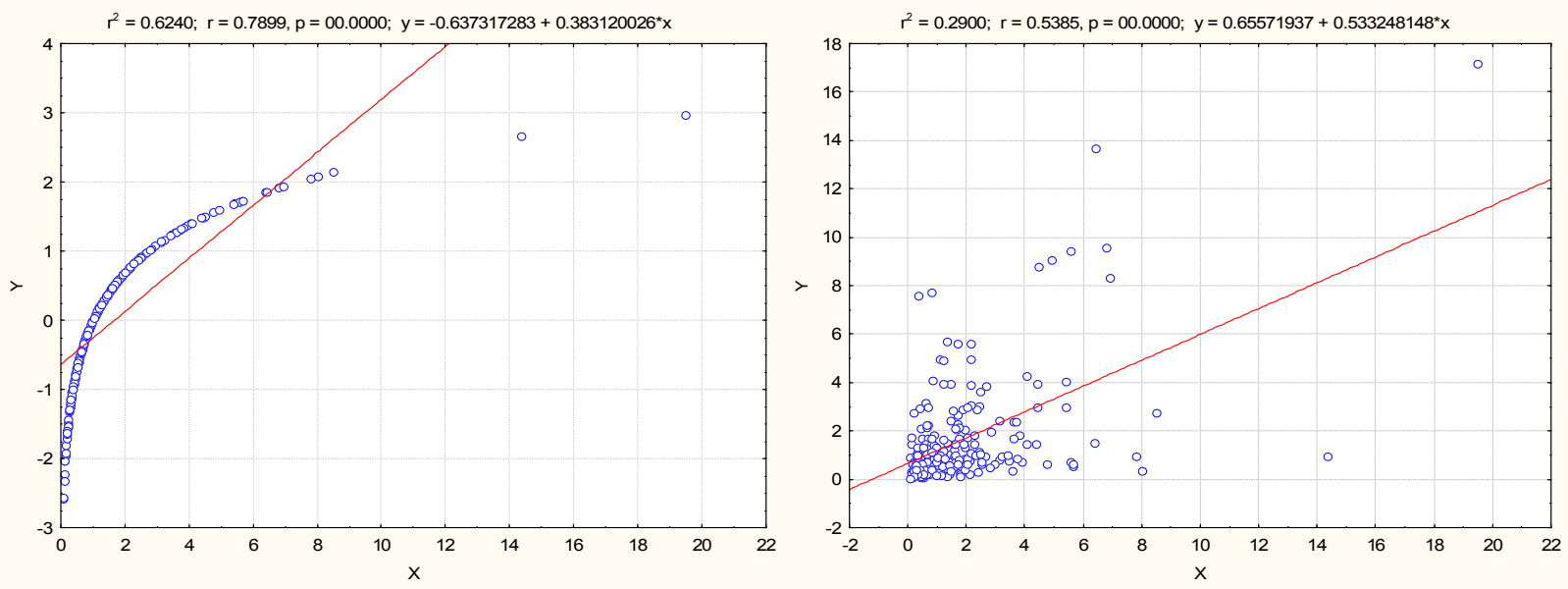


Pearson Correlation and Regression



Pearson correlation is a measure of goodness of fit in a *linear* regression.

