Reserving methods: future trends

Greg Taylor

Taylor Fry Consulting Actuaries
University of Melbourne
University of New South Wales
Overview

• Discussion focuses on stochastic reserving models
• Some comments on current stochastic reserving practices
• Discussion of some of the more advanced models currently available
• Examination of some extensions of these that are within reach in the near future
Stochastic models

Opening observations
General framework

• Data vector $Y$
• Model $Y = f(\beta) + \epsilon$

  ![Diagram](Diagram.png)

- Parameter vector
- Stochastic error vector

• Estimate $\hat{\beta}$ of $\beta$
• Future observation vector $Z = g(\beta) + \eta$
• Forecast $Z^* = g(\hat{\beta})$ of $Z$
• Prediction error $Z - Z^* = [g(\beta) - g(\hat{\beta})] + \eta$
Estimating prediction error

- Prediction error $Z - Z^* = [g(\beta) - g(\hat{\beta})] + \eta$

- Both errors estimated in terms of residuals of data with respect to model $R = Y - \hat{Y} = Y - f(\hat{\beta})$

- Ultimately distributional properties of $Z^*$ depend on $R(f)$ and $g$
Pictorially

Data $\rightarrow$ Model $f$ $\rightarrow$ Fitted $\rightarrow$ Forecast $g$ $\rightarrow$ Forecast
Pictorially

Data $\xrightarrow{\text{Model } f} \text{Fitted} \xrightarrow{\text{Forecast } g} \text{Forecast}

\text{Residual} \xrightarrow{\text{Forecast } g} \text{Forecast error}
Pictorially

Data \rightarrow \text{Model } f \rightarrow \text{Fitted} \rightarrow \text{Forecast } g

\text{Residual} \rightarrow \text{Forecast } g

\text{Forecast error}

Call this case of forecast error estimation \textcolor{red}{\textbf{coherent}}
Pictorially

Data \rightarrow \text{Model } f \rightarrow \text{Fitted} \rightarrow \text{Forecast } g \rightarrow \text{Forecast}

\text{Residual} \rightarrow \text{Forecast } h??? \rightarrow \text{Forecast error}
For example

Data → Model PPCI → Fitted → Forecast PPCI → Forecast

Residual → Forecast chain ladder (Mack) → Forecast error
Pictorially

Data → Model $f$ → Fitted → Forecast $g$ → Forecast

Residual → Forecast $h???$ → Forecast error

Call this case of forecast error estimation **incoherent**
Pictorially

Data → Model f → Fitted → Forecast g → Forecast

Residual → Forecast h???
Conclusion 1

• Any incoherent estimation of stochastic properties of a loss reserve is meaningless
Available options for forecast error estimation

• Only two
  – **Internal estimation**
    • Based on measured error between data and model (such as just illustrated)
    • Good for capturing features inherent in the model
      – Parameter error
      – Process error
  – **External estimation**
    • Based on
      – Identification of specific components of forecast error (see O’Dowd, Smith & Hardy, 2005) e.g.
        » Future changes in superimposed inflation
        » Generally systemic changes that are not well represented in past data
      – Judgmental assessment of their contributions
Conclusion 2

• Ideally, forecast error should be composed of
  – Internal estimates
    • Parameter error
    • Process error
  – External estimates
    • Model specification error
    • Errors due to other systemic effects
Internal estimation of forecast error

Large residuals imply large forecast error
Conclusion 3

- Good models produce low forecast error (CoV)
  - Economic in use of capital
- Poor models produce high forecast error
  - Uneconomic in use of capital
Internal estimation of forecast error
Internal estimation of forecast error

This is what the bootstrap does
Bootstrapping

• One internal form of forecast error estimation
• Are there others?
• Very rarely
  – Due to intractable mathematical complexity in mapping residuals to forecast error
• So need to make the bootstrap work
Bootstrap

Data → Model f → Fitted → Forecast g → Forecast

Residual → Forecast g → Forecast error
Bootstrap

Data → Model f → Fitted → Forecast g → Forecast

Pseudo-residual → Resample → Residual → Forecast g → Forecast error
Bootstrap

- Data
  - Model f
    - Fitted
      - Forecast g
        - Forecast
          - Forecast error