Shortcomings of Pearson



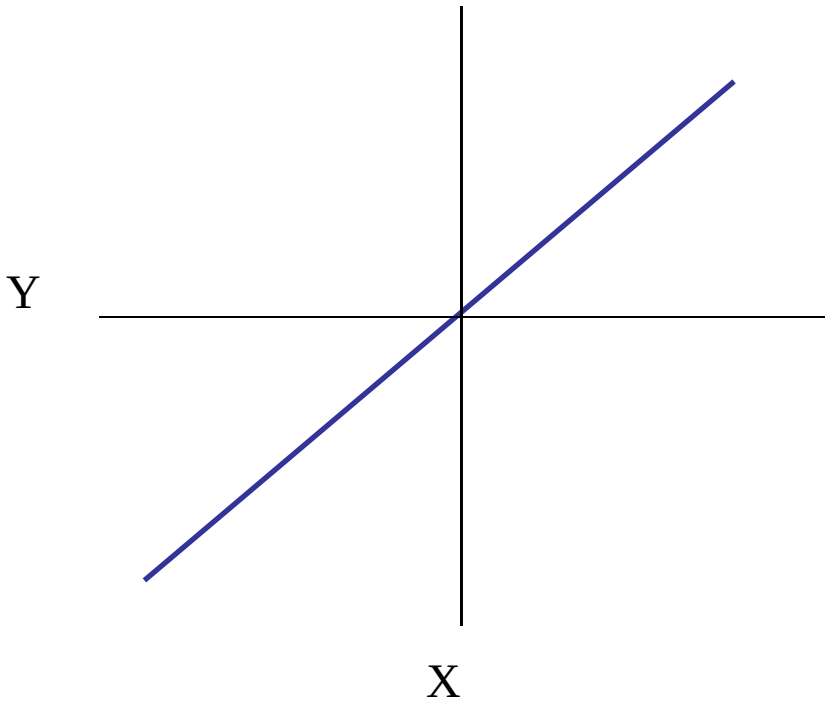
Pearson correlation is a bad measure of goodness of fit when:

- The relationship is not linear;
- The distribution is not elliptic.

$$Cov(X,Y) = E\left[\left(X - \overline{X}\right)\left(Y - \overline{Y}\right)\right]$$

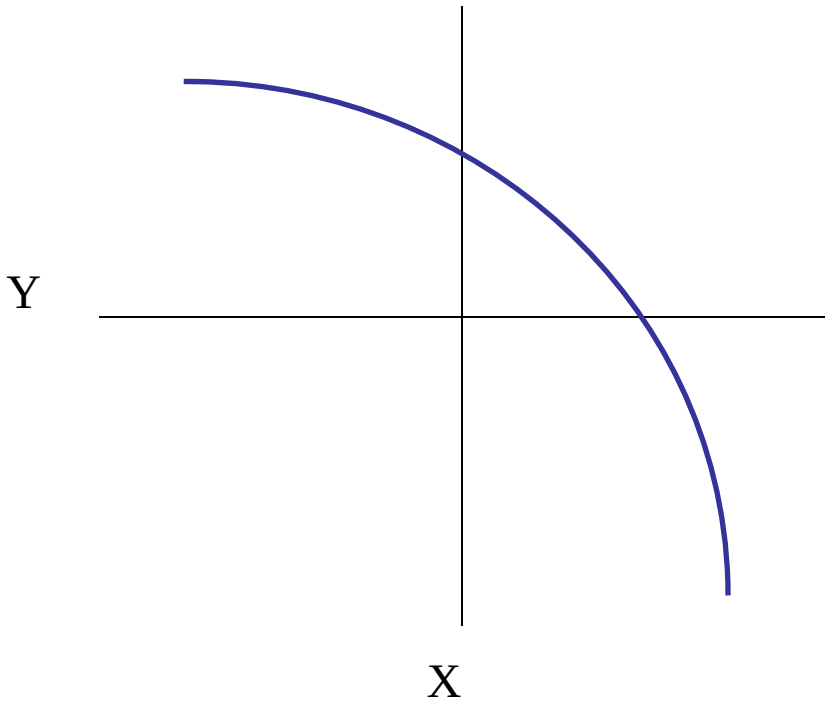
Perfect dependence : Know one, know the other

$$P(X \le x) = P(Y \le y)$$



Comonotonicity

$$P(X \le x) = 1 - P(Y \le y)$$

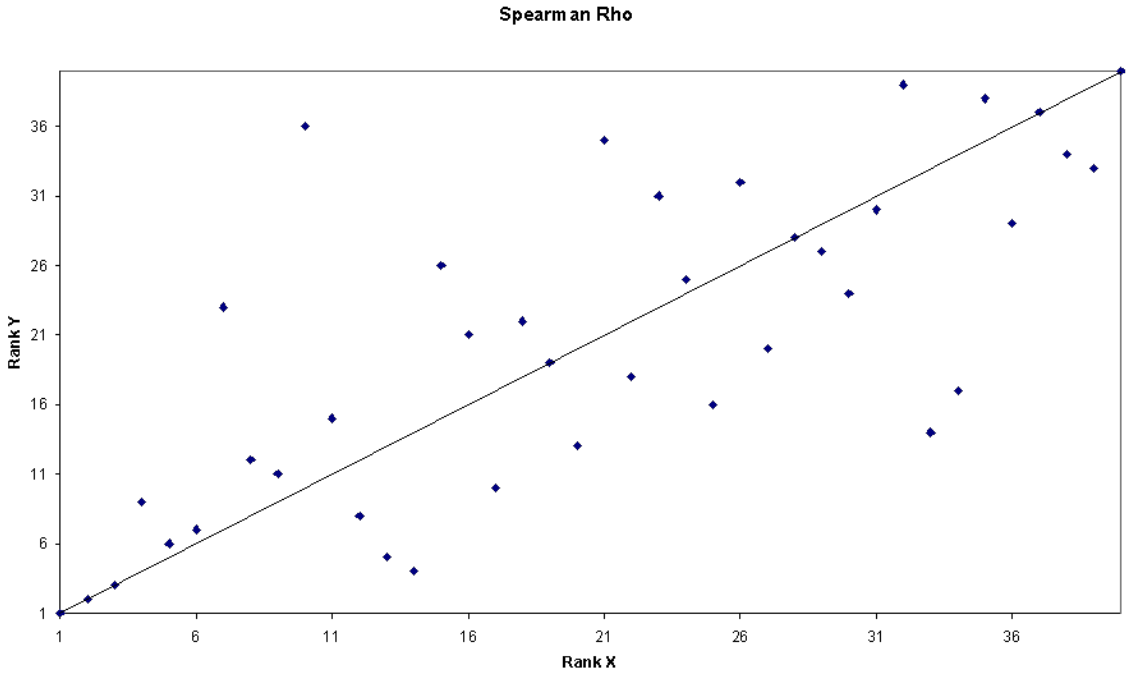


Countermonotonicity

Properties of Dependence

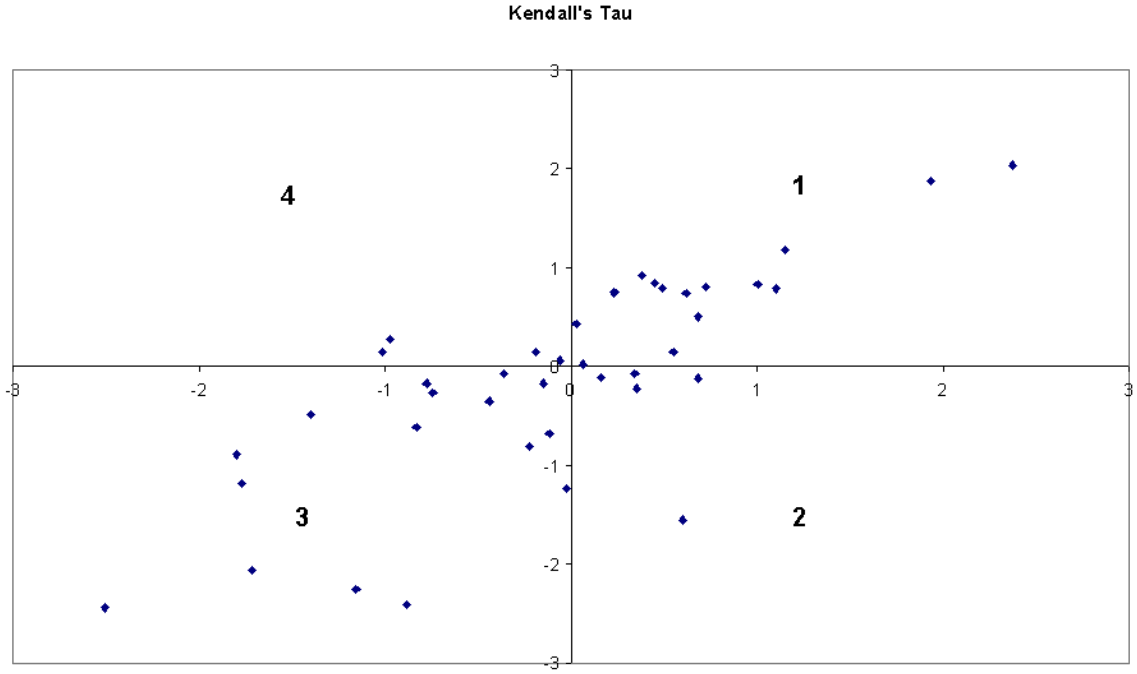
P1	$\rho(X,Y) = \rho(Y,X)$	Symmetry
P2	$-1 \leq \rho(X,Y) \leq 1$	Normalisation
P3	$\rho(X,Y) = 1$ $\rho(X,Y) = -1$	Comonotonicity Countermonotonicity
P4	$\rho(T(X),Y) = \rho(X,Y)$ $\rho(T(X),Y) = -\rho(X,Y)$	Invariance under monotonic transformation