- Generate pseudo-data
  - Pseudo-residual
    - Resample
      - Residual
        - Forecast g
Bootstrap

Data \xrightarrow{Model \ f} Fitted \xrightarrow{Forecast \ g} Forecast

Generate pseudo-data

Assumed iid

Pseudo-residual

Resample

Residual \xrightarrow{Forecast \ g} Forecast error
Bootstrap

- Generate pseudo-data
- Resample
- Assumed iid
- Pseudo-residual
- Data
  - Model f
  - Fitted
  - Forecast g
  - Forecast error
  - Forecast
What happens when residuals not iid? - example

Residuals assume more influential positions – can distort model and forecast

Actually large residuals

Resample

Assumed iid
Conclusion 4

- Particular care is needed to ensure that model residuals are consistent with iid assumption if ludicrous bootstrap results are to be avoided
Individual claim reserving and Statistical case estimation
Reserving data treatment

<table>
<thead>
<tr>
<th>Raw data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Claim 1</td>
</tr>
<tr>
<td>Claim 2</td>
</tr>
<tr>
<td>Claim 3</td>
</tr>
<tr>
<td>:</td>
</tr>
<tr>
<td>:</td>
</tr>
<tr>
<td>:</td>
</tr>
<tr>
<td>Claim n</td>
</tr>
</tbody>
</table>

- Date of accident
- Date of notification
- Age
- Gender
- Income
- etc
Reserving data treatment

<table>
<thead>
<tr>
<th>Claim 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Claim 2</td>
<td></td>
</tr>
<tr>
<td>Claim 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Claim n</td>
<td></td>
</tr>
</tbody>
</table>

Raw data

Summary data

- Accident period
- Development period

Information lost
Why does quantity of data matter?
Individual claim modelling

• Data vector $Y$
• Model $Y = f(\beta) + \varepsilon$
• Let components $Y_i$ of $Y$ relate to individual claims
  – $Y_i$ denotes some outcome for the i-th claim, e.g. finalised size, paid to date, etc.
• Call this model an individual claim model
• Call a reserve based on such a model an individual claim reserve
Example

\( Y_i = \) finalised individual claim size
\( Y_i \sim \) Gamma

\[ E[Y_i] = \exp \{ \text{function of operational time} \]
\[ + \text{function of accident period (legislative change)} \]
\[ + \text{function of finalisation period (superimposed inflation)} \]
\[ + \text{joint function (interaction)of operational time & accident period} \]
\[ \text{(change in payment pattern attributable to legislative change)} \} \]

Pictorially

No longer

Data

Model \ f

Fitted

Model 

Y=\ f(\beta)+\varepsilon

Forecast

Claim 1
Claim 2
Claim 3
:\ : :
:\ : :
:\ : :
Claim n

Forecast

(reserve)

Claim 1
Claim 2
Claim 3
:\ : :
:\ : :
:\ : :
Claim n

Forecast

\ g(\hat{\beta})
For example

\[ Y = f(\beta) + \varepsilon \]

Claim 1
Claim 2
Claim 3
Claim n

Model

Forecast

\[ g(\hat{\beta}) \]

Accident period 1
Accident period 2
Accident period 3
Accident period m

Claim 1
Claim 2
Claim 3
Claim n
Alternatively

Special case of individual claim reserving – **statistical case estimation**

\[ Y = f(\beta) + \varepsilon \]

\[ \hat{\beta} \]
Can bootstrap individual claim reserve

Just in the usual way

Forecast (reserve)

Model $Y = f(\beta) + \epsilon$

Residuals

Pseudo-residuals

Generate pseudo-data

Re-sample
Adaptive reserving
Static and dynamic models

- Return for a while to models based on aggregate (not individual claim) data
- Model form is still $Y = f(\beta) + \epsilon$
- Example
  - $i =$ accident quarter
  - $j =$ development quarter
  - $E[Y_{ij}] = a_j^b \exp(-cj) = \exp [\alpha + \beta \ln j - \gamma j]$
    - (Hoerl curve PPCI for each accident period)
Static and dynamic models (cont’d)

• Example

\[ E[Y_{ij}] = a j^b \exp(-cj) = \exp[\alpha + \beta \ln j - \gamma j] \]