Introducing Mr Spearman



- Non-parametric method
- Pearson correlation of rank
- Spearman = 0.744.

Introducing Mr Kendall



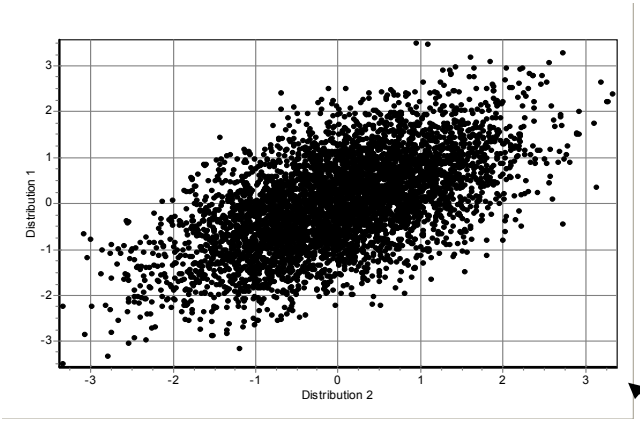
- Values in quadrants 1 and 3 are concordant (26 of these – 65%);
- Values in quadrants 2 and 4 are discordant (14 of these – 35%).
- Kendall's Tau = 0.65 – 0.35 = 0.3.

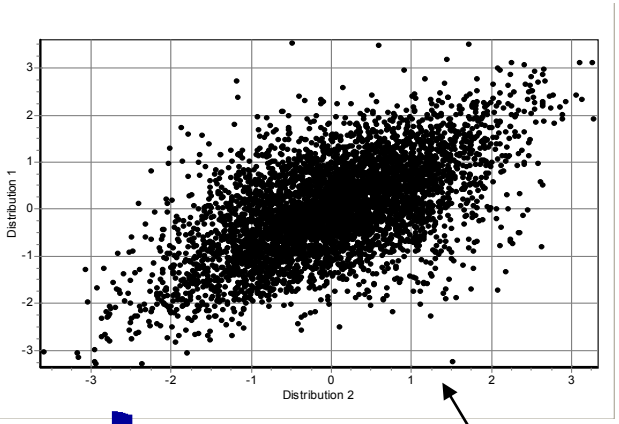
Dependency Measure Scorecard

		Pearson	Spearman	Kendall
P1	Symmetry	✓	✓	✓
P2	Normalisation	✓^	✓	✓
P3	Perfect Dependence	✗	✓	✓
P4	Invariance	✗	✓	✓

^ For elliptic distributions

Copulas





Gaussian

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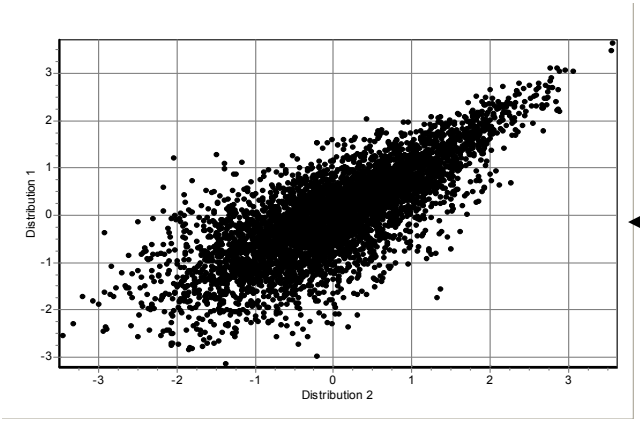

Gumbel

Structural modelling