– Parameters are fixed
– This is a static model

But parameters \(\alpha, \beta, \gamma\) may vary (evolve) over time, e.g. with accident period

Then

– \( E[Y_{ij}] = \exp[\alpha(i) + \beta(i) \ln j - \gamma(i) j] \)
– This is a dynamic model, or adaptive model
Illustrative example of evolving parameters

Separate curves represent different accident periods
Formal statement of dynamic model

• Suppose parameter evolution takes place over accident periods
  \[ Y(i) = f(\beta(i)) + \varepsilon(i) \]  [observation equation]
  \[ \beta(i) = u(\beta(i-1)) + \xi(i) \]  [system equation]

• Let \( \hat{\beta}(i|s) \) denote an estimate of \( \beta(i) \) based on only information up to time \( s \)

Some function

Centred stochastic perturbation
Adaptive reserving

\[ \hat{\beta}^{(1|k)} \]

\[ \hat{\beta}^{(2|k)} \]

\[ \hat{\beta}^{(k|k)} \]

k-th diagonal

Forecast at valuation date k
Adaptive reserving (cont’d)

- Reserving by means of an adaptive model is **adaptive reserving**
- Parameter estimates evolve over time
- Fitted model evolves over time
- The objective here is “robotic reserving” in which the fitted model changes to match changes in the data
  - This would replace the famous actuarial “judgmental selection” of model
Special case of dynamic model: DGLM

- $Y(i) = f(\beta(i)) + \varepsilon(i)$ [observation equation]
- $\beta(i) = u(\beta(i-1)) + \xi(i)$ [system equation]
- Special case:
  - $f(\beta(i)) = h^{-1}(X(i) \beta(i))$ for matrix $X(i)$
  - $\varepsilon(i)$ has a distribution from the exponential dispersion family
- Each observation equation denotes a GLM
  - Link function $h$
  - Design matrix $X(i)$
- Whole system called a Dynamic Generalised Linear Model (DGLM)
Special case of DGLM: Kalman filter

- \( Y(i) = f(\beta(i)) + \varepsilon(i) \) [observation equation]
- \( \beta(i) = u(\beta(i-1)) + \xi(i) \) [system equation]
- Special case:
  - \( f(\beta(i)) = h^{-1}(X(i) \beta(i)) \) for matrix \( X(i) \)
  - \( \varepsilon(i) \) has a distribution from the exponential dispersion family
- Further specialised
  - \( h(.) = \) identity function
    - So \( f(.) \) is linear
  - \( u(.) \) is linear
  - \( \varepsilon(i), \xi(i) \sim N(0,.) \)
  - This is the model underlying the **Kalman filter** (see De Jong & Zehnwirth, 1983)
Form of Kalman filter

- Let \( \hat{Y}(i|s) \) be a fitted value, or forecast, of \( Y(i) \) on the basis of data to time \( s \)
- Take \( \hat{Y}(i|s) = X(i) \hat{\beta}(i|s) \)
- Kalman filter estimates
  \[
  \hat{\beta}(i|i) = \hat{\beta}(i|i-1) + K(i) [Y(i) - X(i) \hat{\beta}(i|i-1)]
  \]
  Kalman gain (credibility) matrix
Implementation of DGLMs

• The restrictions of the Kalman filter may not always be convenient
  – Linear relation between response variate and covariates
  – Normal distribution of claim observations
• Implementation of a more general DGLM is more difficult
• Can be done using an MCMC (Markov Chain Monte Carlo) approach
• Would be useful to have a simple updating formula similar to that of the Kalman filter (a GLM filter)
  – See Taylor, 2005
Bootstrapping DGLMs

• Recursive nature of the GLM filter creates correlations between residuals
• So conventional bootstrapping is wrong
  – It assumes independence between residuals
• Necessary to modify the bootstrap to take account of the correlations
  – Say how
  – See Stoffer & Wall (1991)
Adaptive individual claim reserving
We began with…
moved to **GLM** modelling...
changed to adaptive GLM modelling...
also considered individual claim modelling...

\[ Y = f(\beta) + \varepsilon \]
which can be individual claim GLM modelling...
and could be adaptive individual claim GLM modelling...
References


Reserving methods: future trends

Greg Taylor

Taylor Fry Consulting Actuaries
University of Melbourne
University of New South Wales